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Required Share Returns in a Tobin's  $q$  Model**

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# The Equity Risk Premium and the Required Share Returns in a Tobin's $q$ Model<sup>1</sup>

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**Abstract.** Based on the Tobin's  $q$  principle this paper shows that earnings per unit of capital and the output capital ratio are excellent measures of the required share returns because they are only temporarily affected by earnings shocks but are driven permanently by changes in required share returns. Evidence for the US over the period from 1889 to 2002 suggests that real required share returns and the equity risk premium climbed to extraordinarily high levels from the late 1930s, to the end of the 1940s, and have since declined. The risk premium is currently somewhere between 4 and 6%. (*JEL* G12, E22, E44).

**Key words:** Expected share returns, equity risk premium, Tobin's  $q$ , share valuation, and macroeconomic factors.

## 1 Introduction

A recent controversy in the finance literature is whether the high *ex post* equity premium experienced in the 20<sup>th</sup> century will remain high in this century or whether it has been permanently reduced in the postwar period and, therefore, that shareholders can expect a substantial reduction in returns to shares relative to bonds. Some argue that there has been a reduction in the required returns to equity and the equity premium in the postwar period (Arnott and Bernstein, 2002, Arnott and Ryan, 2001, Blanchard, 1993, Claus and Thomas, 2001, Fama and French, 2002, Heaton and Lucas, 1999, Jagannathan *et al.*, 2001). Others argue that share returns are relatively constant in the long run and therefore that the share prices will eventually converge to a level that is compatible with historical real share returns (Shiller, 2001, Smithers and Wright, 2000).

Unfortunately, the *ex post* cannot be used for *ex ante* equity premiums and share returns, as repeatedly stressed at the Equity Premium Forum (2002), predominantly because *ex ante* and *ex post* returns go in opposite directions in response to changes in the required returns. Share returns are consequently misleading indicators of the required share returns in periods when required share returns are changing, and, therefore, in the periods when the identification of the time-profile of the required share returns is most needed. A common approach is to use Gordon's growth model, or its dynamic counterpart, where the expected or required share returns are equal to the expected dividend price ratio plus the expected growth in dividends or earnings per share. Whereas the expected

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dividend price ratio is approximately equal to the actual dividend price ratio and is, therefore, readily available, expected growth in dividends and earnings are not.

Arnott and Bernstein (2002) and Arnott and Ryan (2001), among others, proxy expected growth in dividends as the expected per capita GDP growth rate minus a fixed delusion rate. Others base expected growth in earnings on survey earnings forecasts. The difficulty associated with this method is that the prediction horizon in survey forecasts does not go beyond five years, and the results are consequently highly sensitive to the assumptions regarding expected growth in dividends beyond the five-year forecast horizon. Using the same survey data for the US, Harris and Marston (2001) find the equity premium to be 7.14% whereas Claus and Thomas (2001) find the equity premium to be of around 3%. This discrepancy suggests that the measured equity premium is highly sensitive to the underlying assumptions about the *post* five-year forecast horizon, particularly. Another problem that is associated with the use of survey data is that expected earnings growth exceeded realised earnings growth by 8.5 percentage points in the US over the period from 1979 to 2000 (Sharpe, 2002); thus furthering the difficulties that are associated with the use of survey data for recovering expected returns.

Based on rational pricing Fama and French (2002) argue that the average historical growth in dividends is the relevant measure of expected dividend growth because it is largely unpredictable. Using the postwar average growth in dividends and earnings per share, Fama and French (2002) find the equity premium, which is unadjusted for the variance in returns, to be 2.55% and 4.32%, respectively. On the other hand, Barsky and De Long (1993) argue that rational pricing should be based on a weighted moving average scheme of dividend growth, where the recent past carries the highest weight. Given the strong growth in dividends over the past decade, the model of Barsky and De Long (1993) implies an expected growth in dividends that is well in excess of the figures presented by Fama and French (2002).

