

# **EPRU**

## **Economic Policy Research Unit**

Institute of Economics

University of Copenhagen

Studiestræde 6

DK-1455 Copenhagen K

DENMARK

Tel: (+45) 3532 4411

Fax: (+45) 3532 4444

Web: <http://www.econ.ku.dk/epru/>

## **Cross-border Shopping and Tax Structure**

Vidar Christiansen

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# Cross-border shopping and tax structure<sup>\*</sup>.

Vidar Christiansen (University of Oslo)\*\*

## Abstract

A simple benchmark case is established to derive the optimum commodity tax on an externality-generating good that may be purchased abroad directly by domestic consumers. As the tax should under-cut the Pigovian level, this simple case supports the intuition that taxes should be moderated when commodities are cross-border traded. The analysis is extended in two directions. i) It is demonstrated that a fiscal tax, levied on a cross-border traded good, has more complex effects, weakening, or conceivably eliminating, the case for lowering the tax in response to cross-border shopping. Even if a tax cut alleviates distortions in favour of cross-border shopping, it would also induce a further distortion of consumption in the border region. ii) It is shown that when considering the taxation of *more than one* commodity purchased abroad, we should not neglect *joint* purchases, as done in previous research. To contain cross-border shopping it may be the more efficient reform to lower the tax on a commodity that is purchased only partially abroad.

\*\* Department of Economics, University of Oslo, P.O. Box 1095 Blindern, N-0307 Oslo, Norway. E-mail: vidar.christiansen@econ.uio.no

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## 1. Introduction.

Cross-border shopping has at various times been a hot issue in the tax debate in a number of countries and has presumably played a part in promoting tax reforms on several occasions. The case for tax reforms is not always made explicit, and reforms are seldom driven by a single concern, but it appears that Danish and Irish reforms of indirect taxation in the late eighties and the Norwegian lowering of VAT on food and excises on alcoholic drinks in recent years, and possibly the Swedish lowering of certain excises after joining the EU were at least in part supported by arguments related to cross-border shopping.

Concerns with cross-border shopping take various forms. In popular debate the concern is often with the effects on the domestic retailing business and employment in border regions. However, such effects need not be the most important ones, or indeed at all significant, and in any case there is a need to consider the wider implications for the economy as a whole<sup>1</sup>. Economists' approaches to cross-border shopping have sometimes focused on the international tax competition aspect (e.g. Kanbur and Keen (1993), Ohsawa, (1999), Nielsen and Hvidt (2001)), and at other times been concerned with the best choice of taxes when domestic tax-payers have the option to shop abroad. Christiansen (1994) provided an inverse elasticity characterisation of the optimum tax for a single cross-border traded good and focused on the role of imperfectly competitive markets for the cross-border traded goods. Scharf (1999) investigated the effects of scale economies in cross-border shopping.

In this paper I will focus on implications of cross-border shopping for the tax *structure*. The question is how one would like to differentiate commodity taxes when some commodities are purchased abroad directly by consumers paying foreign prices and foreign commodity taxes. One purpose is to clarify in more detail than previously the trade-offs that should govern the tax structure in the presence of cross-border shopping. Another extension of previous research is to take into account that more than one commodity may be purchased abroad. Not all commodities are purchased to the same extent abroad. Some goods are purchased abroad mainly by people living close to the border, others to a large extent also by people living rather far from the border. (Interesting empirical surveys are reported for instance in Bygvrå et al. (1987), Fitz Gerald (1989) and Keen (2002)). A question is then whether there should

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when I also benefited from presenting a previous version of this paper.

<sup>1</sup> E.g. casual observations of relevance for the employment issue indicate that there are not only cross-border shoppers, but also border-crossing shop assistants.

be, if at all, a uniform adjustment of taxes on all goods purchased abroad, or whether some commodity taxes should be singled out for special treatment. How should the politicians respond to the claims of lobbying campaigns that commodities purchased abroad should be subject to tax cuts?

The kind of country to be studied is a high-price country that is exposed to cross-border shopping by its own residents. Foreign prices and taxes will be considered as fixed. This setting may cover a range of cases. Two (or more) countries may be playing a game, which is a Nash game, or a Stackelberg game with the foreign country as a leader<sup>2</sup>, but we only consider the policy (response function) of a single country, called the home country. As barriers to border crossing have been dismantled there appears to have been a domino effect with countries bordering very low price countries being the first to respond by changing taxes. In doing so they may anticipate the responses of even more expensive countries and in that sense play the role of a leader vis-à-vis those countries. Alternatively, the policy abroad may be perceived as strictly exogenous and not as part of any strategic interaction with the home country. The considerations underlying the tax policy of the foreign country may be dominated by other factors than a possible wish to attract cross-border shoppers from the home country. A major concern may be not to lose too much trade to third countries.

The country under survey will not be assumed to consider lowering taxes to attract foreign cross-border shoppers. This is a more plausible scenario the larger the initial price difference between the countries and the more sparsely populated the foreign border region happens to be. Kanbur and Keen (1993) consider a game between countries of different size in terms of population. The paper highlights the result that the smaller country will impose the higher tax rate in a Nash game between revenue maximising countries. Essential assumptions are that the countries are of equal physical size and have a uniform distribution of the population across area, so that the country with the larger population also has the higher population density in the border regions. Hence the potential for attracting foreign shoppers is larger for the small country, which is induced to keep a low tax rate. In practice many small countries are high tax countries as emphasised by Nielsen (2001) who discusses differences in marginal cost of taxation as a conceivable explanation. In addition, if there are economies of scale in

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<sup>2</sup> While the cross-border shopping literature has mostly considered Nash equilibria, the Stackelberg equilibrium was analysed by Hvidt and Nielsen (2001).

providing public goods, some of which may be “necessities”, a smaller country needs higher taxes to achieve the same standard of public good provision.

In general, and also in a cross-border context, a discussion of commodity taxes should take into account the purposes that these taxes are to serve. Even though there are conditions under which revenue needs and distributional targets can be adequately met by relying solely on income taxes, there are several good reasons for levying indirect taxes in terms of value added and excise taxes.

- (i) A well-established case is the Pigovian argument for imposing commodity taxes to correct for negative externalities in consumption.
- (ii) A value added tax has many of the appealing properties of an expenditure tax<sup>3</sup>. In particular it does not distort the inter-temporal consumption trade-off.
- (iii) Taxes on commodities that are complementary with leisure may increase the tax base and mitigate other tax distortions.
- (iv) Under asymmetric information about taxpayers taxes on properly selected commodities may alleviate self-selection constraints and enhance economic efficiency.
- (v) Taxes on luxury goods may promote distributional objectives when income is an inadequate measure of welfare for instance due to under-reporting or imperfect income measures.

