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## Lobbying for Subsidies with Heterogeneous Firms

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# Lobbying for Subsidies with Heterogeneous Firms\*

Hannes Kammerer<sup>†</sup>

*Work in progress*

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## Abstract

Recent empirical evidence shows that the few firms that receive subsidies are large, and that large firms take a prominent role in shaping public policy by lobbying. In this paper, I present a theoretical framework that accounts for these empirical facts in a unified way. I study the role of firm heterogeneity in productivity for within-industry lobby formation when receiving subsidies and lobbying is costly. Due to firm heterogeneity, a within-industry conflict between receiving and non-receiving firms arises. This conflict creates lobbying incentives for large firms and delivers novel results. Surprisingly, increasing the barriers to lobby or lower firm heterogeneity amplifies this within-industry conflict such that a smaller lobby can attain a higher subsidy rate. Even if barriers to participate are modest, introducing a subsidy program harms particularly the smallest firms in a market.

**JEL Codes:** H25, D72

**Keywords:** Lobbying, firm heterogeneity, subsidies

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# 1 Introduction

The influence of special interest groups on politics has always been of great interest to economists. Surprisingly, the public and scientific debate on lobbying is still dominated by an *inter-industry* view. Accordingly, special interest groups are supposed to lobby for an entire industry and compete against each other for government favors, which then become available to all firms of the represented industry (e.g. an import tariff for the steel industry). However, this traditional way of thinking neglects potential within-industry differences in lobbying incentives and benefits across firms. In this paper, I take a complementary *intra-industry* view on lobbying, which provides novel insights into the within-industry consequences of lobbying.

My approach is motivated by recent empirical studies, which show that large firms take a prominent role in shaping public policy by lobbying (Bombardini, 2008; Kerr et al., 2011). However, in particular those large firms are the ones that benefit heavily from government programs and subsidies. Given that the public-good character of subsidies can be very limited, lobbying for subsidies can generate firm-specific benefits (Rodrik, 1986). Even if subsidies are targeted at a narrowly defined industry, not all firms necessarily receive payments. In the heavily subsidized US agricultural sector, more than 60% of all farms do not receive any government payments (USDA, 2009).<sup>1</sup> A positive relationship between participation in R&D subsidy programs and firm size has also been documented for many countries.<sup>2</sup> For West Germany, Wagner (2010) shows that the few manufacturers that receive government payments are systematically more productive and larger.

To account for these empirical facts, I focus on the impact of government payments across firms within a single industry, when firms can decide to influence public policy by joining an interest group. I incorporate a production subsidy in a monopolistic competition framework where firms are heterogeneous with respect to their productivity (Melitz, 2003). As a novel feature of my model, the subsidy, which is modeled as a reduction of a firm's variable production costs, is only granted to firms that bear the associated *administrative* fixed costs to become eligible.<sup>3</sup> This assumption allows me to make explicit use of heterogeneity in firm productivity to endogenously determine firm participation in the subsidy program. Because receiving firms benefit from the subsidy and sell at a lower price, non-receiving firms suffer from tougher market conditions and a within-industry

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<sup>1</sup>This pattern prevails even at the narrowly defined 5-digit NAICS level (USDA, 2009, table 62). The distribution of US farm subsidies is also highly skewed: 20% (8%) of the farms receive 80% (58%) of the payments (Kirwan, 2007). Given that agricultural subsidies usually depend on the amount of crops produced, this skewness may not surprise. However, it is remarkable that the vast majority of farms are not subsidized at all.

<sup>2</sup>e.g. Duguet (2004) for France, Blanes and Busom (2004) for Spain and Hussinger (2008); Aschhoff (2010) for Germany.

<sup>3</sup>The administrative burden of applying for government programs is not negligible and is of great interests for policymakers. A recent example is the report by the "Farming Regulation Task Force" to the UK Government, which finds more than 200 unnecessary "red tape" burdens and highlights the importance of reducing paperwork for farmers (DEFRA, 2011).

conflict arises. When the subsidy is endogenously determined in a lobbying game à la Grossman and Helpman (1994), this conflict creates incentives for large firms to lobby, such that the size and composition of the lobby is also endogenously determined.

In particular, I extend the standard two stage “Protection for Sale” lobbying game of Grossman and Helpman (1994) by an additional first stage where each firm decides to join a special interest group. Besides determining eligibility, the administrative fixed costs then also reflect a firm’s lobby entry costs. Only firms with productivity above the *lobby cutoff* decide to become eligible and to join the lobby. This feature of the model is consistent with recent empirical evidence on lobbying, which shows that there are considerable fixed costs associated with lobbying and that within an industry only few and large firms lobby (Kerr et al., 2011).

The optimal subsidy rate in the model depends on the trade-off between the markup distortion from monopolistic competition and a novel distortion caused by the administrative fixed costs. If these costs are negligible, the ex-ante welfare maximizing subsidy rate exactly compensates for the markup distortion in the economy. If the fixed costs distortion is large, an ex-ante welfare maximizing government should neglect the markup distortion from monopolistic competition and it should not introduce a subsidy. In contrast, a government that is influenced by lobby contributions will introduce a subsidy in equilibrium.

One popular justification for subsidies is to help small firms. The results of my paper show that even under modest barriers to participate, the introduction of a subsidy program harms particularly the smallest firms in a market. Thus, ignoring these barriers, policymakers may obtain results directly opposing their intention.

Comparative statics of the lobbying equilibrium depend on the within-industry conflict that drives lobbying. If the government values lobby contributions more than general welfare, increasing firm heterogeneity leads to a decline in the subsidy rate. This result qualifies findings in the literature that firm size dispersion is positively related to the political power of a lobby (e.g. Bombardini, 2008). Similarly, if the government values lobby contributions more than general welfare, higher lobby entry costs, while unambiguously reducing the (relative) size of the lobby, increase the equilibrium subsidy rate. This finding contrasts with the standard assumption in the literature that lobby size is positively related to the political power of an interest group (e.g. Acemoglu and Robinson, 2001).

The theoretical framework and the key mechanism underlying this paper are closely related to the heterogeneous firm literature of international trade. The monopolistic competition model of Melitz (2003) is a landmark in this literature.<sup>4</sup> A key feature of Melitz-type models is that with sufficiently high trade costs only very efficient firms decide to export. Similarly, in my model, with sufficiently high administrative fixed costs only

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<sup>4</sup>Melitz (2003) extends the framework in Hopenhayn (1992) to monopolistic competition in a general equilibrium setting. See Redding (2011) for a comprehensive review on international trade theory and firm heterogeneity. Complementary, Bernard et al. (2012) review empirical evidence on firm heterogeneity in the context of international trade.

the most efficient firms decide to receive subsidies and lobby.

Recent papers study the impact of public policies in heterogeneous firm models (e.g. Chor, 2009; Demidova and Rodríguez-Clare, 2009; Pflüger and Russek, 2011; Pflüger and Suedekum, 2013). In these papers, policy instruments still affect all firms in the market in the same way. To the best of my knowledge, my paper is the first one that makes explicit use of firm heterogeneity to endogenize the set of firms that benefit from a policy instrument.

Due to data availability, there is still little empirical evidence on the firm-level impact of production subsidies. Using a panel data set for German manufactures in the period 1999-2006, Wagner (2010) shows that while the fraction of subsidized firms is low, receiving firms in Western Germany are larger and were already more profitable before receiving subsidies.<sup>5</sup> These patterns confirm the theoretical predictions of my model.

Some authors also introduce lobbying in Melitz-type models. Abel-Koch (2010) and Rebeyrol and Vauday (2008) analyze fixed costs of production or entry as policy instruments. Chang and Willmann (2006) make use of the opposing interests of domestic and exporting firms to model lobbying for an import tariff. In contrast to my approach, these papers take the lobby itself or the mass of lobbying firms as exogenously given and firm heterogeneity is not exploited to endogenize lobby formation.

Endogenous lobby formation has been studied by Mitra (1999). While his paper looks at lobby formation across industries, I focus on lobby formation within an industry. In a related study, Bombardini (2008) analyzes lobby participation across firms. In her paper, firms can participate in lobbying for sector specific trade policies, which benefit all domestic firms within the sector. In contrast, in my model firms participate in lobbying to receive benefits at the expense of other firms within the sector. This within-industry conflict leads to novel and complementary insights into lobby participation across firms.<sup>6</sup>

A growing body of studies employ US firm-level lobbying data, which became available through the 1995 Lobbying Disclosure Act (e.g. Ansolabehere et al. (2002); Bombardini and Trebbi (2009); Chen et al. (2010); Tressel et al. (2009); Ludema et al. (2010) and Igan et al. (2011)). As one of the most recent papers in this literature, Kerr et al. (2011) provide novel firm-level evidence on lobbying behavior of publicly traded US firms. While only few firms are politically active, lobby participation and lobbying expenditures are positively correlated with firm size. As a main result, Kerr et al. (2011) find evidence that there are fixed entry costs to lobbying. This supports the assumption made in my

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<sup>5</sup>Given the particular historical and economic situation in Germany, the results of Wagner (2010) differ for Western and Eastern Germany. In his dataset, only 3.35% of the manufacturing firms in Western Germany and 17.27% in Eastern Germany received subsidies in 2006. Subsidized manufacturers in Eastern Germany are also less productive and less human capital intensive firms.

<sup>6</sup>My modeling approach differs in several other dimensions from Bombardini (2008). In Bombardini (2008), lobby entry of an additional firm raises the benefits of all members proportional to the entrant's firm size. This requires that each firm is of positive mass, such that individual contributions can change the political equilibrium. If the joint benefits from lobby entry of an additional firm lie below the lobby entry costs, the firm is not allowed to join the lobby. Bombardini (2008) uses specific factor model with a finite set of goods. Heterogeneity in firm size is due to different endowments of the specific factor. Therefore, firm-level differences in productivity are absent.

paper that there are barriers to start lobbying.

The rest of the paper is organized as follows. In Section 2, after presenting a baseline model with heterogeneous firms, I introduce a production subsidy and lobbying in the model. In Section 3 and Section 4, I work through the two emerging cases with low and high administrative fixed costs. For both cases, I first derive the equilibrium with an ex-ante welfare maximizing government, before analyzing the equilibrium of the lobbying game. Section 5 concludes.

## 2 Theoretical framework

In this section, I first lay out a baseline model with heterogeneous firms. Subsequently, I introduce a production subsidy and lobbying in the model.

### 2.1 Baseline model

**Preferences** There are two sectors in the economy: a differentiated goods sector and a sector where a homogenous numéraire good,  $X$ , is produced. One unit of this outside good is produced by one unit of labor input, such that the wage rate is fixed to one. To simplify notation, the mass of labor in the economy is also normalized to one, such that total labor income is fixed to unity as well.<sup>7</sup> The quasilinear utility function of the representative consumer is given by

$$U(X, Q) = y \ln(Q) + X,$$

where  $y > 0$ . By utility maximization,  $y$  is the constant aggregate expenditure on all available differentiated varieties,  $y = PQ$ . The CES composite good  $Q$  consists of a continuum of available varieties  $\omega \in \Omega$ :

$$Q = \left[ \int_{\omega \in \Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}},$$

where  $\sigma > 1$  denotes the elasticity of substitution between varieties, and  $q(\omega)$  is the consumed quantity of variety  $\omega$ . The sum of aggregate profits and labor income defines total income  $Y = \Pi + 1$ , which is spent on the differentiated goods and the outside good, such that  $Y = y + X$ . Utility maximization yields the standard CES-demand for variety  $\omega$ :

$$q(\omega) = Ap(\omega)^{-\sigma}, \quad A = yP^{\sigma-1}, \quad (1)$$

where  $p(\omega)$  denotes the price of variety  $\omega$ , and  $P$  is the dual price index defined by

$$P = \left[ \int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}. \quad (2)$$

**Technology and firm behavior** Firms use labor as input to produce their unique variety in a market with monopolistic competition. A firm draws its productivity  $\varphi$  (i.e. the inverse of its variable per-unit labor requirement) from a Pareto distribution with

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<sup>7</sup>I implicitly assume that aggregate labor demand in the differentiated goods sector is less than one, such that the differentiated goods are produced in equilibrium. All results hold if the mass of labor in the economy is greater than one.

shape parameter  $\theta > \sigma$  and scale parameter  $b > 0$ .<sup>8</sup> The cumulative distribution function is given by  $G(\varphi) = 1 - \left(\frac{b}{\varphi}\right)^\theta$  such that the probability density function is  $g(\varphi) = \frac{\theta b^\theta}{\varphi^{\theta+1}}$  for  $\varphi \geq b$ .<sup>9</sup> Low values of  $\theta$  correspond to “fat tails” of the productivity distribution and therefore to greater firm heterogeneity. Following Chaney (2008), the set of *possible entrants*,  $J$ , is a fixed measure. Only a subset of those firms will be active in equilibrium. The economy is in a steady state, such that firm entry equals firm exit. After a firm knows its productivity draw, it has to pay production fixed costs  $f$  to be an active producer. The sum of these fixed costs and variable costs  $l^{var}(\varphi) = \frac{q(\varphi)}{\varphi}$  are a firm’s total costs (i.e. total labor requirement):

$$l(\varphi) = \frac{q(\varphi)}{\varphi} + f.$$

Each firm chooses the price of its variety to maximize profits,  $\pi = p(\varphi)q(\varphi) - \frac{q(\varphi)}{\varphi} - f$ . With CES demand (equation (1)), profit maximization leads to the standard constant markup pricing rule in the Dixit and Stiglitz (1977) framework:

$$p(\varphi) = \frac{\sigma}{\sigma - 1} \frac{1}{\varphi}.$$

Equilibrium revenues of a firm are

$$r(\varphi) = A [p(\varphi)]^{1-\sigma} = A \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma-1} \varphi^{\sigma-1}.$$

Variable profits are proportional to revenues,  $\pi^{var}(\varphi) = \frac{r(\varphi)}{\sigma}$ , such that equilibrium profits of a firm are

$$\pi(\varphi) = \frac{r(\varphi)}{\sigma} - f = B\varphi^{\sigma-1} - f, \quad B = \frac{(\sigma - 1)^{\sigma-1}}{\sigma^\sigma} A. \quad (3)$$

The productivity level of the marginal firm that makes zero profits is implicitly defined by  $\pi(\varphi_{base}^*) = 0$ . Using equation (3), the resulting *product market cutoff* is given by

$$\varphi_{base}^* = \left(\frac{f}{B}\right)^{\frac{1}{\sigma-1}}. \quad (4)$$

Therefore, the mass of active firms is  $J_A = J(1 - G(\varphi_{base}^*))$ . Note that henceforth the subscript “*base*” refers to variables of the baseline model. Using equations (2) and (4), the baseline price index and the baseline product market cutoff can be rewritten as functions of the model parameters:

$$P_{base} = \tilde{\kappa} f^{\frac{1}{\theta} \frac{\theta - \sigma + 1}{\sigma - 1}} \quad (5)$$

and

$$\varphi_{base}^* = \kappa f^{\frac{1}{\theta}},$$

where  $\kappa = \left(J \frac{\theta b^\theta}{\theta - \sigma + 1} \frac{\sigma}{y}\right)^{\frac{1}{\theta}}$  and  $\tilde{\kappa} = \kappa^{-1} \left(\frac{\sigma}{\sigma - 1}\right) \left(\frac{\sigma}{y}\right)^{\frac{1}{\sigma - 1}}$ . The price index is increasing in  $\theta$ , but the product market cutoff is decreasing in  $\theta$ . Less dispersion in firm heterogeneity and therefore relatively less high productive firms makes the composite good more expensive (i.e. the price index rises). Therefore, the product market cutoff declines and the marginal

<sup>8</sup>Note that for aggregate sales (quantity) to be well defined, it must hold that  $\theta > \sigma$ .

<sup>9</sup>Given its productivity draw, each firm produces a single variety. However, several firms can have identical productivity draws.

firm at the new cutoff has a lower productivity level. Aggregate profits in the economy are only a function of the parameters  $\sigma$ ,  $\theta$  and  $y$ :

$$\Pi = \int_{\omega \in \Omega} \pi(\varphi) d\omega = \frac{\sigma - 1}{\sigma} \frac{y}{\theta}.$$

Due to quasilinear preferences, welfare is given by  $W = Y + CS$ , where  $CS = u(Q) - PQ = y \ln(\frac{y}{P}) - y$  denotes consumer surplus. Using the expression for aggregate profits and consumer surplus, welfare in the baseline model can be rewritten as

$$W_{base} = \underbrace{\frac{\sigma - 1}{\sigma} \frac{y}{\theta}}_{\text{agg. profits } \Pi} + \underbrace{+ 1}_{\text{labor income}} + \underbrace{y \ln(y) - y \ln(\tilde{\kappa} f^{\frac{1}{\theta}} \frac{\theta - \sigma + 1}{\sigma - 1}) - y}_{\text{consumer surplus } CS}. \quad (6)$$

## 2.2 Introducing a production subsidy

The model presented so far is a simple autarky version of the Melitz (2003) and Chaney (2008) heterogeneous firms framework. I will now extend this baseline model by introducing a production subsidy, which firms can only receive after paying additional administrative fixed costs,  $f_s$ . Ex-ante no firm is excluded from the subsidy or directly picked by the government. However, depending on the level of these additional fixed costs, ex-post not all firms will necessarily receive subsidy payments.

I assume that the administrative fixed costs  $f_s$  contain two parts. On the one hand, for a given subsidy rate, they are *bureaucratic fixed costs* that have to be payed to receive government payments. The bureaucratic burden due to applications for government programs is not negligible. In particular for small firms, filling in paperwork and applying for government grants and subsidies can be very costly.<sup>10</sup> Similarly, acquiring information on the existence of suitable subsidy programs or uncertainty to receive payments after a long and cumbersome application process can also be seen as a part of these bureaucratic fixed costs. On the other hand, when the subsidy is endogenously determined, the administrative fixed costs  $f_s$  are considered to be *political fixed costs* that are necessary to enter a lobby. Fixed costs to lobby have been frequently used in the theoretical models of lobby formation (e.g. Mitra, 1999; Bombardini, 2008). Recently, Kerr et al. (2011) provide also first empirical evidence for lobby fixed costs.

Of course, lobbying and applying for subsidies are two different – although not mutually exclusive – firm activities. The assumption that there is a single fixed cost for both activities may seem strong. Taking this assumption literally, one should think of the fixed costs as allowing firms to apply for subsidies that are only granted to lobbying firms. A prominent and controversially discussed examples for government payments that allow for such discrimination across firms are Congressional Earmarks in the US. However, there are many examples where lobbying produces spillovers to non-lobbying firms. In Appendix C, I therefore relax the assumption that only lobbying firms can receive subsidies, and consider a much more complex model with two distinct fixed costs for

<sup>10</sup>For instance, the report by the UK “Farming Regulation Task Force” finds more than 200 unnecessary “red tape” burdens and highlights the importance of reducing paperwork for UK farmers (DEFRA, 2011).



lobbying and becoming eligible for subsidy payments. Even if the subsidy is not perfectly targetable to lobbying firms, I derive parameter conditions such that only lobbying firms receive subsidies. Therefore, I consider this is a robust assumption that simplifies a much more complex model.

I assume that the production subsidy  $s > 1$  reduces a firm's variable costs by a factor of  $\frac{1}{s}$ .<sup>11</sup> Similar to iceberg trade costs in international trade theory, using a subsidy on variable costs keeps the model relatively tractable. However, beyond pure technical reasons, there are various real world examples of government policies that reduce firms' variable production costs. For instance, low interest government loans, wage subsidies or business tax credits reduce at least partially variable input costs. While subsidy agreements within the WTO framework try to limit the use of "specific" subsidies, not all firm-specific subsidies are ruled out. In particular, if a specific subsidy involves research activities, it is even considered to be "non-actionable" subsidy that cannot be challenged in front of the WTO.<sup>12</sup> In an international trade context, a variable costs subsidy is equivalent to an export promoting policy that lowers variable transportation costs. Such policies, like export credits and export insurances, could be preferred by governments because they are less likely to be identified as forbidden export subsidies. Therefore, modeling a subsidy on variable costs describes the nature of many government subsidy programs very well.

In particular, given a subsidy rate  $s$ , the subsidized variable costs that a firm takes into account when maximizing its profits are

$$l_s^{var}(\varphi) = \frac{q_s(\varphi)}{s\varphi}. \quad (7)$$

Let  $f_s$  denote the administrative costs that have to be payed by each firm to receive the subsidy. Total costs of a subsidized firm are

$$l_s(\varphi) = \frac{q_s(\varphi)}{s\varphi} + f + f_s,$$

such that firm profits are  $\pi_s = p_s(\varphi)q_s(\varphi) - \frac{q_s(\varphi)}{s\varphi} - f - f_s$ .<sup>13</sup> As will be shown in detail below, the combination of firm heterogeneity and additional administrative fixed costs leads to self-selection of firms into subsidized production, depending on firm productivity (i.e. only large and more efficient firms receive payments). The key mechanism is that subsidy payments per firm (not the subsidy rate!) increase with firm productivity and therefore with firm size, while the fixed costs to receive payments are the same for all firms.<sup>14</sup> As a consequence, there exists an eligibility cutoff with respect to firm productivity below which firms will not become eligible to receive subsidies. With the set of subsidized

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<sup>11</sup>In Appendix B, I consider an ad-valorem output subsidy as an alternative policy instrument. Because subsidy payments per firm still increase in firm sales and firm productivity, the selection mechanism that determines eligibility in my model still works. In general, more productive firms will select into subsidized production, if firm profits are supermodular in firm productivity and the subsidy rate.

<sup>12</sup>Within the WTO framework the "Agreement on Subsidies and Countervailing Measures" (ASCM) is the juridical basis of international rules concerning export and domestic subsidies. See Article 8 of the ASCM for rules on non-actionable subsidies.

<sup>13</sup>Here, I take the subsidy rate as exogenously given. When the subsidy is endogenously determined (Section 2.3), there are additional lobbying contributions that firms have to pay.

<sup>14</sup>With a variable costs subsidy, equilibrium subsidy payments per unit of output even decrease with firm size.

varieties given by  $\Omega^s$ , the price index is defined by

$$P_s = \left[ \int_{\omega \in \Omega^s} p(\omega)^{1-\sigma} d\omega + \int_{\omega \in \Omega^s} p_s(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}. \quad (8)$$

Given that a firm pays the fixed costs to receive subsidies, it will maximize its profits by setting the market price of its variety to

$$p_s(\varphi) = \frac{\sigma}{\sigma - 1} \frac{1}{s\varphi}.$$

Equilibrium revenues and profits of a subsidized firm are then respectively,

$$r_s(\varphi) = A_s [p_s(\varphi)]^{1-\sigma} = A_s \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma-1} (\varphi s)^{\sigma-1}, \quad A_s = y P_s^{\sigma-1}$$

and

$$\pi_s(\varphi) = \frac{r_s(\varphi)}{\sigma} - f - f_s = B_s (\varphi s)^{\sigma-1} - f - f_s, \quad B_s = \frac{(\sigma - 1)^{\sigma-1}}{\sigma^\sigma} A_s.$$

Aggregate revenues and aggregate profits can be split up into revenues and profits of **E**ligible firms and **N**on-**E**ligible firms:  $y = R_E + R_{NE}$  and  $\Pi = \frac{\sigma-1}{\sigma} \frac{y}{\theta} = \Pi_E + \Pi_{NE}$ .