Finally, Blanchard (1993) uses the conditional forecast of dividend growth to recover the expected returns to equity. This method requires that shareholders use the information contained in the conditional variables when they forecast dividend growth. Whereas this assumption may be valid for short-run forecasts, it is unlikely to be useful for long run forecasting given that dividend growth is dominated by fluctuations on business cycle frequencies. Overall, the difficulty associated with the Gordon model approach is that the results are overly sensitive to the assumptions underlying the model of earnings growth expectations.

This paper suggests a complementary method of measuring the level and the historical path of the required share returns that requires *no* assumptions about the underlying process that generates expected growth in dividends and only requires data that are readily available from national accounts. It is shown that the required share returns vary proportionally with earnings per unit of

capital and the output capital ratio. Earnings per unit of capital and the output capital ratio will temporarily deviate from their steady state values following demand shocks, technology shocks and supply shocks, but will, within a short period in time, return to their long run equilibrium due to an endogenous adjustment of the capital stock. It is shown that the steady state values of earnings per unit of capital and the output capital ratio are unaffected by all shocks except shocks to the required returns provided that the tax structure remains unaltered. From this it follows that a change in the required share returns is reflected by a proportional change in earnings per unit of capital and the output capital ratio. This explains the apparent empirical paradox that share returns are negatively related to the growth in earnings per unit of capital and the output capital ratio, as shown in the empirical section.

Using long historical data for the US it is shown that both the required share returns and the equity premium peaked in the 1940s and first reached their *pre*-Depression levels in the beginning of the 1980s. It is, furthermore, shown that the post-war decline in the required share returns explains a large proportion of the increase in share prices over the same period. The required share returns have today reached a plateau of about 7% and the equity premium over the past two decades has fluctuated between 4 and 6%, which is close to the level that prevailed before the Great Depression.

## 2 A macroeconomic model of equilibrium expected returns

The investment decision of the shareholder and the firm's investment decision are determined jointly. The shareholder determines the required share returns and the firm continues to undertake investment projects until the expected returns to investment equal the required returns to equity. A positive shock that increases the returns to investment in excess of the required returns initiates investment that terminates when the gap between returns to investment and the required returns has been closed and, therefore, when earnings per unit of capital of the last investment project equals its initial value. In this section this principle is used to derive equilibrium conditions for firms and investors.

The objective function of the firm under Cobb-Douglas technology is given by:

$$\max \Omega = \int_{t=0}^{\infty} e^{-\rho t} \left[ (1-\tau) \{ P_t \cdot A L_t^\alpha K_t^{1-\alpha} - W_t L_t - P_t^i C(I_t) \} - P_t^i I_t (1-\Gamma_t) \right] dt,$$

st.

$$K_{t+1} = I_t + (1-\delta)K_t,$$

where  $\Omega$  is the present value of nominal profits,  $P$  is the value-added price-deflator,  $K$  is capital services,  $I$  is net investment,  $W$  is the wage rate,  $L$  is labour services,  $\rho$  is the required returns to equity,  $C(I)$  is the adjustment cost of investment as a positive function of investment,  $\delta$  is the rate of capital depreciation,  $\tau$  is the corporate tax rate,  $P^i$  is the investment deflator,  $A$  and  $\alpha$  are fixed

parameters, and  $\Gamma$  is the sum of investment tax credits and the present value of expected future depreciation allowances for tax purposes as a percentage of acquisition costs.<sup>2</sup> The subscript  $i$  is omitted for simplicity.

The current-value Hamiltonian of this optimisation problem is given by:

$$H = (1 - \tau)[P_t A L_t^\alpha K_t^{1-\alpha} - W_t L_t - P_t^i C(I_t)] - P_t^i I_t (1 - \Gamma_t) + Q_t [I_t - \Delta K_t - \delta K_t], \quad (1)$$

where  $Q$  is the shadow price of capital or Tobin's  $q$ . The first order conditions for optimum under the assumption of perfect competition in the goods market are:

$$(1 - \tau)(1 - \alpha)(Y_t / K_t) = (\rho + \delta)Q_t - \dot{Q}_t, \quad (2)$$

$$\lim_{t \rightarrow \infty} e^{-\rho t} Q_t K_t = 0. \quad (3)$$

$$1 - \Gamma_t + C'(I_t)(1 - \tau) = Q_t / P_t^i, \quad (4)$$

where Equation (3) is the transversality condition, which rules out explosive bubbles and Equation (2) is equity market equilibrium in which the required returns to equity equal expected capital gains plus after tax earnings per unit of capital. After-tax earnings per unit of capital are given by the term  $(1 - \tau)(1 - \alpha)(Y_t / K_t)$ , where the marginal productivity of capital is  $(1 - \alpha)(Y_t / K_t)$  under the Cobb-Douglas technology assumption.

