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Think, but Not Too Much: A Dual-Process Model of Willpower and Self-Control^{*}

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Abstract

Dual-process theories view decisions as the result of the interaction of two qualitatively different types of processes, automatic/impulsive and controlled/deliberative. This paper considers a model of self-control where each decision can be taken by either an automatic process or a deliberative one. In line with recent evidence from psychology, effortful self-control (willpower) is modeled as a limited resource, i.e. exercising self-control for an initial decision limits the amount of self-control available for persevering later. Automatic decisions follow a reinforcement-based process, while controlled ones are utility-maximizing. A "personal evolution" approach shows that agents might fall into self-control traps: for instance, although exercising full self-control might be efficient, decision makers might be caught in a "personal optimum" where no self-control is exercised. Reciprocally, agents might also fall prey to excessive self-control, where they waste willpower in initial decisions only to give in to temptation later.

Keywords: Dual processes, self-control, reinforcement, willpower. JEL Codes: D03, D83.

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1. INTRODUCTION

Self-control problems come in different flavors. Consider any task or project requiring perseverance, as dieting, sticking to an exercise plan, or writing a referee report. The typical self-control failure which comes to mind involves giving in to temptation, that is, failing to complete a task which has already been started when confronted with an alternative which appears more tempting in the short run, like eating the chocolate pie, sitting down to watch a movie, or working on your own, more interesting research. This failure can be easily conceptualized as an implementation failure of deliberative, longrun planning and a triumph of shortsighted, impulsive behavior. There are, however, self-control problems of a completely different kind. For instance, many decision makers, who have become acquainted with their own self-control capabilities over the years, argue to themselves that giving in to temptation is too likely, and hence it is better not to start at all. In this case, the problem arises from deliberation, and not in spite of it. Further, there is also some puzzling evidence that certain self-control problems might be best tackled by avoiding the conscious exercise of self-control. An extreme example can be found in the smoke-cessation literature, where it has recently been found that unplanned smoke-quitting attempts are twice as likely to be successful as planned ones (Ferguson, Shiffman, Gitchell, Sembower, and West, 2009; West and Sohal, 2006).

The fact that there are different kinds of self-control problems is well-known in applied psychology, which differentiates the shielding of an ongoing goal striving from getting started with goal striving (see e.g. Gollwitzer and Sheeran, 2006). Also well-known are some possible solutions or "strategies". Specifically, motivation psychology has developed surprising tactics to improve self-control, whose characteristics shed light on the essence of self-control problems (see Fujita, 2011, for a recent review). The first problem, giving in to temptation, can be fought by shielding the personal goal-striving from unwanted thoughts (Achtziger, Gollwitzer, and Sheeran, 2008; Bayer, Gollwitzer, and Achtziger, 2010). The second problem, the failure to get started, can sometimes be counteracted by automatizing the desired behavior—in other words, thinking less about it. Compact pieces of popular wisdom also encompass this insight, from "Don't be a quitter!" to "Just do it!".

As an illustrative example, consider so-called "implementation intentions" (Gollwitzer, 1999). Those refer to a family of tactics arising in social psychology which can be adapted to fight different kinds of self-control problems, An implementation intention is merely an action trigger where the decision maker forms a plan in an "if/when... then..." format, specifying an anticipated cue as a condition for starting a course of action, e.g. "when I finish this proof, then I'll start with that referee report." Extensive evidence (see Achtziger and Gollwitzer, 2010, for a review) indicates that this produces an automatization of behavior which actually helps attain the desired goals, by counteracting the failure-to-get-started phenomenon.¹ In a second step, implementation intentions can also be used to protect ongoing goal striving from temptation by specifying the anticipated temptations as cues, e.g. "if I am offered a snack, I'll ask for a glass of water instead". In the first case, the action trigger is avoiding the onset of a (possibly short-sighted) deliberative process. In the second case, it is fighting fire with fire, deliberately replacing a predictable impulsive reaction with a different one; in other words, in this case the action trigger acts as a psychological commitment mechanism, implementing the action that a previous deliberative process found optimal.²

For an economist, it might be surprising to consider that a larger involvement of deliberation versus impulsive or automatized actions might be the solution in some cases and the problem in others. Following Thaler and Shefrin (1981) and Schelling (1984), multiple selves models in economics typically address self-control problems by postulating a long-run, farsighted planner and one or several shortsighted doers. For instance, in the realm of intertemporal choice (Bernheim and Rangel, 2004; Benhabib and Bisin, 2005; Fudenberg and Levine, 2006; Ali, 2011), different utility functions are assigned to different selves, each modeling one particular motivation of a given, fixed agent. There is no question, however, that within this approach optimality would always ensue if the long-run planner could have her say. The problem is the lack of commitment mechanisms: a fully rational decision maker would like to commit to the planner's chosen course of action.

The view from psychology is different. Dual-process theories postulate that the human mind is mainly influenced by two kinds of processes, called *automatic* and *controlled* (e.g., Bargh, 1989; Loewenstein and O'Donoghue, 2005; Evans, 2008; Weber and Johnson, 2009). Automatic processes capture e.g. impulsive reactions and are defined as efficient, fast, and unconscious. They rely on learned associations (brake if the traffic light is red) and often respond to simple reinforcement mechanisms. Controlled processes are the basic ingredient of deliberation and are defined as slow, partially reflected upon consciously, and consuming cognitive resources. The distinction between automatic and controlled processes is often analogous to the economists' distinction between a heuristic/intuitive and a rationality benchmark (Kahneman, 2003). In a second step, dual-process accounts of human behavior are often extrapolated to "dual-system" theories, which postulate that different information-processing systems are responsible

¹Implementation intentions are also helpful when the desired behavior requires a punctual action which the decision maker might delay or fail to execute under normal circumstances. For instance, in a recent field application, Milkman, Beshears, Choi, Laibson, and Madrian (2011) used implementation intentions to enhance influenza vaccination rates. Nickerson and Rogers (2010) used implementation intentions to increase voter turnout in the 2008 US presidential elections.

²Other psychological manipulations follow different strategies. For instance, "cognitive reconstrual" aims for a reappraisal of the decision framework to promote a more abstract evaluation which should reduce impulsive behavior. Examples range from Moore, Mischel, and Zeiss (1976) to Fujita and Han (2009).

for handling different types of processes, e.g. an impulsive system and a reflective one (Strack and Deutsch, 2004). Loewenstein and O'Donoghue (2005) label the two systems "affective" and "deliberative", while Epstein (1994) calls them "experiential" and "rational".

There are two key differences between dual-process theories and the multiple selves approach. The first is that, in spite of the occasional use of the adjective "rational" to refer to controlled processes, neither kind is assumed to correspond to an all-knowing rational planner. Indeed, if a fully rational planner decides that it is better not to start a task because of anticipated temptation, quitting must certainly be optimal. This is not the case if the decision to quit is the result of deliberation but not as farsighted as it should. The second difference is that in multiple-selves models, every self is "rational" in the sense of being a utility-maximizer given a certain utility function; in contrast, automatic processes might not be rationalizable as utility-maximizing actors, whether shortsighted or not. Rather, they might capture much simpler processes as e.g. pure reinforcement.

In this paper, we propose a simple model of self-control based on ideas from the dual-process literature which can account for different self-control failures, including both "impulsive" giving in to temptations and "deliberative" failures to get started. In doing so, we rely on intuitions and insights from multiple-selves models but depart from the planner-doer formalism in favor of a dual-process approach. The basic ingredients of the model are as follows.