Define average productivity of eligible firms as

$$\tilde{\varphi}_L = \left[ \frac{1}{1 - G(\varphi_L^*)} \int_{\varphi_L^*}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}},$$

where  $\varphi_L^*$  denotes the productivity cutoff above which all firms receive subsidies. Then,  $J_L = J(1 - G(\varphi_L^*))$  denotes the mass of firms that receives subsidies and aggregate revenues of all eligible firms can be written as

$$R_E = J_L r_s(\tilde{\varphi}_L). \quad (9)$$

Similarly, aggregate profits of eligible firms are

$$\Pi_E = J_L \pi_s(\tilde{\varphi}_L) = \frac{R_E}{\sigma} - J_L (f + f_s).$$

Using the Pareto distribution, the relative mass of active firms that receive subsidy payments,  $J_R = \frac{J_L}{J_A}$ , depends only on the Pareto shape parameter and the ratio of the cutoffs:

$$J_R = \left( \frac{\varphi^*}{\varphi_L^*} \right)^\theta.$$

Subsidy payments per firm are the difference between the true variable costs for output  $q_s(\varphi)$  and the subsidized variable costs, equation (7):

$$\Delta l^{var}(\varphi) = \frac{s-1}{s} \frac{q_s(\varphi)}{\varphi} = (s-1) \left( \frac{\sigma-1}{\sigma} \right) r_s(\varphi).$$

Aggregating over all receiving firms gives the government's total subsidy payments:

$$S = (s-1) \frac{\sigma-1}{\sigma} R_E.$$

For simplicity, I assume that the subsidy is financed by a lump-sum tax on labor income, such that the upper bound of the government budget is  $S \leq 1$ . Welfare is then given by

$$W_s = \Pi + (1 - S) + (y \ln(\frac{y}{P_s}) - y).$$

The welfare channels of the subsidy can already be seen from this formula. Aggregate subsidy payments reduce net labor income and therefore welfare. Consumer surplus is

also affected by the subsidy, because the price index changes. However, because aggregate profits remain constant, the subsidy only shifts profits among firms within the sector.

### 2.3 Introducing lobbying

In this section, I present a lobbying game in which the subsidy rate is endogenously determined. Although the policy instrument of interest is a production subsidy in monopolistic competition, the lobbying framework I build on follows the menu auction approach by Bernheim and Whinston (1986) and is well known from the “Protection for Sale” model by Grossman and Helpman (1994). Recently, lobbying has been introduced in models with heterogeneous firms (e.g. Abel-Koch (2010), Rebeyrol and Vauday (2008), and Chang and Willmann (2006)). In contrast to these papers, the novel feature of my approach is to make explicit use of firm heterogeneity to determine the size and composition of the lobby endogenously. I extend the standard two stage “Protection for Sale” lobbying game by an additional first stage where each firm decides to join a special interest group that lobbies for a subsidy. Additionally to determining the set of eligible firms, the fixed costs  $f_s$  are now also considered to be *political* fixed costs that allow firms to join a lobby and to benefit from its lobbying achievements.<sup>15</sup> Therefore, in the lobbying game the eligibility cutoff is also the *lobby cutoff*. Because all firm with productivity above this cutoff join the lobby, the size and the composition of the lobby is an equilibrium object. Consequently, in an equilibrium of the lobbying game, the mass of lobbying firms has to induce a lobby contribution schedule that is consistent with the equilibrium subsidy rate.<sup>16</sup>

The driving force behind lobbying is a distributional conflict between receiving and non-receiving firms. By lobbying for an increase in the subsidy rate, receiving firms can benefit at the expense of non-receiving firms by selling at lower price and increasing their profits. This leads to a drop in the price index such that non-receiving firms lose profits. If the administrative fixed costs are low, all firms receive subsidy payments and this within-industry conflict is absent. Therefore, receiving firms have no incentive to offer positive contributions to the government. I term this the *low costs case*, which will be extensively discussed in Section 3. In contrast, if the administrative fixed costs are high and only a subset of active firms receives payments, the arising within-industry conflict gives firms an incentive to influence the government by lobbying. In Section 4 this *high costs case* is analyzed in detail.

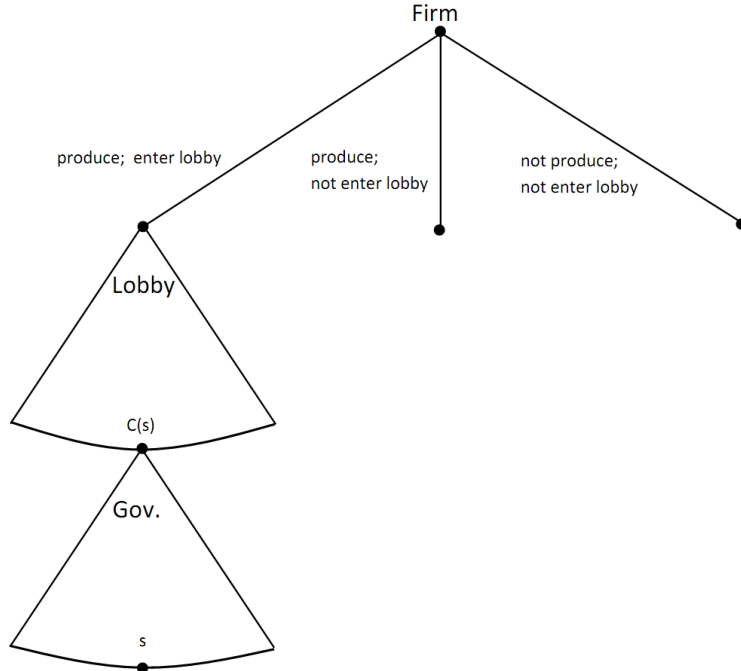
#### 2.3.1 Timing and structure of the lobbying game

The timing of the lobbying game is as follows. In first stage, each firm decides whether to produce and to join the lobby (pay  $f_s$  and  $f$ ); to produce but not to join the lobby (pay only  $f$ ); or neither to produce nor to lobby. In the second stage, the lobby offers

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<sup>15</sup>The assumption that there is only a single fixed costs  $f_s$  for being eligible *and* for lobbying, is not as restrictive as it might seem. In Appendix C, I relax this assumption.

<sup>16</sup>Non-receiving firms would benefit from a decline in the subsidy rate. However, due to the lobby fixed costs the interests of small firms are not recognized by the government.



**Figure 1:** Game tree of the lobbying game

a contribution schedule  $C(s)$ . In the third stage, the government chooses the subsidy  $s$  (given  $C(s)$ ) and firms set the profit maximizing price and produce either with or without the subsidy. Figure 1 shows the timing of the lobbying game graphically.

The government objective function is

$$G = \alpha W(s) + C(s),$$

where  $C(s)$  is the contribution schedule offered by the lobby and  $\alpha$  is the relative weight that the government puts on general welfare. Given the timing of the game, when the government decides about the subsidy rate, firms already joined the lobby and the lobby determined its contribution schedule. Therefore, the government takes the number of lobbying firms and the contribution schedule as given. Similarly, when the lobby determines its contribution schedule, it maximizes the profits of its current members, taking the number of lobby members as given. Anticipating the optimal behavior of the lobby and the government, only firms with productivity above the product market cutoff,  $\varphi^*$ , decide to be active producers and only firms with productivity above the lobby cutoff,  $\varphi_L^*$ , will decide to join the lobby.<sup>17</sup>

The joint contribution schedule offered by the lobby,  $C(s)$ , has to be financed by individual member contributions  $c(\varphi, s)$  such that  $C(s) = \int_{\varphi_L^*} c(\varphi, s) dG(\varphi)$ . I assume that these individual contributions are such that each lobby member still gains from joining the lobby:

**Assumption 1.** *If  $\varphi \geq \varphi_L^*$  then  $c(\varphi, s) \leq \pi_s(\varphi) - \pi(\varphi)$ .*

With this assumption, it can never be the case that a firm with net-benefits from receiving the subsidy (after paying lobby entry costs) would like to exit the lobby because

<sup>17</sup>Note that for low levels of  $f_s$  there is only a single cutoff.

of additional individual contributions. In other words, even with additional individual lobby contributions, the functional form of the productivity cutoffs is the same as with an exogenous subsidy rate.<sup>18</sup> Note that I do not impose any further restrictions on how the lobby collects the individual contributions from its member firms.

### 2.3.2 Equilibrium in the general case

Before analyzing different cases of lobbying game with explicit functional forms, I derive the equilibrium of the lobbying game in the general case. With a single lobby within the sector, a modified version of the second lemma in Bernheim and Whinston (1986) can be stated:

**Proposition 1.** *A set  $\{C^o, s^o, \varphi^o, \varphi_L^o\}$  is a subgame-perfect Nash equilibrium of the lobbying game if and only if:*

1. *only firms with  $\varphi > \varphi^o$  produce and only firms with  $\varphi > \varphi_L^o$  enter the lobby,*
2.  *$C^o \geq 0$  is feasible for the lobby,*
3.  *$s^o \in \arg \max_{s \in [1, \bar{s}]} G = \{\alpha W(s) + C^o(s)\},$*
4.  *$\{\alpha W(s^o) + C^o(s^o)\} + \{\Pi_E(s^o) - C^o(s^o)\} \geq \{\alpha W(s) + C^o(s)\} + \{\Pi_E(s) - C^o(s)\}$   
 $\forall s \in [1, \bar{s}],$*
5.  *$\exists s^* \in [1, \bar{s}],$  such that  $s^* \in \arg \max_{s \in [1, \bar{s}]} \{\alpha W + C^o(s)\}$  and  $C^o(s^*) = 0.$*

Condition 1 is directly related to the additional first stage of the lobbying game, where heterogeneous firms select into producing and lobbying. Condition 2 states that the offered contribution schedule is non-negative and fulfills Assumption 1. The equilibrium subsidy must also maximize the government's objective (condition 3) and the joint welfare of government and the lobby (condition 4) on the set of feasible subsidy rates. For condition 5 to hold, there must exist a feasible subsidy rate that maximizes the government's objective, given that the contributions of the lobby are zero.

As a refinement of the set of all Nash Equilibria, I assume that the contribution schedules are truthful in the sense that they represent the true preferences of the lobby:

**Assumption 2.** *Aggregate contribution schedules are truthful:*

$$C^T = \max [\Pi_E - B_L, 0],$$

*where  $\Pi_E$  are aggregate profits of the lobby members and  $B_L$  denotes the additional aggregate surplus of all lobbying firms, determined in equilibrium.*

Bernheim and Whinston (1986) argue that truthful strategies may be focal within the Nash set, and they show that every best-response set contains a truthful strategy. With truthful contribution schedules the following corollary can be stated:<sup>19</sup>

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<sup>18</sup>To get an intuition for this assumption, consider the marginal firm that joins the lobby. This firm makes zero additional profits from subsidized production, and any additional contribution would force the firm to exit the lobby.

<sup>19</sup>The corollary and its proof is similar to the one in Grossman and Helpman (1994), p.840.

**Corollary 1.** *Under truthful contribution schedules, the equilibrium subsidy satisfies*

$$s^o = \arg \max_{s \in [1, \bar{s}]} \{\alpha W(s) + \Pi_E(s)\}.$$

*Proof.* See Appendix A.1. □

Thus, with truthful contributions, the government behaves *as if* it maximizes a weighted sum of general welfare and joint lobby profits. The equilibrium contributions compensate the government for the weighted welfare loss induced from deviating from the subsidy rate that would maximize general welfare,  $s^*$ . It must therefore hold that  $C^T(s^o) = \alpha [W(s^*) - W(s^o)]$ . With truthful contributions (Assumption 2), the equilibrium lobby surplus is  $B_L = \Pi_E(s^o) - \alpha [W(s^*) - W(s^o)]$ . With welfare defined by  $W = \Pi + 1 - S + CS$ , where aggregate profits,  $\Pi$ , are constant, the first-order condition of the government maximization problem is given by

$$\frac{\partial G}{\partial s} = \alpha \left( \frac{\partial CS}{\partial s} - \frac{\partial S}{\partial s} \right) + \frac{\partial \Pi_E}{\partial s} = 0. \quad (10)$$

Note, when solving equation (10) the government takes the mass of active firms and the mass of lobby members as given.

To analyze the equilibrium of the lobbying game in detail, I will distinguish in the following between the low fixed costs case and the high fixed costs case. In both cases, I will first state the equilibrium expressions for a given subsidy rate, before deriving the optimal subsidy rate for a government that maximized *ex-ante* general welfare and that takes firm entry and exit into account. Subsequently, I derive the equilibrium of the lobbying game, where the government takes the mass of firms as given, when setting the subsidy rate.

### 3 The low administrative fixed costs case

If the administrative fixed costs are sufficiently low, all active firms find it profitable to receive subsidy payments. Henceforth, I call this the *low costs case*.

#### 3.1 Cutoff, price index and aggregate variables

For a given subsidy rate  $s$ , if the administrative fixed costs are sufficiently low, all active firms will be subsidized, such that  $\Omega = \Omega^s$ .<sup>20</sup> There is only one cutoff, the *eligibility and product market cutoff*<sup>21</sup>, defined by  $\pi_s(\varphi_{L,low}^*) = 0$ :

$$\varphi_{L,low}^* = s^{-1} \left( \frac{f + f_s}{B_s} \right)^{\frac{1}{\sigma-1}}. \quad (11)$$

Using this expression of the cutoff, the price index in the low costs case can be rewritten in terms of the model parameters:

$$P_{s,low} = \tilde{\kappa} (f + f_s)^{\frac{1}{\theta} \frac{\theta - \sigma + 1}{\sigma - 1}} s^{-1}, \quad (12)$$

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<sup>20</sup>The precise parameter condition that separates the low costs case from the high costs case is derived in Section 4.1.

<sup>21</sup>In the lobbying game, I will call this cutoff also the *lobby cutoff*.

where  $\tilde{\kappa} = \kappa^{-1} \left( \frac{\sigma}{\sigma-1} \right) \left( \frac{\sigma}{y} \right)^{\frac{1}{\sigma-1}}$ . Note that the elasticity of the price index with respect to the subsidy rate is given by  $\epsilon_{P_s, low} = -1$ , such that there is a perfect “pass-through” of the subsidy on the price index. The cutoff  $\varphi_{L, low}^*$  can also be rewritten in terms of model parameters:

$$\varphi_{L, low}^* = \kappa [f + f_s]^{\frac{1}{\theta}}, \quad (13)$$

where  $\kappa = \left( J \frac{\theta b^\theta}{\theta - \sigma + 1} \frac{\sigma}{y} \right)^{\frac{1}{\theta}}$ . In contrast to the administrative fixed costs, the subsidy rate does not appear in equation (13). For rising administrative fixed costs, the cutoff increases such that less firms are subsidized and active. Therefore, if  $f_s \rightarrow 0$ , the mass of active firms increases and converges to the value of the baseline model. The following lemma summarizes the results for the low costs case with a given subsidy rate:

**Lemma 1.** *In the low costs case with a given subsidy rate  $s$ , there is a unique eligibility and product market cutoff  $\varphi_{L, low}^*$  and*

1. *the price index lies below the baseline value,  $P_{s, low} < P_{base}$ ;*
2.  *$\varphi_{L, low}^*$  lies above the baseline value,  $\varphi_{L, low}^* > \varphi_{base}^*$ ;*
3.  *$\varphi_{L, low}^*$  is invariant to a change in the subsidy rate, but a rise in  $f_s$ , in  $f$ , in  $\sigma$  or in firm heterogeneity leads to an increase of  $\varphi_{L, low}^*$  and to a decline of  $J_A = J_L$ .*

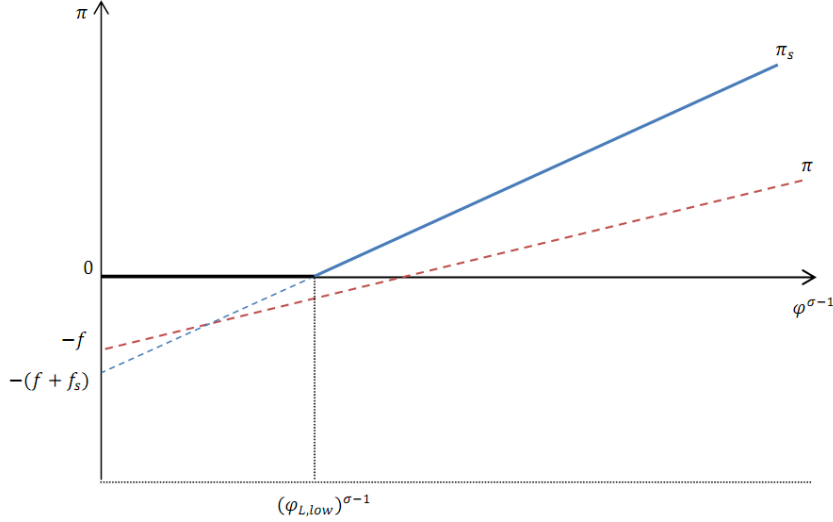
*Proof.* See Appendix A.2. □

The first statement of Lemma 1 follows directly from the comparison of equation (5) and equation (12). It shows that, despite the additional administrative fixed costs, the introduction of the subsidy has a positive effect on consumer surplus. However, due to the perfect pass-through of the subsidy on the price index, the only reason why introducing the subsidy program reduces available varieties (statement (2)) is the presence of the administrative fixed costs.

To give an intuition for statement 3 of Lemma 1, in Figure 2 firms’ profits are plotted as a function of firm productivity. The subsidy increases variable profits, such that bearing the relatively low administrative fixed cost is profitable for all active firms. Consequently, for all productivity levels associated with positive profits, the  $\pi_s$ -line lies above the  $\pi$ -line. Thus, there is only a single productivity cutoff,  $\varphi_{L, low}^*$ , at which the marginal (subsidized) firm makes zero profits.<sup>22</sup> A lower subsidy rate or higher fixed costs have a direct effect as well as counteracting indirect effect on profits. The price index increases (indirect effect), such that the  $\pi_s$ -line (ceteris paribus) rotates upwards. However, firm profits also decrease directly through higher fixed costs (shifts  $\pi_s$ -line downwards) or through a lower subsidy rate (rotates  $\pi_s$ -line downwards). The net effect on the cutoff is exactly zero for a decreasing subsidy rate, while the net effect of increasing fixed costs is positive.

Given that in the low costs case all active firms are subsidized and the upper bound of the government’s budget is one, for total aggregate subsidy payments it must hold that

<sup>22</sup>Note that the intersection of the dashed-brown line and the x-axis is not the baseline cutoff, which would lie to the left of  $\varphi_{L, low}^*$  (Lemma 1, statement 2). The decline of the price index, induced by the introduction of the subsidy, leads to a downward rotation of the  $\pi$ -line.



**Figure 2:** Firm profits in the low costs case

$S_{low} = (s - 1) \frac{\sigma - 1}{\sigma} y \leq 1$ . Therefore, the highest subsidy rate that the government is able to finance is  $\bar{s}_{low} = 1 + \frac{\sigma - 1}{\sigma - 1} \frac{1}{y}$  and the derivative of  $S_{low}$  with respect to  $s$  is a constant:

$$\frac{\partial S_{low}}{\partial s} = \frac{\sigma - 1}{\sigma} y. \quad (14)$$

Using the expression for the price index (equation (12)), the marginal gain in consumer surplus is convex and decreasing in  $s$ :

$$\frac{\partial CS_{low}}{\partial s} = \frac{y}{s}. \quad (15)$$

### 3.2 Ex-ante welfare optimum in the low costs case

As a benchmark, consider a government that does not take into account any lobby contributions, but maximizes only general welfare from an *ex-ante* perspective. The objective function of the government is then  $G = W_s$  and it chooses to the ex-ante optimal subsidy rate before firms enter the market and claim eligibility (i.e. taking firm entry behavior into account). Combining the derivatives of total subsidy payments and of consumer surplus (equation (14) and equation (15)), the first-order condition for an interior welfare optimum is

$$\frac{\partial W_{low}}{\partial s} = \frac{\partial CS_{low}}{\partial s} - \frac{\partial S_{low}}{\partial s} = \frac{y}{s} - \frac{\sigma - 1}{\sigma} y = 0.$$

The unique interior solution is given by  $s^* = \frac{\sigma}{\sigma - 1}$ , such that the optimal subsidy exactly compensates for the markup distortion. Total subsidy payments evaluated at the interior optimum are  $S_{low}^* = \frac{y}{\sigma}$ .<sup>23</sup> Note that the interior welfare optimum is identical to the global welfare optimum in a model without any administrative fixed costs. However, the interior solution in the low costs case may be welfare dominated by a corner solution at  $s = 1$ . To see this more explicitly, consider the difference between the interior welfare optimum in the low costs case and welfare in the baseline case (equation (6)):

$$W_{low}^* - W_{base} = y \ln\left(\frac{P_{base}}{P_{s^*,low}^*}\right) - \frac{y}{\sigma}, \quad (16)$$

<sup>23</sup>With labor income normalized to unity, this solution is always feasible if  $\sigma > y$ .



where  $P_{s^*,low} = P_{base} \frac{\sigma-1}{\sigma} \left(1 + \frac{f_s}{f}\right)^{\frac{1}{\theta} \frac{\theta-\sigma+1}{\sigma-1}}$ . While the second term of equation (16) (total subsidy payments) is constant, the first term (difference in consumer surplus) is decreasing in  $f_s$ . Therefore, if the administrative fixed costs are sufficiently high, the interior optimum might not longer be a global optimum. The following lemma gives the precise condition when this is the case:

**Lemma 2.** *In the low costs case, the unique **interior** (ex-ante) welfare optimum is given by  $s^* = \frac{\sigma}{\sigma-1}$ . For administrative fixed costs above  $f_s = f \left( \exp \left[ \frac{\ln(\frac{\sigma}{\sigma-1}) - \frac{1}{\sigma}}{\frac{1}{\theta} \frac{\theta-\sigma+1}{\sigma-1}} \right] - 1 \right)$ , the **global** (ex-ante) welfare optimum is given by  $s^* = 1$ .*

*Proof.* See Appendix A.3. □

Lemma 2 is driven by the trade-off between the markup distortion and a novel distortion associated with the administrative fixed costs. To get rid of the markup distortion, the government would like to introduce a subsidy. However, the administrative fixed costs associated with this subsidy cause an additional distortion in the economy. If the negative welfare impact of the administrative fixed costs distortion is too large, a welfare maximizing government should not introduce a subsidy. For low values of  $f_s$ , the positive impact of the administrative fixed costs on the price index is modest and the difference between baseline and low costs price index is quite large. Therefore, the resulting increase in consumer surplus is large enough to compensate for the financing of the subsidy (i.e. equation (16) is positive). However, if  $f_s$  increases above a certain threshold, the price index is too close to its baseline value. Therefore, consumer surplus increases little, and the introduction of any subsidy reduces welfare. In the low costs case, consumer surplus is the only channel through which an increase in  $f_s$  affects welfare. Therefore, the next corollary follows directly from Lemma 2:

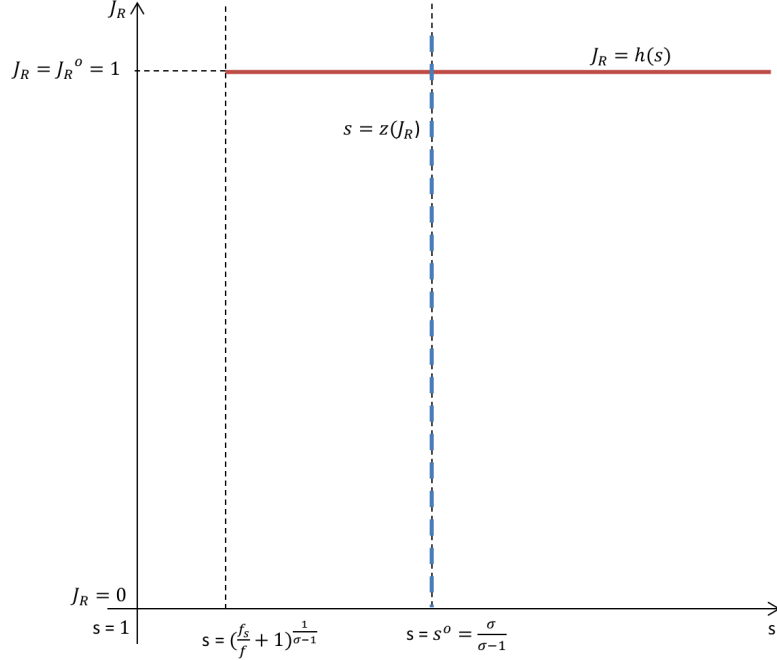
**Corollary 2.** *In the low costs case, (ex-ante) welfare is decreasing in  $f_s$ .*

*Proof.* See Appendix A.4. □

For the special case where the administrative fixed costs converge to zero, the low costs case nests the “traditional” inter-industry view on production subsidies. Accordingly, all firms in the market receive the subsidy without a cost and a markup-compensating subsidy rate is optimal. Consequently, without administrative fixed costs, introducing a subsidy would not have an effect on the cutoff and on firms’ profits. Consumer surplus, however, would substantially increase because all varieties are sold at marginal costs. However, even for positive but modest levels of administrative fixed costs, the induced anti-variety effect makes the interior optimal subsidy rate welfare inferior to a corner solution without a subsidy (Lemma 2).