Equation (4) can be rewritten as the Tobin's  $q$  model of investment:

$$\dot{K}_t = F[(Q_t / P_t^i + \Gamma_t - 1) / (1 - \tau)], \quad (5)$$

where  $I = 0$  for  $(Q_t / P_t^i + \Gamma_t - 1) = 0$ ,  $I > 0$  for  $(Q_t / P_t^i + \Gamma_t - 1) > 0$ , and  $I < 0$  for  $(Q_t / P_t^i + \Gamma_t - 1) < 0$ . In the absence of taxes Tobin's  $q$  is one. However, as shown by Summers (1981) and Hayashi (1982), Tobin's  $q$  is not one in the presence of investment tax credits and depreciation of capital for tax purposes, because they lower the effective acquisition cost of investment and therefore lower the benchmark level of  $Q/P^i$  at which investment is undertaken.

### 3 Equilibrium required returns

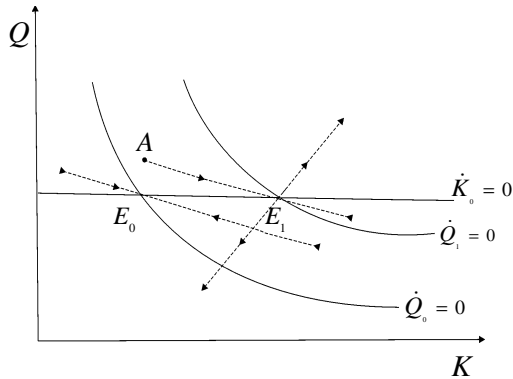
Equations (2) and (5) form a simultaneous first-order differential system that will be used in this section to show that the equilibrium  $Y/K$  and earnings per unit of capital are only affected by

<sup>2</sup> The required returns to equity are assumed constant from the *decision period* to infinity. A varying discount rate can be readily incorporated into the model. However, it is probably most realistic to assume that investors have approximately constant required returns to infinity *when* they make the investment decision. The discount factor may then unexpectedly change in the periods between investment decisions.

permanent shifts in the required returns and the composition of corporate taxes and tax credits. The implication of this is that steady state movements in the  $Y/K$  ratio will echo movements in required returns provided that the composition of taxes remains constant. The dynamics of the system are displayed in Figure 1. The  $\dot{Q}_t = 0$  curve slopes downwards due to diminishing returns to capital.

Consider an unanticipated shock to the economy that enhances earnings per unit of capital (Figure 1). This could be a technology innovation, a reduction in union power, a reduction in real oil or import prices, or a positive demand shock. The slope of the  $\dot{Q} = 0$  curve steepens because earnings per unit of capital have increased. The perfect foresight share market jumps from  $E_0$  to  $A$  on impact to capitalise on the temporarily higher earnings. Since  $(Q_t / P_t^i + \Gamma_t - 1) > 0$ , a capital accumulation process is initiated, which reduces earnings per unit of capital due to diminishing returns to capital. The capital accumulation terminates when  $Q$  is back at its initial level at the point  $E_1$ , at which earnings per unit of capital, and hence the  $Y/K$  ratio, are equal to their initial levels. Share prices have consequently only been temporarily affected by the shock.

**Figure 1. Dynamics of  $Q$  and  $K$**



The dynamic relationship between earnings shocks, share prices and capital accumulation as displayed in Figure 1, can be observed in an historical context. Many, but not all, significant share market booms that have been triggered by positive demand and supply shocks have been associated with investment booms. These are, for instance, the investment and share market booms over the following periods: 1900-1907, 1921-29, 1970-73, and 1993-2000. In all these instances the extraordinarily high earnings per unit of capital have lasted for a short period in time because capital accumulation drove earnings per unit of capital down to their initial levels.

