

# Wage Formation in a Cointegrated VAR Model: A Demand and Supply Approach\*

Thórarinn G. Pétursson<sup>†</sup> and Torsten Sløk<sup>‡</sup>

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## Abstract

Usually cointegrated VAR models of wage formation are analysed in a wage-price setup. However, theoretical wage bargaining models provide the background for a wage-employment setup. The two relations of interest are the labour demand equation from the profit maximizing firms and the (bargained) wage equation from maximizing the Nash product of the wage bargaining process. From the underlying economic model we derive explicit parameter restrictions which are analysed using a multivariate cointegration approach, using quarterly data from Denmark. These restrictions are not rejected and the theoretical model with maximizing agents can be said to give a good description of wage formation in Denmark.

*Keywords:* Wage formation, labour demand, cointegration.

*JEL Classifications:* E24, C12, C13, C32

## 1. Introduction

In this paper we set up a theoretical model of utility maximizing consumers that form trade unions, and profit maximizing firms bargaining over the real wage in

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<sup>†</sup> Central Bank of Iceland, Economics Department, and University of Iceland. Address: Central Bank of Iceland, Kalkofnsvegi 1, 150 Reykjavik, Iceland. e-mail: thorar@centbk.is.

<sup>‡</sup> University of Copenhagen and Economic Policy Research Unit. Address: EPRU, Nansensgade 19, 5th floor, DK-1366 Copenhagen K, Denmark. e-mail: nats@hp4.econ.cbs.dk.

markets characterised by monopolistic competition, of the Blanchard and Kiyotaki (1987) type. From this model we derive two long-run solutions, one determining (labour) demand, and the other determining supply (wage bargaining). We then use a cointegrated vector autoregressive (VAR) approach to identify these relations using quarterly Danish data and test the restrictions implied by the theoretical model. These restrictions suggest a "main course" wage relation and long run ineffectiveness of demand management. Both these restrictions are accepted by the data. Further, we find evidence of real wage rigidity with an elasticity of real wages, in terms of the unemployment rate, of -0.25. Finally, we find empirical support for a log specification of utility and a Cobb-Douglas specification of production.

Other studies of wage formation within the cointegrated VAR approach, typically apply a wage-price setup, where wages are determined by the bargaining process and prices determined as a mark-up over wages. However, micro founded macro theory more naturally suggests a wage-quantity setup, where the quantity is employment. In addition, we feel that estimating a wage equation alone or jointly with a mark-up rule does not provide much insight into the relation between wages and employment, as it does not provide information on how sensitive employment is to wages. Both a wage and an employment equation are therefore needed. This is the approach we adopt here. Andersen and Risager (1990) also adopt this approach on Danish data, but without using the cointegration approach. Further, they do not test parameter restrictions, implied by the underlying model.

Another important difference between this study and many other studies of wage formation within the cointegrated VAR framework, is that we derive explicit restrictions implied by the wage bargaining solution from well defined objective functions of consumers and firms and test these restrictions within the cointegration approach. This is very important, as using a "data led" approach often makes it difficult to make meaningful economic interpretations of empirical results from cointegration analysis. These explicit links with the underlying economic model also facilitates policy analysis, such as policies directed towards the unemployment problem.

In a study, in some ways similar to ours, Bårdsen and Fisher (1995) and Bårdsen *et al.* (1995) use cointegrating restrictions from a wage bargaining model, using Norwegian and UK data. They do not, however, derive their restrictions explicitly from an underlying economic model with utility maximizing agents, and further, they use a wage-price setup instead of using a wage-employment setup. The wage-price setup is also used in Juselius (1992) on Danish data. Danish data is also used in Hansen and Warne (1995) who estimate cointegrating relations on annual data, however for a much longer period, namely from 1905 to 1992. Neither of these studies use explicit objective functions for the agents in the economy.

Engsted and Haldrup (1994) estimate a forward looking, rational expectations model of labour demand, assuming quadratic adjustment costs. They use sectoral data for Danish manufacturing, but are not able to find economically interpretable long-run relations.

The paper is organised as follows. In Section 2 we describe the underlying theoretical model of wage formation and labour demand, and derive the two long-run relations for real wages and employment. In section 3 we estimate a cointegrated VAR model and identify the two long-run relations and test the over-identifying restrictions implied by our model. Section 4 concludes.

## 2. The Model

### 2.1. Consumers

In this economy there are a number of monopolistically competing firms,  $i = 1, \dots, m$ , each producing their own product, as in Blanchard and Kiyotaki (1987). Consumers derive utility only from consumption and supply labour inelastically. Consumption is defined as an index of all the consumption goods in the economy. The utility function is given by

$$U(c_{1h}, \dots, c_{mh}) = (C_h)^\sigma / \sigma \quad (2.1)$$

where  $\sigma$  measures the curvature of utility. Note that  $\sigma = 0$  indicates a log utility function,  $U = \ln C_h$ . The consumption good index is defined as

$$C_h = \left(\frac{1}{m}\right)^{\frac{1}{1-E}} \left(\sum_{i=1}^m (c_{ih})^{\frac{(E-1)}{E}}\right)^{\frac{E}{(E-1)}}$$

and the consumer price index is defined as

$$Q(Q_1, \dots, Q_m) = \left(\frac{1}{m} \sum_{i=1}^m Q_i^{(1-E)}\right)^{\frac{1}{(1-E)}}$$

where  $E > 1$  is the elasticity of substitution between consumption goods. Consumer  $h$  maximizes utility subject to his budget constraint

$$\sum_{i=1}^m Q_i c_{ih} \leq W_h$$

where  $W_h$  are wages.<sup>1</sup> Solving the consumer's problem gives the following demand for each individual product in the economy

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<sup>1</sup>To keep the notation simple we abstract from profit payments that consumers receive from the firms. This does not change the results in any significant way.

$$C_i = \frac{W}{mQ} \left( \frac{Q_i}{Q} \right)^{-E} \quad (2.2)$$

Inserting this optimal choice of consumption goods into the utility function gives an expression for the indirect utility function of the consumer

$$S(W, Q) = (W_h/Q)^\sigma / \sigma$$

which is used when forming the trade unions.

## 2.2. Trade unions

We assume that each firm has an associated trade union (thus, assuming industry trade unions). To form the trade unions we use the indirect utility function for each member. Since there exists unemployment benefits the local union may find it optimal to create unemployment. It is assumed that all members face the same risk of being unemployed and that all agents are risk neutral. Thus the expected utility of a union member is

$$E(S(W, Q)) = \frac{N_i}{M_i} (W_i^c)^\sigma / \sigma + \frac{M_i - N_i}{M_i} U_0 \quad (2.3)$$

where  $M$  is the number of members of the trade union,  $N$  is employment in the trade union and  $W_i^c \equiv \frac{W_i}{Q}$  is the real consumption wage.  $U_0$  is the fall-back utility, i.e. the utility of a worker who fails to become employed in sector  $i$ .