The general framework starts by distinguishing two broad types of processes, controlled/deliberative and automatic/impulsive. In contrast to multiple-selves models, we propose to explicitly incorporate a differential modeling for both kinds of processes. Controlled processes will be associated to fully rational agents who optimize a given utility function under correctly updated beliefs. Even more, we will consider a single, fixed utility function for all controlled processes, i.e. controlled processes are always mutually consistent. As a model for automatic processes, we consider an explicit stimulusresponse approach. More specifically, automatic processes correspond to the behavior of boundedly rational behavioral rules after an implicit dynamic process has "settled". That is, an automatic process acts as if in the steady state of an appropriately defined dynamical process which itself models the evolution of behavioral rules of thumb. In this paper, we will focus on *reinforcement* behavior, one of the basic motors of human learning (Bush and Mosteller, 1951, 1955), which was first incorporated into economic modeling by Cross (1973, 1983) and has been often incorporated in models of learning in games (Börgers and Sarin, 1997; Erev and Roth, 1998; Camerer and Ho, 1999).³

³There is abundant psychophysiological evidence showing that reinforcement processes have an automatic nature, being associated with extremely fast and unconscious brain responses. See e.g. Holroyd

We will build the analysis in two steps. In the first step, we consider an isolated instance of a self-control problem, simplified to the essence: an opportunity to start a desirable behavior and a later situation where temptation appears. At each possible decision point, as postulated by (a simplified version of) dual-process theories, the actual decision of a decision maker might be made by either a controlled or an automatic process, according to probabilities exogenous to the decision maker. The model will be analyzed using the novel concept of *Dual-Process Equilibrium*, which takes into account the utility-maximizing character of controlled processes and the steady-state nature of automatic ones.

The exercise of willpower can be identified with the inhibition of automatic behavior (impulses). Hence, the probability of automatic vs. controlled behavior is the essence of self-control. Many factors can influence this personal parameter, from individual characteristics to training or the nature of the problem. In the second step, we will introduce a "personal evolution" approach, internalizing this probability. To keep the analysis simple, personal evolution is identified with a local-adjustment dynamics where the probability increases or decreases following the gradient of individual payoffs. Implicitly, the decision maker learns to adjust the equilibrium of the various processes determining behavior in a way which is rationally responsive to the anticipated results. However, this leaves the door open to myopic behavior in the form of local maxima. Hence, even in the personal evolution extension of the model, there is no all-knowing, long-run planner.

We find that decision makers can become trapped in suboptimal situations with frequent self-control failures. Depending on situational and personal factors to be described below, those can be of different kinds. It is possible that a decision maker becomes trapped in an "excessive deliberation" state, where he consumes his cognitive resources deliberating on whether to get started, but frequently fails to do so, even though a lower reliance on deliberation would result in a higher rate of success.⁴ Decision makers in this situation will not face temptation often, because their main problem is a (rationalized) failure to get started. On the opposite side of the behavioral spectrum, it is possible that deliberation always leads to getting started, but still self-control problems appear. One problem is that a decision maker might end up relying excessively on automatic processes, even though more frequent deliberation would be optimal. A second and more surprising problem is a mirror-image of the first. In this case, decision makers might be caught in an "excessive self-control" trap, where they frequently exercise effortful self-control, and then end up giving in to temptation too often after having started a

and Coles (2002).

 $^{^{4}}$ An additional negative effect of excessive deliberation pointed out in psychology is that it might also lead to inaction and procrastination simply because a decision is not reached in time. We abstract from this added complication here.

goal-directed plan of action. In both cases, decision makers will identify self-control failures with an inability to resist temptation or shield already-initiated goal-striving.

The paper is structured as follows. Section 2 discusses evidence on self-control problems and presents the basic model. Section 3 presents the concept of Dual Process Equilibrium and characterizes the equilibria of the model. Section 4 tackles the personal evolution approach. Section 5 briefly discusses alternative evolutionary approaches. Section 6 concludes. Proofs are relegated to the Appendix.

2. A Self-Control Model

In order to illustrate the approach, in this paper we will focus on a specific example which aims to capture the essence of self-control problems. To avoid confusion, we will distinguish between *self-control* and *willpower*. We will use the word *self-control* in a general sense refering to all self-regulation strategies that a decision-maker might use, including e.g. adopting self-commitment mechanisms, psychological interventions as the implemention intentions mentioned above, or the avoidance of future temptations (Thaler and Shefrin, 1981). The word *willpower* will be used in a specific sense, namely the effortful inhibition of impulses as part of a self-control act.⁵ In particular, instances of self-control without conscious deliberation (e.g. Bargh and Chartrand, 1999) correspond to effortless self-control, i.e. self-control not requiring willpower. The existence of this possibility is the key to the success of implementation intentions.

Psychological models of self-control (Baumeister, 2002) postulate that the same mental resource is used for different tasks requiring willpower, ranging from the inhibition of impulses to persistence on complex cognitive tasks, etc. This resource is viewed as being limited; acts of willpower consume it. In other words, decision makers are on a "cognitive budget" regarding willpower. Limiting this resource by exerting effortful selfcontrol induces a state of "ego-depletion" that impairs the performance of subsequent tasks that also require willpower. Accordingly, ego-depletion is a state in which decision makers' capacity to exert effortful self-control is reduced (Baumeister, 2002; Baumeister, Vohs, and Tice, 2007; Muraven, Tice, and Baumeister, 1998).⁶ Dozens of empirical studies have found that depleted subjects have more difficulties exercising willpower than non-depleted controls, and are hence more likely to give up in tasks requiring persis-

⁵Self-control is often defined in the second, narrower sense (e.g. Baumeister, 2002). Recently, Fujita (2011) has argued in favor of a broader conceptualization, defining self-control as the process of advancing distal, abstract motives over more proximal, concrete ones when the two motives conflict. Under this view, for instance, the deliberate use of a self-commitment mechanism is a self-control strategy, whether the mechanism is external (as e.g. committing to savings plans) or internal (as in e.g. Ali, 2011).

⁶This is often called "the strength model of self-control". This model views self-control analogously to a muscle: behavior that requires high willpower consumes strength and energy. Willpower becomes "fatigued" after being exerted in one task, leading to a reduced self-control capacity in a subsequent task.

tence (Baumeister, Bratslavsky, Muraven, and Tice, 1998), overeat (Vohs and Heatherton, 2000), shop impulsively (Vohs and Faber, 2007), and even cheat when reporting their own performance (Mead, Baumeister, Gino, Schweitzer, and Ariely, 2009). Hagger, Wood, Stiff, and Chatzisarantis (2010) present an extensive review of experimental studies in this area.⁷

We integrate these insights in our model as follows. A decision maker (DM) faces a task which requires perseverance to complete. Initially (t = 1), the DM can simply *Shirk*, not getting started at all, resulting in a neutral outcome, also called *Shirk* for simplicity. If the DM actually manages to *Start* with the task, further down the road (t = 2) he will be faced with temptation. The DM will then have a choice between exercising self-control (*Persevere*), resulting in *Success*, or alternatively to *Give Up*, resulting in failure. A graphical representation of the decision tree is given in the left-hand part of Figure 1.



FIGURE 1: Schematic representation of the basic model.

This setting is similar to the basic problem described in e.g. the multiple-selves model of Bénabou and Tirole (2004), who use it to study internal commitments based on self-

⁷Ozdenoren, Salant, and Silverman (forthcoming) consider a formal model where an agent with limited willpower has to allocate this resource optimally in order to regulate consumption over time.

reputation over the own, uncertain willpower. Here, we postulate a model explicitly based on dual-process ideas instead. At each of the two decision nodes, two different processes compete for determining the DM's actual decision, an automatic process and a controlled one. Controlled processes in both decision nodes correspond to rational utility maximization given the appropriate beliefs (which in equilibrium will be further required to be correct, given the behavior prescribed by other processes). Given preferences $Success \succ Shirk \succ Failure$, we can normalize utilities to u(Success) = 1, u(Shirk) = 0, and u(Failure) = -d with d > 0.

Note that controlled processes maximize the same, shared utility function and hence are consistent with each other. This is in stark contrast to models of multiple-selves or time-inconsistent preferences, where decisions taken at different decision nodes by otherwise rational selves follow different utility functions.

The automatic processes which play a role at each decision node capture impulsive behavioral factors. At the second decision node (t = 2), the process corresponds to a "breakdown of will" and simply leads to giving up. Of course, this could be rationalized by an alternative utility function or generalized to a probability distribution. This simple formulation is meant to capture the possible failure of self-control at this point without introducing unnecessary complications.