Next consider a reduction in the required share returns to equity that is expected to be permanent. This results in a steepening of the  $\dot{Q} = 0$  curve and we get the dynamic path that is displayed in Figure 1 and the economy again ends up at the point  $E_1$ . The dynamic path is similar to

the positive earnings shock that was analysed above. However, in contrast to the earnings shock, where the steady state  $Y/K$  ratio remains unaffected, the reduction in the required returns has permanently reduced the  $Y/K$  ratio and earnings per unit of capital. This is because  $Y$  changes by less than  $K$  in response to changes in the required returns. An earnings shock, however, will lead to an adjustment in the capital stock that exactly matches the earnings-shock-induced change in  $Y$ .

More formally, earnings per unit of capital are *only* affected by the required returns to capital and the tax structure in the steady state as can be seen from the following expression of the steady-state after-tax-earnings per unit of capital:

$$\Psi^* = (1-\tau)(1-\alpha)\left(\frac{Y}{K}\right)^* = (\rho + \delta)(1-\Gamma), \quad (6)$$

where time subscripts are omitted from the equations for simplicity and asterisks signify steady-state values. Here,  $\Psi$  is the after-tax earnings per unit of capital. This equation shows that the *after-tax* earnings per unit of capital are only affected by changes in  $\rho$  and  $\Gamma$  in the steady state. The steady-state value of the after tax earnings per unit of capital is unaffected by changes in other variables.

Rewriting (6) yields the two key equations for  $\rho$ :

$$\rho^{Y/K} = (1-\alpha)\left(\frac{1-\tau}{1-\Gamma}\right)\left(\frac{Y}{K}\right)^* - \delta, \quad (7)$$

and

$$\rho^E = \left(\frac{1-\tau}{1-\Gamma}\right)\Pi^*, \quad (8)$$

where  $\Pi$  is the pre-tax post-depreciation earnings per unit of capital,  $\rho^{Y/K}$  is the  $Y/K$ -based measure of required share returns and  $\rho^E$  is the earnings-based measure of the required returns. Given the parameters  $\alpha$  and  $\delta$  it is straightforward to compute the required returns since data for  $Y/K$  and  $\Pi$  are readily available from national accounts and firms' balance sheets and  $(1-\tau)/(1-\Gamma)$  is close to unity and shows little variation over time, as discussed below. Equation (8) is modified to allow for debt in the empirical section below.

Although Equations (7) and (8) are based on Tobin's  $q$  model, the validity of the models do not rest on the assumption that firms are all-equity provided that company managers have the same return requirements as shareholders. Firms based on non-equity finance will, as long as they maximise profits, also obey the Tobin's  $q$  rule in their investment decisions. They will invest until the expected returns to investment are equal to their required returns. Hence, Equations (7) and (8) are as applicable a century ago as they are today.

The capital stock in the model encompasses both tangible and intangible capital stock. On an aggregate level, however, it is important to distinguish between different items of intangible capital stock. As a general principle the items that increase value of aggregate earnings should be included in the aggregate capital stock. As Smithers and Wright (2000) forcefully argue, the value of goodwill, brand names, managers' ability and franchise value net out in the aggregate. That some companies do well due to these factors is counterbalanced by the below-average performances of other companies.

In contrast to other intangibles the value of R&D capital stock needs to be included in the aggregate capital stock because it increases aggregate earnings. The argument is essentially that R&D affects earnings positively in both the R&D intensive sector and other sectors of the economy despite widespread creative destruction within the R&D intensive sector. The potential costs of creative destruction are counterbalanced by higher earnings in the R&D intensive sector relative to other sectors in the economy, to compensate for the potential risk that is associated with creative destruction; otherwise investors would not hold shares in the R&D intensive sector. The same principle applies to tangible investment in the sense that the new technologies embodied in investment render old capital stock less productive and, therefore, reduce the value of older vintages of capital stock.