When any of these commodity taxes are in place opportunities for cross-border shopping may create distortions and economic inefficiencies as real resources are being used up in pursuit of price savings. It appears to be a widespread perception that cross-border shopping is caused by tax differences. This may or may not be true. Cross-border shopping is caused by *price* differences between countries and, albeit an important one, taxation is only one reason why prices may differ across borders. Production costs, regulation, the competitiveness of markets and exchange rates are other factors that determine the prices in the respective countries. However, it is important to recognise that the tax of a particular country is crucial for the deadweight loss of cross-border shopping inflicted on that country, whatever the factors inducing people to shop abroad. To illustrate, suppose that the production cost of a commodity is higher by *ten* in country H than in country A, while the tax is *five* in both countries. As a result the price is higher by *ten* in country H. Moreover, suppose that the travel and transport cost of acquiring the good abroad is *seven*. Residents of country H will

then find it in their private self-interest to shop abroad to save *three*, even though there is an extra net social cost of *two* since there is a travel cost of *seven* and a *foreign* tax to be paid of *five* to be set against the real cost saving of *ten*. When cross-border shopping takes place there is a reason to be concerned with the domestic tax wedge whatever the reasons for cross-border shopping.

As it is of interest to examine taxes that serve different purposes I will consider two kinds of indirect taxes: externality correcting (Pigovian) taxes and a value added tax. Indeed, many of the excises discussed in the context of cross-border shopping are taxes that are supported by externality arguments, with taxes on alcoholic drinks, tobacco, and petrol as prominent examples<sup>4</sup>. The VAT is a purely fiscal tax. The case to be considered is one in which a uniform VAT rate would be desirable in the absence of cross-border shopping. The issue to be addressed is whether it may be desirable to differentiate VAT rates when cross-border shopping occurs. There are various conceivable reasons why this may be a relevant issue; the other country may for some reason have differentiated VAT rates<sup>5</sup>, some commodities are more likely to be purchased abroad than others, and relative prices may be different in the two countries for other than tax reasons. The models will be kept as simple as possible in other respects in order to focus on only two concerns – a clear-cut domestic tax purpose and the concern with cross-border shopping. I will abstract from a number of second-best and informational problems and distributional concerns that would significantly complicate the analysis and blur the insight we seek on how cross-border shopping should be allowed for alongside some basic domestic tax purposes such as efficient revenue-raising and correcting for externalities.

The present paper differs from several previous papers in two essential respects. The consumption of each commodity is elastic rather than fixed, and more than one commodity may be purchased on each shopping trip across the border. It turns out that these assumptions make a significant difference. With respect to the cost of importing goods the paper shares some important features with Scharf (1999). The costs of acquiring the goods abroad differ and it differs between commodities what is the critical distance from the border beyond which it is not worthwhile buying the commodity abroad.

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<sup>3</sup> An expenditure tax is normally defined as a direct tax on consumption.

<sup>4</sup> It can be discussed to what extent the case is a standard externality argument or a demerit good argument, but in the current context this distinction may be of minor importance.

<sup>5</sup> A Scandinavian example was Sweden lowering her VAT rate on food to be followed by Norway.

Section 2 establishes as a benchmark the case where one externality-generating commodity is purchased abroad. An extension to two such commodities is considered in Section 3, while the special case that one of the commodities is petrol used for travelling abroad is briefly discussed in Section 4. Fiscal taxes inducing cross-border shopping are discussed in Section 5, and Section 6 concludes.

## 2. One externality-generating commodity purchased abroad.

To establish a benchmark I start by considering a very simple case. There is one externality-generating commodity, the consumption of which is  $y$  being composed of an amount  $y_a$  purchased abroad and  $y_h$  purchased at home. The cost of buying the commodity abroad is the foreign price  $q$  and a travel/transport cost of  $c(d)$  at a distance  $d$  from the border. The home country price is denoted by  $Q=1+t$ , where  $t$  is the commodity tax rate, and the pre-tax price is normalised to unity. Other goods are lumped together as one aggregate commodity, which is purchased at home, and  $B$  denotes disposable income. The tax revenue is recycled to the consumers as a lump-sum transfer. The country is condensed to a straight line. The population, normalised to unity, is distributed across the country according to a density function  $f(d)$ . The total length of the country is  $D$ . There is a negative externality of  $E$  for the economy as a whole. The tax revenue, which is recycled as a transfer payment to the population, is denoted by  $T$ . The utility function of a consumer living at a distance  $d$  from the border is given by

$$U = B + u(y) - (q + c(d))y_a - Qy_h + T - E \quad (1)$$

The externality-generating commodity will be purchased abroad if and only if

$$q + c(d) \leq Q \quad (2)$$

The critical distance from the border at which shopping abroad just ceases to be worthwhile is denoted by  $\bar{d}$  and is defined by

$$q + c(\bar{d}) = Q \quad (3)$$

The demand is

$$y_a = y(q + c(d)) \quad \text{for } d \leq \bar{d} \quad (4)$$

and

$$y_h = y(Q) \quad \text{for } d \geq \bar{d} \quad (5)$$

The indirect utility at a distance  $d$  from the border is given by the function

$$V(B, 1+t, E, d). \quad (6)$$

The externality is a consumption externality that is simply proportional by a factor  $e$  to the total consumption of the good in question

$$E = e \int_0^{\bar{d}} y_a f(s) ds + e \int_{\bar{d}}^D y_h f(s) ds \quad (7)$$

The tax revenue to be recycled to the taxpayers through a lump-sum transfer is

$$T = t \int_{\bar{d}}^D y_h f(s) ds \quad (8)$$

Total welfare is

$$W = \int_0^{\bar{d}} V f(s) ds + \int_{\bar{d}}^D V f(s) ds \quad (9)$$

The welfare effect of changing  $t$  is

$$W_t = \int_0^{\bar{d}} [T_t - E_t] f(s) ds + \int_{\bar{d}}^D [-y_h + T_t - E_t] f(s) ds \quad (10)$$

The impact on the externality level is

$$E_t = e \int_{\bar{d}}^D y'(\bar{Q}) f(s) ds \quad (11)$$

and the tax revenue is affected in the following way

$$T_t = Y_h + t \int_{\bar{d}}^D y'(\bar{Q}) f(s) ds - \bar{t} \bar{y} f(\bar{d}) \bar{d}' \quad (12)$$

where  $\bar{d}' = \partial \bar{d} / \partial Q$ , which is positive as seen from (3). Inserting the tax and externality effects we find that

$$W_t = (t - e) Y'_h - \bar{t} \bar{y} f(\bar{d}) \bar{d}' \quad (13)$$

For  $t=e$

$$W_t = -\bar{t} \bar{y} f(\bar{d}) \bar{d}' < 0. \quad (14)$$

By setting  $W_t = 0$  the optimum tax rate is characterised by

$$(e - t)(-Y'_h) = \bar{t} \bar{y} f(\bar{d}) \bar{d}' \quad (15)$$

which implies that  $t < e$ .



