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CASE STUDY

Open Access

Forecasting time and place of earthquakes using a Semi-Markov model (with case study in Tehran province)

Ramin Sadeghian

Abstract

The paper examines the application of semi-Markov models to the phenomenon of earthquakes in Tehran province. Generally, earthquakes are not independent of each other, and time and place of earthquakes are related to previous earthquakes; moreover, the time between earthquakes affects the pattern of their occurrence; thus, this occurrence can be likened to semi-Markov models. In our work, we divided the province of Tehran into six regions and grouped the earthquakes regarding their magnitude into three classes. Using a semi-Markov model, it proceeds to predict the likelihood of the time and place of occurrence of earthquakes in the province.

Keywords: Semi-Markov model, Earthquake occurrence, Forecasting, Transition matrix

Background

Forecasting the time and place of earthquakes is considered to be very important in science (Sadeghian 2007). Many researchers in the fields of mathematics, physics, and geology have tried to forecast the time and place of earthquake occurrence. Most of these are one-dimensional studies, looking at either the time or place of occurrence, and some of them have studied the prediction of occurrence as two-dimensional (Papazachos 1992; Rikitake 1976; Kelleher et al. 1973; Nanjo et al. 2006; Qin et al. 1999). While forecasting time or place alone cannot help prevent devastation caused by earthquakes, it is helpful if both dimensions are looked at together; it is useful to consider the dimension of magnitude as the third dimension along them. Some researchers have explained that the occurrence of earthquakes can be considered as memoryless models such as Markov models (Altinok and Kolcak 1999; Patwardhan et al. 1980; Nava et al. 2005). Some researchers have studied on earthquake prediction using a Markov chain model (such as Di Luccio et al. 1997; Console et al. 2002; Console 2001), and few of them have considered the model as semi-Markov (Altinok and Kolcak 1999; Patwardhan et al. 1980). These papers using a semi-

Markov model don't have either much analysis or any evaluation on obtained results. Altinok and Kolcak (1999) and Patwardhan et al. (1980) didn't use scientific techniques for selecting used regions and magnitudes, while we apply a clustering method for regions and magnitudes. Altinok and Kolcak (1999) didn't discuss about selecting time unit. Nava et al. (2005) used a Markov model for the validation of earthquake occurrence, while the proposed model is a semi-Markov model. Patwardhan et al. (1980) and Altinok and Kolcak (1999) didn't forecast time, place, and magnitude of earthquake occurrence simultaneously, while we obtain joint functions for forecasting all dimensions simultaneously. The paper here tries to improve weaknesses of the papers and apply different methods for determining regions, magnitudes, and time unit. Using a semi-Markov model to forecast time, place, and magnitude of occurrence of earthquakes in the province, it is possible to improve the precautionary measures. Some advantages of semi-Markov models in comparison to other models were explained by Patwardhan et al. (1980). Because energies saved under faults are freed when an earthquake occurs, the time and magnitude of the earthquake within a specified region depend on the previous earthquake in the region. Consequently, earthquake occurrence can be explained by Markov models. Note that future periods can be forecasted by previous periods in Markov models. Nava et al. (2005) used a Markov model for the validation of earthquake,

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while the proposed model is a semi-Markov model. Also, Sadeghian and Jalali-Naini (2008a,b) applied a semi-Markov model and probability distributions on the field of earthquakes. In the next sections, we describe definitions of a semi-Markov model and its necessary parameters such as holding time, core matrix, interval transition probability matrix, and their analyses. A semi-Markov model is used for Tehran province. Next, the data corresponding to the province and its magnitudes during 1966 to 2004 are clustered, and then, both region-to-region and magnitude-to-magnitude transition matrices are obtained. Forecasting is performed by matrices obtained during next 3 years.

Case description

Semi-Markov model

Consider a continuous time chain $\{X(t)\}_{t \in [0, \infty)}$, taking value in a discrete state space S :

1. $T_n (n \geq 1)$ is the time of the n th transition after $t = 0$. (Note that the chain does not have to change states at each transition, as at time T_4 in Figure 1).
2. The duration between two consecutive transitional epochs T_{n-1}, T_n is defined as the chain's n th holding time.
3. If the chain's sample path is right continuous (i.e., the time is considerable as piecemeal), then we write $X_n \equiv X(T_n)$.