### 3.3 Lobbying in the low costs case

Consider a government that takes lobby contribution into account and maximizes its objective function,  $G = \alpha W_s + C(s)$ , by choosing the optimal subsidy rate within the



**Figure 3:** Equilibrium low costs case

lobbying game (i.e. taking entry behavior of firms as given).

**Equilibrium in the low costs case** Given that the lobby cutoff (equation (13)) is invariant with respect to  $s$ , the derivatives of consumer surplus and total subsidy payments in the lobbying game are given by equation (14) and equation (15). Therefore, with constant aggregate profits,  $\frac{\partial \Pi_E}{\partial s} = \frac{\partial \Pi}{\partial s} = 0$ , the first-order condition of the lobbying game, equation (10), leads to the interior solution  $s^o = \frac{\sigma}{\sigma-1}$ . Because the government implements the subsidy rate that maximizes *ex-ante* general welfare,  $s^* = s^o = \frac{\sigma}{\sigma-1}$ , lobby contributions are zero in equilibrium. The following Lemma summarized the equilibrium of the lobbying game in the low costs case:

**Lemma 3.** *In the low costs case with lobbying, there exists a unique equilibrium of the lobbying game, such that*

1. *all firms with  $\varphi > \varphi_{L,low}^o = \kappa [f + f_s]^{\frac{1}{\theta}}$  produce and enter the lobby,*
2. *lobby contributions are  $C^T(s^o) = 0$ ,*
3. *the government implements the interior (welfare) optimum  $s^o = s^* = \frac{\sigma}{\sigma-1}$ .*

Given that all active firms join the lobby, the relative mass of lobbying firms  $J_R = \frac{J_L}{J_A}$  is one. In Figure 3, which shows the equilibrium of the low cost case graphically,  $J_R$  is depicted by the function  $h(s) = 1$ . The optimal subsidy rate is depicted by the function  $z(J_R) = \frac{\sigma}{\sigma-1}$ , such that the unique equilibrium is given by the intersection ( $s = \frac{\sigma}{\sigma-1}, J_R = 1$ ). In the low costs case, an increase in the subsidy rate is not particularly beneficial for some active firms at the expense of others, such that there is no within-industry conflict across firms. Without this conflict, lobbying incentives are limited and the government implements the interior *ex-ante* welfare maximizing subsidy rate.

## 4 The high administrative fixed costs case

If the administrative fixed costs are sufficiently high, only a subset of active firms decide to receive the subsidy. Henceforth, I call this the *high costs case*.

### 4.1 Cutoffs, price index and aggregate variables

For a given subsidy rate  $s$ , the marginal firm that enters the product market makes zero profits,  $\pi(\varphi^*) = 0$ . For sufficiently high administrative fixed costs this marginal firm will not receive subsidies. The corresponding *product market cutoff* is

$$\varphi^* = \left( \frac{f}{B_s} \right)^{\frac{1}{\sigma-1}}. \quad (17)$$

For the marginal firm that decides to receive subsidies, profits from subsidized production equal profits from non-subsidized production:  $\pi_s(\varphi_L^*) = \pi(\varphi_L^*)$ . The *eligibility cutoff*<sup>24</sup> is therefore

$$\varphi_L^* = \left( \frac{f_s}{(s^{\sigma-1} - 1)B_s} \right)^{\frac{1}{\sigma-1}}. \quad (18)$$

The ratio of the cutoffs depends only on the fixed costs, the subsidy rate and the elasticity of substitution,  $\frac{\varphi_L^*}{\varphi^*} = \left( \frac{f_s}{f(s^{\sigma-1} - 1)} \right)^{\frac{1}{\sigma-1}}$ . The relative mass of lobbying firms is therefore  $J_R = \frac{J_L}{J_A} = \left( \frac{f_s}{f(s^{\sigma-1} - 1)} \right)^{-\frac{\theta}{\sigma-1}}$ . There are active firms that do not receive subsidies if the administrative fixed costs are sufficiently high:<sup>25</sup>

$$f_s > f(s^{\sigma-1} - 1).$$

Using the cutoffs (equation (17) and (18)), the price index in the high costs case can be rewritten as a function of the model parameters:

$$P_{s,high} = \tilde{\kappa} [f^{\frac{\sigma-\theta-1}{\sigma-1}} + (s^{\sigma-1} - 1)^{\frac{\theta}{\sigma-1}} f_s^{\frac{\sigma-\theta-1}{\sigma-1}}]^{-\frac{1}{\theta}}, \quad (19)$$

where  $\tilde{\kappa} = \kappa^{-1} \left( \frac{\sigma}{\sigma-1} \right) \left( \frac{\sigma}{y} \right)^{\frac{1}{\sigma-1}}$ . For  $f_s \rightarrow \infty$ , the price index in the high costs case converges (from below) to the baseline value. An increase in the subsidy rate leads to a decline in the price index. However, with only a subset of active firms subsidized, the “pass-through” of the subsidy on the price index is now incomplete (i.e.  $|\epsilon_{P_{s,high}}| < 1$ ).<sup>26</sup> Using the expression of the price index (equation (19)) together with equation (17) and equation (18), the cutoffs can also be expressed in terms of model parameters:

$$\varphi^* = \kappa [f + f_s \left( \frac{f_s}{f(s^{\sigma-1} - 1)} \right)^{-\frac{\theta}{\sigma-1}}]^{\frac{1}{\theta}} \quad (20)$$

and

$$\varphi_L^* = \kappa [f \left( \frac{f_s}{f(s^{\sigma-1} - 1)} \right)^{\frac{\theta}{\sigma-1}} + f_s]^{\frac{1}{\theta}}, \quad (21)$$

where  $\kappa = \left( J \frac{\theta b \theta}{\theta - \sigma + 1} \frac{\sigma}{y} \right)^{\frac{1}{\theta}}$ . The following lemma summarizes the results for the high costs case with a given subsidy rate  $s$ .

<sup>24</sup>In the lobbying game, this will be the *lobby cutoff*.

<sup>25</sup>This condition implies reasonable levels of administrative fixed costs. For instance, if  $s = 1.05$  and  $\sigma = 2$ , administrative fixed costs have to be at least 5% of production fixed costs.

<sup>26</sup>See Appendix A.5 for an explicit expression of  $\epsilon_{P_{s,high}}$ .

**Lemma 4.** *In the high costs case with a given subsidy rate  $s$ , there is both a product market cutoff  $\varphi^*$  and a eligibility cutoff  $\varphi_L^*$  and*

1. *the price index lies below the baseline value  $P_{s,high} < P_{base}$ ;*
2.  *$\varphi^*$  lies above the baseline value  $\varphi^* > \varphi_{base}^*$ ;*
3. *a rise in  $s$  or  $f$ , or a decline in  $f_s$  increases  $\varphi^*$  and  $J_L$  but decreases  $\varphi_L^*$  and  $J_A$ ;*
4. *a rise in firm heterogeneity increases both  $\varphi^*$  and  $\varphi_L^*$ .*

*Proof.* See Appendix A.8. □

From statement 1 of Lemma 4 it follows directly that the introduction of the subsidy has still a positive impact on consumer surplus. Similar to the low costs case, the introduction of the subsidy reduces available varieties (statement (2)). However, in contrast to the low costs case, the cutoffs depend not only on the fixed costs but also on the subsidy rate. While an increase in the subsidy leads to more eligible firms, it reduces the total mass of active firms. Because the least efficient firms exit, there are less varieties available.

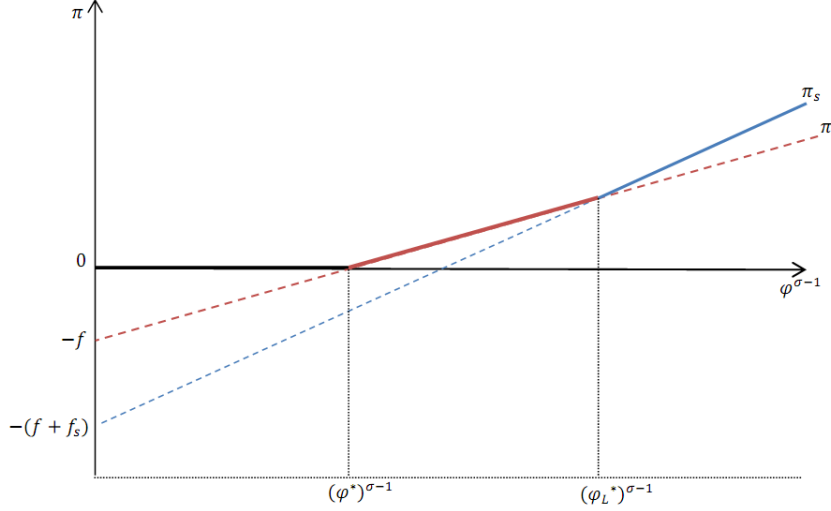
To get an intuition for statement 3 of Lemma 4, in Figure 4 firms' profits are plotted as a function of firm productivity. Bearing the relatively high administrative fixed costs is not profitable for all active firms such that some relatively less efficient firms produce without the subsidy ( $\pi$ -line). The product market cutoff lies at the intersection of the  $\pi$ -line and the x-axis, while the eligibility cutoff lies at the intersection of the  $\pi$ -line and the  $\pi_s$ -line. An increase in the subsidy rate rotates the  $\pi_s$ -line upwards, because the positive direct effect on firms' profits is stronger than the negative indirect effect from the decreasing price index. The  $\pi$ -line rotates downwards, because non-eligible firms' profits are only negatively affected through the decreasing price index. As a result, the product market cutoff increases, and the eligibility cutoff decreases. An increase in the administrative fixed costs has the opposite effect. It shifts the  $\pi_s$ -line downwards and makes subsidized production for the marginal firm at the eligibility cutoff unprofitable (eligibility cutoff increases). Because of the associated increase in the price index, the  $\pi$ -line rotates upwards and more firms find it profitable to be active (product market cutoff declines).

Even though both cutoffs increase in firm heterogeneity (statement 4 of Lemma 4), the mass of firms that receive subsidies may increase:

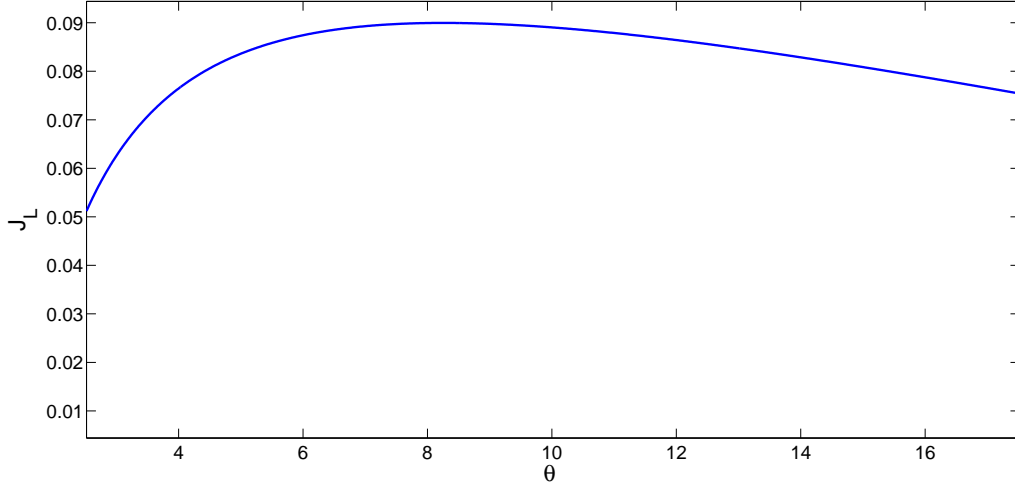
**Lemma 5.** *A rise in firm heterogeneity decreases  $J_A$  and increases  $J_R$ . If the subsidy rate is sufficiently high, there is a hump-shaped relationship between firm heterogeneity and  $J_L$ .*

*Proof.* See Appendix A.9 □

There are two counteracting effects that determine the impact of an increase in firm heterogeneity on the mass of firms. First, more dispersion in firm productivity implies more high productive firms (i.e. fatter tail of the Pareto distribution) in the economy. Therefore, conditional on the relevant cutoff, the mass of firms to the right of this cutoff



**Figure 4:** Firm profits in the high costs case  
Mass of receiving firms,  $s=2$ ,  $\sigma=2.5$

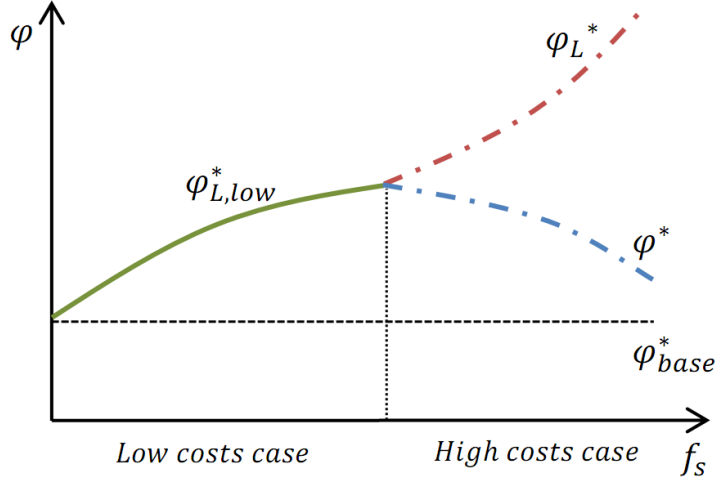


**Figure 5:** Mass of firms that receive subsidy and firm heterogeneity

increases. Second, because of the associated decline in the price index, the relevant cutoff increases and there is a negative effect on the mass of firms. For the total mass of active firms  $J_A$ , the second effect always dominates. However, for the mass of firms that receive subsidies  $J_L$ , the first effect can dominate the second one. In particular, if the subsidy is sufficiently high, starting from a low value a rise in firm heterogeneity increases  $J_L$ .<sup>27</sup> For high values of firm dispersion, however, more heterogeneity leads to a decline of  $J_L$ . Figure 5 shows this result graphically.

To compare the high costs case with the low costs case, Figure 6 shows the cutoffs as a function of the administrative fixed costs. For low levels of  $f_s$ , the eligibility and product market cutoff,  $\varphi_{L,low}^*$ , starts at the baseline value,  $\varphi_{base}^*$ , and is increasing in  $f_s$ . For high fixed costs,  $f_s > f(s^{\sigma-1} - 1)$ , there are two cutoffs. The product market cutoff,  $\varphi^*$ , is decreasing in  $f_s$  and converges to the baseline cutoff for  $f_s \rightarrow \infty$ . However, the eligibility cutoff,  $\varphi_L^*$ , is increasing in  $f_s$ . For  $f_s \rightarrow f(s^{\sigma-1} - 1)$ , the two cutoffs of the high costs case converge to the single cutoff of the low costs case. In the high costs case, the subsidy

<sup>27</sup>See proof in Appendix A.9 for the exact threshold condition on  $s$ . If  $s$  lies below this threshold, the mass of firms that receive subsidies is always increasing in firm heterogeneity,  $\frac{\partial J_L}{\partial \theta} < 0$ .



**Figure 6:** Cutoffs and administrative fixed costs

causes an within-industry conflict between receiving and non-receiving firms. Figure 6 demonstrates the extensive margin component of this conflict: lower administrative fixed costs (or a higher subsidy rate) reduce the mass of active firms (i.e.  $\varphi^*$  increases) and increase the mass of receiving firms (i.e.  $\varphi_L^*$  declines).

The subsidy relocates profits from less efficient firms to more efficient firms, because the latter are able to pay the administrative fixed costs. Thus, receiving firms do not just gain from the subsidy, they also benefit at the expense of non-receiving firms. Through the associated drop in the price index, an increase in the subsidy rate results in a negative externality for other firms, especially for non-receiving competitors.

In the high-costs case, total subsidy payments are  $S_{high} = (s - 1) \frac{\sigma - 1}{\sigma} R_E$ .<sup>28</sup> Given the budget constraint of the government, the upper bound on the subsidy rate is implicitly defined by  $\bar{s}_{high} = 1 + \frac{\sigma}{\sigma - 1} \frac{1}{R_E}$ . Thus, any subsidy rate that can be financed in the low costs case could also be financed in the high costs case. Intuitively, high fixed costs allow to highly subsidize a small mass of receiving firms. Because aggregate revenues of receiving firms are now a function of the subsidy rate, the derivative of  $S_{high}$  with respect to  $s$  is

$$\frac{\partial S_{high}}{\partial s} = \frac{\sigma - 1}{\sigma} R_E + (s - 1) \frac{\sigma - 1}{\sigma} \frac{\partial R_E}{\partial s}. \quad (22)$$

The second term of this expression contains two additional channels that affect  $S_{high}$ . First, at the intensive margin, already receiving firms increase their revenues. Second, at the extensive margin, there are some firms that start subsidized production ( $\varphi_L^*$  decreases). In contrast to the low costs case, total subsidy payments depend on the administrative fixed costs, on the production fixed costs and on firm heterogeneity (i.e. the Pareto shape parameter  $\theta$ ):

**Corollary 3.** *In the high costs case, total subsidy payments decrease in the administrative fixed costs,  $\frac{\partial S_{high}}{\partial f_s} < 0$ . However, total subsidy payments increase in the production fixed costs,  $\frac{\partial S_{high}}{\partial f} > 0$  and in firm heterogeneity (i.e. lower  $\theta$ )  $\frac{\partial S_{high}}{\partial \theta} < 0$ .*

*Proof.* See Appendix A.6. □

<sup>28</sup>Explicit expressions for  $R_E$ ,  $\frac{\partial R_E}{\partial s}$ ,  $\Pi_E$  and  $\Pi_{NE}$  are given in Appendix A.5.

Higher administrative fixed costs lead to less firms that receive the subsidies, but the receiving firms are on average more productive. Facing a higher price index, these firms increase their sales and the subsidy payments per firm also increase. However, the negative effect on the extensive margin (less firms subsidized) is stronger, such that total subsidy payments decrease,  $\frac{\partial S_{high}}{\partial f_s} < 0$ . In contrast, higher production fixed costs increase total subsidy payments,  $\frac{\partial S_{high}}{\partial f} > 0$ . Due to the associated rise in the price index, there are more firms receive subsidies and these firms also increase their sales. An increase in the Pareto shape parameter leads to less firm heterogeneity (i.e. thinner tails of the Pareto distribution). Aggregate revenues of receiving firms decrease and total subsidy payments also decline,  $\frac{\partial S_{high}}{\partial \theta} < 0$ .

Using the expression of the price index in the high costs case (equation (19)), the derivative of consumer surplus with respect to  $s$  is

$$\frac{\partial CS_{high}}{\partial s} = \frac{R_E}{s}. \quad (23)$$

Note that  $R_E$  is not constant but converges to zero for  $s \rightarrow 1$ . Thus, the derivative of consumer welfare is not longer a convex and decreasing function in  $s$ .

#### 4.2 Ex-ante welfare optimum in the high costs case

Consider the benchmark case where the government chooses the *ex-ante* welfare maximizing subsidy rate before firms enter the market and claim eligibility (i.e. taking firm entry behavior into account). Using the derivatives of consumer surplus and of total subsidy payments (equation (23) and equation (22)), the first-order condition for an interior welfare optimum is

$$\frac{\partial W_{high}}{\partial s} = \frac{\partial CS_{high}}{\partial s} - \frac{\partial S_{high}}{\partial s} = \frac{R_E}{s} - \frac{\sigma - 1}{\sigma} R_E - (s - 1) \frac{\sigma - 1}{\sigma} \frac{\partial R_E}{\partial s} = 0. \quad (24)$$

If the third term of the first-order condition was zero, the markup compensating subsidy rate would be an interior optimum. Thus, the optimal subsidy rate crucially depends on the marginal effect on receiving firms' aggregate revenues  $\frac{\partial R_E}{\partial s}$ . To analyze the properties of the optimal subsidy rate, the first-order condition can be rewritten by using the elasticity of  $R_E$  with respect to  $s$ ,  $\epsilon_{R_E,s} = \frac{\partial R_E}{\partial s} \frac{s}{R_E} \geq 0$ .<sup>29</sup> The interior optimum is then implicitly defined by

$$s^* = \frac{\frac{\sigma}{\sigma-1} + \epsilon_{R_E,s}}{1 + \epsilon_{R_E,s}}. \quad (25)$$

Note that the right hand side of this equation is also a function of  $s$ . Thus, an interior solution would be a fixed point that solves equation (25). From equation (25), two properties of the optimal subsidy rate are immediately apparent. First, the markup compensating subsidy rate,  $s = \frac{\sigma}{\sigma-1}$ , can be obtained only if  $\epsilon_{R_E,s} = 0$ . Second, it is never optimal to set the subsidy above the markup compensating level, because  $\epsilon_{R_E,s} \geq 0$ . In

<sup>29</sup>Note that  $\epsilon_{R_E,s}$  can be decomposed into an intensive margin and an extensive margin:

$$\epsilon_{R_E,s} = \underbrace{(\sigma - 1) \left( 1 - \frac{R_E}{y} \right)}_{\text{intensive margin}} + \underbrace{(\theta - \sigma + 1) \frac{R_E}{J_{RY}} \frac{f}{f_s}}_{\text{extensive margin}}.$$

comparison to the low costs case, the marginal loss is relatively higher than the marginal gain from an increase in the subsidy rate, because of the additional third term in equation (24). Therefore, the optimal subsidy rate cannot be greater than the interior optimum of the low costs case,  $s = \frac{\sigma}{\sigma-1}$ . Thus, even though a higher subsidy rate would lead to additional subsidized varieties, it is never optimal to set the subsidy rate above the markup compensating level. A next step is to analyze if an interior solution with  $s \leq \frac{\sigma}{\sigma-1}$  exists. As the following Lemma shows, because  $\epsilon_{R_E,s}$  is strictly increasing and unbounded in  $\theta$ , there does not exist an interior solution.<sup>30</sup>

**Lemma 6.** *In the high costs case, for any  $\theta > \sigma$  there does not exist an interior solution to the government's first-order condition, and (ex-ante) welfare is maximized at  $s^* = 1$ .*

*Proof.* See Appendix A.10. □

Lemma 6 is quite different from the corresponding welfare result in the low costs case (Lemma 2). Recall, in the low costs case there is always an interior solution, which for increasing administrative fixed costs is dominated by a corner solution at  $s = 1$ . In contrast, Lemma 6 states that in the high costs case, there does not even exist any interior solution. This is due to the fact that aggregate revenues of receiving firms increase in the subsidy rate, which increases the marginal loss in subsidy payments.

The welfare impact of an increase in the administrative fixed costs also differs from the low costs case. The following corollary can be stated:

**Corollary 4.** *In the high costs case, if  $\theta$  is sufficiently high, welfare is increasing in the administrative fixed costs,  $\frac{\partial W_{high}}{\partial f_s} > 0$ .*

*Proof.* See Appendix A.7. □

In comparison to the low costs case, an increase in the administrative fixed costs affects welfare through two channels. First, positively through decreasing total subsidy payments (Corollary 3). Second, negatively through a decline in consumer surplus, because the price index rises. However, the latter channel will be dampened through an effect on the extensive margin, because the product market cutoff decreases and more varieties are available. This positive variety effect will be more pronounced, the thinner the tails of the Pareto distribution (i.e. high  $\theta$ ). Thus, in contrast to the low costs case, welfare in the high costs case is not necessarily decreasing in the administrative fixed costs.

The impact of an increase in the administrative fixed costs on receiving firms' individual profits differs with firm size and productivity. While all receiving firms suffer directly from increasing fixed costs, the largest and most efficient firms benefit indirectly from the associated increase in the price index. For these firms, the increase in variable profits overcompensates the loss from additional fixed costs.<sup>31</sup> Therefore, an interest-

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<sup>30</sup>From equation (25), it is apparent that an interior solution requires that for any value of  $s$ ,  $\epsilon_{R_E,s}$  is sufficiently low. Otherwise, the left hand side of equation (25) would always be strictly larger than the right hand side, which converges to 1 for  $s \rightarrow 1$ . Because  $\epsilon_{R_E,s}$  is strictly increasing in  $\theta$ , the elasticity can be so large that equation (25) does not hold at any interior point. Note that  $\theta > \sigma$  is necessary for aggregate quantity to be well-defined.