In general, automatic processes determine behavior on the basis of associations created in previous instances of the problem. We make this explicit for the key automatic process operating at the first decision node (t = 1). This process corresponds to an effortless *reinforcement* process. Hence, while we can adopt a static approach for the controlled processes, modeling this automatic process requires an implicitly dynamic approach. Formally, this process specifies a probability to start, z_n , which is increased each time that the two-stage decision situation is faced and a decision to start (independently of which process determined it) results in success. Analogously, z_n is decreased when such a decision results in a failure. The subindex n refers to occurrences of the two-stage problem, i.e. to an orthogonal temporal dimension. In equilibrium, and following a reduced-form dynamic formulation, the process will simply be required to have arrived at a (stable) steady state, $E(z_{n+1}|z_n) = z_n$, given the behavior prescribed by other processes.

The last element of the model specifies which process actually determines the decision at each decision node. We adopt a probabilistic formulation. Let λ_t denote the probability that the decision at node t = 1, 2 is determined by the automatic process. Hence, a large λ_1 indicates low self-control at t = 1. A schematic representation of the postulated processes and their probabilities is given in the right-hand part of Figure 1.

In this framework, willpower is the act of inhibiting an automatic decision and adopting a controlled one instead. Exercising willpower is costly and reduces the selfcontrol budget available for subsequent decisions. This is modeled as follows. While λ_1 is fixed at t = 1, there are two possible values of λ_2 , which depend on whether willpower has been already exercised or not. More specifically, if the automatic process took the decision at t = 1, then $\lambda_2 = \lambda_2(A)$; if it was the controlled process which took the decision at t = 1, then $\lambda_2 = \lambda_2(C) > \lambda_2(A)$. That is, the exercise of willpower at t = 1, resulting in a controlled decision, reduces the probability that the DM will be able to exercise self-control again at t = 2. Note that an automatic decision might result in an "automatic start" (and hence be viewed as effortless self-control) and still consume no self-control resources. Symmetrically, a controlled decision might lead to a rational failure to start and still consume those resources. What matters is the adoption of a controlled decision, and not the particular content of that decision; to put it simply, effort is spent independently of success.

3. DUAL-PROCESS EQUILIBRIA

The key concept for the analysis of dual-process models as sketched here is that of a *Dual-Process Equilibrium*, which is defined to be a profile formed by optimal strategies for the controlled processes and steady states for the automatic processes. In general, optimality of the strategies of the former might depend on the actions of the latter, and the steady-state computation for the latter might depend on the strategies of the former. The key difference between this equilibrium concept and, say, a Perfect Bayesian Equilibrium, is that automatic processes are not rationalized as optimizing agents, but rather they are required to act as in a stable state of an implicit dynamical process Equilibrium controlled processes behave in a more "classical" way, essentially corresponding to standard, optimizing players who take the behavior of other processes (controlled or automatic) fully into account.

Let us turn to the self-control problem described above. Behavior at t = 2 can be characterized in a straightforward way. Since u(success) > u(failure), the controlled process always chooses to Persevere, while the automatic process, by construction, always chooses to Give Up. The equilibrium behavior of the controlled process at t = 1corresponds to *sequential rationality*, i.e. utility maximization given correct beliefs.⁸ The computation is as follows.

Start
$$\succeq$$
 Shirk $\iff 1 - \lambda_2(C) - d\lambda_2(C) \ge 0 \iff \lambda_2(C) \le \frac{1}{1+d}$ (1)

⁸Note that an extensive form capturing the model would include a move by nature after the controlled process decides to start, with fixed probabilities $\lambda_2(C)$, $1-\lambda_2(C)$ leading to (decisions themselves leading to) failure and success, respectively. Hence the implicit condition on beliefs is that the probabilities of nature's decisions are "known" by the rational processes.

Last, we discuss the behavior prescribed by the reinforcement process at t = 1. Suppose that the two-stage problem is faced repeatedly, instances being given by n = 1, 2, ...The reinforcement process determines whether to start or not following a simple probability distribution $(z_n, 1 - z_n)$. The probability of starting is updated every time this action is adopted: it is increased if it previously led to a success and decreased if it previously led to a failure. Following a classic formulation (Cross, 1973),

$$z_{n+1} = \begin{cases} \theta + (1-\theta)z_n \text{ if success}\\ (1-\theta)z_n & \text{if failure} \end{cases}$$

where $0 < \theta < 1$ is an adjustment factor (which will be irrelevant for steady-state computations). If the DM shirks, the payoffs of *start* are not experienced and z_n is not updated. If the DM starts, whether the process which led to starting the task was an automatic or a controlled one is irrelevant for the purposes of updating z_n : the probability is based on actual experiences and not on internal processes.

Given the behavior prescribed by the other three processes, it is possible to compute the expected value of z_{n+1} after the *n*-th instance of the problem. In equilibrium we will require the reinforcement process to have reached a probabilistic steady state, i.e. $z = z^*$ with

$$E(z_{n+1}|z_n = z^*) = z^*.$$

Further, we will also require that the dynamics points towards z^* in a probabilistic sense, i.e. $E(z_{n+1}|z_n < z^*) > z_n$ and $E(z_{n+1}|z_n > z^*) < z_n$ in an interval around z^* . If these conditions are met, we refer to z^* as a stable steady state for the reinforcement process.

In view of these considerations, and in particular since behavior at t = 2 is already determined, we can provide a simpler definition for the model at hand.

Definition 1. A **Dual-Process Equilibrium** (DPE) of the self-control model is a pair (x, z) such that

- (i) the strategy prescribing to start with probability x is optimal for the controlled process at t = 1 given $\lambda_2(C)$, and
- (ii) z is a stable steady state for the reinforcement process at t = 1, given the behavior of the controlled process at t = 1.

In general, one might want to allow the increase in λ_2 due to an initial exercise of willpower to depend on λ_1 , the probability of impulsive decision-making at the beginning. For instance, it would be sensible to assume that increased self-control possibilities at t = 1 (reduced λ_1) result in a larger loss in the self-control budget at t = 2 (increased $\lambda_2(C) - \lambda_2(A)$). For this reason, and also in order to minimize the use of subindices in the subsequent analysis, we now adapt our notation as follows.

$$\lambda_1 = \Delta, \quad \lambda_2(A) = \lambda, \quad \lambda_2(C) = \lambda + g(\Delta)$$

with $0 < g(\Delta) < 1 - \lambda$. Note that, however, at this point we do not exclude the possibility of a constant $g(\Delta)$.

With this reformulation, the parameter λ can be interpreted as the intensity of the temptation at t = 2, while Δ (or, rather, $1 - \Delta$) can be interpreted as the intensity with which willpower is exercised. From now on, we assume that $1 - \lambda(1 + d) > 0$, or

$$\lambda < \frac{1}{1+d}.\tag{2}$$

This condition serves to exclude trivial cases. If it were violated, then the controlled process would never consider starting, even in the absence of depletion effects.

3.1 The Reinforcement Process

Consider a Dual-Process Equilibrium where the controlled process at t = 1 starts with probability $x \in [0, 1]$. This probability will affect the probability of an automatic start, z, since a controlled start leads to a decision at t = 2 with reduced probability of perseverance.