The chain of $\{X(t)\}_{t \in [0, \infty)}$ is said to be semi-Markov if its development after each transition is independent of its behavior before that time. This means that the distribution of its holding time $T_n - T_{n-1}$ is independent of its behavior before T_{n-1} but may be a function of $X_{n-1} = X(T_{n-1}), X_n = X(T_n)$. If $X_{n-1} = i, X_n = j$, we denote the holding time $T_n - T_{n-1}$ by t_{ij} .

If all holding times in a semi-Markov chain are equal to a constant value, the chain can be studied as a discrete-time Markov chain. To describe it completely, we need only to have all transition probabilities.

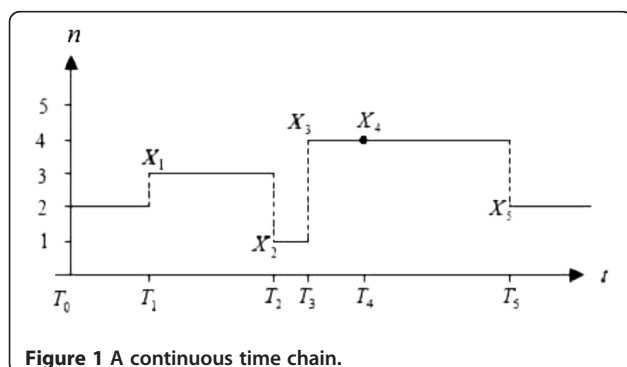


Figure 1 A continuous time chain.

If a semi-Markov chain has only one state, all its holding times can only be a function of this one state; hence, they are independent and identically distributed. The chain therefore can be studied as a renewal stream, with a renewal at each transition; we only need its holding time distribution as the life distribution of the stream (Jalali-Naini 1997; Minh 2001). See previous studies of Kulkarni (1995), Cotea and Stein (2006), Bradley et al. (2006), Bradley and Wilson (2005), and Jenamani et al. (2003) for some general references about semi-Markov models.

Transition matrix

The transition probability G_{ij} is the probability that a semi-Markov process which has entered state i on its last transition will enter state j on its next transition. These probabilities must satisfy the conditions:

$$\begin{aligned} G_{ij} &\geq 0 ; \quad i = 1, \dots, N, \quad j = 1, \dots, N \\ \sum_{j=1}^N G_{ij} &= 1 ; \quad i = 1, \dots, N \end{aligned} \quad (1)$$

where N is the total number of states in the system (Minh 2001; Altinok and Kolcak. 1999; Kulkarni 1995; Jenamani et al. 2003).

Holding time

Before the transition from state i to state j , the process remains in state i for a time t_{ij} . The holding times t_{ij} are positive, integer-valued, random variables. All holding times are finite, and each is at least equal to one time unit. The probability mass function T_{ij} in t_{ij} is called the holding time mass function for a transition from state i to j :

$$\Pr\{t_{ij} = m\} = T_{ij}(m); \quad m = 1, \dots, n \quad (2)$$

Here, n is the number of time intervals. We must specify both the holding time mass functions and the transition probabilities to describe a discrete-time semi-Markov process completely (see Minh 2001; Altinok and Kolcak 1999; Kulkarni 1995; Jenamani et al. 2003).

Core matrix

The ij th element of the core matrix $C(m)$ is the probability of the joint event in which a system that entered state i at time 0 makes its next transition to state j and makes that transition after a holding time m (Altinok and Kolcak 1999).

$$\begin{aligned} C_{ij}(m) &= \Pr((X_n = j | X_{n-1} = i), (t_{ij} = m)) \\ &= \Pr(X_n = j | X_{n-1} = i) \cdot \Pr(t_{ij} = m) \\ &= G_{ij} \cdot T_{ij}(m) \Rightarrow C_{ij}(m) = G_{ij} T_{ij}(m); \quad i, j \\ &= 1, \dots, N, \quad m = 1, \dots, n \end{aligned} \quad (3)$$

We denote the above equation in congruent matrix multiplication form by,

$$C(m) = G \otimes T(m), \quad (4)$$

where the operator \otimes denotes multiplication of corresponding elements. If we sum the elements of $C(m)$ across the i throw, we obtain the waiting time mass function $w_i(m)$ for the i th state:

$$\sum_{j=1}^N C_{ij}(m) = \sum_{j=1}^N G_{ij} T_{ij}(m) = w_i(m), \quad (5)$$

Indeed, $w_i(m)$ is the probability that waiting time for the i th state is equal to m .