<sup>31</sup>Variable profits are increasing in the price index, which is monotonically increasing in  $f_s$ . Therefore, there



ing policy implication emerges: large receiving firms have an interest in increasing the administrative fixed costs, such that less efficient firms decide to be no longer subsidized.

Hence, the analysis of the high costs case delivers new insights into the sensitivity of the ex-ante optimal subsidy rate. If high administrative fixed costs induce only some active firms to claim eligibility for the subsidy, any subsidy reduces welfare. Therefore, an *ex-ante* welfare maximizing government should completely neglect the initial markup distortion. However, an increase of the subsidy is associated with a within-industry conflict between receiving and non-receiving firms. While general welfare is reduced by the introduction and an increase of a subsidy, large firms still gain. These results motivate the analysis of lobbying for the subsidy in the following section.

### 4.3 Lobbying in the high costs case

Consider a government that is influenced by lobbying and sets the optimal subsidy rate within the lobbying game (i.e. taking entry behavior of firms as given).

#### 4.3.1 Equilibrium in the high costs case

Taking the mass of lobbying and active firms as given, the government chooses a subsidy rate that solves equation (10). Define  $\tilde{\epsilon}_{R_E,s}$  as the elasticity of aggregate lobby revenues with respect to the subsidy rate, holding the mass of lobbying and active firms fixed. The following Lemma describes the optimal behavior of the government:

**Lemma 7.** *In the high costs, the government implements  $s^o = \frac{\frac{\sigma}{\sigma-1} + (1 + \frac{1}{\sigma-1} \frac{1}{\alpha}) \tilde{\epsilon}_{R_E,s}}{1 + \tilde{\epsilon}_{R_E,s}}$  in the lobbying game.*

*Proof.* See Appendix A.11. □

The optimal choice of the subsidy rate depends crucially on the elasticity  $\tilde{\epsilon}_{R_E,s}$ . In Appendix A.11, I show that this elasticity can be written as  $\tilde{\epsilon}_{R_E,s} = (\sigma - 1) \left(1 - \frac{R_E}{y}\right)$ . This expression shows very intuitively that  $\tilde{\epsilon}_{R_E,s}$  is a decreasing function of the relative aggregated lobby revenues  $\frac{R_E}{y}$  and that it is therefore also a decreasing function of the relative mass of lobbying firms  $J_R$ . If the relative mass of lobbying firms converges to one (i.e.  $R_E \rightarrow y$ ), the positive effect on aggregate lobby revenues due to an increase in the subsidy rate is exactly compensated by the associated drop in the price index (i.e. the economy converges to the low costs case). Therefore, the elasticity  $\tilde{\epsilon}_{R_E,s}$  converges to zero and the mark-up compensating subsidy rate is optimal. However, if the relative mass of lobbying firms converges to zero (i.e.  $R_E \rightarrow 0$ ), the impact of the subsidy on the price index is negligible and the elasticity  $\tilde{\epsilon}_{R_E,s}$  converges to  $(\sigma - 1)$ . In this case, the optimal subsidy is  $s^o = \frac{\sigma}{\sigma-1} + \frac{1}{\sigma} \left(\frac{1-\alpha}{\alpha}\right)$ . Therefore, we have established the following Corollary:

**Corollary 5.** *The optimal subsidy rate  $s^o$  lies above (below) the mark-up compensating rate  $s = \frac{\sigma}{\sigma-1}$ , if and only if  $\alpha < 1$  ( $\alpha > 1$ ).*

If the government puts less weight on general welfare than on lobby contributions,

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exists a productivity cutoff  $\varphi_{f_s}$ , defined by  $\frac{\partial \pi_s(\varphi_{f_s})}{\partial f_s} = 0$ , such that for all firms with  $\varphi > \varphi_{f_s}$ , the gain in variable profits dominates the (direct) loss from an increase in  $f_s$ .

$\alpha < 1$ , the optimal subsidy is greater than the mark-up compensating level. A government that does not put any weight on general welfare (i.e.  $\alpha \rightarrow 0$ ) will choose the highest feasible subsidy rate it is able to finance,  $s^o = \bar{s}$ . If the government puts more weight on welfare than on lobby contributions,  $\alpha > 1$ , the subsidy will be below the mark-up compensating rate. For a government that maximizes only general welfare *within* the lobbying game, and that does not take the lobby contributions into account (i.e.  $\alpha \rightarrow \infty$ ), we get:

$$\lim_{\alpha \rightarrow \infty} s^o = \frac{\frac{\sigma}{\sigma-1} + \tilde{\epsilon}_{R_E,s}}{1 + \tilde{\epsilon}_{R_E,s}}. \quad (26)$$

With  $\tilde{\epsilon}_{R_E,s} \in (0, \sigma - 1)$ , it follows from equation (26) that  $\lim_{\alpha \rightarrow \infty} s^o \in (\frac{\sigma}{\sigma-1} - \frac{1}{\sigma}, \frac{\sigma}{\sigma-1})$ . Therefore, the subsidy that maximizes general welfare within the subsidy game lies strictly below the mark-up compensating rate and above the corner solution  $s = 1$ . The *ex-post* welfare maximizing subsidy rate within the lobbying game, defined by equation (26), differs considerably from the *ex-ante* welfare maximizing subsidy rate implied by equation (25). Once firms have payed the administrative fixed costs, it is no longer optimal to implement the corner solution as Lemma 6 would suggest for an ex-ante welfare maximizing government.

Denote  $\varphi^o$  and  $\varphi_L^o$  the equilibrium product market cutoff and lobby cutoff, respectively (i.e. equation (20) and equation (21) evaluated at  $s^o$ ). Anticipating the optimal behavior of the lobby and the government, a firm with productivity  $\varphi$  decides to be an active producer only if  $\varphi > \varphi^o$ , and to join the lobby only if  $\varphi > \varphi_L^o$ . Let the function  $h(s)$  denote the relative mass of lobby members as a function of the subsidy rate:

$$h(s) = J_R = \left( \frac{f_s}{f(s^{\sigma-1} - 1)} \right)^{-\frac{\theta}{\sigma-1}}. \quad (27)$$

Let  $z(J_R)$  denote the subsidy rate set by the government as a function of the relative mass of lobby members:

$$z(J_R) = s = \frac{\frac{\sigma}{\sigma-1} + \left(1 + \frac{1}{\sigma-1} \frac{1}{\alpha}\right) \tilde{\epsilon}_{R_E,s}}{1 + \tilde{\epsilon}_{R_E,s}}. \quad (28)$$

At an equilibrium of the lobbying game,  $z(J_R)$  and  $h(s)$  intersect in the  $(s, J_R)$  space. Figure 7 depicts the equilibrium graphically for the case  $\alpha < 1$ , and the following lemma shows that the equilibrium of the lobbying game is unique.

**Lemma 8.** *If  $f_s > f((\frac{\sigma}{\sigma-1})^{\sigma-1} - 1)$  (high costs case), there exists a unique equilibrium of the lobbying game, such that*

1. *all firms with  $\varphi > \varphi^o$  produce and all firms with  $\varphi > \varphi_L^o$  enter the lobby,*
2. *lobby contributions are  $C^T(s^o) = \alpha(y \ln(\frac{P_{s^o}}{P_{s^*}}) + S_{s^o} - S_{s^*})$ ,*
3. *the government implements  $s^o = \frac{\frac{\sigma}{\sigma-1} + (1 + \frac{1}{\sigma-1} \frac{1}{\alpha}) \tilde{\epsilon}_{R_E,s}}{1 + \tilde{\epsilon}_{R_E,s}}$ .*

*Proof.* See Appendix A.12. □

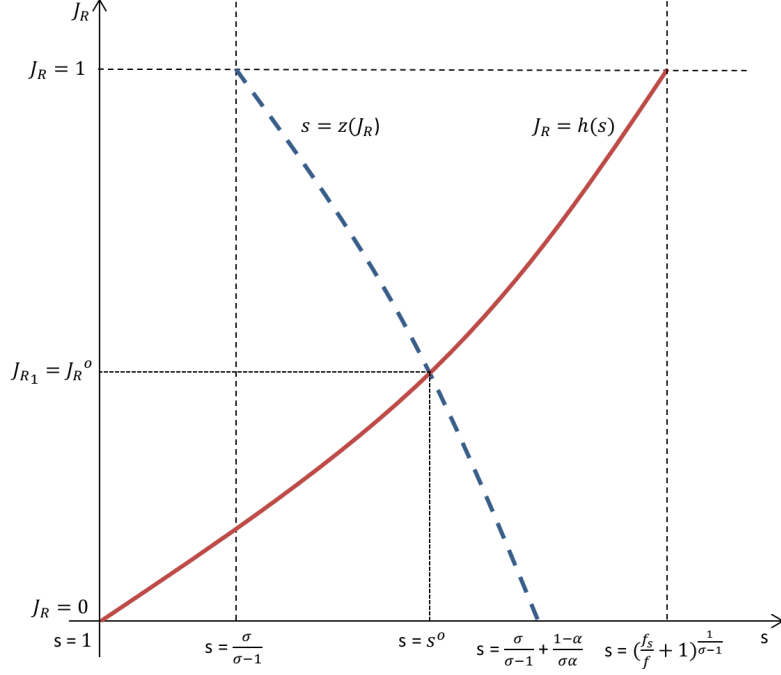


Figure 7: Equilibrium high costs case, ( $\alpha < 1$ )

### 4.3.2 Comparative statics of the lobbying game

**Varying the welfare weight  $\alpha$**  The relative number of lobbying firms  $J_R$  does not directly depend on  $\alpha$ , only indirectly through a change in the subsidy rate. An increase in the welfare weight, however, shifts the  $z(J_R)$  curve (equation (28)) to the left, such that for any given level of  $J_R$  the optimal subsidy rate declines. This leads to a lower equilibrium subsidy rate and therefore to a decline in the relative mass of lobbying firms. The following lemma summarizes the effects of an increase in  $\alpha$ , while Figure 8 shows the result graphically for the case where  $\alpha < 1$ .

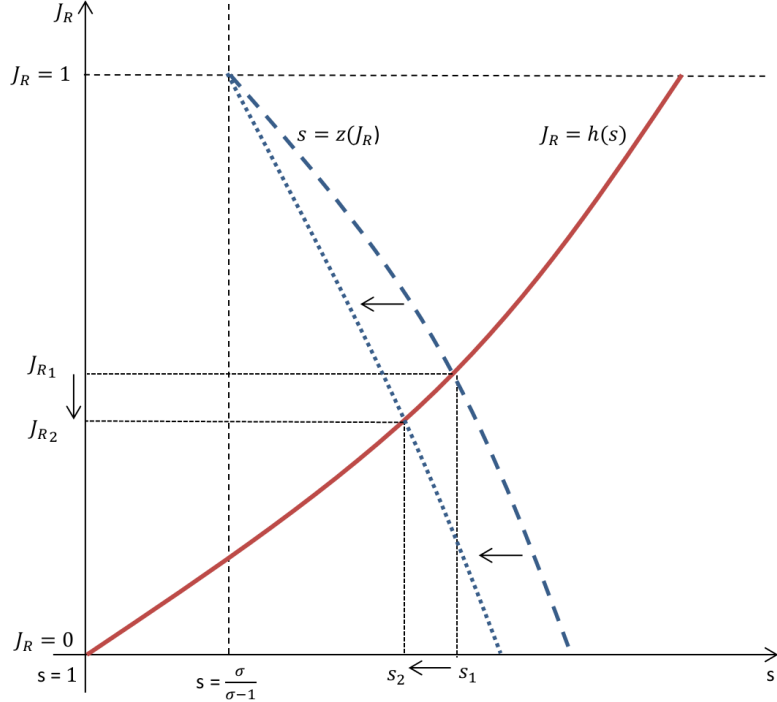
**Lemma 9.** *Increasing the welfare weight  $\alpha$  decreases both the relative mass of lobbying firms and the optimal subsidy rate.*

*Proof.* See Appendix A.13. □

**Varying the administrative fixed costs  $f_s$**  Because the government moves after firms joined the lobby, a change in the fixed costs  $f_s$  does not have a direct effect on the optimal decision of the government  $\frac{\partial z(J_R; f_s)}{\partial f_s} = 0$ . However, there is an indirect effect on the subsidy rate via the relative mass of lobbying firms:  $\frac{ds}{df_s} = \frac{\partial z(J_R; f_s)}{\partial J_R} \frac{\partial h(s; f_s)}{\partial f_s}$ . The direct negative effect on the relative mass of lobbying firms is  $\frac{\partial h(s; f_s)}{\partial f_s} = -\frac{\theta}{\sigma-1} \frac{J_R}{f_s} < 0$ . Therefore, the  $h(s; f_s)$  curve shifts downwards and only the sign of  $\frac{\partial z(J_R; f_s)}{\partial J_R}$  determines the total equilibrium effect on the subsidy rate. The following Lemma summarizes the effects of an increase in the administrative fixed costs:

**Lemma 10.** *Increasing the administrative fixed costs  $f_s$  decreases always the relative mass of lobbying firms, but increases the optimal subsidy rate if  $\alpha < 1$  and decreases the optimal subsidy rate if  $\alpha > 1$ .*

*Proof.* See Appendix A.14. □



**Figure 8:** Comparative statics: increasing  $\alpha$ , ( $\alpha < 1$ )

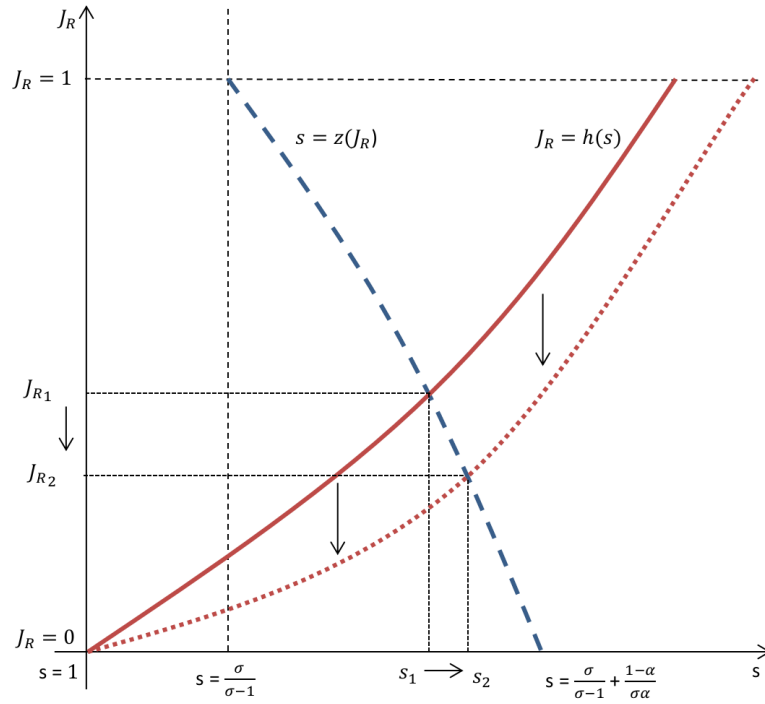
If the government puts a relatively low weight on general welfare,  $\alpha < 1$ , a decline in the relative number of lobbying firms leads to an increase in the subsidy rate. This effect is entirely due to the change in the relative number of lobbying firms. In this case, making lobbying harder by increasing the barriers to lobby, leads to less firms that lobby but to a higher subsidy rate. Figure 9 shows the result graphically for the case where  $\alpha < 1$ .

**Varying firm heterogeneity  $\theta$**  An increase in  $\theta$  is associated with a thinner tail of the Pareto distribution and less firm heterogeneity. This has a direct negative effect on the relative mass of lobbying firms, shifting the  $h(s)$  curve downwards. However, holding the (relative) mass of lobbying firms constant, less firm heterogeneity leads to a decline of average productivity and therefore to a decline in average revenues of lobbying firms. Therefore,  $\tilde{\epsilon}_{R_E,s}$  increases even if the mass of lobbying firms is constant. This effect shifts the  $z(J_R)$  curve to the right if  $\alpha < 1$ , and to the left if  $\alpha > 1$ . The following lemma summarizes the equilibrium effects of a decline in firm heterogeneity and Figure 10 shows the result graphically for the case where  $\alpha < 1$ .

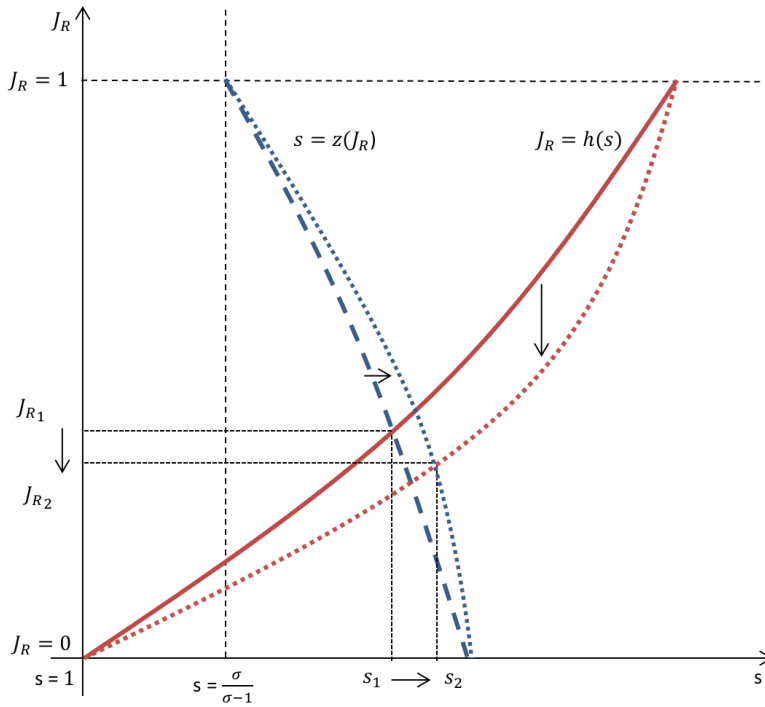
**Lemma 11.** *If  $\alpha > 1$ , decreasing firm heterogeneity (i.e. increasing  $\theta$ ) decreases both the relative mass of lobbying firms and the optimal subsidy rate. However, if  $\alpha < 1$ , decreasing firm heterogeneity increases the optimal subsidy rate and has an ambiguous effect on the relative mass of lobbying firms.*

*Proof.* See Appendix A.15 □

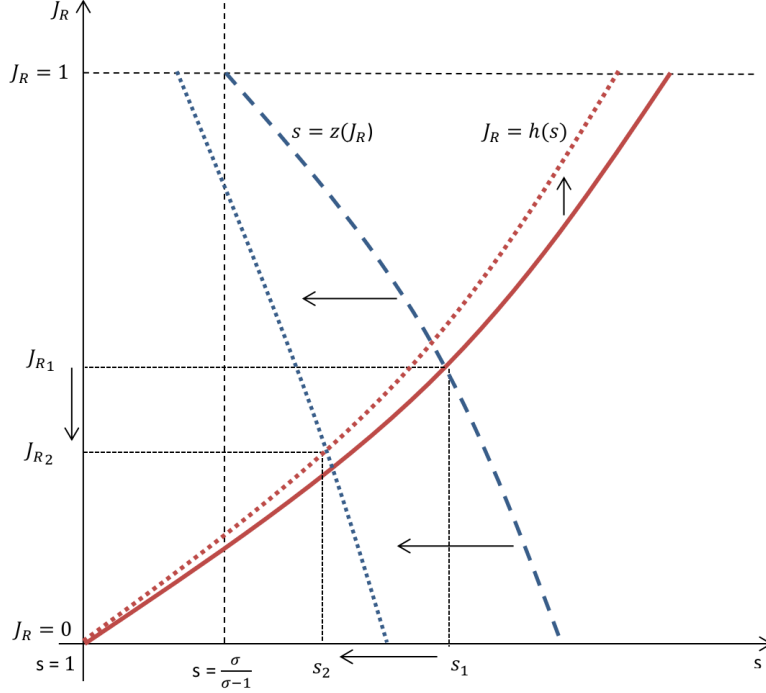
**Varying the elasticity of substitution  $\sigma$**  An increase in  $\sigma$  leads to more heterogeneity in the sales distribution, because high productive firms benefit more from a higher



**Figure 9:** Comparative statics: increasing  $f_s$ , ( $\alpha < 1$ )



**Figure 10:** Comparative statics: increasing  $\theta$ , ( $\alpha < 1$ )



**Figure 11:** Comparative statics: increasing  $\sigma$ , ( $\alpha < 1$ )

elasticity of substitution. Therefore, there will be relatively more firms in the lobby, such that the  $h(s)$  curve shifts upwards. Considering the limits of the  $z(J_R)$  function, we see that  $\lim_{J_R \rightarrow 1} z(J_R) = \frac{\sigma}{\sigma-1}$  and  $\lim_{J_R \rightarrow 0} z(J_R) = \frac{\sigma}{\sigma-1} + \frac{1}{\sigma} \frac{1-\alpha}{\alpha}$ . Thus, the  $z(J_R)$  curve shifts to the left. The following lemma summarizes the equilibrium effects of an increase in  $\sigma$ , while Figure 11 shows the result graphically for the case where  $\alpha < 1$ .

**Lemma 12.** *If  $\alpha < 1$ , increasing the elasticity of substitution  $\sigma$  decreases the optimal subsidy rate and has an ambiguous effect on the relative mass of lobbying firms.*

*Proof.* See Appendix A.16. □

### 4.3.3 Simulation results

To visualize the comparative static results for various values of the welfare weight  $\alpha$ , I simulate the model using MATLAB. In particular, I set the following parameter values:  $\theta = 3$ ,  $\sigma = 2.5$ ,  $f_s = 2$ ,  $f = 1$ ,  $J = 1$ ,  $b = 1$ ,  $y = 1$ . Figure 12, 13 and 14 show the impact of an increase in  $f_s$ ,  $\theta$  and  $\sigma$  respectively, on the optimal subsidy rate in the lobbying game and on relative lobby size. In Figure 12, relative lobby size is always decreasing in the fixed costs to lobby, while the optimal subsidy rate increases in  $f_s$  if  $\alpha < 1$  (Lemma 10). In Figure 13, both the subsidy and relative lobby size decrease in  $\theta$  if  $\alpha > 1$ . For  $\alpha < 1$ , however, the optimal subsidy rate increases in  $\theta$ , while the relative size of the lobby can both increase (e.g.  $\alpha = 0.001$ ) or decrease (e.g.  $\alpha = 0.5$ ) (Lemma 11). In Figure 14, the optimal subsidy rate decreases in  $\sigma$ , if  $\alpha < 1$  but can increase if  $\alpha > 1$  (e.g.  $\alpha = 1000$ ) (Lemma 12).