Consider the *n*-th instance of the two-stage problem. If the automatic process takes the decision at t = 1, then the expected value of z_{n+1} is given by

$$z_n \left[(1 - \lambda)(\theta + (1 - \theta)z_n) + \lambda(1 - \theta)z_n \right] + (1 - z_n)z_n = z_n \left[1 + \theta \left(1 - \lambda - z_n \right) \right]$$
(3)

If the controlled process decides instead, the expected value of z_{n+1} becomes

$$x \left[(1 - \lambda - g(\Delta))(\theta + (1 - \theta)z_n) + (\lambda + g(\Delta))(1 - \theta)z_n \right] + (1 - x)z_n =$$
$$= z_n + x\theta \left[1 - \lambda - g(\Delta) - z_n \right] \quad (4)$$

Recall that if either the automatic or the controlled process determine the decision but lead to shirking, z_{n+1} is not updated. We obtain

$$E(z_{n+1}) = \Delta z_n \left[1 + \theta \left(1 - \lambda - z_n \right) \right] + (1 - \Delta) \left[z_n + x \theta \left(1 - \lambda - g(\Delta) - z_n \right) \right] = z_n + \theta \left[(1 - \Delta) x \left(1 - \lambda - g(\Delta) \right) - z_n \left((1 - \Delta) x - \Delta \left(1 - \lambda - z_n \right) \right) \right]$$
(5)

The condition for a (stable) steady state of the reinforcement process becomes

$$E(z_{n+1}) \stackrel{\geq}{\equiv} z_n \iff \Delta z_n^2 + z_n \left((1-\Delta)x - \Delta(1-\lambda) \right) - (1-\Delta)x(1-\lambda - g(\Delta)) \stackrel{\leq}{\equiv} 0$$

The left-hand-side of the inequality is a polynomial of second degree with positive discriminant, hence its value is positive below the first root and above the second one, and negative between the roots. Thus z_n increases in expected value between the roots and decreases otherwise. However, the value of the polynomial at $z_n = 0$ is negative and can be zero only if x = 0. Hence the first root is either strictly negative and can be ignored (if x > 0) or it yields a steady state z = 0 which is then unstable (if x = 0). The value of the polynomial at z = 1 is strictly positive for any value of x. Hence, it follows that there exists a unique steady state, which is (globally) stable and is (after some simplifications) given by

$$z^* = \frac{1}{2\Delta} \left[-\left((1-\Delta)x - \Delta(1-\lambda) \right) + \sqrt{((1-\Delta)x + \Delta(1-\lambda))^2 - 4x\Delta(1-\Delta)g(\Delta)} \right]$$
(6)

3.2 Equilibria

We start by analyzing pure strategy equilibria⁹ where the controlled process at t = 1 always shirks, i.e. x = 0. By condition (1), this occurs if and only if

$$\lambda + g(\Delta) \ge \frac{1}{1+d}.$$

If the controlled process always shirks, equation (5) reduces to

$$E(z_{n+1}) = z_n + \Delta \theta z_n \left(1 - \lambda - z_n\right) \tag{7}$$

and there are two steady states, z = 0 and $z = 1 - \lambda$. The first one is unstable and the second one is stable. Hence, we obtain a Shirking DPE where the controlled process at t = 1 always shirks and the reinforcement process starts automatically with probability $z = 1 - \lambda$.

The interpretation of this equilibrium is straightforward. The controlled process is discouraged by a too large probability of breakdown of will at t = 2, i.e. whenever the DM "thinks about it", he gives up since he does not trust himself to continue. Given this, the action to start occurs only automatically ("just do it"), and the model becomes a classic reinforcement example. Hence, in equilibrium the probability to start is numerically equal to the probability that the task is successfully completed, corresponding to a simple *probability matching* result as known from the reinforcement learning literature (Bush and Mosteller, 1951; Siegel and Goldstein, 1959).¹⁰

⁹Whether a Dual-Process Equilibrium is in mixed or pure strategies refers to the strategies of the controlled processes only.

¹⁰Suppose a decision maker repeatedly chooses among two options, A and B, only one of them leading to a reward. Suppose that the probability that A is the winning option is constant among periods and

We now turn to pure strategy equilibria where the controlled process at t = 1 always starts, i.e. x = 1. By condition (1), this occurs if and only if

$$\lambda + g(\Delta) \le \frac{1}{1+d}.$$

If the controlled process always starts, equation (5) reduces to

$$E(z_{n+1}) = z_n + \theta \left[(1 - \Delta)(1 - \lambda - g(\Delta)) - z_n \left(1 - 2\Delta + \Delta \lambda + \Delta z_n\right) \right]$$
(8)

There is a unique steady state, which is globally stable and given by (taking x = 1 in (6))

$$z^* = 1 + \frac{1}{2\Delta} \left[-(1 + \Delta\lambda) + \sqrt{(1 - \Delta\lambda)^2 - 4\Delta(1 - \Delta)g(\Delta)} \right]$$
(9)

Finally, consider mixed-strategy equilibria where the controlled process at t = 1 randomizes, starting with probability 0 < x < 1. By condition (1), this can occur if and only if

$$\lambda + g(\Delta) = \frac{1}{1+d}$$

Since this equation involves only exogenous parameters at this point, we conclude that mixed-strategy equilibria are possible but correspond to knife-edge situations. If the equality holds, we obtain a continuum of mixed-strategy equilibria with $x \in [0, 1]$ and z = z(x) given by (6).

In summary,

Theorem 1. A Dual-Process Equilibrium of the self-control model always exists.

- (a) If $\lambda + g(\Delta) > \frac{1}{1+d}$, there exists a unique DPE, where the controlled process at t = 1 always shirks (x = 0) and the reinforcement process starts automatically with probability $z = 1 \lambda$ (Shirking DPE).
- (b) If $\lambda + g(\Delta) < \frac{1}{1+d}$, there exists a unique DPE, where the controlled process at t = 1 always starts (x = 1) and the reinforcement process starts automatically with probability z^* given by (9) (Willpower DPE).
- (c) If $\lambda + g(\Delta) = \frac{1}{1+d}$, there exists a continuum of DPE, indexed by $x \in [0, 1]$. At each one of these DPE, the controlled process at t = 1 starts with probability x and the reinforcement process starts automatically with probability z^* given by (6).

equal to p > 1/2. In the long run, reinforcement learning will lead to the decision maker choosing A a percentage p of the time, while a rational decision maker should simply choose A all the time.

3.3 Rational Failures to Get Started

The existence of Shirking equilibria in Theorem 1 captures the phenomenon of "deliberative failures to get started". Case (a) indicates that this phenomenon might appear when depletion effects are strong or the disutility of failure is large. For instance, imagine a student who suddenly realizes that there are only a few weeks left until the exam for a difficult, boring subject (and let us abstract from the reasons for the student's procrastination). In order to pass the exam, the student needs to tackle an intensive study plan with several multi-hour study sessions per day. Every given day, it is likely that the first session will cause a strong depletion effect leading to a failure of perseverance as the time slot for the second session approaches. The student might well anticipate this and never get started at all, giving up on the subject. A different example are tasks where there is a high social or subjective cost of failure, e.g. if giving up carries a (subjective or objective) stigma. For instance, starting with but failing to complete an ambitious task might be damaging for the DM's self-image, and hence the DM might rationally decide not to risk failure and never attempt the task.

A shirking equilibrium, however, is an equilibrium, and under the condition given in Theorem 1(a), it is the only equilibrium. It would be futile to attempt to bring the DM to exercise willpower and deliberately start the self-control task. The effects of depletion, which the DM anticipates, will offset the benefits of deliberately starting more often. On the one hand, depletion will lead to increased failure and hence increase disutility. On the other hand, increased failure will feed back to the automatic process, which will correspondingly reduce the share of automatic starts.

This outcome is, in a sense, inefficient. In a shirking equilibrium, the automatic process is caught in an "irrational" probability-matching outcome. By (2), it would be optimal to start at t = 1 if depletion effects were absent. Then, it would be more efficient to bring the automatic process to start more often, without incurring in depletion effects. Indeed, the expected utility in Shirking equilibria (where x = 0 and $z = 1 - \lambda$) is

$$\Pi_0 = \Delta(1 - \lambda)(1 - \lambda(1 + d)) > 0$$
(10)

and, if the automatic process would operate at $z > 1 - \lambda$, the expected payoff would be $\Delta z(1 - \lambda(1 + d)) > \Pi_0$. Moreover, increasing Δ (thinking less) would also result in an increased utility, even if z remained unchanged.

Increasing Δz without engaging active deliberation is precisely the objective of psychological manipulations to counteract the failure to get started, as e.g. implementation intentions. By setting a just-do-it automatized action trigger, an implementation intention simultaneously increases the likelihood of automatic behavior (reducing willpower) and the probability that the automatic process starts with the task. This explains why, in situations where we can rationally talk ourselves out of an action plan, a seemingly irrational commitment to give up willpower acts might be welfare-improving.