We can conclude that if there is a single externality-generating good that can be purchased abroad, the externality correcting tax should be set below the Pigovian level. The intuition for the result is simple. The desire to internalise the external effect must be traded off against the inefficiency resulting from inducing cross-border shopping. This is a very simple example of the more general principle that the concern with inefficient cross-border shopping must be traded off against the domestic benefits from imposing taxes.

### 3. Two externality-generating goods purchased abroad.

Let us now consider the case in which two externality-generating goods, labelled 1 and 2, may be purchased abroad. It is assumed that it is possible to buy a fixed amount of each commodity per trip. This may be so because a consumer is only entitled to bring into the country a certain allowance on each journey, or the reasons may have to do with transport technology or possibilities for storing the goods between journeys. The approach opted for is the simplest possible one, but similar mechanisms may arise even with somewhat different assumptions. (For a further discussion, see footnote 5 below. ) However, I do not integrate the consumption technological features of Scharf (1999).

For simplicity we normalise by assuming that one unit of a commodity can be purchased abroad per trip. We assume there is a general cost of going to the border (and back) to do shopping. This cost is for simplicity assumed to be proportional to the distance to the border and is expressed as  $Kd$  where  $K$  is a positive parameter. In addition there may be a good specific and distance related cost  $c_i(d)$ , which may capture the cost of preserving perishable and fragile items during the journey by means of thermo-bags, packaging, etc., or reflect a certain decay of the commodity. The consumption of commodity  $i$  generates an externality  $e_i$  per unit of the good. A commodity tax  $t_i$  is imposed. We assume that there is a fixed production cost implying that  $dQ_i/dt_i = 1$ . Let  $C$  denote a third commodity that causes no externality and is purchased at home, let  $y_1$  and  $y_2$  be quantities of the respective externality-generating goods, and let subscripts  $a$  and  $h$  indicate purchases abroad and at home, respectively.

The consumer's budget constraint is

$$B + T = C + Q_1 y_{1h} + (q_1 + c_1) y_{1a} + Q_2 y_{2h} + (q_2 + c_2) y_{2a} \quad (16)$$

The utility function, substituting for C from the budget constraint, is

$$U = B + u_1(y_1) + u_2(y_2) - (q_1 + c_1(d))y_{1a} - Q_1 y_{1h} - (q_2 + c_2(d))y_{2a} - Q_2 y_{2h} + T - E \quad (17)$$

I will consider the following pattern of cross-border and domestic shopping :

Assume that

for  $d \leq \hat{d}$  neither commodity is purchased at home,

for  $\hat{d} \leq d \leq \bar{d}$  commodity 1 is purchased at home and abroad while commodity 2 is only purchased abroad,

for  $\bar{d} \leq d \leq D$  neither commodity is purchased abroad.

In general the pattern of trade is endogenous, contingent on the taxes that are levied, but I shall assume the tax optimum to be such that all three intervals are of strictly positive length, since this is regime that, to my knowledge, has not been discussed in the literature on cross-border shopping.

The transfer to the consumers is equal to the tax revenue

$$T = t_1 \int_0^{\hat{d}} y_{1h} f(s) ds + t_1 \int_{\hat{d}}^{\bar{d}} y_{1h} f(s) ds + t_1 \int_{\bar{d}}^D y_{1h} f(s) ds + t_2 \int_0^{\hat{d}} y_{2h} f(s) ds + t_2 \int_{\hat{d}}^{\bar{d}} y_{2h} f(s) ds + t_2 \int_{\bar{d}}^D y_{2h} f(s) ds \quad (18)$$

The total externality is

$$E = e_1 \int_0^{\hat{d}} y_1 f(s) ds + e_1 \int_{\hat{d}}^{\bar{d}} y_1 f(s) ds + e_1 \int_{\bar{d}}^D y_1 f(s) ds + e_2 \int_0^{\hat{d}} y_2 f(s) ds + e_2 \int_{\hat{d}}^{\bar{d}} y_2 f(s) ds + e_2 \int_{\bar{d}}^D y_2 f(s) ds \quad (19)$$

The total welfare of the population is

$$U = \int_0^{\hat{d}} U(s) f(s) ds + \int_{\hat{d}}^{\bar{d}} U(s) f(s) ds + \int_{\bar{d}}^D U(s) f(s) ds, \quad (20)$$

where  $U(d)$  is the utility obtained by a person living at a distance  $d$  from the border.

At location  $d$  a consumer will buy the commodities abroad only if each good is cheaper abroad inclusive of transport

$$q_i + c_i(d) \leq Q_i \quad i=1,2 \quad (21)$$

and the cost inclusive of transport and travel costs is lower than the cost at home

$$q_1 + c_1(d) + q_2 + c_2(d) + Kd \leq Q_1 + Q_2 \quad (22)$$

Note that since there is a travel cost, at least one of the constraints in (21) must hold as a strict inequality.

In the border region  $d \leq \hat{d}$  both commodities are purchased abroad because that is cheaper:

$$Kd + q_1 + c_1(d) + q_2 + c_2(d) < Q_1 + Q_2 \quad (23)$$

$$q_i + c_i(d) < Q_i \quad i=1,2 \quad (24)$$

The number of trips,  $n$ , made from a distance  $d$  from the border is determined by the equation

$$u_1'(n) + u_2'(n) = Kd + q_1 + c_1(d) + q_2 + c_2(d) \quad (25)$$

The benefit from a marginal shopping trip is equal to its cost. At  $\hat{d}$  the number of trips is  $\hat{n}$  and

$$u_1'(\hat{n}) = Q_1 \quad (26)$$

The marginal benefit from consuming commodity 1 is just equal to the domestic price, which implies that some of the commodity will be purchased at home beyond  $\hat{d}$ . Thus  $\hat{n}$  and  $\hat{d}$  are determined by (25) and (26).