#### **4 Empirical evidence on the historical path of expected share returns and the equity premium**

Based on Equations (7) and (8), in this section, the required returns are calculated for the US over the period from 1889 to 2002. The equations are modified to allow for debt and R&D capital, and are cyclically adjusted. The data are cyclically adjusted to allow for the influence of temporary shocks on the  $Y/K$  ratio and earnings per unit of capital. The cyclical adjustment filters out the cyclical movements in required share returns, which gives the advantage of assessing the slow movements in the required returns free of business cycle frequencies.

##### **4.1 Required share returns**

The empirical counterparts of Equations (7) and (8) are the following:

$$\rho_t^{c,Y/K} = 0.3 \left( \frac{Y_t}{K_t^T + K_t^{R\&D}} \right) - \delta_t, \quad (9)$$

and

$$\rho_t^{c,E} = \frac{NOS_t + R \& D_t - \delta^{R\&D} K_t^{R\&D}}{(K_t^T P_t^T + K_t^{R\&D} P_t^{R\&D})(1-b)}, \quad (10)$$



where  $b$  is the leverage ratio,  $K^T$  is tangible capital stock,  $K^{R\&D}$  is R&D capital stock,  $NOS$  is net operating surplus, R&D is expenditures on R&D,  $P^T$  is a price index of tangible capital stock,  $P^{R\&D}$  is a price index of R&D capital stock,  $\delta$  is the overall depreciation rate,  $\delta^{R\&D}$  is the depreciation rate of R&D capital stock, and the superscript  $c$  stands for cyclically unadjusted. The leverage ratio is set to 0.25. The depreciation rate,  $\delta$ , is a weighted average of the depreciation rate for R&D capital stock (20%) and tangible capital stock (8%). The R&D capital stock is calculated using the perpetual inventory method based on R&D expenditures in the corporate sector after 1953 and patents in the corporate sector before that period, as detailed in the data appendix.

Capital's income share,  $(1 - \alpha)$ , is set to 0.3, which is standard in the literature, and  $(1 - \tau)/(1 - \Gamma)$  is set to one since it is only available over a short time-span. Based on the Summers' (1981) calculations of  $\Gamma$  and standard corporate tax rates, the term  $(1 - \tau)/(1 - \Gamma)$  is close to one and shows little variation over time. Earnings per unit of capital are adjusted for cyclical movements in income shares and output.<sup>3</sup> The  $Y/K$  ratio is adjusted for cyclical movements in income using the same principle. Both measures of the required share returns are based on data for the US corporate sector.

The estimates of the required share returns are displayed in Figure 2 over the period from 1889 to 2002. The earnings-based measure starts first in 1909 because no corporate earnings data are available before that date. The earnings-based measure fluctuates more than the  $Y/K$ -based measure, particularly in the pre-war period; otherwise the figures track each other closely. The path of required share returns confirms the suspicion in the literature that required share returns have declined since 1950 and that the high post-war share returns were, to a large degree, unexpected (Fama and French, 2002).

Perhaps the most intriguing feature of the figure is that the time-profiles in the estimated required returns, to a very high degree, track the time-profile of the marginal tax rates on dividend income as calculated by McGrattan and Prescott (2003) and shown in their Figure 1. McGrattan and Prescott (2003) argue that the strong increase in the dividend tax rate during WWII that peaked at 50%, and thereafter declined to a plateau of 15-20% in the mid 1980s, can explain a large fraction of the decline in the required returns and also the equity premium because bond returns were taxed