4. The Optimal Degree of Willpower

Many decision makers struggle for self-control in certain aspects of their lives. Here we propose a "personal evolution" approach as a shortcut for *intrapersonal* parameter adjustment, as (over the course of one's life) the reliance on certain processes becomes established.

More specifically, suppose the decision maker faces self-control problems as described above along his or her personal life. The quest for optimal self-control reduces to adjustments in the parameter Δ , the probability of impulsive behavior in the presence of full self-control resources. Imagine that the decision maker attempts to slowly build self-control (reducing Δ) but repeatedly experiences failure, in the form of giving in to temptation, after exercising willpower. Given the disutility of failure, this might result in an overall reduction in utility, discouraging the decision maker from further reducing Δ . Reciprocally, if the reduction in Δ leads to an overall increase in utility due to an increase in the success rate, the decision maker will further reduce Δ .

In this section, we internalize the parameter Δ following a further implicitly dynamic approach. Our interpretation is that, for classes of decision problems which are encountered relatively often, this parameter is "adjusted" over one's life in a local way, i.e. it will increase or decrease slightly whenever doing so results in a utility gain. In other words, we adopt a gradient dynamics leading to *local* optimality as a criteria for personal optimality. In the framework of the self-control model analyzed in the previous sections, we then ask the question of whether relying on automatic processes beyond a given, unavoidable background level (presumably founded on biological evolution, childhood experiences, etc) can become a local optimum for a decision maker.

Consider the following framework. We consider the particular case with $g(\Delta) = g$, i.e. there is a fixed, physiological depletion effect. The utilities of shirking, success, and failure are fixed. The parameter λ is also fixed and, as commented above, taken to represent a measure of the "temptation" associated to a given self-control problem. However, the parameter Δ might vary as follows:

$$\Delta = \Delta_0 + \Delta_p$$

where Δ_0 is a fixed "background automaticity rate" (you cannot avoid a certain probability of reacting automatically), but Δ_p is a "personal" parameter which becomes trained over time. An appropriate criterion to evaluate "personal optimality" is simply expected utility.

$$\Pi(x,z) = \Delta z \left[-\lambda d + (1-\lambda) \right] + (1-\Delta)x \left[-(\lambda+g)d + (1-\lambda-g) \right] = \Delta z \left[1 - \lambda(1+d) \right] + (1-\Delta)x \left[1 - (\lambda+g)(1+d) \right]$$
(11)

We focus on the case where Willpower equilibria exist, since else there is little to study. The formalism is particularly convenient here, since for this particular problem the parameter Δ does not affect whether the DPE is of one or the other type (this needs not be true for more general models).

The utility in Willpower equilibria (where x = 1 and z is given by (9)) is

$$\Pi_{1} = (1 - \lambda(1 + d)) \left[1 + \frac{1}{2} \left[-(1 + \Delta\lambda) + \sqrt{(1 - \Delta\lambda)^{2} - 4\Delta(1 - \Delta)g} \right] -(1 - \Delta) \left(\frac{g(1 + d)}{(1 - \lambda(1 + d))} \right) \right]$$
(12)

Say that Δ is a *personal optimum* if it is a local maximum of Π_1 under the constraint that $\Delta \geq \Delta_0$. Say that it is *efficient* if it is a global optimum of Π_1 under the same constraint. The following lemmata (whose proofs are relegated to the appendix) explore the properties of Π_1 .

Lemma 1. Π_1 is a strictly convex function of Δ .

Lemma 2. There exist g, \overline{g} with $0 < g < \overline{g} < 1$ such that

- (a) if $g \geq \overline{g}$, Π_1 is strictly increasing in Δ ;
- (b) if $g \leq \underline{g}$, Π_1 is strictly decreasing in Δ ; and
- (c) if $\underline{g} < g < \overline{g}$, Π_1 has a unique interior minimum $\Delta(g)$.

The convex shape of Π_1 has an intuitive interpretation. Consider case (c) in the last lemma. The utility function Π_1 has two local maxima, with minimal and maximal willpower, respectively. The reason for this U-shaped utility is the tradeoff between the direct and indirect effects of willpower exercise. Increasing willpower (reducing Δ_p) leads to a larger share of deliberate starts. However, each deliberate start causes depletion and reduces the chances of later perseverance. Further, failed perseverance "teaches" the automatic process (through reinforcement) to reduce the probability of an automatic start.

Suppose that the DM is exercising very low willpower (Δ close to 1). Then, most of the successful runs (starting, then persevering) come from automatic starts, which occur with a given probability z > 0. If the percentage of the decisions taken by the controlled process is slightly increased, the associated successes will deliver positive utility, but the increased failures due to the effects of depletion will feed back in an overall reduced share of automatic starts. Since most of the successful runs actually come from automatic starts, this will have a large negative effect on overall utility.

Suppose, on the contrary, that the share of automatic decisions is small (Δ close to 0). Then, the positive effect of increasing the number of deliberative starts will easily offset the negative effect of reduced automatic starts due to failed attempts. However, if depletion effects are too strong, as in case (a), the direct disutility of failure is too large and this offsetting effect will never materialize. On the other hand, if the effects of depletion are small enough, as in case (b), increasing the share of deliberate starts will always pay off.

Since there are at most two personal optima, efficiency becomes a matter of pairwise comparison. We conclude that, depending on personal and situational factors (the strength of the temptation, the personal disutility of failure, the background automaticity, and the depletion associated to the task), either maximum exercise of willpower or full automatization of behavior will be efficient. However, the DM might well be trapped in the inefficient personal optimum. The following theorem spells out the circumstances under each personal optimum is efficient.

Theorem 2. Let $\lambda + g < \frac{1}{1+d}$. If the probability of automatic decisions at $t = 1, \Delta$, is constrained by $\Delta \geq \Delta_0 > 0$ then there exist $\underline{g}, \overline{g}, g^*$ with $0 < \underline{g} < g^* < \overline{g} < 1$, and continuous functions $\Delta(g), \Delta'(g)$ with $\Delta'(\underline{g}) = \Delta(\underline{g}) = 1, 0 \leq \Delta'(g) < \Delta(g)$ on $g \in]\underline{g}, \overline{g}[$, $\Delta'(g^*) = 0$, and $\Delta(\overline{g}) = 0$, such that exactly one of the following cases occur:

- (I) Efficient Automaticity. Full automaticity ($\Delta = 1$) is the only personal optimum and also efficient, if and only if one of the following possibilities occur:
 - (Ia) High Depletion: $g \geq \overline{g}$;
 - (*Ib*) High Background Automaticity: $\Delta_0 \ge \Delta(g)$ (and $g < g < \overline{g}$).
- (II) Efficient Self-Control. Maximum Self-Control ($\Delta = \Delta_0$) is the only personal optimum and also efficient, if and only if $g \leq g$ (Low Depletion);
- (III) Self-Control Failure. Full automaticity is a personal inefficient optimum (with maximum control being the efficient optimum) if and only if $\underline{g} < g < g^*$ (moderately low depletion) and $\Delta_0 < \Delta'(g)$ (low background automaticity).
- (IV) Excessive Self-Control. Maximum Self-Control is a personal inefficient optimum (with full automaticity being the efficient optimum) if and only if $\Delta'(g) < \Delta_0 < \Delta(g)$ (moderate background automaticity) (and $\underline{g} < g < \overline{g}$). In case $g \ge g^*$, then $\Delta'(g) = 0$ and the condition $\Delta'(g) < \Delta_0$ is void.

(V) In the knife-edge case with $\Delta_0 = \Delta'(g)$ and $\underline{g} < g < g^*$, both maximum self-control and full automaticity are personal efficient optima.

The different cases in the last theorem are qualitatively illustrated in Figure 2. We can summarize the classification in words as follows. First, a task causing next to no depletion (Case II) does not pose a problem (since you know you can trust yourself in the future) and will efficiently be undertaken under maximum control. The latter kind of tasks will typically not be perceived as self-control problems at all.