Since both commodities are purchased abroad domestic prices have no demand effects within the border region, but will affect the extension of the region. It is easy to recognise that

$$\frac{\partial \hat{d}}{\partial Q_1} = \hat{d}' > 0 \quad (27)$$

In the region  $\hat{d} \leq d \leq \bar{d}$  commodity 1 is purchased at home and abroad while commodity 2 is only purchased abroad. The demand pattern is characterised by

$$u_1'(y_1) = Q_1 \quad (28)$$

$$u_1'(n) > Q_1 \quad (29)$$

$$Kd + q_1 + c_1(d) + q_2 + c_2(d) < Q_1 + Q_2 \quad (30)$$

$$u_1'(y_1) + u_2'(n) = Kd + q_1 + c_1(d) + q_2 + c_2(d), \quad (31)$$

which is equivalent to

$$u_2'(n) = Kd + q_2 + c_2(d) - (Q_1 - q_1 - c_1(d)), \quad (32)$$

The marginal utility from consuming commodity one is equated to the domestic price (eq. (28)) as only purchasing  $n$  units abroad would imply a marginal utility above the domestic

price (formula (29)). The cost of a consumption bundle is cheaper abroad than at home (eq. (30)). The number of trips is determined by equating the marginal benefit from a consumption bundle to the marginal cost (eq. (31)). Alternatively the rule can be expressed as the marginal benefit from buying commodity 2 abroad being equated to the net marginal cost where the price saving from buying a unit of commodity one abroad is interpreted as a gain reducing the cost of purchasing commodity two abroad (eq. (32)).

It is easy to acknowledge from the conditions above that

$$dy_1 / dQ_1 < 0, dy_1 / dQ_2 = 0, dy_2 / dQ_2 = dn / dQ_2 = 0, dy_2 / dQ_1 = dn / dQ_1 > 0.$$

We may note in particular that it is only the domestic price for commodity 1 that affects demand for commodity 2. There is more to save from shopping abroad the higher the domestic price of commodity 1, and more trips to the border are worthwhile<sup>6</sup>. The price for commodity 2 does not have a similar effect as no purchases of commodity 2 are made at home and its domestic price becomes irrelevant.

At  $\bar{d}$

$$K\bar{d} + q_1 + c_1(\bar{d}) + q_2 + c_2(\bar{d}) = Q_1 + Q_2 \quad (33)$$

The cost of buying abroad and at home is the same and the consumer is just indifferent and will switch to domestic purchases as the cost of cross-border shopping increases beyond  $\bar{d}$ .

In the region  $\bar{d} \leq d \leq D$  neither commodity is purchased abroad and for each commodity the marginal benefit is equated to the domestic price.

It is easy to recognise that

$$\frac{\partial \bar{d}}{\partial Q_1} = \frac{\partial \bar{d}}{\partial Q_2} = \bar{d}' > 0 \quad (34)$$

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<sup>6</sup> When two or more goods are purchased abroad, there are several ways to model joint purchases of the goods. Suppose one good is highly perishable while the other can be stored for a rather long time. Examples might be food and tobacco. Suppose the goods can be stored for, respectively,  $\tau_1$  and  $\tau_2$  units of time, and  $\tau_1 < \tau_2$ . Then if the consumer buys the latter commodity abroad, he may also buy the former good when cross-border shopping. However,  $\tau_1$  units of time after his shopping trip, he will need to replenish his supply of good 1. As he will now need to buy only commodity 1, going abroad for that purpose may not be worthwhile, and he may top up his purchases in the domestic market between shopping trips abroad. The domestic price of good 1 may have cross effects on the demand for the other good, and the number of trips may be affected. We get a mechanism rather similar to the one considered above.

A ‘bar’ and a ‘hat’ are used, respectively, to indicate variables evaluated at distances  $\hat{d}$  and  $\bar{d}$  from the border. Let us also introduce the notation

$$\hat{Y}_i = \int_{\hat{d}}^{\bar{d}} y_i f(s) ds, \quad (35)$$

which is the total purchase of commodity  $i$  in the interval  $(\hat{d}, \bar{d})$ ,

$$\bar{Y}_i = \int_{\bar{d}}^D y_i f(s) ds, \quad (36)$$

which is the total purchase of commodity  $i$  in the interval  $(\bar{d}, D)$ . Corresponding notation is used for purchases abroad and domestically and the total number of trips to the border.

The effects of the tax rates on the key variables are derived in an appendix. The following welfare effects are obtained.

$$\frac{\partial W}{\partial t_1} = t_1 \frac{\partial \hat{Y}_{1h}}{\partial Q_1} + t_1 \frac{\partial \bar{Y}_1}{\partial Q_1} - t_1 \bar{n}f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_1} - t_2 \bar{n}f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_1} - e_1 \frac{\partial \hat{Y}_{1h}}{\partial Q_1} - e_1 \frac{\partial \hat{N}_1}{\partial Q_1} - e_1 \frac{\partial \bar{Y}_1}{\partial Q_1} - e_2 \frac{\partial \hat{Y}_2}{\partial Q_1} \quad (37)$$

$$\frac{\partial W}{\partial t_2} = t_2 \frac{\partial \bar{Y}_2}{\partial Q_2} - t_1 \bar{n}f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_2} - t_2 \bar{n}f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_2} - e_2 \frac{\partial \bar{Y}_2}{\partial Q_2} \quad (38)$$

To examine how cross-border shopping may modify the Pigovian rule let us consider the effects of deviating marginally from the Pigovian tax setting. For  $t_1 = e_1$  and  $t_2 = e_2$

$$\frac{\partial W}{\partial t_1} = -e_1 \frac{\partial \hat{N}}{\partial Q_1} - e_2 \frac{\partial \hat{Y}_2}{\partial Q_1} - t_1 \bar{n}f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_1} - t_2 \bar{n}f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_1} \quad (39)$$

$$\frac{\partial W}{\partial t_2} = -t_1 \bar{n}f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_2} - t_2 \bar{n}f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_2} \quad (40)$$

Since

$$\frac{\partial \bar{d}}{\partial Q_1} = \frac{\partial \bar{d}}{\partial Q_2} = \bar{d}',$$

$$-\frac{\partial W}{\partial t_1} = -\frac{\partial W}{\partial t_2} + e_1 \frac{\partial \hat{N}}{\partial Q_1} + e_2 \frac{\partial \hat{Y}_2}{\partial Q_1} \quad (41)$$

We note that  $\hat{N}$  and  $\hat{Y}_2$  increase when  $Q_1$  increases so that the second last and the last term are both positive. We see that lowering the tax on commodity one has an effect beyond that of lowering the tax on commodity two. Lowering the former price is thus a more efficient instrument at the initial point. The reason is that the price of commodity one can be used to reduce the non-internalised parts of the externalities, i.e. the externalities that can be attributed to the consumption of goods purchased abroad. When making purchases abroad the quantity purchased of commodity one enables the consumer to make a saving equal to the price difference across the border. This saving will shrink if the domestic price is lowered and the consumers are induced to do less shopping abroad buying less of *both* commodities abroad.