From this can be subtracted the utility in the case of a break down  $U_0$  and then we have the expression for the Nash bargaining product. Multiplying out and re-organizing leads us to a utility function of the trade union, where the union has preferences over employment and real wages

$$T_i = N_i [(W_i^c)^\sigma / \sigma - U_0] \quad (2.4)$$

## 2.3. Firms

Firms in the economy maximize profits. We solve the firms problem and find their demand for labour, conditional on the other production factor,  $K_i$

$$\max_{N_i, P_i, Y_i} \Pi_i = \frac{P_i Y_i - W_i N_i}{P} \quad (2.5)$$

subject to

$$\begin{aligned}
X_i &= C \left( \frac{P_i}{P} \right)^{-E} \\
Y_i &= (\alpha N_i^\rho - (1 - \alpha) K_i^\rho)^{\frac{1}{\rho}}; \quad \rho \leq 0 \\
X_i &= Y_i
\end{aligned}$$

where  $P_i$  is the output price of firm  $i$ .  $P$  is an aggregate index of these prices. Maximization of real profits  $\frac{(P_i Y_i - W_i N_i)}{P}$  with respect to  $N_i$  subject to product demand,  $X_i$ , the production function, and the market clearing condition  $X_i = Y_i$ , gives the following (conditional) labour demand schedule for the firm

$$\alpha \left( \frac{E - 1}{E} \right) \left( \frac{Y_i}{N_i} \right)^{1-\rho} = \frac{W_i}{P} \quad (2.6)$$

This standard condition implies that firms employ workers until the marginal revenue product equals the real product wage. This can be rewritten as

$$N_i = \left( \frac{W_i}{P} \left( \frac{E}{\alpha(E-1)} \right) Y_i^{\rho-1} \right)^{\frac{1}{\rho-1}} \quad (2.7)$$

In symmetric equilibrium, where  $N_i = N$ , we can rewrite log employment as

$$n = y - \frac{1}{1-\rho} (w - p) + \xi \quad (2.8)$$

where lower case letters denote logs and  $\xi \equiv \left( \frac{1}{1-\rho} \right) \log \left( \frac{\alpha(E-1)}{E} \right)$ .  $\rho < 0$  indicates a CES technology, whereas  $\rho = 0$  indicates a Cobb-Douglas technology. Notice that the other production factor  $K_i$  is assumed exogenous in the maximization of profits with respect to employment. But  $K_i$  is of course still reflected in  $Y_i$ . The labour demand schedule is therefore better understood as being conditional on output demand. Another reason for using (2.8) is the lack of data for the capital stock and we want to avoid the problems of constructing such data.

## 2.4. Wage bargaining

We now let firms and unions bargain, applying the asymmetric Nash-bargaining solution and assuming the bargaining is of the right-to-manage type. The outcome of the bargaining process is found by solving

$$\max_{W_i} T_i^\lambda \Pi_i \quad (2.9)$$

subject to labour demand given in (2.7), where  $\lambda$  parameterizes union bargaining power.

The solution to the wage bargaining process implies that the wage is set as a mark-up on fall-back utility. To solve for the mark-up we need an expression for the elasticity of profit and labour demand with respect to real wages,  $\epsilon_\pi$  and  $\epsilon_N$  (see Lockwood *et al.* 1995). For this purpose we define

$$\begin{aligned}\eta_i &\equiv \frac{N_i W_i^c}{Y_i V} \\ &= \frac{W_i/P}{Z_i}\end{aligned}\tag{2.10}$$

where  $V \equiv \frac{P}{Q}$  is the price wedge. Note that  $\eta_i$  is simply the real unit labour cost, or the wage share in value added, of firm  $i$ , where  $Z_i \equiv \frac{Y_i}{N_i}$  is labour productivity. The two elasticities can now be written as

$$\epsilon_\pi = \frac{\eta_i}{1 - \eta_i}\tag{2.11}$$

and

$$\epsilon_N = (1 - \rho)^{-1}\tag{2.12}$$

The solution to the Nash bargaining can now be written as

$$(W_i^c)^\sigma / \sigma = (1 - \sigma \kappa_i)^{-1} U_0\tag{2.13}$$

where  $\kappa_i \equiv (\epsilon_\pi/\lambda + \epsilon_N)^{-1}$ , see e.g. Manning (1993). The bargained wage is therefore equal to a mark-up on fall-back utility, where the mark-up depends on the bargaining power of the union and the elasticities of labour demand and profit with respect to the real wage.

In equilibrium  $U_0$  will be endogenous. Standard search theory implies that  $U_0$  equals a weighted average of the utility of real unemployment benefits,  $\frac{B}{Q}$ , and the expected utility of getting a job elsewhere, where the weight on  $\frac{B}{Q}$ ,  $\varphi(u)$ , is an increasing function of the unemployment rate. So in symmetric equilibrium, where  $W_i = W$ , we have

$$U_0 = [1 - \varphi(u)] (W^c)^\sigma / \sigma + \varphi(u) (B/Q)^\sigma / \sigma$$

By inserting this into (2.13) and re-organizing we obtain

$$(W^c)^\sigma = \frac{(B/Q)^\sigma}{1 - \sigma \kappa / \varphi(u)}$$

or

$$R^\sigma = 1 - \sigma \kappa / \varphi(u)$$

where  $R \equiv \frac{B/Q}{W^c}$  is the replacement ratio. This equation is however not easy to estimate. The simplest approach is to assume that technology is Cobb-Douglas so that  $\rho = 0$ . Then  $\eta = \frac{\alpha(E-1)}{E}$  and  $\kappa = \left(\frac{\lambda E + \alpha(1-\lambda)(E-1)}{\lambda(E-\alpha(E-1))}\right)^{-1} \equiv \bar{\kappa}$  is simply a constant. Then taking logs we get

$$w^c = (b - q) - \frac{1}{\sigma} \log \left( 1 - \frac{\sigma \bar{\kappa}}{\varphi(u)} \right) \quad (2.14)$$

This is the equation that forms the basis for a real wage equation in many papers that estimate macro wage equations, e.g. Holmlund and Kolm (1995) and Lockwood and Manning (1993). However, the Cobb-Douglas specification is restrictive as it rules out some variables, such as productivity,  $Z$ , and the price wedge,  $V$ . If we approximate around  $\bar{\kappa}$  as  $\kappa = \bar{\kappa} \left(\frac{\eta}{\alpha(E-1)/E}\right)^{-\psi}$ , so when  $\psi = 0$ ,  $\eta = \frac{\alpha(E-1)}{E}$  and  $\kappa = \bar{\kappa}$ , we obtain

$$w^c = z - \frac{1}{\psi} \log u + v - \frac{1}{\psi} \log(1 - R^\sigma) + \zeta \quad (2.15)$$

where we assume that  $\varphi(u) = \varphi u$ , as implied by standard search theory, see Manning (1990, 1993), and  $\zeta \equiv \log \left[ \left(\frac{\alpha(E-1)}{E}\right) \left(\frac{\sigma \bar{\kappa}}{\varphi}\right)^{\frac{1}{\psi}} \right]$ . The Cobb-Douglas functional form is a special case as  $\psi \rightarrow 0$ . We also approximate  $\log(1 - R^\sigma)$ , which is non-linear in parameters, with  $-\theta \log R^\sigma$ , which is simply a constant  $(-\sigma\theta)$  times  $\log R$ . The bargaining solution now becomes

$$(w - q) = z - \frac{1}{\psi} \log u + v + \frac{\sigma\theta}{\psi} r + \zeta \quad (2.16)$$

where again lower case letters denote logs.