The cumulative probability distribution of the waiting time is obtained from

$$LEw_i(n) = \sum_{m=1}^n w_i(m), \quad (6)$$

$LEw_i(m)$ is the probability that the waiting time for the i th state is less than or equal to n . The complementary of LEw_i is then given by

$$Gw_i(n) = \sum_{m=n+1}^{\infty} w_i(m), \quad (7)$$

$Gw_i(m)$ is the probability that waiting time for the i th state is greater than n .

Interval transition probability matrix

The most important statistics of the semi-Markov process are the interval transition probabilities. The probability of a transition from state i to state j in the interval $(0, n)$ requires that the process makes at least one transition during that interval. The process could have made its first transition from state i to some state at time m , where $(0 \leq m \leq n)$, and then by the sum of a succession of transitions, it could have made its way to state j at time n .

$$\begin{aligned} F(n) &= GW(n) + \sum_{m=0}^n G \otimes T(m) F(n-m) \\ &= GW(n) + \sum_{m=0}^n C(m) F(n-m); \quad n \\ &= 0, 1, 2, \dots \end{aligned} \quad (8)$$

Here, $GW(n)$ is a diagonal matrix with its i th element equal to $Gw_i(n)$. The interval transition probability $F(n)$ is obtained by a recursive procedure. Because $T(0)$ is equal to 0, $F(n)$ is obtained for the interval $1 \leq m \leq n$. In

the case $n = 0$, $F(n)$ is equal to the Kronecker Delta (Equation 9) or identity matrix.

$$F_{ij}(0) = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \quad (9)$$

In the earthquake phenomenon, these probabilities can be used for studying earthquake hazards (Altinok and Kolcak 1999; Patwardhan et al. 1980) and evaluating seismic hazard (Nava et al. 2005).

Discussion and evaluation

Application of the model

Tehran province is selected as an area of investigation. This area is bounded by longitudes 50.42°E , 53.16°E and latitudes 35.23°N , 36.33°N (Figure 2). Earthquake data are obtained from a catalog in the USGS website (USGS 2011).

After filtering and removing unsuitable data, the data wherein 44 earthquakes occurred from 1966 to 2004 are used. Note that if there are sufficient data such as a large number, accuracy, and long time interval, we expect that the accuracy of forecasting is increased and that the proposed model is validated mathematically.

We consider two transitions: from region-to-region and magnitude-to-magnitude.

Region-to-region transition

In the first stage, regions are chosen as states. We divided Tehran province into six regions $R_1, R_2, R_3, R_4, R_5, R_6$ (Figure 2). The transition probability matrix of region-to-region transitions was determined in Equation 10. Selecting 1 year for time unit, the largest time interval for the region-to-region transitions is found to be 5 years; therefore, holding time mass functions $T_R(m)$ are obtained by taking into account the time lapse between successive earthquakes (Equation 11). Also, interval transition probabilities $F_R(n)$ are calculated using Equation 8. They are shown in Equations 11 and 12.

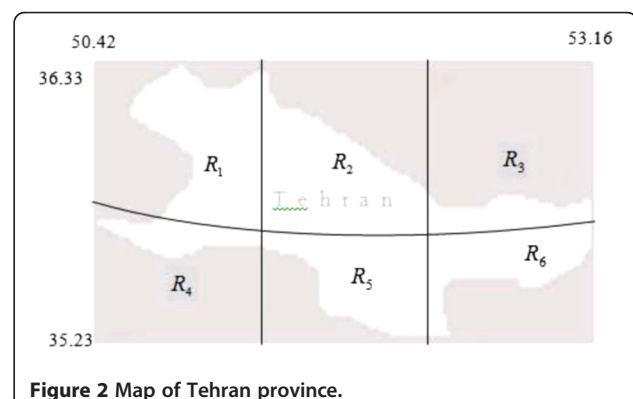


Figure 2 Map of Tehran province.