We note that one should give priority to reduce the price of the good that is purchased only partially abroad by a number of consumers, that is, those living in the middle region. This may seem paradoxical as one might think that it would be more urgent to deal directly with the price of the good that is purchased entirely abroad by these consumers. However, lowering the price of that commodity is less potent in reducing cross-border trade since those purchasing a commodity entirely abroad are not responsive to the domestic price at all.

The first order conditions obtained by setting the marginal welfare effects of the taxes equal to zero imply the following characterisation of optimal taxes:

$$(t_1 - e_1) \left( \frac{\partial \hat{Y}_{1h}}{\partial Q_1} + \frac{\partial \bar{Y}_1}{\partial Q_1} \right) = t_1 \bar{n}f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_1} + t_2 \bar{n}f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_1} + e_1 \frac{\partial \hat{N}}{\partial Q_1} + e_2 \frac{\partial \hat{Y}_2}{\partial Q_1} \quad (42)$$

$$(t_2 - e_2) \frac{\partial \bar{Y}_2}{\partial Q_2} = t_1 \bar{n}f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_2} + t_2 \bar{n}f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_2} \quad (43)$$

The discrepancy between the optimal tax and the Pigovian tax may be interpreted as a tax

wedge.  $(t_1 - e_1) \left( \frac{\partial \hat{Y}_{1h}}{\partial Q_1} + \frac{\partial \bar{Y}_1}{\partial Q_1} \right)$  and  $(t_2 - e_2) \frac{\partial \bar{Y}_2}{\partial Q_2}$  may be interpreted as marginal tax wedge

effects on domestic purchases, i.e. they measure the marginal efficiency losses from under-taxing the domestic purchases relative to the Pigovian benchmark..

To *summarise* the implications of the optimality conditions, we note that :

- Both tax rates should under-cut the Pigovian ones.

- One should trade off incomplete taxation of externalities against the social cost of cross-border shopping
- The marginal effect on the incompletely taxed externality of domestic purchases should be equated to the marginal cost of cross-border shopping and the marginal externalities of purchases abroad
- The marginal efficiency losses from under-taxing domestic purchases (relative to the Pigovian benchmark) should be larger for the commodity partially purchased abroad.

#### 4. Petrol as a special case.

Petrol (or similar fuel) is one of the commodities that consumers in some countries have found it cheaper to buy abroad. It is also an example of a commodity that is taxed on externality grounds to internalise the various costs of using roads and polluting the environment. This is a special case as petrol is used as an input by cross-border shoppers travelling by car (and indirectly by those going on coach trips). Let us consider how consumers may be affected by the petrol price depending on their location measured by distance from the border. We assume that some other commodity, called commodity two, is purchased abroad by consumers living closer than  $\bar{d}$  to the border, but purchased at home by consumers living farther from the border.

As the formal analysis is analogous to the one carried out above I shall confine myself to a verbal discussion of the relevant effects. Let us consider the pattern of trade in the following regions defined by distance from the border:

I The region  $[0, \hat{d}]$

Commodity two and petrol are both purchased entirely abroad

II The region  $[\hat{d}, \tilde{d}]$

Commodity two is purchased abroad. The petrol tank is filled up on each trip abroad. The petrol purchased abroad and not used for cross-border trips is used for other petrol consumption. The foreign purchases of petrol are supplemented by domestic purchases.

III The region  $[\tilde{d}, \bar{d}]$

Commodity two is purchased abroad. The petrol tank is filled up on each trip abroad, but the amount of petrol purchased abroad is not sufficient for cross-border trips. Some of the petrol needed for cross-border travel (and other consumption) is purchased at home.

IV The region  $[\bar{d}, D]$

No cross-border shopping takes place.

Let us consider the effects in each region of increasing the price of petrol. The effects will in some respects deviate from those of increasing the price for commodity one above since petrol is used as an input for the cross-border travel.

Region I: A marginal change of the domestic petrol price has no effect.

Region II: The price saving of buying petrol abroad increases and induces further cross-border shopping. Less is purchased at home.

Region III: The cost of cross-border shopping increases and cross-border shopping is discouraged as some of the petrol needed becomes more expensive.

Region IV: The region will expand as cross-border travel becomes more costly, and there is less cross-border shopping. This implies a discrete change in the purchases of petrol abroad and at home, as a number of shopping trips no longer take place. Within the region domestic purchases will decrease in response to the price increase.

Thus there are conflicting effects on cross-border shopping and demand for petrol of increasing its price. Those living neither too far from nor too close to the border are induced to do more cross-border shopping than before, whilst those living farther away are discouraged. Considering marginal welfare effects at the Pigovian starting point we get effects similar to those of (39) and (40) of the previous section. As the externalities of domestic purchases are internalised it is only changes in purchases abroad that affect welfare. The difference from the previous case is the conflicting effects on cross-border purchases of petrol from the mid region. The case for lowering the tax on petrol is weakened or even reversed.

## **5. VAT and one commodity purchased abroad.**

We will now consider purely fiscal commodity taxes, which may be perceived as value added taxes. We assume no external effects, but in other respects we retain the main features of the model above.

Let us assume that each consumer has a fixed income (endowment), which is expended on two commodities labelled 0,1. Domestic prices are  $Q_i = k_i + t_i$ , where  $k_i$  is the production cost and pre-tax price. Commodity 0 is always purchased at home. Commodity 1 is purchased at



home, or abroad. Let  $c_i(d)$  be the travel and transport cost of acquiring one unit of commodity one abroad. A necessary condition for purchases abroad is that  $q_i + c_i(d) < Q_i$ .