We rewrite this equation in terms of real product wages. This is in accordance with other studies of wage formation in the Nordic countries, who find that the real product wage is more important than the real consumption wage. See, for example, Calmfors and Nymoen (1990) for the Nordic countries, Forslund and Risager (1994) for Sweden, and Nymoen (1992) for Finland. This is simply a re-normalization of (2.16)

$$(w - p) = z - \frac{1}{\psi} \log u + \frac{\sigma\theta}{\psi} r + \zeta \quad (2.17)$$

The wage bargaining equation indicates that an increase in labour productivity will lead to higher real wages, by increasing the profitability of the firms, thus increasing the probability of accepting higher real wages on their behalf. The theoretical model indicates that the coefficient on productivity should be unity, implying that our right hand side variable is the real unit labour cost,  $(w - p - z)$ .

An alternative interpretation is that the right hand side variable is the wage share of value added, indicating that wage formation can be described by the "main course" theory, cf. Nymoen (1992).<sup>2</sup>

The wage bargaining solution implies that an increase in the unemployment rate will weaken the bargaining position of the trade unions in their claims for higher real wages. The opposite occurs when the replacement ratio increases, due to e.g. a rise in real unemployment benefits which increases fall-back utility.

Finally, a rise in the price wedge,  $v = (p - q)$ , will in general improve the bargaining position of unions and lead to higher real consumption wages. The theoretical model implies that a rise in the price wedge will have a proportional affect on real consumption wages, or equivalently, that the price wedge does not affect the real product wage. Thus, there is no scope for domestic demand policy to affect employment in the long run, cf. Andersen (1989). This is, of course, a testable implication of the model.

### 3. Empirical Analysis<sup>3</sup>

#### 3.1. The data

In this section, we estimate the theoretical wage and labour demand relations, (2.8) and (2.17) obtained in the previous section. We use quarterly, seasonally adjusted data from Denmark, for the period 1975(1) to 1993(4). The data is described in more detail in the appendix. The data essentially builds on the quarterly national accounts from Statistics Denmark and is obtained from the MONA database maintained at the Danish Central Bank. A documentation of the MONA model can be found in Christensen and Knudsen (1992).

The data are reported in Figure A.1 in the appendix. As seen from the figure, it seems appropriate to treat the series as non-stationary for the period analysed here. This also applies to the unemployment rate, although this could be a small sample problem. However, Hansen and Warne (1995) who analyse annual data from 1905 to 1992 also find evidence of non-stationarity of the unemployment rate, or at least, a very high level of persistence.

Table A.6 in the appendix reports the results of more formal unit root tests, using the Phillips and Perron (1988) univariate unit root test. Real wages, employment, the unemployment rate, and output seem to contain a single unit root, whereas the price wedge might be stationary and productivity and the replace-

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<sup>2</sup>The bargaining solution for the wage share can also be justified by the idea that trade unions take social distribution into account in the bargaining process.

<sup>3</sup>The computer programs PC-FIML 8.1 and CATS for RATS 4.2 were used to obtain all numerical results.



ment ratio could be stationary around a linear time trend, although with roots close to unity. A second unit root is, however, strongly rejected in all cases. We therefore conclude that all the series contain a single unit root, or at least a root very close to unity. This implies that the series can be adequately approximated as  $I(1)$  processes.

### 3.2. Complete VAR analysis

To estimate the model we use a multivariate approach to capture the simultaneous nature of the decision problem in question. The advantage of this approach is that all the information in the data is used, thereby increasing estimation efficiency. This approach also provides a natural framework for analysing the long-run behaviour of the data, by using its inherent non-stationarity to estimate the long-run relations of the underlying economic model.

As a starting point of our analysis, as a tentative description of the underlying data generation process, we estimate a "complete" VAR, where all variables in the information set  $\mathcal{I}_{wnt} = \{(w - p)_t, n_t, z_t, \log u_t, v_t, r_t, y_t\}$  are treated as endogenous. This information set contains seven variables, and is therefore quite large relative to our data set. It would therefore be useful to work with a simplified information set, which corresponds to a "partial" VAR where the system is conditioned upon some of the variables. We will return to this below.

To estimate the model we start with an unrestricted, Gaussian VAR( $k$ ) model, rewritten in the familiar error correction form, in terms of stationary variables in  $I(0)$  space

$$\Delta \mathbf{X}_t = \sum_{i=1}^{k-1} \Gamma_i \Delta \mathbf{X}_{t-i} + \alpha \beta' \mathbf{X}_{t-1} + \Upsilon \mathbf{D}_t + \varepsilon_t; \quad t = 1, \dots, T \quad (3.1)$$

where  $\mathbf{X}_t$  is a  $(p \times 1)$  vector containing the  $p$  variables of the model, and  $\mathbf{D}_t$  is a  $(s \times 1)$  vector containing the deterministic variables of the model, namely a constant and dummy variables. We assume that the initial values  $\{\mathbf{X}_{-k+1}, \dots, \mathbf{X}_0\}$  can be taken as given, and that  $\varepsilon_t \sim \text{IN}_p(\mathbf{0}, \Omega)$ . Finally, the long-run part of the model allows for linear feedback from the lagged stationary equilibrium errors,  $\alpha \beta' \mathbf{X}_{t-1}$ , where  $\beta$  is a  $(p \times r)$  matrix of full rank, containing the  $r$  cointegration vectors and  $\alpha$  is a  $(p \times r)$  matrix of full rank, containing the adjustment coefficients (the factor loadings). This formulation assumes that  $\mathbf{X}_t$  is  $I(1)$ , i.e. that the components of the vectors are at most  $I(0)$  or  $I(1)$ . To ensure that  $\mathbf{X}_t$  is not  $I(2)$  it is further required that  $\alpha'_{\perp} \Gamma \beta_{\perp}$  has full rank, where  $\Gamma = \mathbf{I} - \sum_{i=1}^{k-1} \Gamma_i$ , and  $\alpha_{\perp}$  and  $\beta_{\perp}$  are the  $(p \times (p - r))$  orthogonal complements of  $\alpha$  and  $\beta$  (see Johansen, 1988, 1991, 1995a).

Table A.1 contains the cointegration analysis for the complete VAR. For the

lag length we choose  $k = 2$ , which is found sufficient to capture the main features of the data.<sup>4</sup> There are three dummy variables in the VAR, designed to capture residual outliers in the initial unrestricted system. The dummy variables are  $d802_t$ , which is unity in 1980(2) and zero elsewhere,  $d871_t$ , which is unity in 1987(1) and zero elsewhere, and  $d922_t$ , which is unity in 1992(2) and zero elsewhere. These variables are included to capture exogenous shocks not captured by the VAR model and are restricted to the short-run dynamics. It should be emphasized that the inclusion of the impulse dummy variables did not affect in any way the estimation and identification of the cointegration vectors.

The table reports the  $\lambda_{\text{trace}}$  test from Johansen and Juselius (1990) for the cointegration rank, with and without the small sample corrections suggested by Reimers (1992). At the 95% critical level, the cointegration rank test indicates one or two cointegration vectors, depending on whether the small sample corrections are applied. At the 90% critical level the test indicates one or four cointegrating vectors, although two cointegrating vectors are not far from being rejected, applying the small sample adjusted critical values. Further, the first two eigenvalues are quite large, and as our theoretical model implies two cointegrating relations, we assume that there are two long-run relations in our data set.