Second, if a task causes a high depletion (but not so high that the equilibrium becomes of the shirking type), then it is optimal to automatize the associated decisions as much as possible. Note that this is different from the reasoning behind a Shirking DPE. In the situation described in case Ia, the controlled process at t = 1 will still find it optimal to start, given that this process is required to make the decisions. However, the DM (viewed as a planner) would have preferred that this process does not make the decision at all. The reason is that, for the controlled process, the costs of depletion are sunk. Ex ante, however, it would have been preferable to avoid them.

For intermediate depletion values, the outcome depends on the background automaticity rate. If that rate is high, then in practice a full-control personal optimum becomes unattainable (independently of whether it *would* have been efficient or not), and hence again decision makers efficiently rely on full automaticity (case Ib). For instance, if you want to get rid of an habitual behavior, but that behavior happens to be aligned with your inborn (or childhood-learned) reflexes, you will find that consciously fighting it is next to impossible.

Trouble arises systematically for intermediate depletion values and moderate or low background automaticity rates, for in this case there are always two personal optima but only one of them (except in knife-edge cases) is efficient. If background automaticity is relatively low (case III), we reach a situation with which many decision makers frequently identify. In this case, although exercising maximum self-control would be efficient, there exist personal optima with full automaticity. In this case, the decision-maker is caught in a trap. Although it might be clear that exercising full self-control would be beneficial, the road to heaven is paved with very slippery stones, for, since the inefficient personal optimum is a local maximum, any small increase in self-control will lead to a *decrease* in utility. Hence, slowly increasing your exercised self-control will not lead to immediate rewards; quite the opposite.

This situation, with moderate depletion and low background automaticity, corresponds to the quintessential self-control problem where people fail to exercise willpower when they should. The insight of the model presented here is that decision-makers might still be at a personal optimum, where small increases in the exercise of willpower are counterproductive. Hence, we are led to the conclusion that the most promising course of action is a radical shift, which typically will fail in the absence of external help, e.g. in the form of a commitment mechanism or psychological interventions as those described in the introduction.

The second problematic situation is an "excessive self-control trap" (case IV). If the background automaticity is relatively high but not extreme, full automaticity is efficient, but decision makers may be caught in personal optima with maximum exercise of willpower. Ultimately, this is inefficient because those attempts remain relatively unsuccessful compared with what could be achieved under full automaticity. Decision makers exercise too much willpower, constantly trying to complete a task or bring a certain habitual behavior in line, more or less successfully. Slightly lowering his or her efforts is counterproductive. The irony is that spontaneous, effortless, unplanned behavior would actually result in an improvement in this case. Further, it stands to reason that this phenomenon will typically not be fully recognized, since, after all, the decision maker seems to be doing his or her best.

It is conceivable that clinically overweight people, or people exhibiting mild forms of compulsive behavior, might fall in this case. In both examples, the behavior in question is likely to be relatively automatized (a moderately high, probably learned background automaticity rate) and the depletion associated to every act of willpower is moderate. Although addiction problems are most certainly not merely willpower problems (Bechara, Noel, and Crone, 2006), this might also be the case some non-compulsive smokers fall in, hence yielding an explanation why *unplanned* smoke-quitting attempts are twice as likely to be successful as planned ones (Ferguson, Shiffman, Gitchell, Sembower, and West, 2009; West and Sohal, 2006). Indeed, it appears that each year more smokers quit smoking unassisted ("cold turkey", i.e., simply stop overnight; Chapman and MacKenzie, 2010) than by all assisted methods combined. In terms of the model, this might correspond to smokers previously caught in the excessive self-control trap (since large numbers of smokers report wanting to give up smoking) and suddenly finding out that there is a minimum-resistance path to accomplish their objectives.

This last observation, which is admittedly speculative, points to a possible asymmetry in the concept of personal evolution introduced in this paper. We have identified personal evolution with the gradual adjustment of a self-control parameter, following the direction of increased payoffs. It might, however, be reasonable to postulate that "jumps" in the direction of reduced self-control are easier than those in the opposite direction. This might be an additional explanation why some of the most successful psychological interventions in the realm of self-control operate by automatizing behavior, rather than calling for the exercise of additional willpower.



FIGURE 2: Illustration of the cases in Theorem 2.

5. Evolutionary Optimality and Automatic Rationality

Suppose we considered a more biological concept of evolutionary stability. In an implicit evolutionary approach, as in the literature on the evolution of preferences (e.g. Samuelson, 2004; Netzer, 2009), "nature" acts as a shortcut for the evolutionary process, choosing certain parameters in order to maximize appropriate evolutionary criteria.¹¹

Let us start with a simple consideration. A first, *naïve* criterion would be to maximize the probability of an automatic start, z. This objective is often implicitly or explicitly cited in psychology as a beneficial effect of various forms of interventions. The intuition for why a large proportion of automatic starts is desirable is simple. Since an automatic start does not consume self-control resources, it would be optimal to start automatically as often as possible, hence economizing self-control resources to avert possible breakdowns of will later on.

 $^{^{11}{\}rm For}$ instance, Netzer (2009) has offered an evolutionary rationale for the existence of different utility functions for short-run and long-run decisions.

Since automatic starts are dynamically "trained" by a reinforcement process, intuitively every controlled start, which hence leads to a decision with limited self-control at t = 2, will reduce the probability of an automatic start. Thus, the probability of an automatic start will be maximal at the unappealing shirking equilibria. The following result formalizes this intuition.

Proposition 1. Fix the temptation probability λ . Suppose nature's objective is to maximize the probability of an automatic start, z. If either

- (a) Δ and $g(\Delta)$ are also fixed and nature selects the disutility of failure only, d;
- (b) or if nature is able to select any values of, d > 0, $\Delta > 0$, and $g(\Delta) \in]0, 1 \lambda[$,

then the optimal values are such that a Shirking equilibrium with x = 0 and $z = 1 - \lambda$ results.

Fixing the temptation parameter λ is the most reasonable approach in this situation. This parameter measures the extent of the basic decision problem we are modeling, which results e.g. due to nature's tinkering with an immediate consumption bias due to other, more basic survival needs (e.g. the need to store fat for a long winter in a hunter-gatherer society). The remaining parameters have more involved interpretations: Δ is related to the availability of self-control when faced with long-term decisions, $g(\Delta)$ is the reduction in self-control due to a previous exercise thereof, and d is the perceived disutility of failure. Obviously, nature would not be able to set $g(\Delta)$ arbitrarily close to zero due to basic neurophysiological constraints. However, the last result holds independently of such additional constraints as long as $g(\Delta) > 0$.¹²

Proposition 1 shows that the mere automatization of behavior cannot have been an evolutionary criterion for the human mind. If we consider Willpower equilibria, Lemma 1 shows that such an evolutionary approach for the automaticity rate Δ is not appropriate, since convexity of Π_1 implies that duality, i.e. $\Delta \in [0, 1]$ cannot be optimal in this sense. This is, however, not surprising. The reasons for evolutionary optimality of dual processes have to be looked for elsewhere. The main evolutionary advantage of a dual-process decision maker is in *decision speed*, an aspect totally ignored here. Automatic reactions are far quicker than controlled ones. They allowed our ancestors to run first (hence avoiding being eaten) and think later. If you are in the middle of the street and suddenly discover a truck is about to hit you, it is not a good idea to stop and think carefully whether jumping right or left is better. Just jump. Following the evolution parable, nature has probably found it optimal to endow us with a background

¹²Indeed, an all-powerful nature capable of "designing humans" who experience no depletion effects would find that setting $g(\Delta) = 0$ yields $z^*(1) = z^*(0)$, i.e. Willpower equilibria would deliver the same probability of automatic start as Shirking equilibria.

automaticity rate $\Delta_0 > 0$ to be applied over a variety of different tasks. Once we particularize to a specific task, though, that background automaticity becomes an added constraint and we enter the realm of personal evolution.