The budget constraint is

$$B = Q_0 y_0 + Q_1 y_1 + (q_1 + c_1(d)) y_{1a}. \quad (44)$$

Let  $\bar{d}$  be the threshold distance from the border at which the consumers stop buying abroad:

$$q_1 + c_1(\bar{d}) + K\bar{d} = Q_1 \quad (45)$$

The utility function is  $U(y_0, y_1, y_2)$ . The indirect utility function is:

$$\begin{aligned} V(Q_0, q_1 + c_1(d), B) & \text{ for } d \leq \bar{d} \\ V(Q_0, Q_1, B) & \text{ for } d > \bar{d} \end{aligned} \quad (46), (47)$$

We then have the demand functions

$$y_0 = y_0(Q_0, q_1 + c_1(d), B) \text{ for } d \leq \bar{d} \quad (48)$$

$$y_0 = y_0(Q_0, Q_1, B) \text{ for } d > \bar{d} \quad (49)$$

$$y_1 = y_{1a} = y_1(Q_0, q_1 + c_1(d), B) \text{ for } d \leq \bar{d} \quad (50), (51)$$

$$y_1 = y_{1h} = y_1(Q_0, Q_1, B) \text{ for } d > \bar{d}$$

To simplify notation the following aggregates are introduced:

$$\begin{aligned} Y_{1h} &= \int_{\bar{d}}^D y_1 f(s) ds \\ Y_0 &= \int_0^D y_0 f(s) ds \end{aligned} \quad (52), (53)$$

We note that

$$\frac{\partial Y_{1h}}{\partial Q_1} = \int_{\bar{d}}^D \frac{\partial y_1}{\partial Q_1} f(s) ds - \bar{y}_1 f(\bar{d}) \bar{d}' \quad (54)$$

$$\text{where } \bar{y}_1 = y_1(Q_0, q_1 + c_1(\bar{d}), B) = y_1(Q_0, Q_1, B) \quad (55)$$

$$\bar{d}' = \frac{\partial \bar{d}}{\partial Q_1} \quad (56)$$

The tax revenue is

$$T = t_0 Y_0 + t_1 Y_{1h} \quad (57)$$

Total welfare is

$$W = \int_0^{\bar{d}} V(Q_0, q_1 + c_1(s), B) f(s) ds + \int_{\bar{d}}^D V(Q_0, Q_1, B) f(s) ds \quad (58)$$

Since it is not the aim of this paper to discuss distributional issues, I will assume that the same welfare weight is given to a marginal income unit for everybody and set the marginal utility of income equal to a constant, which for simplicity may unity. A further simplification is to assume that the marginal propensity to consume a commodity is the same for all consumers, which serves the purpose of avoiding otherwise cumbersome formulae.

Let  $T_i$  and  $W_i$  denote the respective derivatives with respect to  $t_i$ . To assess a marginal tax reform we need to consider the additional tax revenue per unit of real income foregone by the consumers. If higher for commodity 0 than for commodity 1, there is a case for a marginal tax reform shifting some of the tax burden from commodity 1 to commodity 0. One can then reduce the loss of real income and increase welfare, while maintaining the total tax revenue. At the optimum the ratio of additional tax revenue to the marginal burden on the consumer should be equated across taxes.

Let  $\sigma_{ij}$  denote the aggregate Slutsky elasticity of commodity  $i$  with respect to price  $j$ .

Moreover, a ‘bar’ is used to denote aggregates referring to the region between the limits  $\bar{d}$  and  $D$ , and a superscript  $\theta$  is used to indicate aggregates in the region between 0 and  $\bar{d}$ .

Thus for example  $\bar{\sigma}_{00}$  denotes the aggregate direct Slutsky elasticity of good 0 in the region beyond  $\bar{d}$ , and  $Y_1^\theta$  is the aggregate consumption of commodity 1 in the region  $[0, \bar{d}]$ .

If initially there is uniform percentage taxation

$$\frac{T_0}{-W_0} - \frac{T_1}{-W_1} = \frac{t_0}{Q_0} \frac{Y_0^\theta}{Y_0} \sigma_{00}^\theta + \frac{t_1}{Q_1} Q_1 y_{1B} \frac{Y_0^\theta}{Y_0} + t_1 \frac{\bar{y}_1}{\bar{Y}_1} f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_1} \quad (59)$$

as derived in the appendix. To interpret the effects we should note which are the tax distortions prior to a tax reform. Firstly, there is a distortion in the border region  $[0, \bar{d}]$  discriminating against the commodity purchased domestically (and labelled  $\theta$ ). There is a tax on commodity  $\theta$ , while commodity 1 is virtually untaxed in this region for the reason that it is purchased abroad and subject to no domestic tax. The actual marginal resource cost of the commodity is the cost of acquiring it abroad, and that is exactly the cost facing domestic

consumers. There is no tax wedge<sup>7</sup>. Since only commodity 0 is taxed, its consumption is tax discouraged, and this distortion is being aggravated when the tax is increased. This is the first effect on the right hand side of (59). The effect is stronger the more elastic the demand for the taxed commodity, and the larger the share of consumption of commodity 0 that takes place in the border region.

Taxation is higher in the region further away from the border since both goods are in effect subject to domestic taxes there. An effect of shifting taxes from commodity 1 to commodity 0 is that disposable income decreases in the border region where people, not paying *domestic* tax on commodity one, do not benefit from the tax cut. On the other hand there is a net tax relief in the high tax region where people pay all domestic taxes. The total effect is to shift disposable income and demand to the high-tax region stimulating taxed consumption, which is good for efficiency as taxed commodities are being under-consumed (by first-best standards). The magnitude of pre-existing distortions is mitigated, and there is an efficiency gain expressed by the second term on the right hand side of (59). We note that the effect is stronger the larger the marginal propensity to spend income on commodity one.

Finally, there is a distortion because real costs are incurred in pursuit of the price saving obtained by cross-border shopping, as discussed already in the introduction. Lowering the tax on commodity 1 will induce marginal cross-border shoppers to shift their purchases to the domestic market, which has a beneficial effect on economic efficiency. The effect is captured by the third term on the right hand side of (59).

As the tax reform proceeds beyond a marginal change, a distortion will be created in the non-cross-border shopping region as commodity 0 becomes more heavily taxed than commodity 1, and also this further harmful effect must be allowed for at the optimum, characterised by:

$$\frac{T_0}{-W_0} - \frac{T_1}{-W_1} = \frac{t_0}{Q_0} \frac{Y_0^0}{Y_0} \sigma_{00}^0 + \left( \frac{t_0}{Q_0} - \frac{t_1}{Q_1} \right) \left( \bar{\sigma}_{00} \frac{\bar{Y}_0}{Y_0} + \bar{\sigma}_{11} \right) + \frac{t_1}{Q_1} Q_{1Y_{1B}} \frac{Y_0^0}{Y_0} + t_1 \frac{\bar{y}_1}{\bar{Y}_1} f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_1} = 0 \quad (60)$$

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<sup>7</sup> The foreign tax is part of the real resource cost of the home country and has no distortionary effect.