Table A.1 also reports tests for weak exogeneity of a given variable for both cointegration vectors.<sup>5</sup> The test indicates that weak exogeneity of all variables can be rejected, except for  $z_t$  and  $y_t$ . However, when weak exogeneity of both these variables is tested jointly, we obtain  $\chi^2(4) = 15.5$  ( $\mathbf{p} = 0.00$ ), indicating that these variables cannot jointly be treated as weakly exogenous for both cointegration vectors. In our partial analysis below we therefore choose to condition on  $y_t$ .

### 3.3. Partial VAR analysis

To estimate the partial VAR system, we need to find a valid conditioning of the joint density of the data into a conditional density and a marginal density. The weak exogeneity tests above implied that we can either condition on  $z_t$  or  $y_t$  but not on both. We therefore choose  $y_t$  as our conditional variable. Other marginalizations did not prove successful. Our partial information set is therefore

$$\mathcal{P}_{wnt} = \{(w - p)_t, n_t, z_t, \log u_t, v_t, r_t \mid y_t\}$$

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<sup>4</sup>A likelihood ratio (LR) test for lag 3 against lag 4 gives a test statistic of  $F(49, 197) = 0.6$  ( $\mathbf{p} = 0.98$ ), using Rao's  $F$ -approximations of the LR test (see Doornik and Hendry, 1995). The LR test for lag 2 against 4 gives a test statistic of  $F(98, 249) = 1.2$  ( $\mathbf{p} = 0.18$ ). The LR test for lag 2 against 3 gives a test statistic of  $F(49, 233) = 1.3$  ( $\mathbf{p} = 0.14$ ). Lag 1 is however rejected against the higher lags. The lag choice of 2 is also supported by Schwartz and Hannan and Quinn information criteria. These results are available on request from the authors.

<sup>5</sup>See Engle *et al.* (1983) and Johansen (1992a).

To estimate the model with this partial information set, we partition the  $\mathbf{X}_t$  vector into  $(\mathbf{X}_{1t} : \mathbf{X}_{2t})$ , where  $\mathbf{X}_{1t}$  is a  $(m \times 1)$  vector of endogenous variables and  $\mathbf{X}_{2t}$  is a  $((p - m) \times 1)$  vector of weakly exogenous variables. The covariance matrix,  $\mathbf{\Omega}$ , can be partitioned accordingly as

$$\mathbf{\Omega} = \begin{pmatrix} \mathbf{\Omega}_{11} & \mathbf{\Omega}_{12} \\ \mathbf{\Omega}_{21} & \mathbf{\Omega}_{22} \end{pmatrix}$$

The conditional system for  $\Delta\mathbf{X}_{1t}$ , given the past and  $\Delta\mathbf{X}_{2t}$ , can now be written as (see Johansen, 1992a, b)

$$\Delta\mathbf{X}_{1t} = \sum_{i=1}^{k-1} \mathbf{\Gamma}_i^* \Delta\mathbf{X}_{t-i} + \mathbf{\alpha}^* \boldsymbol{\beta}' \mathbf{X}_{t-1} + \boldsymbol{\omega} \Delta\mathbf{X}_{2t} + \mathbf{\Upsilon}^* \mathbf{D}_t + \boldsymbol{\varepsilon}_t^*$$

where  $\mathbf{\Gamma}_i^* \equiv (\mathbf{\Gamma}_{1i} - \boldsymbol{\omega} \mathbf{\Gamma}_{2i})$ ,  $\mathbf{\alpha}^* \equiv (\boldsymbol{\alpha}_1 - \boldsymbol{\omega} \boldsymbol{\alpha}_2)$ ,  $\mathbf{\Upsilon}^* \equiv (\mathbf{\Upsilon}_1 - \boldsymbol{\omega} \mathbf{\Upsilon}_2)$ ,  $\boldsymbol{\varepsilon}_t^* \equiv (\boldsymbol{\varepsilon}_{1t} - \boldsymbol{\omega} \boldsymbol{\varepsilon}_{2t})$ , and  $\boldsymbol{\omega} \equiv \mathbf{\Omega}_{12} \mathbf{\Omega}_{22}^{-1}$ . In general a full system analysis is necessary, as the factor loadings,  $\boldsymbol{\alpha}_2$ , will also enter the marginal system, implying cross-equation interrelations. Under weak exogeneity, however,  $\boldsymbol{\alpha}_2 = \mathbf{0}$ , so the partial system

$$\Delta\mathbf{X}_{1t} = \sum_{i=1}^{k-1} \mathbf{\Gamma}_i^* \Delta\mathbf{X}_{t-i} + \boldsymbol{\alpha}_1 \boldsymbol{\beta}' \mathbf{X}_{t-1} + \boldsymbol{\omega} \Delta\mathbf{X}_{2t} + \mathbf{\Upsilon}^* \mathbf{D}_t + \boldsymbol{\varepsilon}_t^* \quad (3.2)$$

will contain the same information on the cointegration vectors and the factor loadings as the complete system, implying that efficient estimates of  $\boldsymbol{\alpha}$  and  $\boldsymbol{\beta}$  can be obtained from the partial analysis.

### 3.3.1. The unrestricted partial system

Table A.2 reports the cointegration analysis of the unrestricted partial system. For rank analysis of partial systems, Harboe *et al.* (1995) have generated asymptotic quantiles of the rank test. As our model has the constant unrestricted, the dynamic part will include a drift term. This implies that the distribution of the trace test will depend on a nuisance parameter that is equal to the ratio of the length of the drift parameters in the partial and marginal systems, standardized by their covariance. We therefore estimate the nuisance parameter and approximate the critical values, as suggested by Harboe *et al.* (1995).<sup>6</sup>

At the 95% critical level, the  $\lambda_{\text{trace}}$  implies two or three cointegrating vectors, depending on whether the small sample adjustments are applied or not. At the

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<sup>6</sup>The drift in the marginal system is found to be insignificant from zero,  $F(1, 59) = 0.4$  ( $p = 0.52$ ), implying that the true critical values are closer to the lower bands, see Harboe *et al.* (1995).

90% critical level, the  $\lambda_{\text{trace}}$  indicates one or two cointegrating vectors, depending on the small sample adjustments. Applying the small sample corrections we therefore accept two long-run relations at the 90% critical level and are not far from accepting it at the 95% level.

The unrestricted cointegrating vectors from the partial and complete systems are similar, but it is hard to give some economically meaningful interpretations to the vectors. This is often the case in practice and suggests the importance of using the structural information derived from a well specified economic model to identify the cointegrating vectors, as argued, for example, by Bårdsen and Fisher (1995) and Söderlind and Vredin (1996). This is the approach we follow in the next section.

Finally, Table A.3 reports the residual analysis of the partial system. These tests do not reject that the residuals are normally distributed, homoscedastic innovation errors for each individual equation and the system as a whole, implying data congruency of our unrestricted partial system.