$$G_R = \begin{matrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_6 \end{matrix} \begin{pmatrix} 0 & 0 & .67 & 0 & .33 & 0 \\ .125 & .375 & .375 & .125 & 0 & 0 \\ 0 & .17 & .33 & 0 & .08 & .42 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ .25 & .25 & 0 & 0 & 0 & .5 \\ 0 & .06 & .22 & 0 & .11 & .61 \end{pmatrix} \quad (10)$$

$$T_R(1) = \begin{pmatrix} 0 & 0 & .5 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & .8 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & .5 \\ 0 & 1 & .5 & 0 & .5 & .91 \end{pmatrix}$$

$$T_R(2) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .5 \\ 0 & 0 & .25 & 0 & .5 & .09 \end{pmatrix}$$

$$T_R(3) = \begin{pmatrix} 0 & 0 & .5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .25 & 0 & 0 & 0 \end{pmatrix} \quad (11)$$

$$T_R(4) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$T_R(5) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$F_R(1) = \begin{pmatrix} .335 & 0 & .335 & 0 & .33 & 0 \\ .125 & .375 & .375 & .125 & 0 & 0 \\ 0 & .17 & .494 & 0 & 0 & .336 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & .25 & 0 & 0 & .5 & .25 \\ 0 & .06 & .11 & 0 & .055 & .775 \end{pmatrix}$$

$$F_R(2) = \begin{pmatrix} .335 & .14 & .165 & 0 & .165 & .195 \\ .089 & .329 & .368 & .047 & .041 & .126 \\ .021 & .14 & .345 & .021 & .018 & .455 \\ .125 & .375 & .375 & .125 & 0 & 0 \\ .031 & .109 & .121 & .031 & .264 & .444 \\ .008 & .088 & .193 & .008 & .113 & .59 \end{pmatrix}$$

$$F_R(3) = \begin{pmatrix} .017 & .083 & .49 & .017 & .093 & .3 \\ .1 & .24 & .334 & .041 & .043 & .242 \\ .025 & .137 & .33 & .018 & .056 & .434 \\ .089 & .329 & .368 & .047 & .041 & .126 \\ .274 & .119 & .168 & .014 & .052 & .373 \\ .014 & .117 & .262 & .011 & .112 & .484 \end{pmatrix}$$

$$F_R(4) = \begin{pmatrix} .1 & .142 & .331 & .01 & .036 & .381 \\ .06 & .193 & .356 & .03 & .054 & .307 \\ .03 & .133 & .35 & .017 & .073 & .397 \\ .1 & .24 & .334 & .041 & .043 & .242 \\ .114 & .11 & .281 & .015 & .15 & .33 \\ .035 & .129 & .274 & .015 & .096 & .451 \end{pmatrix}$$

$$F_R(5) = \begin{pmatrix} .055 & .128 & .325 & .018 & .08 & .394 \\ .058 & .17 & .348 & .025 & .054 & .342 \\ .033 & .13 & .29 & .017 & .155 & .375 \\ .06 & .193 & .356 & .03 & .054 & .307 \\ .111 & .144 & .264 & .014 & .107 & .36 \\ .051 & .132 & .288 & .016 & .086 & .427 \end{pmatrix} \quad (12)$$

Magnitude-to-magnitude transitions

In the second stage, magnitudes are chosen as states. Similar to the previous stage, we apply the AGNES method (Ghazanfari and Rezaei 2006) for the clustering of magnitudes, and three clusters on magnitudes as states are established in the whole regions as follows:

$$\begin{aligned} M1 : MB < 4.5 \\ M2 : 4.5 \leq MB \leq 5 \\ M3 : 5 < MB \end{aligned} \quad (13)$$

where MB is a unit of the magnitude of earthquakes; it is called as body wave magnitude.

The transition matrix of magnitude-to-magnitude transitions is given in Equation 17. The holding time mass functions $T_M(m)$ and interval transition probabilities

$F_M(n)$ are calculated using Equation 2 through Equation 8. These are shown in Equations 15 and 16.