We note that in the special case of no cross-border shopping the last term vanishes as do the first and the third since  $Y_0^0=0$ . Then, unsurprisingly, we have the optimality condition prescribing uniform percentage taxes on the two commodities.

$$\left(\frac{t_0}{Q_0} - \frac{t_1}{Q_1}\right) = 0 \tag{61}$$

We should note that when one envisages a marginal tax reform as a response to cross-border shopping there are both positive and negative first-order effects. Hence it is not even obvious that one should take any step towards containing cross-border shopping. Reforming taxes to contain cross-border shopping is not worthwhile if demand for the non-tradable good is sufficiently elastic. The more elastic the demand the more serious is the aggravation of the pre-existing distortion as a tax reform is introduced and the more serious becomes the new distortion created by differentiating tax rates. We note that whilst the distortion in the non-border-shopping region becomes more serious as the tax rates move apart, the other effects tend to diminish as  $Y_0^0$  falls and  $\bar{Y}_0$  and  $\bar{Y}_1$  increase.

## 6. Conclusion.

When the residents of the home country have the opportunity to shop abroad, domestic commodity taxes distort the shopping behaviour by providing private incentives to use up real resources in pursuit of price savings abroad. In addition, paying foreign rather than domestic taxes, is a real resource cost for the national economy, even if it is a matter of indifference for the individual consumer. Using up real resources in order to acquire goods at a lower tax abroad may appear beneficial for the individual consumer, but is costly for the home country. It follows that one should aim at averting strong incentives to cross-border shopping, which indicate that one should compromise the original aim pursued by the tax. The paper has illustrated such a case by showing that a purely externality-motivated tax should be set below the Pigovian level in order not to induce too much cross-border trade. This result supports the intuition that taxes should be moderated when they induce cross-border shopping.

More, importantly the paper has gone on to discuss that in general further considerations have to be made since cross-border shopping may generate more complex distortions. Addressing fiscal taxes, and assuming that in the first place one would like to have neutral commodity

taxes,<sup>8</sup> two distortions are relevant when considering a marginal tax reform: i. The distorted choice between shopping at home and abroad, as in the externality case, and ii. Consumption in the border region is distorted in disfavour of the commodity purchased at home. Lowering the tax on the cross-border traded good, and increasing other taxes to maintain government revenue, will alleviate the detrimental incentive to go abroad for shopping, but will aggravate the distortion in disfavour of the domestic good. Both these are first order effects when starting from neutral taxes, and one will have to invoke quantitative magnitudes in order to sign the net effect.

Another extension of the simple benchmark case, is to consider more than one cross-border traded commodity. Previous research has shed light on how a commodity should be taxed, conditional on its characteristics, when subject to cross-border trade. However, this approach has dealt separately with each commodity, and it is not clear that single commodity rules generalise to the case of joint purchases abroad of more than one commodity. We may then have combinations of goods purchased entirely abroad and goods purchased partially abroad. This is consistent with empirical evidence that the trade pattern is commodity specific and changes systematically with distance from the border.

When a bundle of commodities is purchased abroad one has to observe how changing the domestic price of a commodity affects the marginal price of purchases abroad. If a consumer purchases his entire amount of a commodity abroad<sup>9</sup>, changing its domestic price will not affect its import price for the consumer. However, changing the price of a commodity *partially* purchased abroad may affect the import price of the whole bundle, and thus the consumer's amount of cross-border shopping. This demonstrates the need to consider the commodities as a jointly purchased bundle of commodities that should not only be taxed according to their individual characteristics, but also according to the effects of their respective taxes on the import price of the bundle.

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<sup>8</sup> Neutrality here means uniform percentage taxation.

<sup>9</sup> It is assumed that the consumer is not himself a marginal cross-border shopper in the sense that he is indifferent between buying his bundle abroad and at home.

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## Appendix

### Section 3

Let us consider the tax effects related to eq. (18)

$$\frac{\partial T}{\partial t_1} = \hat{Y}_{1h} + \bar{Y}_1 + t_1 \frac{\partial \hat{Y}_{1h}}{\partial Q_1} + t_1 \frac{\partial \bar{Y}_1}{\partial Q_1} + t_1 \bar{y}_{1h} f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_1} - t_1 \bar{y}_1 f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_1} - t_2 \bar{y}_2 f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_1} \quad (A1)$$

as  $\hat{y}_{1h} = 0$  and  $y_{2h} = 0$  in  $[0, \bar{d}]$ .

$$\frac{\partial T}{\partial t_1} = \hat{Y}_{1h} + \bar{Y}_1 + t_1 \frac{\partial \hat{Y}_{1h}}{\partial Q_1} + t_1 \frac{\partial \bar{Y}_1}{\partial Q_1} - t_1 \bar{n} f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_1} - t_2 \bar{n} f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_1} \quad (A2)$$

as  $y_{1h} = y_1 - n$ , and  $\bar{y}_2 = \bar{n}$ . We substitute from  $Y_{ih} = Y_i - N$ .

$$\frac{\partial T}{\partial t_1} = \hat{Y}_{1h} + \bar{Y}_1 + t_1 \frac{\partial \hat{Y}_1}{\partial Q_1} - t_1 \frac{\partial \hat{N}}{\partial Q_1} + t_1 \frac{\partial \bar{Y}_1}{\partial Q_1} - t_1 \bar{n} f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_1} - t_2 \bar{n} f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_1} \quad (A3)$$

$$\frac{\partial T}{\partial t_2} = \bar{Y}_2 + t_2 \frac{\partial \bar{Y}_2}{\partial Q_2} - t_1 \bar{n}f(\bar{d})\bar{d}' - t_2 \bar{n}f(\bar{d})\bar{d}' \quad (\text{A4})$$

There is a standard effect of  $t_2$  in the domestic market and in addition tax revenue from both taxes are lost when there is an expansion of the area from which people do cross-border shopping.

$$\frac{\partial E}{\partial t_1} = e_1 \frac{\partial \hat{Y}_1}{\partial Q_1} + e_1 \frac{\partial \bar{Y}_1}{\partial Q_1} + e_2 \frac{\partial \hat{Y}_2}{\partial Q_1} \quad (\text{A5})$$

$$\frac{\partial E}{\partial t_2} = e_2 \frac{\partial \bar{Y}_2}{\partial Q_2} \quad (\text{A6})$$

The welfare effects of the taxes are derived from eq. (20). To find the effect on individual utility we differentiate U making use of the envelope theorem, and then integrate to find the total welfare effects. We easily recognise that the utility level is a continuous function of distance from the border.