Two comments are in order. First, in spite of these considerations, the "automatization of rationality" remains an attractive goal, and some psychological interventions as implementation intentions (Gollwitzer, 1999) have been designed to pursue it. The interpretation of such interventions, however, is not that they aim to increase the probability of starting automatically, z, for its own sake. In our terms, their aim is, given a DPE (z, x), to introduce a new automatic process which ignores the outcome of reinforcement and follows the decision that the controlled process would have taken *without* spending self-control resources. Hence, the objective of such interventions is to increase z by changing the nature of the automatic process involved, moving it away from pure reinforcement, and effectively producing more "rational" decisions with a minimum of cognitive costs.

Second, Proposition 1 does not mean that the model at hand or related ones could not be used to tackle questions of evolutionary optimality. In particular, the background rate Δ_0 and the disutility of failure *d* can only be understood through such an approach.¹³ However, such an approach would be better undertaken in a setting where different tasks appear at random, leading e.g. to different values of the temptation parameter. This can be done extending our basic game to one of imperfect information. This task is left for future research.

6. CONCLUSION

This paper presents a new modelization of the basic idea that decisions are the result of the interaction of different processes within a decision maker. Drawing on dual-process theories from psychology, we propose to analyze decisions through *Dual-Process Equilibria*, where processes of different nature are modeled in different ways. Controlled processes are modeled as rational players from classical game theory. Automatic processes are modeled through techniques and ideas from the literature of learning in games. In the model presented here, and following again insights from psychology, we have identified them with reinforcement processes. The interaction among different processes is captured through an extensive-form formulation analogous to that employed for models with multiple selves.

¹³Evidence from neuroscience, incorporated e.g. in prospect theory and reflected in phenomena like loss aversion, shows that the human brain is hardwired to experience a greater disutility from negative outcomes than the corresponding utility from positive outcomes of equivalent magnitude. In other words, even if the objective (in evolutionary terms) utility of failure is equal in magnitude but opposite in sign to the utility of success, nature seems to have found optimal to set the subjective magnitude of failure's disutility above that of success. This would be an argument for d > 1 in the current setting.

The first finding is that, if depletion effects are too strong or the disutility of failure is too high, Shirking equilibria appear where decision makers rationally decide not to get started at all. However, psychological interventions might improve welfare by automatizing behavior, i.e. avoiding excessive deliberation.

Willpower equilibria involve controlled processes which start rationally but cause depletion and effortless automatic processes which only start with a certain probability. Inhibiting automatic reactions in favor of controlled ones correspond to acts of willpower. In this setting, the idea of *personal evolution* helps explain individual heterogeneity and shows that suboptimal modes of behavior, as e.g. self-control failures, can correspond to cognitive traps.

We identify two such traps. The first one correspond to the classical idea of selfcontrol failure, where the individual relies excessively on automatic processes. However, this situation is a local maximum and any attempt to gradually reduce this dependence results in a utility loss, even though a maximum exercise of willpower would be efficient.

The second trap is more surprising. If depletion effects and/or background automaticity are moderate but not too low, the exact opposite might happen. Decision makers might then be caught in a local optimum involving maximum exercise of willpower, which however leads to overall results which are worse than those which could be obtained by relying on automatic (or automatized) behavior only. Hence, decision makers who appear to be doing their best, without however satisfactorily reaching their objectives, might be well advised to radically rethink their approach.

A. Appendix: Proofs

PROOF OF LEMMA 1. For notational convenience, let $R_{\Delta} = (1 - \Delta \lambda)^2 - 4\Delta (1 - \Delta)g$. R_{Δ} is the discriminant of the polynomial we analyzed to compute z^* (recall (6)), for x = 1. As we argued in the main text, $R_{\Delta} > 0$.

To see that Π_1 is a strictly convex function of Δ , we compute

$$\frac{\partial \Pi_1}{\partial \Delta} = (1 - \lambda(1+d)) \left[-\frac{1}{2} \left[\lambda + \frac{(1 - \Delta\lambda)\lambda + 2(1 - 2\Delta)g}{\sqrt{R_\Delta}} \right] + \left(\frac{g(1+d)}{(1 - \lambda(1+d))} \right) \right]$$
(A1)
$$\frac{\partial^2 \Pi_1}{\partial \Delta^2} = -\frac{1}{2} \frac{(1 - \lambda(1+d))}{R_\Delta} \left[\left(-\lambda^2 - 4g \right) \sqrt{R_\Delta} + \frac{((1 - \Delta\lambda)\lambda + 2(1 - 2\Delta)g)^2}{\sqrt{R_\Delta}} \right]$$

Rcall that, by (2), $\lambda(1+d) < 1$. Then,

$$\begin{split} \frac{\partial^2 \Pi_1}{\partial \Delta^2} &\gtrless 0 \iff \left(-\lambda^2 - 4g \right) \sqrt{R_\Delta} + \frac{\left((1 - \Delta \lambda)\lambda + 2(1 - 2\Delta)g \right)^2}{\sqrt{R_\Delta}} & \leqq 0 \iff \\ \left((1 - \Delta \lambda)\lambda + 2(1 - 2\Delta)g \right)^2 &\lessgtr \left(\lambda^2 + 4g \right) R_\Delta \iff \\ \left(1 - 2\Delta \right)^2 g^2 + (1 - \Delta \lambda)\lambda(1 - 2\Delta)g & \leqq -\lambda^2 \Delta (1 - \Delta)g + g(1 - \Delta \lambda)^2 - 4\Delta (1 - \Delta)g^2 \iff g & \oiint g & \leqq 1 - \lambda \end{split}$$

Since $g < 1 - \lambda$, we conclude that $\frac{\partial^2 \Pi_1}{\partial \Delta^2} > 0$ for all Δ , and the claim holds.

PROOF OF LEMMA 2: First note that if $\frac{\partial \Pi_1}{\partial \Delta}\Big|_{\Delta=0} \ge 0$, then Π_1 is strictly increasing in Δ ny Lemma 1 and reaches a maximum at $\Delta = 1$. This happens if and only if (evaluating (A1) at $\Delta = 0$)

$$\frac{\partial \Pi_1}{\partial \Delta}\Big|_{\Delta=0} \ge 0 \iff \lambda + g \le \left(\frac{g(1+d)}{(1-\lambda(1+d))}\right) \iff g \ge \lambda \frac{1-\lambda(1+d)}{d+\lambda(1+d)} =: \overline{g}$$

Note that $\overline{g} \in]0,1[$.

If $\frac{\partial \Pi_1}{\partial \Delta}\Big|_{\Delta=1} \leq 0$, then Π_1 is strictly decreasing in Δ and reaches a maximum at $\Delta = 0$. This happens if and only if (evaluating (A1) at $\Delta = 1$)

$$\begin{split} \frac{\partial \Pi_1}{\partial \Delta} \Big|_{\Delta=1} &\leq 0 \iff \lambda + \frac{(1-\lambda)\lambda - 2g}{(1-\lambda)} \geq \left(\frac{2g(1+d)}{(1-\lambda(1+d))}\right) \iff \\ & (1-\lambda(1+d))(\lambda(1-\lambda) - g) \geq g(1-\lambda)(1+d) \iff \\ (1-\lambda(1+d))\lambda(1-\lambda) \geq g\left[(1-\lambda)(1+d) + (1-\lambda(1+d))\right] = g\left[1+(1-2\lambda)(1+d)\right] \\ & \iff g \leq \lambda(1-\lambda)\frac{1-\lambda(1+d)}{1+(1-2\lambda)(1+d)} =: \underline{g} \end{split}$$

Notice that $\lambda < \frac{1}{1+d}$ implies that the denominator in the last fraction is always strictly positive.

It can be shown that $\underline{g} < 1 - \lambda$. This holds if and only if

$$\begin{split} \lambda(1-\lambda(1+d)) < 1 + (1-2\lambda)(1+d) & \Longleftrightarrow \ \lambda < 1 + (1-\lambda)^2(1+d) \\ & \Longleftrightarrow \ 1 + (1-\lambda)(1+d) > 0 \end{split}$$

which proves the claim. Notice that $\underline{g} < \overline{g}$ follows by convexity of Π_1 (Lemma 1), but it also can be easily checked directly. If $\underline{g} < g < \overline{g}$, then there exists an interior minimum, but still there is a corner solution.