$$G_M = \begin{matrix} M_1 \\ M_2 \\ M_3 \end{matrix} \begin{pmatrix} .5 & .3 & .2 \\ .45 & .5 & .05 \\ .167 & .5 & .333 \end{pmatrix} \quad (14)$$

$$T_M(1) = \begin{pmatrix} 1 & 1 & .75 \\ .67 & .7 & 0 \\ 0 & 1 & 1 \end{pmatrix} \quad T_M(2) = \begin{pmatrix} 0 & 0 & .25 \\ .11 & .2 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$T_M(3) = \begin{pmatrix} 0 & 0 & 0 \\ .22 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad T_M(4) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$T_M(5) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & .1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (15)$$

$$F_M(1) = \begin{pmatrix} .55 & .3 & .15 \\ .302 & .698 & 0 \\ 0 & .5 & .5 \end{pmatrix} \quad F_M(2) = \begin{pmatrix} .365 & .435 & .2 \\ .322 & .633 & .045 \\ .318 & .515 & .167 \end{pmatrix}$$

$$F_M(3) = \begin{pmatrix} .326 & .51 & .164 \\ .38 & .486 & .134 \\ .359 & .538 & .103 \end{pmatrix} \quad F_M(4) = \begin{pmatrix} .347 & .507 & .146 \\ .336 & .514 & .15 \\ .37 & .495 & .135 \end{pmatrix}$$

$$F_M(5) = \begin{pmatrix} .347 & .51 & .143 \\ .328 & .526 & .146 \\ .346 & .507 & .147 \end{pmatrix} \quad (16)$$

Forecasting

In this section, time and magnitude of earthquake occurrence are forecasted for each region in the next 3 years. Note that some forecasting methods based on probability approaches don't have sufficient accuracy, and others are more accurate. Matrices P_1 through P_6 show region-to-region transition probabilities by time of occurrence without considering the dimension of magnitude. In other words, the sum of each column is equal to 1 (see Tables 1, 2, 3, 4, 5, and 6). For example, the ij th element of matrix P_1 shows the probability of earthquake occurrence in region R_i during year j if the last earthquake has occurred in region R_1 . Similarly, matrices P_7 through P_9 show magnitude-to-magnitude transition probabilities by time of occurrence without considering the spatial dimension (see Tables 7, 8, 9). In these matrices, the sum of each column is also equal to 1.

Here, a semi-Markov model for considering the time of earthquake occurrence is applied. This method requires time, place, and magnitude of previous

Table 1 Matrix P1

		1	2	3	4	5
P1 =	R1 to R1	0.335	0.335	0.017	0.1	0.055
	R1 to R2	0	0.14	0.083	0.142	0.128
	R1 to R3	0.335	0.165	0.49	0.331	0.325
	R1 to R4	0	0	0.017	0.01	0.018
	R1 to R5	0.33	0.165	0.093	0.036	0.08
	R1 to R6	0	0.195	0.3	0.381	0.394

Table 2 Matrix P2

		1	2	3	4	5
P2 =	R2 to R1	0.125	0.089	0.1	0.06	0.058
	R2 to R2	0.0375	0.329	0.24	0.193	0.17
	R2 to R3	0.375	0.368	0.334	0.356	0.348
	R2 to R4	0.125	0.047	0.041	0.03	0.025
	R2 to R5	0	0.041	0.043	0.054	0.054
	R2 to R6	0	0.126	0.242	0.307	0.342

Table 3 Matrix P3

		1	2	3	4	5
P3 =	R3 to R1	0	0.021	0.025	0.03	0.033
	R3 to R2	0.17	0.14	0.137	0.133	0.13
	R3 to R3	0.494	0.345	0.33	0.35	0.29
	R3 to R4	0	0.021	0.018	0.017	0.017
	R3 to R5	0	0.018	0.056	0.073	0.155
	R3 to R6	0.336	0.455	0.434	0.397	0.375

Table 4 Matrix P4

		1	2	3	4	5
P4 =	R4 to R1	0	0.125	0.089	0.1	0.06
	R4 to R2	1	0.375	0.329	0.24	0.193
	R4 to R3	0	0.375	0.368	0.334	0.356
	R4 to R4	0	0.125	0.047	0.041	0.03
	R4 to R5	0	0	0.041	0.043	0.054
	R4 to R6	0	0	0.126	0.242	0.307

Table 5 Matrix P5

		1	2	3	4	5
P5 =	R5 to R1	0	0.031	0.274	0.114	0.111
	R5 to R2	0.25	0.109	0.119	0.11	0.144
	R5 to R3	0	0.121	0.168	0.281	0.264
	R5 to R4	0	0.031	0.014	0.015	0.014
	R5 to R5	0.5	0.264	0.052	0.15	0.107
	R5 to R6	0.25	0.444	0.373	0.33	0.36