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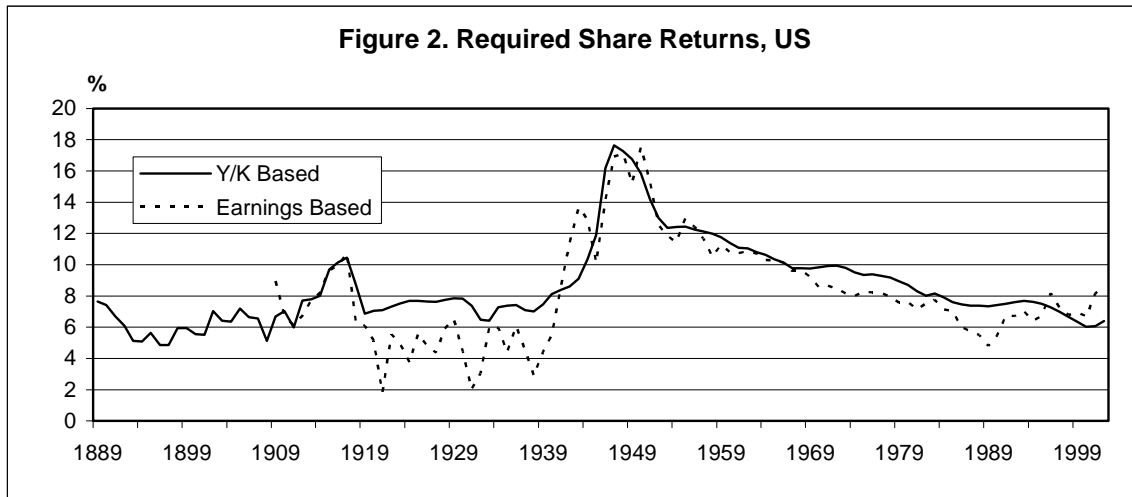
<sup>3</sup> More precisely, the cyclically adjusted earnings per unit of capital are computed as follows. First the following relationship is regressed using the Cochrane-Orcutt iterative procedure that allows for first order serial correlation:

$$\rho_t^{c,E} = a_0 + a_1 y_t^c + a_2 S_t^{L,c} + \varepsilon_t,$$

where  $y^c$  is cyclical income per capita, and  $S^{L,c}$  is the cyclical share of income going to labour in manufacturing. Cyclical income is measured as the difference between the log of per capita income minus an eight-year lagged average of the log of per capita income. The same method is used for cyclical factor shares. Thereafter, the coefficient estimates are used to calculate cyclically adjusted earnings per unit of capital as follows:

$$\rho_t^E = \rho_t^{E,c} - \hat{a}_1 y_t^c - \hat{a}_2 S_t^{L,c}.$$

much more leniently than share returns. Similar arguments are put forward by Siegel (2002b). A casual comparison of Figure 2 above and Figure 1 in McGrattan and Prescott (2003) suggests that the path of dividend taxes has played an important role for the time-path of the required share returns. Furthermore, Siegel (2002b) argues that the required share returns declined in the post-war period due to declining transaction costs, increasing macroeconomic stability, and because investors learned about the risk return relationship on stocks.



The time-path of the required share returns over the past two decades is consistent with Sharpe's (2002) time-series data. Based on the survey evidence on earnings and inflation expectations, Sharpe (2002) constructs the path of the real required share returns over the past two decades under the assumption that earnings expectations beyond year 12 are constant. His method cannot be used to determine the absolute level of required returns; only the relative movements over time. Sharpe finds that required share returns declined from 1983 to the share market crash in 1987, when it increased to a slightly higher plateau before decreasing further in the second half of the 1990s. This path is also evident in the estimates of the required returns in Figure 2, particularly the *Y/K*-based measure.

The hump-shaped required returns over the period from the late 1930s to the mid 1950s explain an apparent puzzle, namely that *ex post* real share returns were almost zero over the period from 1938 to 1948 despite the surge in earnings over the same period, which is evident from Figure 2. The risk premium increased to such an extent over this period that it counterbalanced the effects on real share returns of the increasing earnings over the same period. Similarly the surge in corporate earnings throughout WWI resulted in negative real share returns because the increasing required share return more than counterbalanced the increase in earnings. Conversely, the real accumulated share index increased fourfold from 1948 to 1956 despite the strong reduction in

earnings. Likewise, the real share returns was 4% annually from the end of 1917 to the end of 1919 in spite of a sharp drop in earnings per unit of capital. These examples highlight the sharp contrasts between movements in *ex post* and *ex ante* share returns; an issue that is examined using regression techniques in the next section.