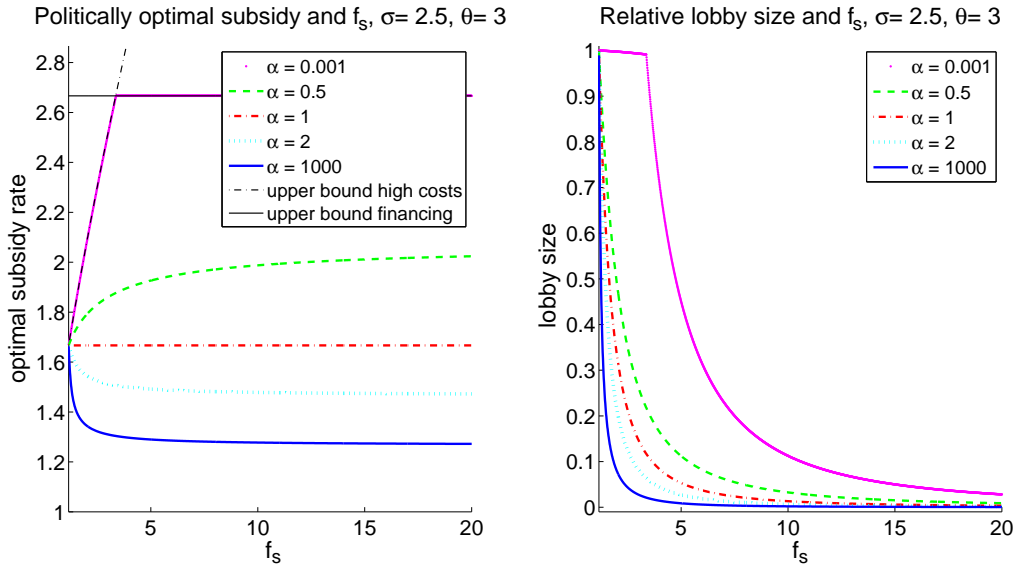


Figure 12: Comparative statics: increasing  $f_s$

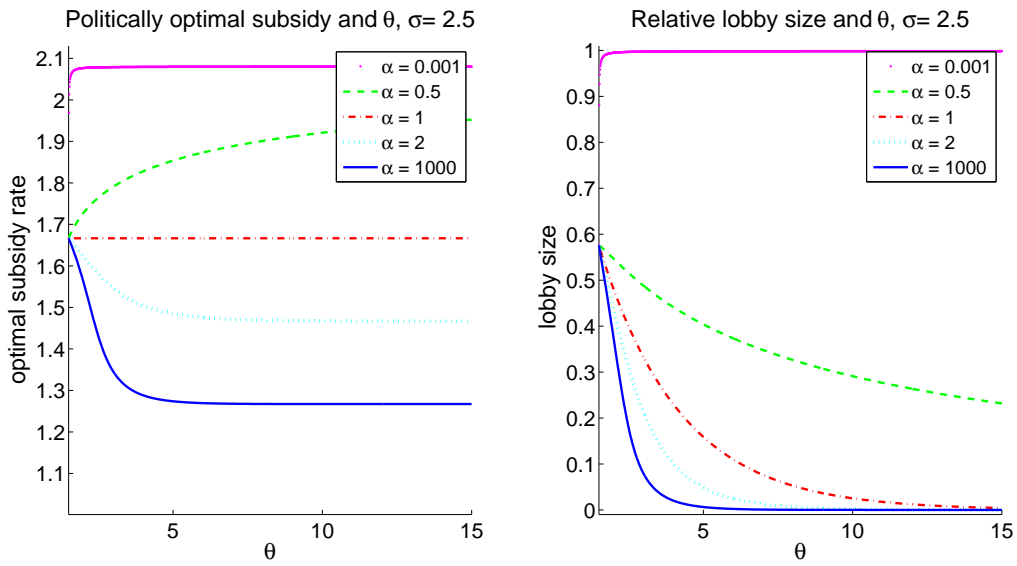


Figure 13: Comparative statics: increasing  $\theta$

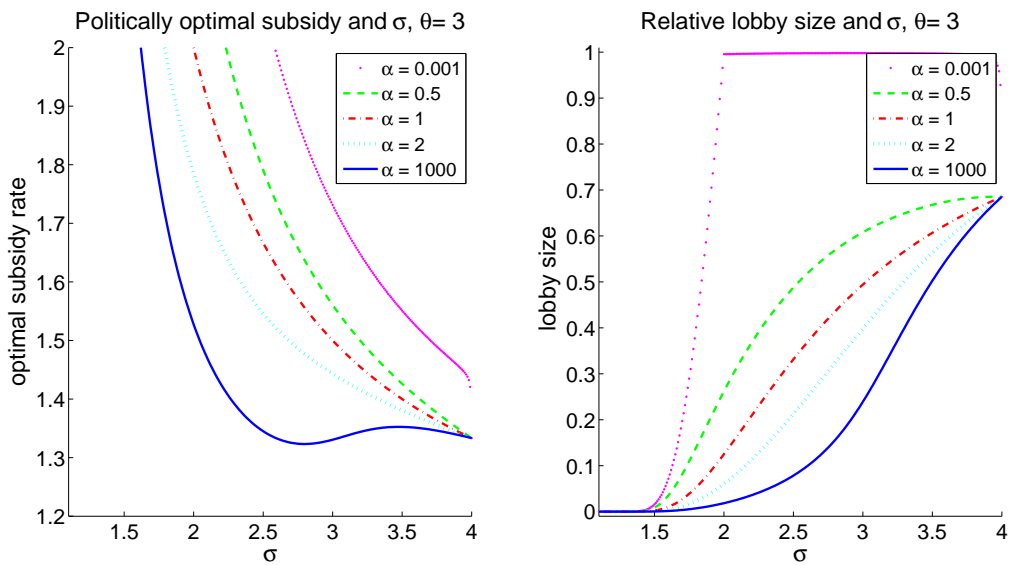


Figure 14: Comparative statics: increasing  $\sigma$

## 5 Conclusion

In this paper, heterogeneous firms have to bear administrative fixed costs to receive a production subsidy. The benefits from the subsidy increase with firm productivity, such that the set of receiving firms is endogenously determined. When receiving firms are allowed to lobby for a higher subsidy rate, this mechanism results in an endogenous set of lobbying firms.

In the model, the welfare impact of a production subsidy depends crucially on the level of the associated administrative fixed costs. If these costs are too high, a welfare maximizing government should neglect the mark-up distortion from monopolistic competition and it should not introduce a subsidy. Moreover, if the fixed costs to receive the subsidy are high, such that only the most efficient firms are subsidized, a rise in the subsidy harms small firms. This creates a distributional within-industry conflict across firms, which is the driving force behind lobbying in the model.

An increase in the barriers to lobby unambiguously reduces the size of the lobby. However, if the government values lobby contributions highly, increasing the barriers to lobby or less firm heterogeneity increases the equilibrium subsidy rate. These results stand in contrast to conventional wisdom that lobby power and lobby size are positively related.

This is the first paper, to the best of my knowledge, that makes explicit use of heterogeneity in firm productivity to endogenize lobby formation. However, this paper goes beyond a pure technical contribution in an important class of economic models. Given the importance of within-sector reallocation, highlighted in the heterogeneous firm literature of international trade, this paper shows that the within-industry impact of firm-specific government policies should no longer be ignored. While the paper takes a first step in explaining within-industry variation in firm eligibility and lobbying theoretically, further research – in particular on the empirical side – will be necessary for a better understanding of the within-industry effects of firm-specific policy instruments.



## References

- ABEL-KOCH, J. (2010): "Endogenous Trade Policy with Heterogeneous Firms," Discussion paper no. 93, Center for Doctoral Studies in Economics, University of Mannheim.
- ACEMOGLU, D. AND J. A. ROBINSON (2001): "Inefficient Redistribution," *The American Political Science Review*, 95, pp. 649–661.
- ANSOLABEHERE, S., M. TRIPATHI, AND J. M. S. JR (2002): "Are PAC Contributions and Lobbying Linked? New Evidence from the 1995 Lobby Disclosure Act," *Business and Politics*, 4.
- ASCHHOFF, B. (2010): "Who Gets the Money? The Dynamics of R&D Project Subsidies in Germany," *Journal of Economics and Statistics (Jahrbuecher fuer Nationaloekonomie und Statistik)*, 230, 522–546.
- BERNARD, A. B., J. B. JENSEN, S. J. REDDING, AND P. K. SCHOTT (2012): "The Empirics of Firm Heterogeneity and International Trade," *Annual Review of Economics*, 4.
- BERNHEIM, B. D. AND M. D. WHINSTON (1986): "Menu Auctions, Resource Allocation, and Economic Influence," *The Quarterly Journal of Economics*, 101, 1–31.
- BLANES, J. V. AND I. BUSOM (2004): "Who participates in R&D subsidy programs?: The case of Spanish manufacturing firms," *Research Policy*, 33, 1459–1476.
- BOMBARDINI, M. (2008): "Firm Heterogeneity and Lobby Participation," *Journal of International Economics*, 75, 329–348.
- BOMBARDINI, M. AND F. TREBBI (2009): "Competition and Political Organization: Together or Alone in Lobbying for Trade Policy?" NBER Working Papers 14771, National Bureau of Economic Research, Inc.
- CHANEY, T. (2008): "Distorted Gravity: The Intensive and Extensive Margins of International Trade," *The American Economic Review*, 98, 1707–1721.
- CHANG, P.-L. AND G. WILLMANN (2006): "Protection for Sale with Heterogenous Interests within Industries," *Working Paper*.
- CHEN, H., D. PARSLEY, AND Y.-W. YANG (2010): "Corporate Lobbying and Financial Performance," MPRA Paper 21114, University Library of Munich, Germany.
- CHOR, D. (2009): "Subsidies for FDI: Implications from a model with heterogeneous firms," *Journal of International Economics*, 78, 113–125.
- DEFRA (2011): "Striking a balance: reducing burdens; increasing responsibility; earning recognition - A report on better regulation in farming and food businesses," Tech. Rep.

- PB13527, Farming Regulation Task Force, Department of Environment Food and Rural Affairs (DEFRA), United Kingdom.
- DEMIDOVA, S. AND A. RODRÍGUEZ-CLARE (2009): “Trade Policy under Firm-level Heterogeneity in a Small Economy,” *Journal of International Economics*, 78, 100–112.
- DIXIT, A. K. AND J. E. STIGLITZ (1977): “Monopolistic Competition and Optimum Product Diversity,” *The American Economic Review*, 67, 297–308.
- DUGUET, E. (2004): “Are R&D subsidies a substitute or a complement to privately funded R&D ?. An econometric analysis at the firm level,” *Revue d’économie politique*, 0, 245–274.
- GROSSMAN, G. M. AND E. HELPMAN (1994): “Protection for Sale,” *American Economic Review*, 84, 833–50.
- HOPENHAYN, H. A. (1992): “Entry, Exit, and firm Dynamics in Long Run Equilibrium,” *Econometrica*, 60, 1127–1150.
- HUSSINGER, K. (2008): “R&D and subsidies at the firm level: an application of parametric and semiparametric two-step selection models,” *Journal of Applied Econometrics*, 23, 729–747.
- IGAN, D., P. MISHRA, AND T. TRESSEL (2011): “A Fistful of Dollars: Lobbying and the Financial Crisis,” in *NBER Macroeconomics Annual 2011, Volume 26*, National Bureau of Economic Research, Inc, NBER Chapters, 195–230.
- KERR, W. R., W. F. LINCOLN, AND P. MISHRA (2011): “The Dynamics of Firm Lobbying,” NBER Working Papers 17577, National Bureau of Economic Research, Inc.
- KIRWAN, B. E. (2007): “The Distribution of U.S. Agricultural Subsidies,” Tech. rep., University of Maryland - Agricultural & Resource Economics (AREC).
- LUDEMA, R. D., A. M. MAYDA, AND P. MISHRA (2010): “Protection for Free? The Political Economy of U.S. Tariff Suspensions,” CEPR Discussion Papers 7926, C.E.P.R. Discussion Papers.
- MELITZ, M. J. (2003): “The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity,” *Econometrica*, 71, 1695–1725.
- MITRA, D. (1999): “Endogenous Lobby Formation and Endogenous Protection: A Long-Run Model of Trade Policy Determination,” *American Economic Review*, 89, 1116–1134.
- PFLÜGER, M. AND S. RUSSEK (2011): “Trade and Industrial Policies with Heterogeneous Firms: The Role of Country Asymmetries,” Working Papers 094, Bavarian Graduate Program in Economics (BGPE).

- PFLÜGER, M. AND J. SUEDEKUM (2013): “Subsidizing firm entry in open economies,” *Journal of Public Economics*, 97, 258 – 271.
- REBEYROL, V. AND J. VAUDAY (2008): “Live or let die: intra-sectoral lobbying on entry,” Documents de travail du Centre d’Economie de la Sorbonne bla08036, Université Panthéon-Sorbonne (Paris 1), Centre d’Economie de la Sorbonne.
- REDDING, S. J. (2011): “Theories of Heterogeneous Firms and Trade,” *Annual Review of Economics*, 3, 77–105.
- RODRIK, D. (1986): “Tariffs, subsidies, and welfare with endogenous policy,” *Journal of International Economics*, 21, 285 – 299.
- TRESSEL, T., P. MISHRA, AND D. IGAN (2009): “A Fistful of Dollars: Lobbying and the Financial Crisis,” IMF Working Papers 09/287, International Monetary Fund.
- USDA (2009): *2007 Census of Agriculture. Volume 1, Part 51, Geographic Area Series. United States Summary and State Data*, U.S. Dept. of Agriculture, National Agricultural Statistics Service, Washington, D.C.
- WAGNER, J. (2010): “Wer wird subventioniert? Subventionen in deutschen Industrieunternehmen 1999-2006,” *Perspektiven der Wirtschaftspolitik*, 11, 47–74.

## Appendix

### A Proofs and explicit expressions

#### A.1 Proof of Corollary 1

**Corollary.** *Under truthful contribution schedules, the equilibrium subsidy satisfies*

$$s^o = \arg \max_{s \in [1, \bar{s}]} \{\alpha W(s) + \Pi_E(s)\}.$$

*Proof.* The proof is similar to the one in Grossman and Helpman (1994)(p. 840, footnote 7). By condition 3 of Proposition 1 we have

$$G^o = \alpha W(s^o) + C(s^o) \geq G = \alpha W(s) + C(s) \forall s \in [1, \bar{s}].$$

By truthfulness, we have

$$C^T(s^o) = \Pi_E(s^o) - B_L^o$$

and

$$C^T(s) \geq \Pi_E(s) - B_L^o \forall s \in [1, \bar{s}].$$

Therefore,

$$\alpha W(s^o) + \Pi_E(s^o) - B_L^o \geq \alpha W(s) + C^T(s) \geq \alpha W(s) + \Pi_E(s) - B_L^o.$$

Hence,

$$\alpha W(s^o) + \Pi_E(s^o) \geq \alpha W(s) + \Pi_E(s) \forall s \in [1, \bar{s}].$$

□

#### A.2 Proof of Lemma 1

**Lemma.** *In the low costs case with a given subsidy rate  $s$ , there is a unique eligibility and product market cutoff  $\varphi_{L,low}^*$  and*

1. *the price index lies below the baseline value,  $P_{s,low} < P_{base}$ ;*
2.  *$\varphi_{L,low}^*$  lies above the baseline value,  $\varphi_{L,low}^* > \varphi_{base}^*$ ;*
3.  *$\varphi_{L,low}^*$  is invariant to a change in the subsidy rate, but a rise in  $f_s$ , in  $f$ , in  $\sigma$  or in firm heterogeneity leads to an increase of  $\varphi_{L,low}^*$  and to a decline of  $J_A = J_L$ .*

*Proof.* Here, I only show that  $\frac{\partial \varphi_{L,low}^*}{\partial \theta} < 0$ ,  $\frac{\partial J_A}{\partial \theta} = \frac{\partial J_L}{\partial \theta} > 0$  and  $\frac{\partial \varphi_{L,low}^*}{\partial \sigma} > 0$ . With

$$J_L = J_A = J \left( \frac{\varphi_{L,low}^*}{b} \right)^{-\theta} = \left( 1 - \frac{\sigma - 1}{\theta} \right) \frac{y}{\sigma} (f + f_s)^{-1}$$

we get  $\frac{\partial J_L}{\partial \theta} = \left( \frac{\sigma - 1}{\theta^2} \right) \left( \frac{\sigma}{y} [f + f_s] \right)^{-1} > 0$ . However, the derivative can also be written as

$$\frac{\partial J_L}{\partial \theta} = -J_L \left( \ln \left( \frac{\varphi_{L,low}^*}{b} \right) + \theta \frac{\partial \ln(\varphi_{L,low}^*)}{\partial \theta} \right) > 0.$$

For  $\frac{\partial J_L}{\partial \theta} > 0$  it is necessary that  $\frac{\partial \ln(\varphi_{L,low}^*)}{\partial \theta} < 0$ . Therefore,  $\frac{\partial \varphi_{L,low}^*}{\partial \theta} < 0$ . Take the derivative of the productivity cutoff to get:

$$\frac{\partial \varphi_{L,low}^*}{\partial \sigma} \frac{\sigma}{\varphi_{L,low}^*} = \frac{1}{\theta} \left( \frac{\theta + 1}{\theta + 1 - \sigma} \right) > 0.$$

□

#### A.3 Proof of Lemma 2

**Lemma.** *In the low costs case, the unique interior (ex-ante) welfare optimum is given by  $s^* = \frac{\sigma}{\sigma - 1}$ . For administrative fixed costs above  $f_s = f \left( \exp \left[ \frac{\ln(\frac{\sigma}{\sigma - 1}) - \frac{1}{\sigma}}{\frac{1}{\theta} \frac{\sigma - 1}{\sigma - 1}} \right] - 1 \right)$ , the global (ex-ante) welfare optimum is given by  $s^* = 1$ .*

*Proof.* The first part of the lemma has already been shown in the paper. For the second part, consider the difference between welfare in the low costs case and in the baseline

case at any  $s$ :

$$W_{low} - W_{base} = y \ln(s) - (s-1) \frac{\sigma-1}{\sigma} y - \frac{y}{\theta} \frac{\theta-\sigma+1}{\sigma-1} \ln\left(\frac{f+f_s}{f}\right) \quad (29)$$

Note that the first term reflects the positive effect of the subsidy on consumer surplus (all varieties are cheaper). The second term are total subsidy payments and therefore the direct costs that the government has to pay for the subsidy. The third term reflects the novel distortion due to administrative fixed costs. For  $f_s = 0$  only the first two terms would remain. At the interior optimum,  $s^* = \frac{\sigma}{\sigma-1}$ , equation (29) can be rewritten to

$$W_{low} - W_{base} = y \ln\left(\frac{\sigma}{\sigma-1}\right) - \frac{y}{\sigma} - \frac{y}{\theta} \frac{\theta-\sigma+1}{\sigma-1} \ln\left(\frac{f+f_s}{f}\right).$$

Then, with  $(\sigma-1) \ln\left(\frac{\sigma}{\sigma-1}\right) < 1$ , in the two limits of the low costs case, we get respectively,

$$\lim_{f_s \rightarrow f\left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} - 1\right)} W_{low} - W_{base} = \frac{\sigma-1}{\theta} y \ln\left(\frac{\sigma}{\sigma-1}\right) - \frac{y}{\sigma} < 0$$

and

$$\lim_{f_s \rightarrow 0} W_{low} - W_{base} = y \ln\left(\frac{\sigma}{\sigma-1}\right) - \frac{y}{\sigma} > 0.$$

Therefore, by monotonicity and continuity of  $W_{low} - W_{base}$  in  $f_s$ , there is a level of fixed costs,  $f_s^* \in (0, f(s^{\sigma-1} - 1))$ , such that  $W_{low} - W_{base} = 0$  at  $s^* = \frac{\sigma}{\sigma-1}$ . By using equation (29) we get for any  $s$ :

$$\begin{aligned} W_{low} - W_{base} &= y \ln(s) - (s-1) \frac{\sigma-1}{\sigma} y - \frac{y}{\theta} \frac{\theta-\sigma+1}{\sigma-1} \ln\left(\frac{f+f_s}{f}\right) = 0 \\ \iff f_s^* &= f \left( \exp \left[ \frac{y \ln(s) - (s-1) \frac{\sigma-1}{\sigma} y}{\frac{y}{\theta} \frac{\theta-\sigma+1}{\sigma-1}} \right] - 1 \right). \end{aligned}$$

Evaluated at  $s^* = \frac{\sigma}{\sigma-1}$ , we get

$$f_s^* = f \left( \exp \left[ \frac{y \ln\left(\frac{\sigma}{\sigma-1}\right) - \frac{y}{\sigma}}{\frac{y}{\theta} \frac{\theta-\sigma+1}{\sigma-1}} \right] - 1 \right).$$

For  $f_s^*$ , welfare of the baseline case and the interior welfare optimum of the low costs case are equal. By strict monotonicity of  $W_{low}$  with respect to  $f_s$  (Corollary 2), it follows that for lower values of  $f_s$ , the interior welfare optimum of the low costs case is above the baseline value of welfare, and vice versa for greater values of  $f_s$ . Thus, only for low values of  $f_s$ , the interior optimal subsidy,  $s^* = \frac{\sigma}{\sigma-1}$ , is welfare improving. For high values, this is not longer the case. Note that in the low costs case, there are always values of  $f_s$  such that  $f_s^* < f_s < f(s^{\sigma-1} - 1)$ . This is,  $f_s^*$  lies never above the value of administrative fixed costs that defines the low costs case. To see this more explicitly, by using the derived expression,  $f_s^* < f(s^{\sigma-1} - 1)$  can be rewritten to  $\frac{\theta}{\sigma} > \frac{\ln(s)}{(s-1)}$ . With  $\frac{\theta}{\sigma} > 1$  and  $\frac{1}{1-s} \ln(s) < 1$ , this inequality is always fulfilled.  $\square$

#### A.4 Proof of Corollary 2

In the low costs case, the derivative of welfare with respect to the administrative fixed costs is always negative and given by

$$\frac{\partial W_{low}}{\partial f_s} = -\frac{\theta-\sigma+1}{\theta(\sigma-1)} \frac{y}{f+f_s} < 0.$$

### A.5 Explicit expressions for Section 4

With  $R_E = J \int_{\varphi_L^*} r_s(\varphi)g(\varphi)d\varphi$ , aggregate revenues of all receiving firms can be rewritten in terms of the model parameters

$$R_E = \frac{y}{1-s^{1-\sigma}} \left[ \left( \frac{f_s}{f(s^{\sigma-1}-1)} \right)^{\frac{\theta}{\sigma-1}} \frac{f}{f_s} + 1 \right]^{-1}, \quad (30)$$

$R_E$  is increasing in  $s$  and its derivative with respect to  $s$  is

$$\frac{\partial R_E}{\partial s} = R_E \frac{s^{-1}}{s^{\sigma-1}-1} \left[ (1-\sigma) + \theta \frac{R_E}{y} \left( \frac{f_s}{f(s^{\sigma-1}-1)} \right)^{\frac{\theta-\sigma+1}{\sigma-1}} \right] > 0.$$

Using the expressions for  $\varphi_L^*$ ,  $\varphi^*$  and  $P_{s,high}$ , aggregate profits of non-receiving and receiving firms are respectively,

$$\Pi_{NE} = \frac{y \theta - \sigma + 1}{\sigma} \frac{\left[ \frac{\sigma-1}{\theta-\sigma+1} \left( \frac{f}{f_s} \right)^{\frac{\sigma-\theta-1}{\sigma-1}} (s^{\sigma-1}-1)^{-\frac{\theta}{\sigma-1}} - \frac{\theta}{\theta-\sigma+1} (s^{\sigma-1}-1)^{-1} + \frac{f}{f_s} \right]}{[(s^{\sigma-1}-1)^{-\frac{\theta}{\sigma-1}} \left( \frac{f}{f_s} \right)^{\frac{\sigma-\theta-1}{\sigma-1}} + 1]}$$

and

$$\Pi_E = \frac{y \theta - \sigma + 1}{\sigma} \frac{\theta}{\theta} \frac{\left[ \frac{1}{1-s^{1-\sigma}} \frac{\theta}{\theta-\sigma+1} - \frac{f+f_s}{f_s} \right]}{[(s^{\sigma-1}-1)^{-\frac{\theta}{\sigma-1}} \left( \frac{f}{f_s} \right)^{\frac{\sigma-\theta-1}{\sigma-1}} + 1]}.$$

The sum of both equations is simply  $\Pi = \frac{\sigma-1}{\sigma} \frac{y}{\theta}$ .

With  $\left( \frac{f_s}{f(s^{\sigma-1}-1)} \right) > 1$  and  $\frac{\theta-\sigma+1}{\sigma-1} > 0$ , the absolute value of the elasticity of the price index with respect to  $s$  is less than 1:

$$|\epsilon_{P_{s,high}}| = \frac{s^{\sigma-1}}{s^{\sigma-1}-1} \left[ (s^{\sigma-1}-1)^{-\frac{\theta}{\sigma-1}} \left( \frac{f_s}{f} \right)^{\frac{\theta-\sigma+1}{\sigma-1}} + 1 \right]^{-1} < 1.$$

Intuitively, if only a subset of firms is subsidized, the price index becomes less sensitive to a change in the subsidy rate. Note that in the limit where all active firms are subsidized, the elasticity converges to 1.