Table 6 Matrix P6

		1	2	3	4	5
P6 =	R6 to R1	0	0.008	0.014	0.035	0.051
	R6 to R2	0.06	0.088	0.117	0.129	0.132
	R6 to R3	0.11	0.193	0.262	0.274	0.288
	R6 to R4	0	0.008	0.011	0.015	0.016
	R6 to R5	0.055	0.113	0.112	0.096	0.086
	R6 to R6	0.775	0.59	0.484	0.451	0.427

Table 7 Matrix P7

		1	2	3	4	5
P7 =	M1 to M1	0.55	0.365	0.326	0.347	0.347
	M1 to M2	0.3	0.435	0.51	0.507	0.51
	M1 to M3	0.15	0.2	0.164	0.146	0.143

Table 8 Matrix P8

		1	2	3	4	5
P8 =	M2 to M1	0.302	0.322	0.38	0.336	0.328
	M2 to M2	0.698	0.633	0.486	0.514	0.526
	M2 to M3	0	0.045	0.134	0.15	0.146

Table 9 Matrix P9

		1	2	3	4	5
P9 =	M3 to M1	0	0.318	0.359	0.37	0.346
	M3 to M2	0.5	0.515	0.538	0.495	0.507
	M3 to M3	0.5	0.167	0.103	0.135	0.147

earthquakes. If the previous earthquake is within region r_0 and its magnitude is m_0 , then the probability of occurrence of the next earthquake within region r_1 and magnitude m_1 after d time periods is a conditional probability defined as follows:

$$P(R = r_1, M = m_1 | r_0, m_0, d) = P(r_1, m_1 | r_0, m_0, d)$$

=====
 region-to-region transition probabilities are independent from
 magnitude-to-magnitude transition probabilities
 =====

$$P(r_1 | r_0, d) \cdot P(m_1 | m_0, d) = F_{R_{r_0 r_1}}(d) \cdot F_{M_{m_0 m_1}}(d)$$

Using the above equations and the list of data, the last earthquake in Tehran province occurred at 28 May 2004 within region R_1 with a magnitude of 6.3 (MB). Consider the current time as November 2006; in this way, the proposed model forecasts that the most probable occurrence of an earthquake within

Tehran province within region R_3 (northeast of Tehran province) with a magnitude of M_2 ($4.5 \leq MB \leq 5$) from May 2006 to May 2007 is 0.2636 (26.36%).

Conclusion

This article attempts to forecast the likelihood of all earthquakes within Tehran province in the next years by a semi-Markov model. The proposed model forecasts that the most probable occurrence of an earthquake within Tehran province within region R_3 (northeast of Tehran province) with a magnitude of M_2 ($4.5 \leq MB \leq 5$) from May 2006 to May 2007 is 0.2636 (26.36%). According to our forecasting, an earthquake occurred in Roudehen (northeast of Tehran province) with a magnitude of 4.2 MB on 20 December 2006. This can be considered as the evaluation of the proposed model.

The average magnitude of earthquake occurrence in Tehran province is 3.1 MB yearly. The most number of earthquake occurrences have occurred in intervals ($4 < MB \leq 4.5$). It includes 32% all of the events. Here, in this paper, the spatial, temporal, and magnitude dimensions are considered, while in past researches, these three dimensions were considered less.

As a conclusion, the amount of forecast errors shows that MAPE (Haj Shirmohammadi 2003) in this model is equal to 4.45%, while in other models, it is more than 10% (Papazachos 1992; Karakaisi 1994).

The obtained results during the next 50 days show that in other models, 42% of earthquakes have been correctly forecasted. The places of 16% of earthquake occurrences have been correctly forecasted, but not their magnitudes. Sixteen percent of the forecasted earthquakes occurred in places adjacent to the forecasted ones, but their magnitudes were true, and 26% of earthquakes could not have been forecasted at all.

In this model, 56% of earthquakes have been correctly forecasted in all the three dimensions. The places of 19% of earthquake occurrences have been correctly forecasted, but not their magnitudes. Nineteen percent of the forecasted earthquakes occurred in places adjacent to the forecasted ones, but their magnitudes were true, and 6% of earthquakes could not have been forecasted at all.

The mentioned statistics above show that this model in comparison to others can be applied in a more accurate forecasting.

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