### 3.3.2. The restricted partial system

The structural hypotheses are formulated as tests about the cointegration space, i.e. we test whether a given over-identified cointegration vector lies in the stationary part of the space spanned by the non-stationary variables. To impose the over-identifying restrictions on these long-run relations, the restrictions

$$\boldsymbol{\beta} = \{\boldsymbol{\beta}_w, \boldsymbol{\beta}_n\} = \{\mathbf{H}_w \boldsymbol{\varphi}_w, \mathbf{H}_n \boldsymbol{\varphi}_n\}$$

are imposed, where  $\boldsymbol{\beta}$  is partitioned into its two long-run components,  $\mathbf{H}_i$ ,  $i = w, n$ , are  $(p \times c_i)$  restriction matrices, and  $\boldsymbol{\varphi}_i$  are  $(c_i \times 1)$  coefficient vectors, where  $c_i$  are the number of freely estimated coefficients in cointegrating vector  $i$  (cf. Johansen, 1991, 1995b and Johansen and Juselius, 1990, 1994). The over-identifying restrictions are those implied by the theoretical model in the last section and are given in equation (3.3) (ignoring the constants)

$$\boldsymbol{\beta}' \mathbf{X}_{t-1} = \begin{pmatrix} 1 & 0 & -1 & \frac{1}{\psi} & 0 & -\frac{\sigma\theta}{\psi} & 0 \\ \frac{1}{1-\rho} & 1 & 0 & 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} (w-p) \\ n \\ z \\ \log u \\ v \\ r \\ y \end{pmatrix}_{t-1} \quad (3.3)$$

In this case the following restricted model is estimated

$$\Delta \mathbf{X}_{1t} = \sum_{i=1}^{k-1} \mathbf{\Gamma}_i^* \Delta \mathbf{X}_{t-i} + \boldsymbol{\alpha}_{1w} \boldsymbol{\varphi}'_w \mathbf{H}'_w \mathbf{X}_{t-1} + \boldsymbol{\alpha}_{1n} \boldsymbol{\varphi}'_n \mathbf{H}'_n \mathbf{X}_{t-1} + \boldsymbol{\omega} \Delta \mathbf{X}_{2t} + \boldsymbol{\Upsilon}^* \mathbf{D}_t + \boldsymbol{\varepsilon}_t^* \quad (3.4)$$

where the factor loading matrix has been partitioned conformably into  $\{\boldsymbol{\alpha}_{1w}, \boldsymbol{\alpha}_{1n}\}$ . The estimated over-identified cointegrating vectors are given in equation (3.5) with asymptotic standard errors calculated as suggested by Johansen (1991, 1995b), and a test statistic for the validity of the over-identifying restrictions, using the switching algorithms of Johansen (1995b) and Doornik (1995)

$$\boldsymbol{\beta}' \mathbf{X}_{t-1} = \begin{pmatrix} 1 & 0 & -0.803 & 0.210 & -0.123 & 0.020 & 0 \\ & 1.096 & 1 & 0 & 0 & 0 & -0.954 \\ & & & (0.12) & (0.05) & (0.16) & (0.15) \\ & & & & & & (0.07) \end{pmatrix} \begin{pmatrix} (w-p) \\ n \\ z \\ \log u \\ v \\ r \\ y \end{pmatrix}_{t-1} \quad (3.5)$$

$$\mathbf{H}_0 : \boldsymbol{\beta} = \{\mathbf{H}_w \boldsymbol{\varphi}_w, \mathbf{H}_n \boldsymbol{\varphi}_n\} : \chi^2(4) = 7.74 \quad (\mathbf{p} = 0.10)$$

The minimal theoretical over-identifying restrictions are accepted. The coefficient on productivity in the long-run wage relation, and on output in the long-run employment relation are not significant from unity, as implied by the theoretical model. The unit coefficient on productivity in the wage bargaining solution implies that long-run wage formation can be described by the "main course" theory. This is in contrast to Calmfors and Nymo en (1990) who conclude that the coefficient is less than unity for Denmark.

Further, we see that the price wedge does not affect real product wages in the long run. This in turn implies that domestic demand management (e.g. a change in consumption taxes) will not affect employment in the long run, see Andersen (1989). The results also indicate that the replacement ratio does not affect real product wages in the long run. This implies that  $\sigma = 0$ , thus supporting the log specification of utility in (2.1).

Finally, we see that real wages have almost a unit coefficient in the long-run labour relation ( $\rho = 0$ ), indicating a Cobb-Douglas specification of production technology. We therefore add these restrictions.<sup>7</sup> The fully restricted over-identifying cointegrating vectors are given in equation (3.6)

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<sup>7</sup>This finding is somewhat inconsistent, as the long-run employment relation indicates Cobb-Douglas technology, whereas the long-run wage relation indicates  $\psi > 0$ , rejecting the Cobb-Douglas specification.

$$\beta' \mathbf{X}_{t-1} = \begin{pmatrix} 1 & 0 & -1 & 0.250 & 0 & 0 & 0 \\ 1 & 1 & 0 & \text{\small (0.02)} & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} (w-p) \\ n \\ z \\ \log u \\ v \\ r \\ y \end{pmatrix}_{t-1} \quad (3.6)$$

$$H_0 : \beta = \{\mathbf{H}_w \boldsymbol{\varphi}_w, \mathbf{H}_n \boldsymbol{\varphi}_n\} : \chi^2(9) = 9.64 \text{ (p} = 0.38)$$

There are now nine over-identifying restrictions on the cointegrating space, derived from the theoretical model, and these are easily accepted by the data.<sup>8</sup> The first cointegrating vector can be identified as a steady state solution for real unit labour cost, or the wage share, depending negatively on the unemployment rate with a coefficient of 0.25, which is close to what other studies have found for other countries but slightly larger than what is traditionally found for Denmark, cf. Layard *et al.* (1991) and Blanchflower and Oswald (1994). The somewhat low elasticity confirms that real wages in Denmark are quite rigid. Nymoén (1992) argues that a low coefficient on the unemployment rate in real wage equations should be interpreted as low real wage responsiveness (hysteresis in the limit where the unemployment rate has no effect on real wages), but real wage flexibility should be measured by whether the wage share,  $(w-p-z)$ , is stationary or not. If the wage share is stationary, real wages are flexible as they adjust to productivity. Non-stationarity of the wage share would however indicate real wage rigidity. As seen in Figure A.1 and Table A.6 the wage share seems non-stationary, giving further support to our finding of real wage rigidity in Denmark.

Alternatively, the bargaining solution can be written in terms of real consumption wages, depending on productivity, the unemployment rate and the price wedge with a unit coefficient. The estimated factor loading of this cointegrating vector in the real product wage equation is  $-0.23$  ( $t$ -value = 5.1).

The second cointegrating vector can be identified as a steady state solution for (conditional) labour demand, depending on output and real product wages with a unit coefficient, thus indicating a Cobb-Douglas technology. The estimated factor loading of this cointegrating vector in the employment equation is  $-0.10$  ( $t$ -value = 5.5).<sup>9</sup>

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<sup>8</sup>The results are robust across different information sets, as can be seen in Table A.5 in the appendix. Note also that the  $\chi^2$  test statistic for the additional restrictions can be calculated as  $\chi^2(5) = 9.64 - 7.74 = 1.9$  (p = 0.86). Thus, the additional restrictions are easily accepted.