$$\frac{\partial W}{\partial t_1} = -\hat{Y}_{1h} - \bar{Y}_1 + \hat{Y}_{1h} + \bar{Y}_1 + t_1 \frac{\partial \hat{Y}_1}{\partial Q_1} - t_1 \frac{\partial \hat{N}}{\partial Q_1} + t_1 \frac{\partial \bar{Y}_1}{\partial Q_1} - t_1 \bar{n}f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_1} - t_2 \bar{n}f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_1} - e_1 \frac{\partial \hat{Y}_1}{\partial Q_1} - e_1 \frac{\partial \bar{Y}_1}{\partial Q_1} - e_2 \frac{\partial \hat{Y}_2}{\partial Q_1} \quad (\text{A7})$$

Eliminating terms that cancel out

$$\frac{\partial W}{\partial t_1} = t_1 \frac{\partial \hat{Y}_{1h}}{\partial Q_1} + t_1 \frac{\partial \bar{Y}_1}{\partial Q_1} - t_1 \bar{n}f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_1} - t_2 \bar{n}f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_1} - e_1 \frac{\partial \hat{Y}_{1h}}{\partial Q_1} - e_1 \frac{\partial \hat{N}_1}{\partial Q_1} - e_1 \frac{\partial \bar{Y}_1}{\partial Q_1} - e_2 \frac{\partial \hat{Y}_2}{\partial Q_1} \quad (\text{A8})$$

Analogously we get

$$\frac{\partial W}{\partial t_2} = t_2 \frac{\partial \bar{Y}_2}{\partial Q_2} - t_1 \bar{n}f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_2} - t_2 \bar{n}f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_2} - e_2 \frac{\partial \bar{Y}_2}{\partial Q_2} \quad (\text{A9})$$

## Section 5

From the main text

$$W = \int_0^{\bar{d}} V(Q_0, q_1 + c_1(s), B) f(s) ds + \int_{\bar{d}}^D V(Q_0, Q_1, B) f(s) ds \quad (\text{A10})$$

and

$$T = t_0 Y_0 + t_1 Y_{1h}. \quad (\text{A11})$$

Let  $T_i$  and  $W_i$  denote the respective derivatives with respect to  $t_i$ . Making use of the assumptions that welfare weights attached to real income changes are unity, and income derivatives are constant, we easily find that

$$W_0 = -Y_0 \quad (\text{A12})$$

$$W_1 = -\bar{Y}_1 \quad (\text{A13})$$

$$T_0 = Y_0 + t_0 S_{00} + t_1 \bar{S}_{10} - t_0 y_{0B} Y_0 - t_1 y_{1B} \bar{Y}_0$$

$$T_1 = \bar{Y}_1 + t_0 \bar{S}_{10} + t_1 \bar{S}_{11} - t_0 y_{0B} \bar{Y}_1 - t_1 y_{1B} \bar{Y}_1 - t_1 \bar{y}_1 f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_1} \quad (\text{A14})$$

where price effects have been subjected to the Slutsky decomposition, and the following notation has been used:  $s_{ij}$  is the Slutsky derivate of the demand for commodity  $i$  with respect to the price of commodity  $j$ , and  $S_{ij}$  is the corresponding aggregate derivative

$$S_{ij} = \int_0^D s_{ij} f(a) da. \text{ Below } \sigma_{ij} \text{ is used to denote the corresponding Slutsky elasticity.}$$

$y_{iB} = \partial y_i / \partial B$  is the income derivate, which is treated as constant by the assumption above. A 'bar' is used to denote aggregates referring to the region between the limits  $\bar{d}$  and  $D$ , and a superscript  $0$  is used to indicate aggregates in the region between  $0$  and  $\bar{d}$ . Thus for example  $\bar{\sigma}_{00}$  denotes the aggregate Slutsky elasticity in the region beyond  $\bar{d}$ .

The following fractions are established

$$\frac{T_0}{-W_0} = 1 + \frac{t_0}{Q_0} \frac{S_{00} Q_0}{Y_0} + \frac{t_1}{Q_1} \frac{\bar{S}_{10} Q_1 \bar{Y}_0}{\bar{Y}_0 Y_0} - \frac{t_0}{Q_0} Q_{0Y_{0B}} - \frac{t_1}{Q_1} Q_{1Y_{1B}} \frac{\bar{Y}_0}{Y_0} \quad (\text{A15})$$

$$\frac{T_1}{-W_1} = 1 + \frac{t_1}{Q_1} \frac{\bar{S}_{11} Q_1}{\bar{Y}_1} + \frac{t_0}{Q_0} \frac{\bar{S}_{10} Q_0}{\bar{Y}_1} - \frac{t_0}{Q_0 \bar{Y}_1} Q_{0Y_{0B}} \bar{Y}_1 - \frac{t_1}{Q_1 \bar{Y}_1} Q_{1Y_{1B}} \bar{Y}_1 - \frac{1}{\bar{Y}_1} t_1 \bar{y}_1 f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_1} \quad (\text{A16})$$

$$\frac{T_0}{-W_0} - \frac{T_1}{-W_1} = \frac{t_0}{Q_0} \frac{Y_0^0}{Y_0} \sigma_{00}^0 + \left( \frac{t_0}{Q_0} - \frac{t_1}{Q_1} \right) \left( \bar{\sigma}_{00} \frac{\bar{Y}_0}{Y_0} + \bar{\sigma}_{11} \right) + \frac{t_1}{Q_1} Q_{1Y_{1B}} \frac{Y_0^0}{Y_0} + t_1 \frac{\bar{y}_1}{\bar{Y}_1} f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_1}, \quad (\text{A17})$$

where the Slutsky symmetri  $S_{ij} = S_{ji}$ , and the homogeneity properties  $\sigma_{i0} + \sigma_{i1} = 0$ , have been employed, and  $\sigma_{00}$  has been decomposed. If initially there is uniform percentage taxation



$$\frac{T_0}{-W_0} - \frac{T_1}{-W_1} = \frac{t_0}{Q_0} \frac{Y_0^0}{Y_0} \sigma_{00}^0 + \frac{t_1}{Q_1} Q_1 y_{1B} \frac{Y_0^0}{Y_0} + t_1 \frac{\bar{y}_1}{\bar{Y}_1} f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_1} \quad (\text{A18})$$

or

$$\frac{T_0}{-W_0} - \frac{T_1}{-W_1} = \frac{t_0}{Q_0} \frac{Y_0^0}{Y_0} (Q_1 y_{1B} - (-\sigma_{00}^0)) + t_1 \frac{\bar{y}_1}{\bar{Y}_1} f(\bar{d}) \frac{\partial \bar{d}}{\partial Q_1}. \quad (\text{A19})$$