### A.6 Proof of Corollary 3

**Corollary.** *In the high costs case, total subsidy payments decrease in the administrative fixed costs,  $\frac{\partial S_{high}}{\partial f_s} < 0$ . However, total subsidy payments increase in the production fixed costs,  $\frac{\partial S_{high}}{\partial f} > 0$  and in firm heterogeneity (i.e. lower  $\theta$ )  $\frac{\partial S_{high}}{\partial \theta} < 0$ .*

*Proof.* Using  $R_E$ ,  $S_{high}$  can be expressed in terms of the model parameters:

$$S_{high} = (s-1) \frac{\sigma-1}{\sigma} y \frac{1}{1-s^{1-\sigma}} \left[ \left( \frac{f_s}{f(s^{\sigma-1}-1)} \right)^{\frac{\theta}{\sigma-1}} \frac{f}{f_s} + 1 \right]^{-1}.$$

Therefore,  $\frac{\partial S_{high}}{\partial \theta} < 0$ . The derivatives with respect to  $f_s$  and  $f$  are

$$\frac{\partial S_{high}}{\partial f_s} = -\frac{\theta-\sigma+1}{\sigma} \left[ \left( \frac{f_s}{f} \right)^{-\frac{\theta-\sigma+1}{\sigma-1}} (s^{\sigma-1}-1)^{\frac{\theta}{\sigma-1}} + 1 \right]^{-1} \frac{S_{high}}{f_s} < 0$$

and

$$\frac{\partial S_{high}}{\partial f} = -\frac{\theta-\sigma+1}{\sigma} \left[ \left( \frac{f_s}{f} \right)^{-\frac{\theta-\sigma+1}{\sigma-1}} (s^{\sigma-1}-1)^{\frac{\theta}{\sigma-1}} + 1 \right]^{-1} \frac{S_{high}}{f} > 0.$$

□

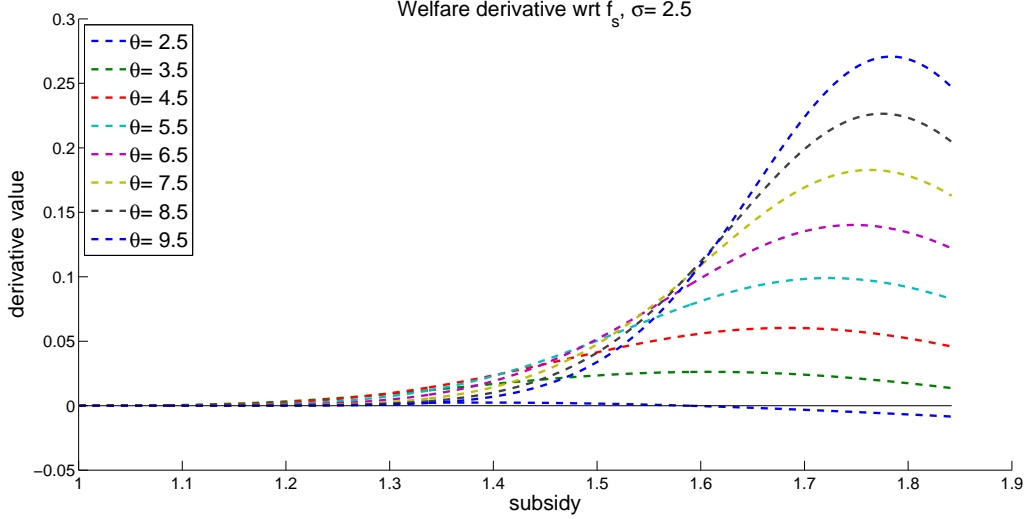


Figure 15:  $\frac{\partial W_{high}}{\partial f_s}$  evaluated at different values of the Pareto shape parameter,  $\theta$

### A.7 Proof of Corollary 4

**Corollary.** *In the high costs case, if  $\theta$  is sufficiently high, welfare is increasing in the administrative fixed costs,  $\frac{\partial W_{high}}{\partial f_s} > 0$ .*

*Proof.* Note that  $\frac{\partial W_{high}}{\partial f_s} = \frac{\partial CS_{high}}{\partial f_s} - \frac{\partial S_{high}}{\partial f_s}$ . Taking the derivative of  $CS_{high}$  gives

$$\frac{\partial CS_{high}}{\partial f_s} = \frac{y \sigma - \theta - 1}{\theta} \frac{\sigma - 1}{\sigma - 1} \frac{(s^{\sigma-1} - 1)^{\frac{\theta}{\sigma-1}} f_s^{\frac{\sigma-\theta-1}{\sigma-1}-1}}{f_s^{\frac{\sigma-\theta-1}{\sigma-1}} + (s^{\sigma-1} - 1)^{\frac{\theta}{\sigma-1}} f_s^{\frac{\sigma-\theta-1}{\sigma-1}}}.$$

Taking the derivative of  $S_{high}$  gives

$$\frac{\partial S_{high}}{\partial f_s} = -(s-1) \frac{\sigma-1}{\sigma} \frac{\theta-\sigma+1}{\sigma-1} (s^{\sigma-1}-1)^{-\frac{\theta}{\sigma-1}} \left(\frac{f_s}{f}\right)^{\frac{\theta-\sigma+1}{\sigma-1}-1} R_E [(s^{\sigma-1}-1)^{-\frac{\theta}{\sigma-1}} \left(\frac{f_s}{f}\right)^{\frac{\theta-\sigma+1}{\sigma-1}} + 1]^{-1}.$$

Therefore,

$$\frac{\partial W_{high}}{\partial f_s} = \left[ (s^{\sigma-1} - 1)^{-\frac{\theta}{\sigma-1}} \left(\frac{f_s}{f}\right)^{\frac{\theta-\sigma+1}{\sigma-1}} + 1 \right]^{-1} \frac{\theta - \sigma + 1}{\sigma - 1} f_s^{-1} \left[ (s-1) \frac{\sigma-1}{\sigma} R_E (s^{\sigma-1} - 1)^{-\frac{\theta}{\sigma-1}} \left(\frac{f_s}{f}\right)^{\frac{\theta-\sigma+1}{\sigma-1}} f - \frac{y}{\theta} \right]$$

Thus  $\frac{\partial W_{high}}{\partial f_s} > 0$  only if

$$(s-1) \frac{\sigma-1}{\sigma} \frac{s^{\sigma-1}}{s^{\sigma-1}-1} f \left[ \left(\frac{f_s}{f(s^{\sigma-1}-1)}\right)^{-\frac{\theta}{\sigma-1}} \frac{f_s}{f} + 1 \right]^{-1} > \frac{1}{\theta}$$

The right hand side of this inequality is strictly decreasing in  $\theta$ , while the left hand side is strictly increasing in  $\theta$ . Therefore, there exists a unique  $\theta$  above which the derivative of welfare with respect to  $f_s$  is positive. In Figure 15,  $\frac{\partial W_{high}}{\partial f_s}$  is plotted against the subsidy rate, for varying values of  $\theta$ . While the derivative could be negative for small values of  $\theta$  (close to  $\sigma$ ), for sufficiently high values of  $\theta$ , an increase in the administrative fixed costs has a positive welfare effect.  $\square$

### A.8 Proof of Lemma 4

**Lemma.** *In the high costs case with a given subsidy rate  $s$ , there is both a product market cutoff  $\varphi^*$  and a eligibility cutoff  $\varphi_L^*$  and*

1. the price index lies below the baseline value  $P_{s,high} < P_{base}$ ;

2.  $\varphi^*$  lies above the baseline value  $\varphi^* > \varphi_{\text{base}}^*$ ;
3. a rise in  $s$  or  $f$ , or a decline in  $f_s$  increases  $\varphi^*$  and  $J_L$  but decreases  $\varphi_L^*$  and  $J_A$ ;
4. a rise in firm heterogeneity increases both  $\varphi^*$  and  $\varphi_L^*$ .

*Proof.* Here, I show statement 3 and 4. In the high costs case, with  $\kappa = \left( J \frac{\theta b^\theta}{\theta - \sigma + 1} \frac{\sigma}{y} \right)^{\frac{1}{\theta}}$ , the derivatives of the cutoffs with respect to  $f_s$  and  $s$  are given by:

$$\begin{aligned} \frac{\partial \varphi_L^*}{\partial f_s} &= \frac{\kappa}{\theta} \left[ \left( \frac{f_s}{f(s^{\sigma-1} - 1)} \right)^{\frac{\theta}{\sigma-1}} f + f_s \right]^{\frac{1-\theta}{\theta}} \left( \frac{\theta}{\sigma-1} \left( \frac{f_s}{f(s^{\sigma-1} - 1)} \right)^{\frac{\theta}{\sigma-1}} \frac{f}{f_s} + 1 \right) > 0, \\ \frac{\partial \varphi^*}{\partial f_s} &= -\frac{\kappa}{\theta} \left[ f + \left( \frac{f_s}{(s^{\sigma-1} - 1)f} \right)^{-\frac{\theta}{\sigma-1}} f_s \right]^{\frac{1-\theta}{\theta}} \frac{\theta - \sigma + 1}{\sigma - 1} \left( \frac{f_s}{(s^{\sigma-1} - 1)f} \right)^{-\frac{\theta}{\sigma-1}} < 0, \\ \frac{\partial \varphi_L^*}{\partial s} &= -(\varphi_L^*)^{1-\theta} \left( \frac{1}{f(s^{\sigma-1} - 1)} \right)^{\frac{\theta + \sigma - 1}{\sigma - 1}} (f_s)^{\frac{\theta}{\sigma-1}} s^{\sigma-2} < 0, \end{aligned}$$

and

$$\frac{\partial \varphi^*}{\partial s} = \kappa \left[ f + \left( \frac{(s^{\sigma-1} - 1)f}{f_s} \right)^{\frac{\theta}{\sigma-1}} f_s \right]^{\frac{1-\theta}{\theta}} s^{\sigma-2} (s^{\sigma-1} - 1)^{\frac{\theta - \sigma + 1}{\sigma - 1}} f^{\frac{\theta}{\sigma-1}} f_s^{\frac{\sigma-1-\theta}{\sigma-1}} > 0.$$

Note that  $\frac{\partial J_A}{\partial \theta} = -J_A \left( \ln \left( \frac{\varphi^*}{b} \right) + \theta \frac{\partial \ln(\varphi^*)}{\partial \theta} \right) > 0$  requires  $\frac{\partial \varphi^*}{\partial \theta} < 0$  and therefore  $\frac{\partial \varphi_L^*}{\partial \theta} = \frac{\partial \varphi^*}{\partial \theta} \left( \frac{f_s}{f(s^{\sigma-1} - 1)} \right)^{\frac{1}{\sigma-1}} < 0$ .  $\square$

## A.9 Proof of Lemma 5

**Lemma.** *A rise in firm heterogeneity decreases  $J_A$  and increases  $J_R$ . If the subsidy rate is sufficiently high, there is a hump-shaped relationship between firm heterogeneity and  $J_L$ .*

*Proof.* The derivative of  $J_L$  with respect to  $\theta$  is

$$\frac{\partial J_L}{\partial \theta} = -J_L \left[ \underbrace{\theta \frac{\partial \ln(\varphi_L^*)}{\partial \theta}}_{<0} + \underbrace{\ln \left( \frac{\varphi_L^*}{b} \right)}_{>0} \right] \quad (31)$$

Therefore,  $\frac{\partial J_L}{\partial \theta} \leq 0$  iff  $\theta \frac{\partial \ln(\varphi_L^*)}{\partial \theta} + \ln \left( \frac{\varphi_L^*}{b} \right) \geq 0$ . In equation (31) the first term in brackets is due to the decline in the lobby cutoff and the second term is due to the change in the density. For  $\ln \left( \frac{\varphi_L^*}{b} \right)$  we get

$$\ln \left( \frac{\varphi_L^*}{b} \right) = \frac{1}{\theta} \ln \left( J \frac{\sigma}{y} \right) + \frac{1}{\theta} \ln \left( \frac{1}{1 - \frac{\sigma-1}{\theta}} \right) + \frac{1}{\theta} \ln \left[ \exp \left[ \frac{\theta}{\sigma-1} \ln \left( \frac{f_s}{f(s^{\sigma-1} - 1)} \right) \right] f + f_s \right] \quad (32)$$

Therefore,

$$\begin{aligned} \theta \frac{\partial \ln(\varphi_L^*)}{\partial \theta} &= -\frac{1}{\theta} \ln \left( J \frac{\sigma}{y} \frac{1}{1 - \frac{\sigma-1}{\theta}} \right) - \frac{1}{\theta} \frac{\frac{\sigma-1}{\theta}}{\theta - \sigma + 1} - \frac{1}{\theta} \ln \left[ \left( \frac{f_s}{f(s^{\sigma-1} - 1)} \right)^{\frac{\theta}{\sigma-1}} f + f_s \right] \\ &\quad + \frac{\left( \frac{f_s}{f(s^{\sigma-1} - 1)} \right)^{\frac{\theta}{\sigma-1}} f}{\left( \frac{f_s}{f(s^{\sigma-1} - 1)} \right)^{\frac{\theta}{\sigma-1}} f + f_s} \left[ \frac{1}{\sigma-1} \ln \left( \frac{f_s}{f(s^{\sigma-1} - 1)} \right) \right]. \end{aligned} \quad (33)$$

Combining equations (33) and equation (32) we can rewrite  $\theta \frac{\partial \ln(\varphi_L^*)}{\partial \theta} + \ln \left( \frac{\varphi_L^*}{b} \right) \geq 0$  as

$$\ln \left( \frac{f_s}{f(s^{\sigma-1} - 1)} \right) \geq \left[ \left( \frac{\sigma-1}{\theta} \right)^2 \frac{1}{\theta - \sigma + 1} \right] \left( 1 + \left( \frac{f_s}{f(s^{\sigma-1} - 1)} \right)^{-\frac{\theta}{\sigma-1}} \frac{f_s}{f} \right). \quad (34)$$

Note that the left hand side of inequality (34) is strictly decreasing in  $s$  but independent



of  $\theta$ , while the right hand side is strictly increasing in  $s$  but decreasing in  $\theta$ . Therefore,  $\forall s \in (1, \left(\frac{f_s}{f} + 1\right)^{\frac{1}{\sigma-1}})$ ,  $\frac{\partial J_L}{\partial \theta} < 0$  if  $\theta$  is sufficiently high.

For any  $s \in (1, \left(\frac{f_s}{f} + 1\right)^{\frac{1}{\sigma-1}})$ , if  $\theta \rightarrow \sigma$  the right hand side of equation (34) converges to its maximum. Define  $\tilde{s} \in (1, \left(\frac{f_s}{f} + 1\right)^{\frac{1}{\sigma-1}})$  as the subsidy rate such that in the limit where  $\theta \rightarrow \sigma$ , equation (34) holds with equality:

$$\ln \left( \frac{f_s}{f(\tilde{s}^{\sigma-1} - 1)} \right) = \left[ \left(1 - \frac{1}{\sigma}\right)^2 \right] \left( 1 + \left( \frac{f_s}{f(\tilde{s}^{\sigma-1} - 1)} \right)^{-\frac{\sigma}{\sigma-1}} \frac{f_s}{f} \right).$$

Given the model restriction  $\theta > \sigma$ , for any  $s \leq \tilde{s}$  it is always the case that  $\frac{\partial J_L}{\partial \theta} \leq 0$  (i.e. inequality (34) holds). Therefore, it is a necessary condition for  $\frac{\partial J_L}{\partial \theta} > 0$  that  $s \in (\tilde{s}, \left(\frac{f_s}{f} + 1\right)^{\frac{1}{\sigma-1}})$ . However, by the definition of  $\tilde{s}$ , for any  $s \in (\tilde{s}, \left(\frac{f_s}{f} + 1\right)^{\frac{1}{\sigma-1}})$  there exists always a value of  $\theta$  sufficiently close to  $\sigma$  such that  $\frac{\partial J_L}{\partial \theta} > 0$  (i.e. inequality (34) does not hold).  $\square$

### A.10 Proof of Lemma 6

**Lemma.** *In the high costs case, for any  $\theta > \sigma$  there does not exist an interior solution to the government's first-order condition, and (ex-ante) welfare is maximized at  $s^* = 1$ .*

*Proof.* Consider the first-order condition:

$$\begin{aligned} \frac{\partial W_{high}}{\partial s} &= \frac{\partial C S_{high}}{\partial s} - \frac{\partial S_{high}}{\partial s} = 0 \\ &= \frac{R_E}{s} - \frac{\sigma-1}{\sigma} R_E - (s-1) \frac{\sigma-1}{\sigma} \frac{\partial R_E}{\partial s} = 0, \end{aligned} \quad (35)$$

where  $R_E = y \frac{1}{1-s^{1-\sigma}} \left[ \left( \frac{f_s}{f(s^{\sigma-1}-1)} \right)^{\frac{\theta}{\sigma-1}} \frac{f}{f_s} + 1 \right]^{-1}$ . Rewrite the first-order condition and define

$$FOC := 1 - \frac{\sigma-1}{\sigma} s - (s-1) \frac{\sigma-1}{\sigma} \epsilon_{R_E,s} = 0, \quad (36)$$

where  $\epsilon_{R_E,s} = \frac{1}{s^{\sigma-1}-1} \left[ (1-\sigma) + \theta s^{\sigma-1} \left[ \left( \frac{f_s}{f(s^{\sigma-1}-1)} \right)^{-\frac{\theta}{\sigma-1}} \frac{f}{f_s} + 1 \right]^{-1} \right]$ .

From equation (36) and with  $\epsilon_{R_E,s} \geq 0$ , it is immediately apparent that any  $s > \frac{\sigma}{\sigma-1}$  can never be a solution to the first-order condition. Moreover, because with  $\frac{f_s}{f(s^{\sigma-1}-1)} > 1$ ,  $\epsilon_{R_E,s}$  is strictly increasing, continuous and unbounded in  $\theta$ , for any  $s \in (1, \frac{\sigma}{\sigma-1}]$ , there will always exist a value of  $\theta$  that implements  $s$  as a solution of the first-order condition. However, by the same argument, for sufficiently high values of  $\theta$  the first-order condition will no longer be fulfilled at any interior point. I will now clarify this last point, by showing that for  $\theta > \sigma$ ,  $FOC < 0 \forall s \in (1, \frac{\sigma}{\sigma-1}]$ .

Rewrite the first-order condition to:

$$\frac{1}{s} - \frac{\sigma-1}{\sigma} - \frac{\sigma-1}{\sigma} \left[ \frac{\left(\frac{s-1}{s}\right)}{s^{\sigma-1}-1} (1-\sigma) + \theta \frac{\left(\frac{s-1}{s}\right)}{s^{\sigma-1}-1} s^{\sigma-1} \left[ \left( \frac{f_s}{f(s^{\sigma-1}-1)} \right)^{-\frac{\theta}{\sigma-1}} \frac{f}{f_s} + 1 \right]^{-1} \right] = 0.$$

Then,  $FOC < 0$  if:

$$\frac{1}{s} < \frac{\sigma-1}{\sigma} + \frac{\sigma-1}{\sigma} \left[ \frac{\left(\frac{s-1}{s}\right)}{s^{\sigma-1}-1} (1-\sigma) + \theta \frac{\left(\frac{s-1}{s}\right)}{s^{\sigma-1}-1} s^{\sigma-1} \left[ \left( \frac{f_s}{f(s^{\sigma-1}-1)} \right)^{-\frac{\theta}{\sigma-1}} \frac{f}{f_s} + 1 \right]^{-1} \right]. \quad (37)$$

That inequality (37) holds, is not immediately apparent. Therefore, I will now construct an auxiliary line that lies weakly above the left hand side of inequality (37). Then, I will

show that the right hand side of inequality (37) lies strictly above this line.

Evaluating the left hand side of this inequality (equation (37)), we see that at  $s = 1$  the left hand side is  $\frac{1}{s} = 1$  and at  $s = \frac{\sigma}{\sigma-1}$  the left hand side is  $\frac{1}{s} = \frac{\sigma-1}{\sigma}$ . A line through the points  $(1, 1)$  and  $(\frac{\sigma}{\sigma-1}, \frac{\sigma-1}{\sigma})$  is defined by

$$l(s) = 1 - (s - 1) \frac{\sigma - 1}{\sigma}.$$

Then, it follows that:

$$\frac{1}{s} \leq 1 - (s - 1) \frac{\sigma - 1}{\sigma},$$

with equality only at  $s = 1$  and  $s = \frac{\sigma}{\sigma-1}$ , because  $\frac{1}{s}$  is a convex function.

Therefore for inequality (37) to hold, it is sufficient to show that

$$\begin{aligned} 1 - (s-1) \frac{\sigma - 1}{\sigma} &< \frac{\sigma - 1}{\sigma} + \frac{\sigma - 1}{\sigma} \left[ \frac{\left(\frac{s-1}{s}\right)}{s^{\sigma-1} - 1} (1 - \sigma) + \theta \frac{\left(\frac{s-1}{s}\right)}{s^{\sigma-1} - 1} s^{\sigma-1} \left[ \left( \frac{f_s}{f(s^{\sigma-1} - 1)} \right)^{-\frac{\theta}{\sigma-1}} \frac{f_s}{f} + 1 \right]^{-1} \right] \\ \Leftrightarrow \frac{\sigma}{\sigma - 1} - s &< \frac{\left(\frac{s-1}{s}\right)}{s^{\sigma-1} - 1} (1 - \sigma) + \theta \frac{\left(\frac{s-1}{s}\right)}{s^{\sigma-1} - 1} s^{\sigma-1} \left[ \left( \frac{f_s}{f(s^{\sigma-1} - 1)} \right)^{-\frac{\theta}{\sigma-1}} \frac{f_s}{f} + 1 \right]^{-1} \\ \Leftrightarrow \frac{\sigma}{\sigma - 1} - s &< \frac{\left(\frac{s-1}{s}\right)}{s^{\sigma-1} - 1} \left[ (1 - \sigma) + \theta s^{\sigma-1} \left[ \left( \frac{f_s}{f(s^{\sigma-1} - 1)} \right)^{-\frac{\theta}{\sigma-1}} \frac{f_s}{f} + 1 \right]^{-1} \right]. \quad (38) \end{aligned}$$

At  $s = \frac{\sigma}{\sigma-1}$ , inequality (38) always holds, because the left hand side is 0 and the right hand side is strictly positive. Taking the limit for  $s \rightarrow 1$  of both sides of inequality (38) we get

$$\begin{aligned} \lim_{s \rightarrow 1} \frac{\sigma}{\sigma - 1} - s &= \frac{1}{\sigma - 1} \leq \quad (39) \\ -1 + \frac{\theta}{\sigma - 1} &= \lim_{s \rightarrow 1} \left( \frac{s-1}{s} \right) \frac{1}{s^{\sigma-1} - 1} \left[ (1 - \sigma) + \theta s^{\sigma-1} \left[ \left( \frac{f_s}{f(s^{\sigma-1} - 1)} \right)^{-\frac{\theta}{\sigma-1}} \frac{f_s}{f} + 1 \right]^{-1} \right]. \end{aligned}$$

Inequality (39) holds strictly only for  $\theta > \sigma$ . For  $\theta = \sigma$  inequality (39) holds with equality and inequality (38) is no longer fulfilled but holds with equality. Thus, for inequality (38) to hold at the limit  $s \rightarrow 1$ , it is necessary and sufficient to have  $\theta > \sigma$ .

So far, I have established that it is necessary and sufficient to have  $\theta > \sigma$  for  $FOC < 0$  at the limit points (i.e. for  $s \rightarrow 1$  and at  $s = \frac{\sigma}{\sigma-1}$ ). I will now show, that  $\theta > \sigma$  is also sufficient for  $FOC < 0$  at any  $s \in (1, \frac{\sigma}{\sigma-1}]$ .