<sup>9</sup>The validity of the weak exogeneity assumption of output can again be tested, given the

The residual diagnostics are given in Table A.4. As before, there is no evidence of autocorrelation, ARCH-effects, or non-normality in individual residuals or the vector residual. Figure A.2 plots the actual and fitted values of the wage and employment equations and the scaled residuals, indicating quite a good fit and orthogonal innovation errors with respect to the information set,  $\mathcal{P}_{wnt}$ .

The final aspect of data congruency is system stability. To establish system stability we estimated the partial system recursively. Figure A.3 reports the recursive one-step residuals for the wage and employment equations, along with their  $\pm 2\hat{\sigma}$  bands, and the recursive one-step and  $N$ -step, decreasing horizon Chow tests, scaled by their 1% significance values. There is no evidence of structural instability in the system.<sup>10</sup> Finally, Figure A.4 plots the two restricted cointegrating relations,  $\hat{\varphi}'_w \mathbf{H}'_w \mathbf{X}_t$  and  $\hat{\varphi}'_n \mathbf{H}'_n \mathbf{X}_t$ , and the recursive eigenvalues, estimated using the switching algorithm of Hansen and Johansen (1993). We can see that the recursive estimates of the two eigenvalues, corresponding to the two stationary combinations, are stable and quite high over the whole sample period.

## 4. Conclusions

In this paper we take a demand and supply approach to analysing the labour market. Instead of analysing wage formation in the commonly used wage-price formulation, we suggest a framework implied by standard macro theory, which analyses wage formation in a wage-employment framework. We set up a model with trade unions and allow the trade unions and firms to bargain over wages, in the right-to-manage manner and find the Nash bargaining solution in symmetric equilibrium. Two long-run, or steady state, relations are derived; one is a conditional labour demand relation linking employment with output and real wages, the other is a real wage relation linking real wages to productivity, the price wedge, the unemployment rate and the replacement ratio.

To test the implications of the model and the parameter restrictions derived from the underlying economic model, we apply a multivariate cointegration approach on quarterly Danish data for the period 1975 to 1993. The cointegrating analysis indicates that two stationary combinations of the non-stationary data

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estimated cointegrated vectors from the partial analysis, by a standard misspecification test (see Johansen, 1992a). When this is done for the complete system we get similar results as before (not surprisingly). The weak exogeneity test for output gives  $\chi^2(2) = 2.2$  ( $p = 0.34$ ) and the joint weak exogeneity test for output and productivity gives  $\chi^2(4) = 21.3$  ( $p = 0.00$ ). When a similar test is conducted in the marginal system (i.e. the output equation), we obtain  $\chi^2(2) = 0.6$  ( $p = 0.73$ ). Thus, the marginalization of the system by conditioning on output seems valid, but further conditioning on productivity as well seems invalid.

<sup>10</sup>This also applies to the other equations residuals (not shown). These results are available on request from the authors.

exist, and that these can be identified as the two underlying theoretical relations. The parameter restrictions derived from the theoretical setup are not rejected by the data.

The wage bargaining relation identifies a steady state solution for real unit labour cost, or alternatively, the wage share in value added, as a negative function of the unemployment rate. Further, the price wedge does not significantly affect real product wages in the long run. This indicates that domestic demand management cannot affect employment in the long run. Finally, the replacement ratio does not affect real product wages in the long run. The unemployment rate has negative effects on real wages. A rise in the unemployment level deteriorates workers' possibility for finding alternative jobs, leading to a fall in expected fall-back utility, and through that, a fall in real, equilibrium wages. The elasticity of real wages with respect to the unemployment rate is 0.25 indicating that a one per cent increase in unemployment will lead to a quarter of per cent decrease in real wages. This also corresponds well with the widely held perception that real wages are quite rigid in Denmark, and in European economies in general, and is close to what other studies of wage formation have found. Finally, our results suggest that log utility and Cobb-Douglas production are not rejected by the data as adequate description of consumer preferences and production technology, respectively. These results were all found robust across different information sets.

The analysis in this paper has illustrated the importance of a clear and explicit link between the underlying economic model and the econometric analysis. A commonly used theoretical macro model with optimizing agents maximizing well defined objective functions was tested and imposing the over-identifying restrictions from the economic model on the cointegration space shows that wage formation in Denmark can be described by this theoretical model.



## A. Appendix

### A.1. Multivariate cointegration tests

<b>Table A.1. Cointegration analysis of the complete VAR</b>							
Eigenvalues	0.48	0.39	0.27	0.23	0.18	0.10	0.02
Hypotheses	$r = 0$	$r \leq 1$	$r \leq 2$	$r \leq 3$	$r \leq 4$	$r \leq 5$	$r \leq 6$
$\lambda_{\text{trace}}$	155.6	105.5	68.0	44.2	24.4	9.6	1.8
$\lambda_{\text{trace}}$ (with adj. df.)	126.9	86.1	55.5	36.1	19.9	7.9	1.5
95% critical values	124.2	94.2	68.5	47.2	29.7	15.4	3.8
90% critical values	117.7	89.4	64.7	43.8	26.7	13.3	2.7
<i>Standardized eigenvectors <math>\beta'</math></i>							
	$(w - p)_t$	$n_t$	$z_t$	$\log u_t$	$v_t$	$r_t$	$y_t$
	1	3.09	1.04	-0.07	0.21	-0.06	-2.73
	-0.02	1	0.03	0.15	0.01	-0.04	-0.74
	-1.23	4.50	1	-0.29	0.63	3.65	-0.94
	70.03	-444.1	-174.3	1	48.10	25.28	361.0
	1.51	-0.69	-2.03	-0.01	1	0.70	1.40
	1.19	1.52	-0.25	0.14	2.32	1	-0.92
	0.18	-0.58	-0.91	0.13	0.52	-0.16	1
<i>Standardized factor loadings <math>\alpha</math></i>							
$(w - p)_t$	0.01	-0.36	-0.01	-0.00	0.02	0.05	0.01
$n_t$	-0.06	0.02	-0.00	0.00	0.01	-0.00	-0.00
$z_t$	0.05	0.17	0.00	-0.00	0.06	0.04	-0.07
$\log u_t$	0.25	-0.50	-0.01	0.00	-0.04	-0.05	-0.19
$v_t$	-0.04	0.30	0.01	-0.00	-0.08	-0.06	-0.01
$r_t$	0.23	0.11	-0.05	0.00	0.09	-0.07	0.08
$y_t$	-0.03	0.18	-0.01	-0.00	0.03	0.04	-0.05
<i>Weak exogeneity test</i>							
	$(w - p)_t$	$n_t$	$z_t$	$\log u_t$	$v_t$	$r_t$	$y_t$
$\beta'_w$ and $\beta'_n$	8.2*	21.0**	2.7	14.9**	7.1*	7.3*	4.0

*Note:* The test statistic  $\lambda_{\text{trace}}$  is the trace eigenvalue statistic for testing cointegration in Johansen and Juselius (1990). The degrees of freedom adjustment is according to Reimers (1992). The critical values are obtained from Osterwald-Lenum (1992). The weak exogeneity test tests whether a given variable is weakly exogenous for both cointegrating vectors, and is asymptotically distributed as  $\chi^2(2)$ . \* (\*\*) denotes rejection of the null hypothesis at the 5% (1%) significance level.