PROOF OF THEOREM 2. The proof relies on Lemmata 1 and 2. First, we claim that there exists g^* such that $\Pi_1|_{\Delta=0} \leq \Pi_1|_{\Delta=1} \iff g \geq g^*$.

To see this, note that since Π_1 is strictly convex by Lemma 1, the function Π_1 has either a global maximum at $\Delta = 0$, or a global maximum at $\Delta = 1$. Whether the first or the second case occur depends only on the following inequality (recall (12)):

$$\Pi_1|_{\Delta=0} \le \Pi_1|_{\Delta=1} \iff \frac{g(1+d)}{(1-\lambda(1+d))} \ge \lambda \iff g \ge \lambda \left[\frac{1}{1+d} - \lambda\right] =: g^*$$

which proves the claim. Note that if $g = g^*$ (hence there are two global maxima), by Lemma 2 we must be in the case $g < g < \overline{g}$, hence $g < g^* < \overline{g}$

We now turn to a case-by-case analysis, systematically following the possible values of the parameters.

Case 1. If $g \geq \overline{g}$, then utility is maximized at full automaticity, $\Delta = 1$; further, utility is strictly increasing in Δ by Lemma 2. Hence full automaticity is the only personal optimum, which is also efficient. This is case (Ia) in the statement.

Case 2. If $g \leq \underline{g}$, then utility would be maximized at full control, $\Delta = 0$; further, utility is strictly decreasing in Δ by Lemma 2. Hence maximum control, $\Delta = \Delta_0$, is the only personal optimum, which is also efficient. This is Case (II) in the statement.

Case 3. If $g^* \leq g < \overline{g}$, then there is an interior *minimum* at $\Delta = \Delta(g)$ by Lemma 2 and $\Delta = 1$ is the global maximum by the claim above.¹⁴ Then, we have to distinguish two subcases.

Case 3a. If $\Delta_0 \geq \Delta(g)$, then utility is strictly increasing at Δ_0 and again full automaticity is the only personal optimum,¹⁵ which is also efficient. This belongs to Case (Ib).

Case 3b. If $\Delta_0 < \Delta(g)$, then utility is strictly decreasing at Δ_0 . It follows that maximum control, $\Delta = \Delta_0$ is a personal optimum but full automaticity is efficient. This belongs to Case (IV).

Case 4. If $\underline{g} < g < g^*$, then there is an interior minimum at $\Delta = \Delta(g)$ by Lemma 2 and $\Delta = 0$ is the global maximum by the claim above. It follows that there exists $\Delta'(g) \in]0, \Delta(g)[$ such that

$$\Pi_1|_{\Delta=\Delta_0} \stackrel{\geq}{\equiv} \Pi_1|_{\Delta=1} \iff \Delta_0 \stackrel{\leq}{\equiv} \Delta'(g)$$

Then, we have to distinguish several subcases.

Case 4a. If $\Delta_0 \geq \Delta(g)$, then utility is strictly increasing at Δ_0 and again full automaticity is the only personal optimum, which is also efficient. This belongs again to Case (Ib).

¹⁴If $g = g^*$, there are two global maxima at $\Delta = 0, 1$, but the first is irrelevant since $\Delta_0 > 0$.

¹⁵In the corner case $\Delta_0 = \Delta(g)$, utility is also strictly increasing given the constraint $\Delta \ge \Delta_0$.

Case 4b. If $\Delta'(g) < \Delta_0 < \Delta(g)$, then utility is strictly decreasing at Δ_0 ; further, the utility at Δ_0 is strictly smaller than that at $\Delta = 1$. Hence, full automaticity is the efficient optimum, but maximum control is an inefficient personal optimum. This belongs again to Case (IV).

Case 4c. If $\Delta_0 = \Delta'(g)$, then utility is strictly decreasing at Δ_0 ; further, the utility at Δ_0 is equal to that at $\Delta = 1$. This is the knife-edge Case (V).

Case 4d. If $\Delta_0 < \Delta'(g)$, then utility is strictly decreasing at Δ_0 , and the utility at Δ_0 is strictly larger than that at $\Delta = 1$. Hence, maximum control is the efficient optimum, but full automaticity is an inefficient personal optimum. This is Case (III).

Continuity of $\Delta(\cdot)$ and the facts that $\Delta(\underline{g}) = 1$ and $\Delta(\overline{g}) = 0$ follow from the definition of $\Delta(\cdot)$ and Lemma 2. Continuity of $\Delta'(g)$ and the facts that $\Delta'(\underline{g}) = 1$, $0 \leq \Delta'(g) < \Delta(g)$ on $g \in]\underline{g}, \overline{g}[$, and $\Delta'(g^*) = 0$ follow from definition of $\Delta'(\cdot)$.

PROOF OF PROPOSITION 1. (a) Notice that d does not appear in any equilibrium computation. It merely determines which type of equilibrium does the model have, Hence, selecting d amounts to selecting the equilibrium type. Given an equilibrium, the relationship between the probability of a controlled start, x, and that of an automatic start, z, is given by equation (6). A simple computation shows that

$$2\Delta \frac{dz^*(x)}{dx} = (1-\Delta) \left[-1 + \frac{(1-\Delta)x + \Delta(1-\lambda) - 2\Delta g(\Delta)}{\sqrt{((1-\Delta)x + \Delta(1-\lambda))^2 - 4x\Delta(1-\Delta)g(\Delta)}} \right]$$
(A2)

Consider the fraction appearing in (A2). The square of this fraction is strictly smaller than one (and hence so is its absolute value) if and only if

$$(((1 - \Delta)x + \Delta(1 - \lambda)) - 2\Delta g(\Delta))^{2} < ((1 - \Delta)x + \Delta(1 - \lambda))^{2} - 4x\Delta(1 - \Delta)g(\Delta)$$

$$\iff 4\Delta^{2}g(\Delta)^{2} - 4((1 - \Delta)x + \Delta(1 - \lambda))\Delta g(\Delta) < -4x\Delta(1 - \Delta)g(\Delta)$$

$$\iff g(\Delta) < 1 - \lambda$$

and this last condition is always fulfilled (recall that $1 - \lambda - g(\Delta) > 0$ is the probability of the controlled process determining the decision at t = 2). Hence, it follows from (A2) that $z^*(x)$ is strictly decreasing and the conclusion follows.

(b) For each combination of the parameters $\Delta, g(\Delta)$, and d, by Theorem 1 three situations might result. If $\lambda + g(\Delta) > \frac{1}{1+d}$, there is a unique DPE of the Shirking type with $z = 1 - \lambda$. If $\lambda + g(\Delta) < \frac{1}{1+d}$, there is a unique DPE of the Willpower type with z given by (9). If $\lambda + g(\Delta) = \frac{1}{1+d}$, there is a continuum of equilibria with $x \in [0, 1]$ and z(x) given by (6).

Consider a situation in the second case and let $z^*(1)$ denote the associated probability of an automatic start. Increasing *d* leaving other parameters constant (for example) leads to an outcome in the first case, with a probability of automatic start $z^*(0) = 1 - \lambda$. But then, using (9),

$$z^{*}(0) > z^{*}(1) \iff$$

$$1 - \lambda > 1 + \frac{1}{2\Delta} \left[-(1 + \Delta\lambda) + \sqrt{(1 - \Delta\lambda)^{2} - 4\Delta(1 - \Delta)g(\Delta)} \right] \iff$$

$$1 - \Delta\lambda > \sqrt{(1 - \Delta\lambda)^{2} - 4\Delta(1 - \Delta)g(\Delta)} \iff$$

$$0 < 4\Delta(1 - \Delta)g(\Delta)$$

which always holds independently of Δ , $g(\Delta)$, and d. Hence, all Willpower equilibria have smaller z than Shirking equilibria.

Suppose now that nature sets $\Delta, g(\Delta)$, and d such that there is a continuum of mixed-strategy equilibria. The computations in part (a) show that the equilibrium which maximizes z is the one with complete shirking, x = 0, and the probability of automatic start is identical to that of shirking equilibria. This completes the proof.

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