Note, that the right hand side of inequality (38) is strictly increasing, continuous and unbounded in  $\theta$ . Therefore, for  $\theta > \sigma$  we get from the right hand side of inequality (38):

$$\begin{aligned} \frac{\left(\frac{s-1}{s}\right)}{s^{\sigma-1} - 1} \left[ (1 - \sigma) + \sigma s^{\sigma-1} \left[ \left( \frac{f_s}{f(s^{\sigma-1} - 1)} \right)^{-\frac{\sigma}{\sigma-1}} \frac{f_s}{f} + 1 \right]^{-1} \right] &< \\ \frac{\left(\frac{s-1}{s}\right)}{s^{\sigma-1} - 1} \left[ (1 - \sigma) + \theta s^{\sigma-1} \left[ \left( \frac{f_s}{f(s^{\sigma-1} - 1)} \right)^{-\frac{\theta}{\sigma-1}} \frac{f_s}{f} + 1 \right]^{-1} \right]. \end{aligned}$$

Then, for inequality (38) to hold, it is sufficient to show that

$$\frac{\sigma}{\sigma - 1} - s \leq \frac{\left(\frac{s-1}{s}\right)}{s^{\sigma-1} - 1} \left[ (1 - \sigma) + \sigma s^{\sigma-1} \left[ \left( \frac{f_s}{f(s^{\sigma-1} - 1)} \right)^{-\frac{\sigma}{\sigma-1}} \frac{f_s}{f} + 1 \right]^{-1} \right], \quad (40)$$

where the inequality holds with equality for  $s \rightarrow 1$ , as shown above. Then, for the

inequality (40) to hold, it is to show that

$$\begin{aligned}
&\Leftrightarrow \frac{\sigma}{\sigma-1} - s \leq \left(\frac{s-1}{s}\right) \frac{1}{s^{\sigma-1}-1} \left[ (1-\sigma) + \sigma s^{\sigma-1} \left[ \left( \frac{f_s}{f(s^{\sigma-1}-1)} \right)^{-\frac{\sigma}{\sigma-1}} \frac{f_s}{f} + 1 \right]^{-1} \right] \\
&\Leftrightarrow \frac{\sigma}{\sigma-1} - s \leq \sigma \frac{\left(\frac{s-1}{s}\right)}{s^{\sigma-1}-1} s^{\sigma-1} \left[ \left( \frac{f_s}{f(s^{\sigma-1}-1)} \right)^{-\frac{\sigma}{\sigma-1}} \frac{f_s}{f} + 1 \right]^{-1} - (\sigma-1) \frac{\left(\frac{s-1}{s}\right)}{s^{\sigma-1}-1} \\
&\Leftrightarrow \frac{1}{\sigma-1} + 1 - s \leq \frac{\left(\frac{s-1}{s}\right)}{s^{\sigma-1}-1} + \sigma \frac{\left(\frac{s-1}{s}\right)}{s^{\sigma-1}-1} s^{\sigma-1} \left[ \left( \frac{f_s}{f(s^{\sigma-1}-1)} \right)^{-\frac{\sigma}{\sigma-1}} \frac{f_s}{f} + 1 \right]^{-1} - \sigma \frac{\left(\frac{s-1}{s}\right)}{s^{\sigma-1}-1}.
\end{aligned} \tag{41}$$

For  $s > 1$  and  $\sigma > 1$ , the term  $\frac{\left(\frac{s-1}{s}\right)}{s^{\sigma-1}-1}$  is continuous and decreasing in  $s$ . With  $\lim_{s \rightarrow 1} \frac{\left(\frac{s-1}{s}\right)}{s^{\sigma-1}-1} = \frac{1}{\sigma-1}$ , it must then hold that  $\frac{1}{\sigma-1} \leq \frac{\left(\frac{s-1}{s}\right)}{s^{\sigma-1}-1}$ , with equality only for  $s \rightarrow 1$ . Then, for inequality (41) to hold, it is left to show that

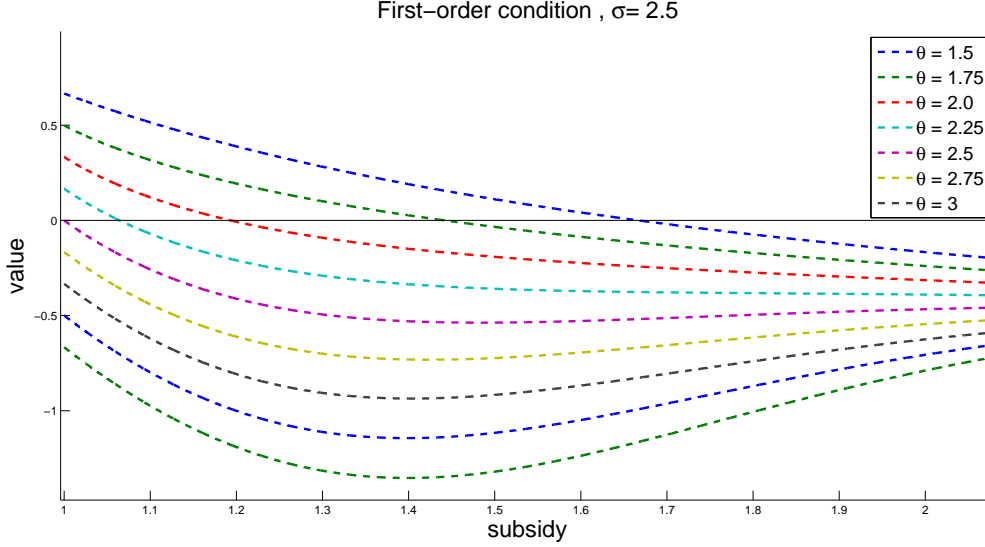
$$\begin{aligned}
&1 - s \leq \sigma \frac{\left(\frac{s-1}{s}\right)}{s^{\sigma-1}-1} \left[ s^{\sigma-1} \left[ \left( \frac{f_s}{f(s^{\sigma-1}-1)} \right)^{-\frac{\sigma}{\sigma-1}} \frac{f_s}{f} + 1 \right]^{-1} - 1 \right] \\
&\Leftrightarrow (s-1) \geq -\sigma \frac{\left(\frac{s-1}{s}\right)}{s^{\sigma-1}-1} \left[ s^{\sigma-1} \left[ \left( \frac{f_s}{f(s^{\sigma-1}-1)} \right)^{-\frac{\sigma}{\sigma-1}} \frac{f_s}{f} + 1 \right]^{-1} - 1 \right] \\
&\Leftrightarrow 1 \geq \frac{\sigma \left(\frac{1}{s}\right)}{s^{\sigma-1}-1} \left[ 1 - s^{\sigma-1} \left[ \left( \frac{f_s}{f(s^{\sigma-1}-1)} \right)^{-\frac{\sigma}{\sigma-1}} \frac{f_s}{f} + 1 \right]^{-1} \right].
\end{aligned}$$

The last inequality will necessarily hold, if the term in brackets is negative:

$$\begin{aligned}
&1 \leq s^{\sigma-1} \left[ \left( \frac{f_s}{f(s^{\sigma-1}-1)} \right)^{-\frac{\sigma}{\sigma-1}} \frac{f_s}{f} + 1 \right]^{-1} \\
&\Leftrightarrow (s^{\sigma-1}-1)^{\frac{\sigma}{\sigma-1}} \left( \frac{f_s}{f} \right)^{-\frac{\sigma}{\sigma-1}} \frac{f_s}{f} \leq (s^{\sigma-1}-1) \\
&\Leftrightarrow \left( \frac{f_s}{f(s^{\sigma-1}-1)} \right)^{-\frac{1}{\sigma-1}} \leq 1.
\end{aligned}$$

The last inequality holds by definition of the high costs case,  $\frac{f_s}{f(s^{\sigma-1}-1)} > 1$ . Thus, for  $\theta > \sigma$  we get  $FOC < 0$ .

Moreover, because  $\epsilon_{R_E, s} = 0$  for  $\theta = \sigma - 1$ , such that the unique solution of the first-order condition is  $s = \frac{\sigma}{\sigma-1}$ , and because I have established that for  $\theta > \sigma$  there cannot be an interior solution, we get that for  $\sigma \geq \theta \geq \sigma - 1$  any interior solution with  $s \in (1, \frac{\sigma}{\sigma-1}]$  is feasible. Figure 16 shows the first-order condition for various values of  $\theta$ . While the first-order condition has a unique solution for  $\sigma \geq \theta \geq \sigma - 1$ , the first-order condition does not have an interior root for any  $\theta > \sigma$ . It can be observed that even for subsidy rates arbitrarily close to 1 the first-order condition is negative if  $\theta > \sigma$ .



**Figure 16:** First-order condition increasing  $\theta$

□

### A.11 Proof of Lemma 7

**Lemma.** *In the high costs, the government implements  $s^o = \frac{\frac{\sigma}{\sigma-1} + (1 + \frac{1}{\sigma-1} \frac{1}{\alpha}) \tilde{c}_{R_E, s}}{1 + \tilde{c}_{R_E, s}}$  in the lobbying game.*

*Proof.* With  $S_{high} = (s-1)^{\frac{\sigma-1}{\sigma}} R_E$  we get

$$\frac{\partial S}{\partial s} = R_E \frac{\sigma-1}{\sigma} - (s-1) \frac{\sigma-1}{\sigma} \frac{\partial R_E}{\partial s}. \quad (42)$$

Consumer surplus changes, because the subsidized firms can sell at lower price and decrease the price index. However, because the cutoffs are already given when the government sets the subsidy, the effect on the price comes only from producers that are in the lobby. With  $CS = y \ln(y_i) - y \ln(P_s) - y$ , by

$$P_s = \kappa_p \left[ (\varphi^*)^{\sigma-\theta-1} + (s^{\sigma-1} - 1) (\varphi_L^*)^{\sigma-\theta-1} \right]^{\frac{1}{1-\sigma}},$$

where  $\kappa_p = \left( J \frac{\theta b^\theta}{\theta - \sigma + 1} \right)^{\frac{1}{1-\sigma}} \left( \frac{\sigma}{\sigma-1} \right)$  we get

$$CS = y \ln(y_i) - y \ln(\kappa_p \left[ (\varphi^*)^{\sigma-\theta-1} + (s^{\sigma-1} - 1) (\varphi_L^*)^{\sigma-\theta-1} \right]^{\frac{1}{1-\sigma}}) - y.$$

Holding the cutoffs fixed, we get:

$$\frac{\partial CS}{\partial s} = \frac{y}{s} s^{\sigma-1} \left[ \left( \frac{\varphi^*}{\varphi_L^*} \right)^{\sigma-\theta-1} + (s^{\sigma-1} - 1) \right]^{-1}.$$

With  $\frac{\varphi_L^*}{\varphi^*} = \left( \frac{f_s}{f(s^{\sigma-1}-1)} \right)^{\frac{1}{\sigma-1}}$  we get:

$$\frac{\partial CS}{\partial s} = \frac{y}{s} \frac{s^{\sigma-1}}{s^{\sigma-1}-1} \left[ \frac{1}{(s^{\sigma-1}-1)} \left( \frac{f_s}{f(s^{\sigma-1}-1)} \right)^{\frac{\theta-\sigma+1}{\sigma-1}} + 1 \right]^{-1}.$$

With  $R_E = y \frac{s^{\sigma-1}}{s^{\sigma-1}-1} \left[ \left( \frac{f_s}{f(s^{\sigma-1}-1)} \right)^{\frac{\theta}{\sigma-1}} \frac{f}{f_s} + 1 \right]^{-1}$  we can rewrite:

$$\frac{\partial CS}{\partial s} = \frac{R_E}{s}. \quad (43)$$

Note that  $\frac{\partial CS}{\partial s}$  is the same even if the cutoffs are allowed to change with the subsidy rate.

Then, the lobby and product cutoffs would change, but the effects would cancel out each other. Moreover, joint profits of the lobby members are given by,

$$\Pi_E = \frac{J_L r_s(\tilde{\varphi}_L)}{\sigma} - J_L(f_s + f),$$

where  $\tilde{\varphi}_L$  denotes average productivity of the lobby members. However, given that the lobby takes the number of members as fixed, we get:

$$\frac{\partial \Pi_E}{\partial s} = \frac{J_L}{\sigma} \frac{\partial r_s(\tilde{\varphi}_L)}{\partial s},$$

or equivalently,

$$\frac{\partial \Pi_E}{\partial s} = \frac{1}{\sigma} \frac{\partial R_E}{\partial s}. \quad (44)$$

Combining equation (42), (43) and (44) gives the FOC:

$$\begin{aligned} \frac{\partial G}{\partial s} &= \alpha \left( \frac{R_E}{s} - R_E \frac{\sigma-1}{\sigma} - (s-1) \frac{\sigma-1}{\sigma} \frac{\partial R_E}{\partial s} \right) + \frac{1}{\sigma} \frac{\partial R_E}{\partial s} = 0. \\ \iff \frac{\partial G}{\partial s} &= \alpha \left( \frac{R_E}{s} - R_E \frac{\sigma-1}{\sigma} - (s-1) \frac{\sigma-1}{\sigma} J_L \frac{\partial r_s(\tilde{\varphi}_L)}{\partial s} \right) + \frac{J_L}{\sigma} \frac{\partial r_s(\tilde{\varphi}_L)}{\partial s} = 0. \\ \iff \frac{\sigma}{\sigma-1} - s - (s-1) &\frac{\partial R_E}{\partial s} \frac{s}{R_E} + \frac{1}{\sigma-1} \frac{1}{\alpha} \frac{\partial R_E}{\partial s} \frac{s}{R_E} = 0. \\ s^o &= \frac{\frac{\sigma}{\sigma-1} + \left(1 + \frac{1}{\sigma-1} \frac{1}{\alpha}\right) \tilde{\epsilon}_{R_E,s}}{1 + \tilde{\epsilon}_{R_E,s}}, \end{aligned} \quad (45)$$

where  $\tilde{\epsilon}_{R_E,s} = \frac{\partial R_E}{\partial s} \frac{s}{R_E}$  is the elasticity of aggregate lobby revenues wrt to the subsidy rate, holding the cutoffs and the number of lobby members fixed. Holding  $J_L$  fixed, we get

$$\begin{aligned} \tilde{\epsilon}_{R_E,s} &= \frac{\partial R_E}{\partial s} \frac{s}{R_E} = \frac{s}{R_E} J_L \frac{\partial r_s(\tilde{\varphi}_L)}{\partial s} \\ &= \frac{s}{R_E} J_L \left( \frac{\sigma}{\sigma-1} \frac{1}{\tilde{\varphi}_L} \right)^{1-\sigma} y \frac{\partial \left( \frac{s^{\sigma-1}}{P_s^{1-\sigma}} \right)}{\partial s}. \end{aligned}$$

With  $P_s = \kappa_p \left[ (\varphi^*)^{\sigma-\theta-1} + (s^{\sigma-1} - 1) (\varphi_L^*)^{\sigma-\theta-1} \right]^{\frac{1}{1-\sigma}}$  we get

$$\frac{s^{\sigma-1}}{P_s^{1-\sigma}} = \kappa_p^{\sigma-1} \left[ (\varphi^*)^{\sigma-\theta-1} + (s^{\sigma-1} - 1) (\varphi_L^*)^{\sigma-\theta-1} \right]^{-1} s^{\sigma-1},$$

such that

$$\begin{aligned} \frac{\partial \left( \frac{s^{\sigma-1}}{P_s^{1-\sigma}} \right)}{\partial s} &= -\kappa_p^{\sigma-1} \left[ (\varphi^*)^{\sigma-\theta-1} + (s^{\sigma-1} - 1) (\varphi_L^*)^{\sigma-\theta-1} \right]^{-2} s^{\sigma-1} (\sigma-1) \frac{1}{s} (s^{\sigma-1}) (\varphi_L^*)^{\sigma-\theta-1} \\ &\quad + \kappa_p^{\sigma-1} \left[ (\varphi^*)^{\sigma-\theta-1} + (s^{\sigma-1} - 1) (\varphi_L^*)^{\sigma-\theta-1} \right]^{-1} (\sigma-1) \frac{1}{s} (s^{\sigma-1}) \\ \iff \frac{\partial \left( \frac{s^{\sigma-1}}{P_s^{1-\sigma}} \right)}{\partial s} &= \kappa_p^{\sigma-1} \left[ (\varphi^*)^{\sigma-\theta-1} + (s^{\sigma-1} - 1) (\varphi_L^*)^{\sigma-\theta-1} \right]^{-1} s^{\sigma-1} \frac{(\sigma-1)}{s} \\ &\quad \left( - (s^{\sigma-1}) (\varphi_L^*)^{\sigma-\theta-1} \left[ (\varphi^*)^{\sigma-\theta-1} + (s^{\sigma-1} - 1) (\varphi_L^*)^{\sigma-\theta-1} \right]^{-1} + 1 \right) \\ \iff \frac{\partial \left( \frac{s^{\sigma-1}}{P_s^{1-\sigma}} \right)}{\partial s} &= \frac{s^{\sigma-1}}{P_s^{1-\sigma}} \frac{(\sigma-1)}{s} \left( 1 - \frac{s^{\sigma-1}}{(s^{\sigma-1} - 1)} \left[ \frac{(\varphi^*)^{\sigma-\theta-1}}{(s^{\sigma-1} - 1) (\varphi_L^*)^{\sigma-\theta-1}} + 1 \right]^{-1} \right). \end{aligned}$$

Therefore,

$$\begin{aligned}\tilde{\epsilon}_{R_E,s} &= \frac{s}{R_E} J_L r_s(\tilde{\varphi}_L) \frac{(\sigma-1)}{s} \left( 1 - \frac{s^{\sigma-1}}{(s^{\sigma-1}-1)} \left[ \frac{(\varphi^*)^{\sigma-\theta-1}}{(s^{\sigma-1}-1)(\varphi_L^*)^{\sigma-\theta-1}} + 1 \right]^{-1} \right) \\ &\iff \tilde{\epsilon}_{R_E,s} = (\sigma-1) \left( 1 - \frac{s^{\sigma-1}}{(s^{\sigma-1}-1)} \left[ \frac{(\varphi^*)^{\sigma-\theta-1}}{(s^{\sigma-1}-1)(\varphi_L^*)^{\sigma-\theta-1}} + 1 \right]^{-1} \right)\end{aligned}$$

With  $\frac{\varphi_L^*}{\varphi^*} = \left( \frac{f_s}{f(s^{\sigma-1}-1)} \right)^{\frac{1}{\sigma-1}}$  we get

$$\iff \tilde{\epsilon}_{R_E,s} = (\sigma-1) \left( 1 - \frac{s^{\sigma-1}}{(s^{\sigma-1}-1)} \left[ \frac{1}{(s^{\sigma-1}-1)} \left( \frac{f_s}{f(s^{\sigma-1}-1)} \right)^{\frac{\theta-\sigma+1}{\sigma-1}} + 1 \right]^{-1} \right)$$

By  $\frac{R_E}{y} = \frac{s^{\sigma-1}}{s^{\sigma-1}-1} \left[ \left( \frac{f_s}{f(s^{\sigma-1}-1)} \right)^{\frac{\theta}{\sigma-1}} \frac{f}{f_s} + 1 \right]^{-1}$  we get

$$\iff \tilde{\epsilon}_{R_E,s} = (\sigma-1) \left( 1 - \frac{R_E}{y} \right) \leq (\sigma-1).$$

□

### A.12 Proof of Lemma 8

**Lemma.** *If  $f_s > f\left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} - 1\right)$  (high costs case), there exists a unique equilibrium of the lobbying game, such that*

1. *all firms with  $\varphi > \varphi^o$  produce and all firms with  $\varphi > \varphi_L^o$  enter the lobby,*
2. *lobby contributions are  $C^T(s^o) = \alpha(y \ln(\frac{P_{s^o}}{P_{s^*}}) + S_{s^o} - S_{s^*})$ ,*
3. *the government implements  $s^o = \frac{\frac{\sigma}{\sigma-1} + (1 + \frac{1}{\sigma-1} \frac{1}{\alpha}) \tilde{\epsilon}_{R_E,s}}{1 + \tilde{\epsilon}_{R_E,s}}$ .*

*Proof.* Condition 1 – 3 are given in text. The relative mass of lobbying firms,  $h(s) = J_R$ , is a strictly increasing function of  $s$ , with  $\lim_{s \rightarrow 1} z(J_R) = 0$  and  $\lim_{s \rightarrow (\frac{f_s}{f} + 1)^{\frac{1}{\sigma-1}}} z(J_R) = 1$ .

Moreover,  $\lim_{J_R \rightarrow 0} z(J_R) = \frac{\sigma}{\sigma-1} + \frac{1}{\sigma} \left( \frac{1-\alpha}{\alpha} \right) > 0$  and  $\lim_{J_R \rightarrow 1} z(J_R) = \frac{\sigma}{\sigma-1} < 1$ . Therefore,  $z(J_R)$  is strictly increasing in  $J_R$  for  $\alpha < 1$  and strictly decreasing in  $J_R$  if  $\infty > \alpha > 1$ , and for any  $\alpha < \infty$  there is a unique interior intersection of  $z(J_R)$  and  $h(s)$ . □

### A.13 Proof of Lemma 9

**Lemma.** *Increasing the welfare weight  $\alpha$  decreases both the relative mass of lobbying firms and the optimal subsidy rate.*

*Proof.* Taking the total derivative of  $s$  with respect to  $\alpha$  gives:

$$\frac{ds}{d\alpha} = \frac{\partial z(J_R; \alpha)}{\partial \alpha} + \frac{\partial z(J_R; \alpha)}{\partial J_R} \frac{\partial h(s; \alpha)}{\partial \alpha}$$

With  $\frac{\partial h(s; \alpha)}{\partial \alpha} = 0$  and  $\frac{\partial z(J_R; \alpha)}{\partial \alpha} = -\frac{1}{\alpha^2} \frac{(\frac{1}{\sigma-1}) \tilde{\epsilon}_{R_E,s}}{1 + \tilde{\epsilon}_{R_E,s}} < 0$  we get  $\frac{ds}{d\alpha} < 0$ . Taking the total derivative of  $J_R$  with respect to  $\alpha$  gives:

$$\frac{dJ_R}{d\alpha} = \frac{\partial h(s; \alpha)}{\partial \alpha} + \frac{\partial h(s; \alpha)}{\partial s} \frac{\partial z(J_R; \alpha)}{\partial \alpha}$$

With  $\frac{\partial h(s; \alpha)}{\partial \alpha} = 0$ ,  $\frac{\partial h(s; \alpha)}{\partial s} > 0$  and  $\frac{\partial z(J_R; \alpha)}{\partial \alpha} < 0$  we get  $\frac{dJ_R}{d\alpha} < 0$ . □

### A.14 Proof of Lemma 10

**Lemma.** *Increasing the administrative fixed costs  $f_s$  decreases always the relative mass of lobbying firms, but increases the optimal subsidy rate if  $\alpha < 1$  and decreases the optimal subsidy rate if  $\alpha > 1$ .*

*Proof.*  $\frac{dJ_R}{df_s} < 0$  was shown in the main text. Note that  $z(J_R; f_s) > \frac{\sigma}{\sigma-1}$  iff  $\alpha < 1$ . Moreover,

$$\frac{\partial z(J_R; f_s)}{\partial \tilde{\epsilon}_{RE,s}} = \left( 1 + \frac{1}{\sigma-1} \frac{1}{\alpha} - \frac{\sigma}{\sigma-1} \frac{1}{1 + \tilde{\epsilon}_{RE,s}} - \left( 1 + \frac{1}{\sigma-1} \frac{1}{\alpha} \right) \frac{\tilde{\epsilon}_{RE,s}}{1 + \tilde{\epsilon}_{RE,s}} \right) [1 + \tilde{\epsilon}_{RE,s}]^{-1} > 0$$

if

$$\left( 1 + \frac{1}{\sigma-1} \frac{1}{\alpha} \right) \left[ \frac{1}{1 + \tilde{\epsilon}_{RE,s}} \right] > \frac{\sigma}{\sigma-1} \frac{1}{1 + \tilde{\epsilon}_{RE,s}},$$

which holds only if  $\alpha < 1$ . With  $\frac{\partial \tilde{\epsilon}_{RE,s}}{\partial J_R} < 0$  iff  $\alpha < 1$ , we get

$$\frac{\partial z(J_R; f_s)}{\partial J_R} = \underbrace{\frac{\partial z(J_R; f_s)}{\partial \tilde{\epsilon}_{RE,s}}}_{>0} \underbrace{\frac{\partial \tilde{\epsilon}_{RE,s}}{\partial J_R}}_{<0} < 0.$$

Therefore,

$$\begin{aligned} \frac{ds}{df_s} &= \frac{\partial z(J_R; f_s)}{\partial f_s} + \frac{\partial z(J_R; f_s)}{\partial J_R} \frac{\partial h(s; f_s)}{\partial f_s} \\ &= \underbrace{\frac{\partial z(J_R; f_s)}{\partial f_s}}_{=0} + \underbrace{\frac{\partial z(J_R; f_s)}{\partial \tilde{\epsilon}_{RE,s}}}_{>0} \underbrace{\frac{\partial \tilde{\epsilon}_{RE,s}}{\partial J_R}}_{<0} \underbrace{\frac{\partial h(s; f_s)}{\partial f_s}}_{<0} > 0 \end{aligned}$$