**Table A.2. Cointegration analysis of the partial VAR**

Eigenvalues	0.48	0.36	0.27	0.19	0.14	0.05
Hypotheses	$r = 0$	$r \leq 1$	$r \leq 2$	$r \leq 3$	$r \leq 4$	$r \leq 5$
$\lambda_{\text{trace}}$	139.1	89.4	55.4	32.0	15.8	3.9
$\lambda_{\text{trace}}$ (with adjusted df.)	117.1	75.3	46.7	27.0	13.3	3.3
95% critical values	106.0	78.5	56.0	35.8	20.5	9.0
90% critical values	101.0	74.3	52.2	32.9	18.2	7.4
<i>Standardized eigenvectors <math>\beta'</math></i>						
$(w - p)_t$	$n_t$	$z_t$	$\log u_t$	$v_t$	$r_t$	$y_t$
1	3.46	1.10	-0.04	0.22	-0.08	-3.02
-0.14	1	-0.53	0.44	0.00	-0.01	-0.51
-0.59	3.30	1	-0.14	0.14	1.25	-1.71
-38.6	92.04	61.10	1	-30.10	-21.31	-88.28
0.18	-2.54	0.21	-0.04	1	0.57	-1.51
3.30	6.93	-4.06	1.15	7.36	1	-1.87
<i>Standardized factor loadings <math>\alpha</math></i>						
$(w - p)_t$	0.00	-0.15	0.02	0.00	0.00	0.01
$n_t$	-0.06	0.00	-0.01	0.00	-0.00	0.00
$z_t$	0.08	-0.02	0.04	-0.00	-0.01	-0.00
$\log u_t$	0.22	-0.15	-0.04	-0.00	0.03	-0.03
$v_t$	-0.03	0.12	0.02	0.00	-0.01	-0.01
$r_t$	0.23	0.03	-0.15	-0.00	-0.10	0.00

*Note:* The test statistic  $\lambda_{\text{trace}}$  is the trace eigenvalue statistic for testing cointegration in Johansen and Juselius (1990). The degrees of freedom adjustment is according to Reimers (1992). The critical values are obtained from Harboe *et al.* (1995) and are designed for rank analysis of partial systems. These critical values are sensitive to a nuisance parameter, which measures the ratio of the length of the drift in the partial system relative to that of the marginal system, standardized by the covariances of the respective systems. The critical values are therefore adjusted to a measure of the nuisance parameter, using the approximation suggested by Harboe *et al.* (1995).

## A.2. Residual analysis

**Table A.3. Residual analysis for the unrestricted partial VAR**

Series	$\hat{\sigma}$ (%)	$F_{ar1-1}(1, 56)$	$F_{ar1-4}(4, 53)$	$F_{arch1}(1, 55)$	$\chi_n^2(2)$
$(w - p)_t$	1.36	0.06	0.23	0.35	0.32
$n_t$	0.32	0.74	1.40	0.17	0.08
$z_t$	0.60	2.85	1.56	2.05	1.09
$\log u_t$	2.75	0.69	0.58	0.09	1.44
$v_t$	1.44	0.71	1.79	2.11	0.90
$r_t$	2.34	1.88	0.78	0.40	2.54
System	—	$F_{ar1-1}(36, 204)$ 1.22	$F_{ar1-4}(144, 171)$ 0.98	—	$\chi_n^2(12)$ 12.37

*Note:*  $\hat{\sigma}$  is the standard error of a given equation.  $F_{ar1-1}$  is a  $F$ -test for first order autocorrelation in a given equation.  $F_{ar1-4}$  is a  $F$ -test for up to fourth order autocorrelation in a given equation.  $F_{arch1}$  is a  $F$ -test for first order autoregressive conditional heteroscedasticity (Engle, 1982) in a given equation.  $\chi_n^2$  is the Doornik and Hansen (1993) normality test for a given equation. The table also shows corresponding system residual tests. \* (\*\*) denotes rejection of the null hypothesis at the 5% (1%) significance level.

**Table A.4. Residual analysis for the over-identified partial VAR**

Series	$\hat{\sigma}$ (%)	$F_{ar1-1}(1, 61)$	$F_{ar1-4}(4, 58)$	$F_{arch}(1, 60)$	$\chi_n^2(2)$
$(w - p)_t$	1.32	0.62	0.38	0.16	0.42
$n_t$	0.33	0.17	0.72	0.01	0.59
$z_t$	0.63	1.07	0.88	1.91	0.91
$\log u_t$	2.68	0.23	0.69	0.25	1.82
$v_t$	1.50	0.01	0.39	3.66	3.26
$r_t$	2.39	1.56	0.54	1.54	3.79
System	—	$F_{ar1-1}(36, 226)$ 1.38	$F_{ar1-4}(144, 200)$ 0.81	—	$\chi_n^2(12)$ 13.94

*Note:* See Table A.3 for explanation.

### A.3. Alternative information sets

Information sets	Cointegration vectors	Test for over-identifying restrictions
$\mathcal{I}_{wt}$	$(w - p)_t = z_t - \underset{(0.04)}{0.286} \log u_t$	$\chi^2(3) = 3.2$ ( $p = 0.36$ )
$\mathcal{I}_{nt}$	$n_t = y_t - (w - p)_t$	$\chi^2(2) = 0.7$ ( $p = 0.72$ )
$\mathcal{I}_{wnt}$	$(w - p)_t = z_t - \underset{(0.02)}{0.227} \log u_t$ $n_t = y_t - (w - p)_t$	$\chi^2(9) = 10.6$ ( $p = 0.30$ )

*Note:* The numbers under the coefficient estimates are asymptotic standard errors.  $\mathcal{I}_{wt} = \{(w - p)_t, z_t, \log u_t, v_t, r_t\}$  is the "wage" data information set.  $\mathcal{I}_{nt} = \{n_t, y_t, (w - p)_t\}$  is the "employment" data set.  $\mathcal{I}_{wnt} = \{(w - p)_t, n_t, z_t, \log u_t, v_t, r_t, y_t\}$  is the joint data set.

### A.4. Unit root tests

Series	H <sub>0</sub> : I(1) vs. H <sub>1</sub> : I(0)		H <sub>0</sub> : I(2) vs. H <sub>1</sub> : I(1)	
	No trend in series	Trend in series	No trend in series	Trend in series
$(w - p)_t$	-1.85	-2.63	-11.76	-11.84
$n_t$	-1.43	-0.39	-5.18	-5.34
$z_t$	0.38	-3.77	-14.74	-14.88
$\log u_t$	-1.86	-2.12	-3.86	-3.89
$v_t$	-3.40	-2.71	-12.69	-14.28
$r_t$	-1.36	-3.73	-12.77	-13.33
$y_t$	-1.18	-2.70	-10.24	-10.26
$(w - p - z)_t$	-0.41	-3.12	-12.33	-12.78

*Note:* The table reports the Phillips and Perron (1988) univariate unit root test on each series. The lag truncation for the Bartlett kernel is three. The number of observations is 76. The 5% critical values are -2.9, for the no trend case, and -3.5, for the trend case. The 1% critical values are -3.6, for the no trend case, and -4.1, for the trend case. The critical values are obtained from MacKinnon (1991).

## A.5. Graphs

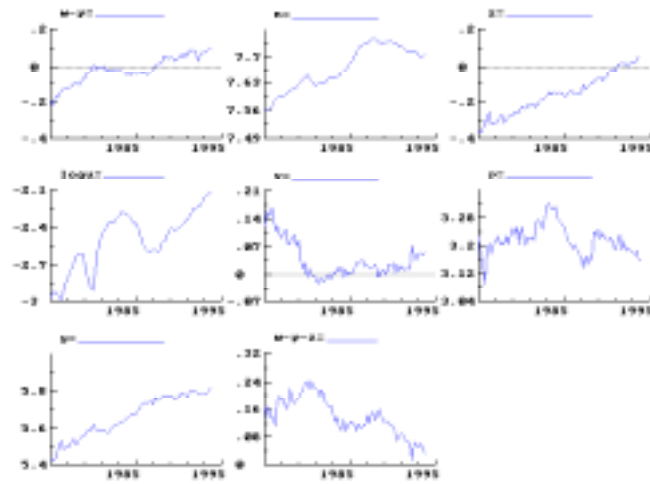


Figure A.1. The data

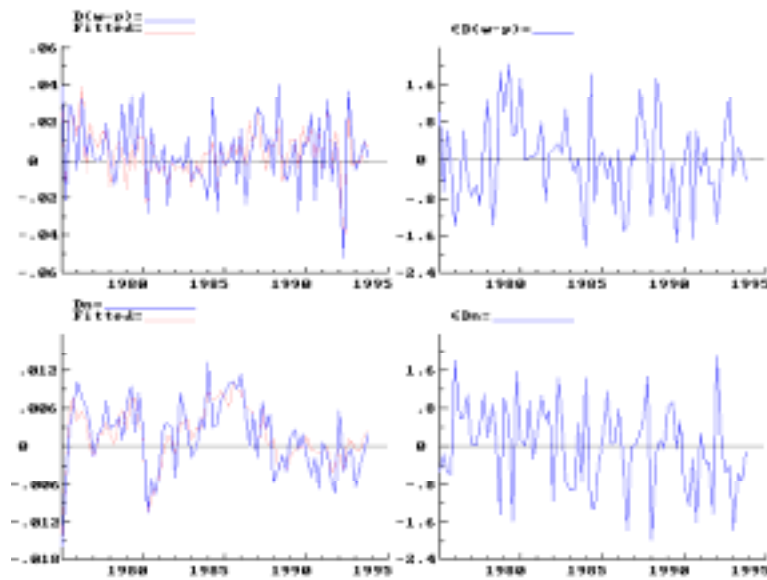


Figure A.2. Actual and fitted values and scaled residuals

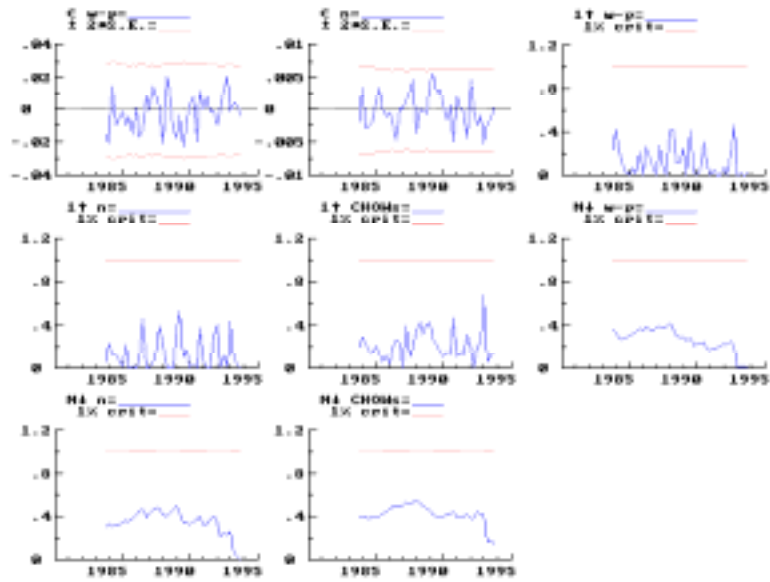


Figure A.3. Recursive diagnostics

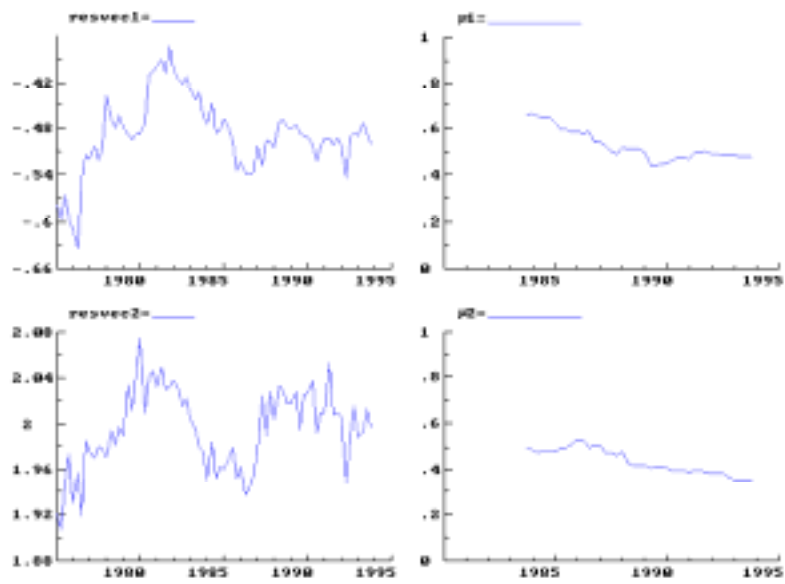


Figure A.4. Cointegrating vectors and recursive eigenvalues

## A.6. Data definitions

The data used in this paper is quarterly, seasonally adjusted, from 1975(1) to 1993(4) and is obtained from MONA, the quarterly macroeconomic model of Danmarks Nationalbank. The exact definitions are (the variables mnemonics in the MONA data are also shown)

$w_t$	$\log(LNA)$	log of the nominal hourly wage in the private non-farm sector
$p_t$	$\log(PYFBX)$	log of the GDP deflator for the private sector excluding agriculture, energy and housing
$q_t$	$\log(PCP)$	log of the private consumption deflator
$n_t$	$\log(QP + QO)$	log of total employment, 1000 fulltime workers
$z_t$	$\log(PROBX)$	log of average hourly productivity in the private sector, excluding agriculture, energy and housing
$\log u_t$	$\log(UL/U)$	log of the unemployment ratio
$b_t$	$\log(UI)$	log of the unemployment benefit after tax
$y_t$	$\log(FYFO + FYFBX)$	log of income in public sector and private sector excluding agriculture, energy and housing in billion 1980 DKK
$v_t$		$(p - q)_t$ log of the price wedge
$r_t$		$(b - w)_t$ log of the replacement ratio
$d802_t$		1 for $t = 1980(2)$ ; 0 otherwise
$d871_t$		1 for $t = 1987(1)$ ; 0 otherwise
$d922_t$		1 for $t = 1992(2)$ ; 0 otherwise

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