By the same argument if  $\alpha > 1$  then  $\frac{ds}{df_s} < 0$ . □

### A.15 Proof of Lemma 11

**Lemma.** *If  $\alpha > 1$ , decreasing firm heterogeneity (i.e. increasing  $\theta$ ) decreases both the relative mass of lobbying firms and the optimal subsidy rate. However, if  $\alpha < 1$ , decreasing firm heterogeneity increases the optimal subsidy rate and has an ambiguous effect on the relative mass of lobbying firms.*

*Proof.* There is a direct negative effect on  $J_R$ :

$$\frac{\partial h(s; \theta)}{\partial \theta} = \frac{J_R}{\theta} \ln(J_R) < 0.$$

However, holding the (relative) mass of firms in the lobby constant, an increase in  $\theta$  leads to a decline in average productivity of lobbying firms,  $\tilde{\varphi}_L = \left( \frac{\theta}{\theta+1-\sigma} \right)^{\frac{1}{\sigma-1}} \varphi_L^*$ , and therefore to a decline in average revenues of lobbying firms. This leads to an increase of  $\tilde{\epsilon}_{RE,s}$  for a constant relative mass of lobbying firms. Therefore,  $\frac{\partial z(J_R; \theta)}{\partial \theta} < 0$  if  $\alpha > 1$  and  $\frac{\partial z(J_R; \theta)}{\partial \theta} > 0$  if

$$\alpha > 1. \text{ With } \frac{\partial z(J_R; \theta)}{\partial \tilde{\epsilon}_{RE,s}} > 0 \text{ if and only if } \alpha < 1, \text{ it holds that } \frac{\partial z(J_R; \theta)}{\partial J_R} = \underbrace{\frac{\partial z(J_R; \theta)}{\partial \tilde{\epsilon}_{RE,s}}}_{>0} \underbrace{\frac{\partial \tilde{\epsilon}_{RE,s}}{\partial J_R}}_{<0} < 0.$$

Thus, the total effect on the subsidy is positive for  $\alpha < 1$ :

$$\frac{ds}{d\theta} = \underbrace{\frac{\partial z(J_R; \theta)}{\partial \theta}}_{>0} + \underbrace{\frac{\partial z(J_R; \theta)}{\partial J_R}}_{<0} \underbrace{\frac{\partial h(s; \theta)}{\partial \theta}}_{<0} > 0.$$

By the same argument if  $\alpha > 1$  then  $\frac{ds}{d\theta} < 0$ . For  $\alpha > 1$ , the total effect on the relative mass of lobbying firms is unambiguously negative:

$$\frac{dJ_R}{d\theta} = \underbrace{\frac{\partial h(s; \theta)}{\partial \theta}}_{<0} + \underbrace{\frac{\partial h(s; \theta)}{\partial s}}_{>0} \underbrace{\frac{\partial z(J_R; \theta)}{\partial \theta}}_{<0} < 0.$$

However, for  $\alpha < 1$  the positive indirect effect through the increase in the subsidy could

dominate the negative direct effect of an increase in  $\theta$ :

$$\frac{dJ_R}{d\theta} = \underbrace{\frac{\partial h(s; \theta)}{\partial \theta}}_{<0} + \underbrace{\frac{\partial h(s; \theta)}{\partial s}}_{>0} \underbrace{\frac{\partial z(J_R; \theta)}{\partial \theta}}_{>0} = ?$$

□

### A.16 Proof of Lemma 12

**Lemma.** *If  $\alpha < 1$ , increasing the elasticity of substitution  $\sigma$  decreases the optimal subsidy rate and has an ambiguous effect on the relative mass of lobbying firms..*

*Proof.* The direct effect of an increase in  $\sigma$  on the relative mass of lobbying firms is given by:

$$\frac{\partial h(s; \sigma)}{\partial \sigma} = J_R \left[ J_R^{-\frac{1}{\sigma-1}} + \frac{\theta}{\sigma-1} \ln(s) \frac{s^{\sigma-1}}{s^{\sigma-1} - 1} \right] > 0.$$

Therefore, the  $h(s)$  curve shifts upwards. Take the limits of the  $z(J_R)$  curve:

$$\lim_{J_R \rightarrow 1} z(J_R) = \frac{\sigma}{\sigma-1}$$

$$\lim_{J_R \rightarrow 0} z(J_R) = \frac{\sigma}{\sigma-1} + \frac{1}{\sigma} \frac{1-\alpha}{\alpha}.$$

Thus, the  $z(J_R)$  curve shifts to the left,  $\frac{\partial z(J_R; \sigma)}{\partial \sigma} < 0$ . For  $\alpha < 1$ ,  $\frac{\partial z(J_R; \sigma)}{\partial J_R} < 0$ , such that  $\frac{\partial z(J_R; \sigma)}{\partial J_R} \frac{\partial h(s; \sigma)}{\partial \sigma} < 0$  and therefore:

$$\frac{ds}{d\sigma} = \underbrace{\frac{\partial z(J_R; \sigma)}{\partial \sigma}}_{<0} + \underbrace{\frac{\partial z(J_R; \sigma)}{\partial J_R} \frac{\partial h(s; \sigma)}{\partial \sigma}}_{<0} < 0.$$

With  $\alpha < 1$  an increase in  $\sigma$  has a negative effect on the equilibrium subsidy rate. For  $\alpha < 1$ , the total effect on the relative mass of lobbying firms is ambiguous:

$$\frac{dJ_R}{d\sigma} = \underbrace{\frac{\partial h(s; \sigma)}{\partial \sigma}}_{>0} + \underbrace{\frac{\partial h(s; \sigma)}{\partial s}}_{>0} \underbrace{\frac{\partial z(J_R; \sigma)}{\partial \sigma}}_{<0} = ?$$

For  $\alpha > 1$  we get  $\frac{\partial z(J_R; \sigma)}{\partial J_R} > 0$  and

$$\frac{ds}{d\sigma} = \underbrace{\frac{\partial z(J_R; \sigma)}{\partial \sigma}}_{<0} + \underbrace{\frac{\partial z(J_R; \sigma)}{\partial J_R} \frac{\partial h(s; \sigma)}{\partial \sigma}}_{>0} = ?$$

For  $\alpha > 1$  the total effect on the relative mass of lobbying firms is also ambiguous:

$$\frac{dJ_R}{d\sigma} = \underbrace{\frac{\partial h(s; \sigma)}{\partial \sigma}}_{>0} + \underbrace{\frac{\partial h(s; \sigma)}{\partial s}}_{>0} \underbrace{\frac{\partial z(J_R; \sigma)}{\partial \sigma}}_{<0} = ?$$

□

## B Alternative instrument: ad-valorem output subsidy

Alternatively to a subsidy on variable costs, one could consider an ad-valorem output subsidy. After paying administrative fixed costs  $f_s$ , a firm becomes eligible for an ad-valorem subsidy  $s > 1$ . At a market price of  $p_s$  a firm receives a producer price per unit sold of  $s \cdot p_s$ . Therefore, a subsidized firm maximizes  $\pi_s = sp_s q_s(\varphi) - \frac{q_s(\varphi)}{\varphi} - f - f_s$  which yields a market price of  $p_s = \frac{\sigma}{(\sigma-1)s\varphi}$ . Hence, the per-unit *producer* price is the same as in the model without a subsidy:  $s \cdot p_s = \frac{\sigma}{\sigma-1} \frac{1}{\varphi}$ . Quantity sold, revenues and profits of a



firm with productivity draw  $\varphi$  are given by:

$$q_s(\varphi) = s^\sigma \frac{\left(\frac{\sigma}{\sigma-1} \frac{1}{\varphi}\right)^{-\sigma}}{P_s^{1-\sigma}} y, \quad r_s(\varphi) = s^\sigma \frac{\left(\frac{\sigma}{\sigma-1} \frac{1}{\varphi}\right)^{1-\sigma}}{P_s^{1-\sigma}} y, \quad \text{and} \quad \pi_s(\varphi) = s^\sigma \frac{\left(\frac{\sigma}{\sigma-1} \frac{1}{\varphi}\right)^{1-\sigma}}{P_s^{1-\sigma}} \frac{y}{\sigma} - f - f_s.$$

Firm revenues can be split up into a “market” and a “government” component:

$$r_s(\varphi) = sp_s q_s(\varphi) = \underbrace{pq_s(\varphi)}_{\text{market revenue}} + \underbrace{(s-1)pq_s(\varphi)}_{\text{subsidy payments}}$$

The subsidy payments per firm can be rewritten as

$$\begin{aligned} (s-1)p_s q_s(\varphi) &= (s-1) \left( \frac{\sigma}{\sigma-1} \frac{1}{s\varphi} \right) s^\sigma \frac{\left(\frac{\sigma}{\sigma-1} \frac{1}{\varphi}\right)^{-\sigma}}{P_s^{1-\sigma}} y \\ &= \frac{s-1}{s} s^\sigma \frac{\left(\frac{\sigma}{\sigma-1} \frac{1}{\varphi}\right)^{1-\sigma}}{P_s^{1-\sigma}} y \\ &= \frac{s-1}{s} r_s(\varphi). \end{aligned}$$

**Low costs case** The product market and eligibility cutoff is defined by  $\pi_s(\varphi_{L,low}^*) = 0$  :

$$\varphi_{L,low}^* = \frac{\sigma}{\sigma-1} \left( \frac{\sigma}{y} \frac{1}{P_s^{\sigma-1}} \frac{f+f_s}{s^\sigma} \right)^{\frac{1}{\sigma-1}}.$$

The price index is given by

$$P_{s,low} = \tilde{\kappa} (f+f_s)^{\frac{1}{\theta}} \frac{\theta+1-\sigma}{\sigma-1} s^{-\frac{1}{\theta}} \frac{\sigma\theta-\sigma+1}{\sigma-1},$$

where  $\tilde{\kappa} = \left( \frac{J\theta b^\theta}{\theta+1-\sigma} \left(\frac{\sigma}{\sigma-1}\right)^{-\theta} \left(\frac{\sigma}{y}\right)^{-\frac{\theta-1+\sigma}{\sigma-1}} \right)^{-\frac{1}{\theta}}$ . The cutoff can then be rewritten as

$$\varphi_{L,low}^* = \kappa \left( \frac{f+f_s}{s} \right)^{\frac{1}{\theta}},$$

where  $\kappa = \left( J \frac{\theta b^\theta}{\theta-\sigma+1} \frac{\sigma}{y} \right)^{\frac{1}{\theta}}$ . Total subsidy payments are given by  $S_{low} = (s-1)y$ . With  $\frac{\partial S_{low}}{\partial s} = y$  and  $\frac{\partial CS}{\partial s} = y \frac{1}{\theta} \frac{\sigma\theta-\sigma+1}{\sigma-1} \frac{1}{s}$ , the FOC for an interior welfare maximum is given by

$$\frac{\partial CS}{\partial s} - \frac{\partial S_{low}}{\partial s} = \frac{1}{\theta} \frac{\sigma\theta-\sigma+1}{\sigma-1} \frac{y}{s} - y = 0.$$

The welfare maximizing subsidy is given by the markup minus the inverse of the Pareto shape parameter:

$$s^* = \left( \frac{\sigma}{\sigma-1} - \frac{1}{\theta} \right).$$

For large values of  $\theta$  (less firm heterogeneity), the subsidy increases, and, for  $\theta \rightarrow \infty$ , the subsidy compensated exactly for the markup distortion. For finite  $\theta$ , the subsidy is always below the markup. More firm heterogeneity (low  $\theta$ ) is associated with a lower optimal subsidy. For  $\sigma-1 = \theta$ , we get  $s^* = 1$ . Thus, for  $\infty > \theta > \sigma-1$ , the optimal subsidy lies in the interval  $(1, \frac{\sigma}{\sigma-1})$ .

**The high costs case** The eligibility cutoff is defined by  $\pi_s(\varphi_L^*) = \pi(\varphi_L^*)$  :

$$\varphi_L^* = \left( \frac{\sigma}{\sigma-1} \right) \left( \frac{\sigma}{P_s^{\sigma-1} y} \frac{f_s}{(s^\sigma-1)} \right)^{\frac{1}{\sigma-1}}.$$

The product market cutoff is defined by  $\pi(\varphi^*) = 0$  :

$$\varphi^* = \left( \frac{\sigma}{\sigma-1} \right) \left( \frac{\sigma}{P_s^{\sigma-1} y} f \right)^{\frac{1}{\sigma-1}}.$$

Hence,  $\varphi_L^* > \varphi^*$  if  $f_s > f(s^\sigma - 1)$ . Further,

$$\varphi_L^* = \varphi^* \left( \frac{f_s}{f(s^\sigma - 1)} \right)^{\frac{1}{\sigma-1}}.$$

The price index is given by

$$P_{s,high} = \tilde{\kappa} [f^{\frac{\sigma-\theta-1}{\sigma-1}} + (s^{\sigma-1} - 1)(s^\sigma - 1)^{\frac{\theta+1-\sigma}{\sigma-1}} f_s^{\frac{\sigma-\theta-1}{\sigma-1}}]^{-\frac{1}{\theta}},$$

where  $\tilde{\kappa} = \left( \frac{J\theta b^\theta}{\theta+1-\sigma} \left(\frac{\sigma}{\sigma-1}\right)^{-\theta} \left(\frac{\sigma}{y}\right)^{\frac{-\theta-1+\sigma}{\sigma-1}} \right)^{-\frac{1}{\theta}}$ . Using this expression, the cutoffs can be rewritten to

$$\varphi_L^* = \kappa \left[ \left( \frac{f_s}{f(s^\sigma - 1)} \right)^{\frac{\theta}{\sigma-1}} f + \left( \frac{s^{\sigma-1} - 1}{s^\sigma - 1} \right) f_s \right]^{\frac{1}{\theta}},$$

where  $\kappa = \left( J \frac{\theta b^\theta}{\theta - \sigma + 1} \frac{\sigma}{y} \right)^{\frac{1}{\theta}}$ . With  $\varphi_L^* = \varphi^* \left( \frac{f_s}{f(s^\sigma - 1)} \right)^{\frac{1}{\sigma-1}}$  we get

$$\varphi^* = \kappa \left[ f + \left( \frac{f_s}{f(s^\sigma - 1)} \right)^{-\frac{\theta}{\sigma-1}} \left( \frac{s^{\sigma-1} - 1}{s^\sigma - 1} \right) f_s \right]^{\frac{1}{\theta}},$$

where  $\kappa = \left( J \frac{\theta b^\theta}{\theta - \sigma + 1} \frac{\sigma}{y} \right)^{\frac{1}{\theta}}$ . Total subsidy payments are given by

$$\begin{aligned} S_{high} &= J \frac{s-1}{s} \int_{\varphi_L} r_s(\varphi) g(\varphi) d\varphi \\ &= \frac{s-1}{s} s^\sigma y \left[ (s^{\sigma-1} - 1) + \left( \frac{f_s}{f(s^\sigma - 1)} \right)^{\frac{\theta-\sigma+1}{\sigma-1}} \right]^{-1}. \end{aligned} \quad (46)$$

## C A model with lobby fixed costs and eligibility fixed costs and an imperfectly targetable subsidy rate

In model of the main paper, fixed costs to receive subsidies and fixed costs to lobby are the same. A firm cannot receive the subsidy without being in the lobby and therefore, there is no free-rider problem. In the following, I relax this assumption. I show that even if the subsidy is not perfectly targetable to lobbying firms, under certain parameter restrictions, only lobbying firms receive the subsidy.

A free-rider problem would arise if a firm could benefit from the subsidy without being in the lobby. For example, one could think of two distinct fixed costs. First, application or eligibility fixed costs  $f_E$  (e.g. paperwork or red-tape). Second, an additional fixed costs,  $f_L$ , to be politically active and to join a lobby that lobbies for an increase in the subsidy rate. If the available subsidy rate is the same for a firm that also pays the additional lobby fixed costs, no firm will find it profitable to do so. In this case, the free-rider problem prevents the lobby from being established. However, if the positive spillovers from lobbying are sufficiently low, such that the subsidy rate for lobbying firms is larger than the subsidy rate for non-lobbying firms, the free-rider problem disappears, even if there are two distinct fixed costs for lobbying and for eligibility. In the following, I will show that a model with two distinct fixed costs for eligibility and lobbying nests my approach with a single fixed costs, even if firm that are eligible but do not lobby receive a fraction of the subsidy. If the subsidy is sufficiently targetable to individual firms, lobbying still generates a sufficiently high benefit, such that there is a lobby cutoff.

In particular, suppose that lobbying for the subsidy does also lead to a benefit for non-lobbying firms, such that lobbying firm benefit from the full subsidy rate  $s$  while

non-lobbying firm that pay only the eligibility fixed costs receive  $\delta s$ , where  $\delta \in [\frac{1}{s}, 1]$ . The parameter  $\delta$  can be interpreted as *targetability* of the subsidy. If  $\delta$  is high, lobbying for the subsidy does not generate much additional benefits for a firm. In this case, the subsidy is not easily targetable to lobbying firms and there are large spillovers to non-lobbying firms. Bearing the additional lobbying fixed costs is then only profitable for a small fraction of high productive firms. In the extreme case where  $\delta = 1$ , the lobby will not be established, because lobbying does not generate additional benefits. For low values of  $\delta$ , however, targetability of the subsidy is high and being eligible without lobbying is relatively unattractive. In the other limit case, where  $\delta \rightarrow \frac{1}{s}$ , all firms that are eligible also lobby, because being eligible without lobbying does not generate much benefit.

In the following, I will derive conditions on  $\delta$ , such that there will be some firms that lobby and others that receive the subsidy without lobbying. I will show that there exist upper and lower bounds  $\underline{\delta} < \delta < \bar{\delta}$ , such that for interior values of  $\delta$  with  $\frac{1}{s} < \underline{\delta} < \delta < \bar{\delta} < 1$  there are three cutoffs: a lobby cutoff, an eligibility cutoff and a product market cutoff. However, for sufficiently low values of  $\delta$ , such that  $\frac{1}{s} \leq \delta \leq \underline{\delta}$  there are only two cutoffs, a lobby and eligibility cutoff and a product market cutoff. Therefore, if  $\frac{1}{s} \leq \delta \leq \underline{\delta}$  the model outlined here simplifies to the main model of the paper.

Profits of a firm that does not receive subsidy payments are given by:

$$\pi(\varphi) = \frac{\left(\frac{\sigma-1}{\sigma}\right)^{1-\sigma} y}{P_s^{1-\sigma}} \frac{y}{\sigma} - f.$$

Profits of a firm that receives a fraction of the subsidy but does not lobby are given by:

$$\pi_{\delta s}(\varphi) = \frac{\left(\frac{\sigma-1}{\sigma} \delta s \varphi\right)^{1-\sigma} y}{P_s^{1-\sigma}} \frac{y}{\sigma} - f - f_E.$$

Profits of a firm that receives the full subsidy rate and lobbies are given by:

$$\pi_s(\varphi) = \frac{\left(\frac{\sigma-1}{\sigma} s \varphi\right)^{1-\sigma} y}{P_s^{1-\sigma}} \frac{y}{\sigma} - f - f_E - f_L.$$

The product market cutoff is defined by  $\pi(\varphi^*) = 0$ :

$$\varphi^* = \left(\frac{\sigma}{\sigma-1}\right) \left(\frac{f\sigma}{y} P_{s,high}^{1-\sigma}\right)^{\frac{1}{\sigma-1}}.$$

The eligibility cutoff is defined by

$$\pi_{s\delta}(\varphi_E^*) = \pi(\varphi_E^*).$$

The lobby cutoff is defined by

$$\pi_s(\varphi_L^*) = \pi_{s\delta}(\varphi_L^*).$$

If  $\delta$  is sufficiently low or  $f_E$  sufficiently high, there will only be a single lobby cutoff defined by:

$$\pi_s(\varphi_{L,old}^*) = \pi(\varphi_{L,old}^*).$$

However, for sufficiently high  $\delta$  (sufficiently low  $f_E$ ), the eligibility cutoff is given by  $\pi_{s\delta}(\varphi_E^*) = \pi(\varphi_E^*)$ :

$$\varphi_E^* = \left(\frac{\sigma}{\sigma-1}\right) \left(\frac{f_E}{((\delta s)^{\sigma-1} - 1) y} \frac{\sigma}{y} P_{s,high}^{1-\sigma}\right)^{\frac{1}{\sigma-1}}.$$

Then,  $\varphi_E^* > \varphi^*$ , if the following condition holds:

$$\delta < \bar{\delta} = \frac{1}{s} \left( \left(\frac{f_E}{f}\right) + 1 \right)^{\frac{1}{\sigma-1}}.$$

Thus, if  $\delta$  is sufficiently low, there exists a production cutoff and an eligibility cutoff. Note

that by  $\frac{1}{s} < \frac{1}{s} \left( \left( \frac{f_E}{f} \right) + 1 \right)^{\frac{1}{\sigma-1}}$  such a value of  $\bar{\delta}$  always exists within the interval  $[\frac{1}{s}, 1]$ . For  $\delta = \frac{1}{s} \left( \left( \frac{f_E}{f} \right) + 1 \right)^{\frac{1}{\sigma-1}}$  we get  $\varphi_E^* = \varphi^*$  and for  $\delta > \frac{1}{s} \left( \left( \frac{f_E}{f} \right) + 1 \right)^{\frac{1}{\sigma-1}}$  we get  $\varphi_E^* < \varphi^*$ .

The lobby cutoff is given by  $\pi_s(\varphi_L^*) = \pi_{s\delta}(\varphi_L^*)$ :

$$\varphi_L^* = \left( \frac{\sigma}{\sigma-1} \right) \left( \frac{f_L}{s^{\sigma-1}(1-\delta^{\sigma-1})} \frac{\sigma}{y} P_{s,high}^{1-\sigma} \right)^{\frac{1}{\sigma-1}}.$$

Then,  $\varphi_L^* > \varphi_E^*$  if

$$\delta > \underline{\delta} = \left( \frac{s^{\sigma-1}f_E + f_L}{s^{\sigma-1}(f_L + f_E)} \right)^{\frac{1}{\sigma-1}} < 1.$$

Note that by  $\left( \frac{s^{\sigma-1}f_E + f_L}{s^{\sigma-1}(f_L + f_E)} \right)^{\frac{1}{\sigma-1}} < 1$  such a value of  $\underline{\delta}$  always exists within the interval  $[\frac{1}{s}, 1]$ . Equivalently, we could write:  $f_E < f_L \frac{(\delta s)^{\sigma-1} - 1}{s^{\sigma-1}(1 - \delta^{\sigma-1})}$ . Thus, if  $\delta > \underline{\delta}$ , the lobby cutoff lies above the eligibility cutoff. Therefore, for  $\delta \in \left( \left( \frac{s^{\sigma-1}f_E + f_L}{s^{\sigma-1}(f_L + f_E)} \right)^{\frac{1}{\sigma-1}}, \frac{1}{s} \left( \left( \frac{f_E}{f} \right) + 1 \right)^{\frac{1}{\sigma-1}} \right)$ , there are three distinct cutoffs: a product market cutoff, an eligibility cutoff and a lobby cutoff. Note that this interval is non-empty, if  $f < \frac{f_L + f_E}{(s^{\sigma-1} - 1)}$ .

However, for  $\delta = \underline{\delta}$  the eligibility cutoff and a lobby cutoff are the same and for  $\delta < \underline{\delta}$ , there is a single lobby and eligibility cutoff defined by  $\pi_s(\varphi_{L,old}^*) = \pi(\varphi_{L,old}^*)$ :

$$\varphi_{L,old}^* = \left( \frac{\sigma}{\sigma-1} \right) \left( \frac{f_L + f_E}{(s^{\sigma-1} - 1)} \frac{\sigma}{y} P_{s,high}^{1-\sigma} \right)^{\frac{1}{\sigma-1}}.$$

Denote  $f_s = f_L + f_E$ , to see that  $\varphi_{L,old}^*$  is identical to the lobby and eligibility cutoff in the main paper. If  $\delta$  is low, then the subsidy is very firm-specific and highly targetable, such that spillovers from lobbying are small. There always exists a  $\underline{\delta} \in [\frac{1}{s}, 1]$  below which all firms that receive the subsidy are also lobbying. In this case we get a single eligibility and lobbying cutoff. Therefore, even in a model with two distinct fixed costs for lobbying and eligibility, for any  $\delta \in [\frac{1}{s}, \underline{\delta}]$  the model reduces to one with a single eligibility and lobbying cutoff. Thus, the model with distinct eligibility and lobbying fixed costs and imperfect targetability nests the model with a single eligibility and lobbying fixed cost and perfect targetability.