The time-profile of the estimated required share returns has implications for the equilibrium P/E ratio. A puzzling issue in finance is 1) whether the P/E ratio is mean reverting; and 2) provided that the P/E ratio is mean reverting, what is the equilibrium P/E ratio? Ross (2002), for instance, argues that “issues involving P/E’s are issues involving whether or not these processes are mean reverting...we know that P/Es revert, but they seem to revert very slowly, and we are able to measure the revision with only great difficulty...our efforts to measure, for example, stock returns – not actual returns but expected returns – have basically been futile...we cannot observe the expected return on stocks even with stationarity in time-series data because volatility and the short periods of time we are able to analyse give us very little hope of pinning down the result” (p 22). Asness (2002) goes a step further by arguing that “an open issue is not mean reversion but whether the right P/E is 15” (p 15).

Based on the model in the previous section the steady-state P/E ratio equals  $1/\rho_t$  since the steady state growth in earnings per unit of capital is zero by implication of the model.<sup>4</sup> Thus, an important implication of the model is that the P/E ratio is mean reverting but that the mean changes over time as  $\rho$  changes. The share market will capitalise on the higher earnings in response to a positive earnings shock and consequently drive the P/E ratio temporarily above its long-run equilibrium. The P/E will return to its long-run equilibrium given by  $\rho^{-1}$ , when the capital stock has reached its new desired level.

The estimates of  $\rho$  above suggest that the equilibrium P/E ratio has been in the range of 12.5 and 16.7 over the past 20 years, which is close to the equilibrium value that has prevailed in most of the pre WWII period. Seen over a very long time-span there has been a tendency for mean-reversion of the P/E ratio because  $\rho$  has, at least over the very long run, also shown mean reverting behaviour. The insight of McGrattan and Prescott (2003) suggests that the dividend tax rate has been a major force behind the historical path of equilibrium required share returns.

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<sup>4</sup> From Gordon’s growth model it follows that

$$\frac{P}{E} = \frac{1-\kappa}{\rho-g} = \frac{1-\kappa}{\rho-r\kappa} = \frac{1-\kappa}{\rho-\rho\kappa} = \frac{1}{\rho},$$

where  $g$  is the expected growth in earnings and  $r$  is the returns to new investment, which may temporarily deviate from  $\rho$ , but is equal to  $\rho$  in the steady state.

## 4.2 The equity premium

Although required share returns are important for the equilibrium P/E ratio and *ex post* and *ex ante* share returns, it is, after all, the equity risk premium that has probably been the most controversial issue in finance and opinions span from a zero risk premium (Arnott and Ryan, 2001) to 7% among finance professors (Welch, 2000). This section sheds light on the current equity risk premium, its historical path and gives a brief discussion of its determinants.

The equity risk premium is calculated as the real expected returns on shares minus the expected real returns on bonds:

$$EP_t = \rho_t - (r_t^B - \pi_t^e),$$

where  $EP$  is the equity premium,  $r^B$  is the expected nominal returns on bonds and  $\pi^e$  is the expected inflation over the maturity period of the bond. The  $Y/K$ -based measure as derived in the previous section, is used for  $\rho$  because it goes further back than the earnings based measure. The expected nominal returns on bonds are approximated by the actual end-year short and long nominal interest rate on government bonds. The expected inflation is the most critical variable. Inflation expectations are not measured as *ex post* inflation because after the abolition of the gold standard in 1933, inflation was largely unexpected. Siegel (2002b), for instance, argues that *ex post* bond returns were biased downwards, and the *ex post* equity premium consequently biased upwards, because investors failed to understand the “inflationary implications of the shift from the gold standard to paper monetary standard” (p 31).

Inflation expectations are measured as follows. The Livingston survey data on expected inflation estimated by professional forecasters over the next 12 months are used over the period from 1947 to 2002.<sup>5</sup> The last quarter forecast of each year is used so that the expected inflation in 1947, for instance, is measured as the survey expected inflation in the 4<sup>th</sup> quarter of 1947.<sup>6</sup> Over the period from 1889 to 1947 the expected inflation is set to zero following the discussion in the literature on the expected inflation. This assumption is also consistent with the assumption of McGrattan and Prescott (2003) that the expected rate of inflation was zero in the gold standard period.

In the most careful study of inflation expectations in the US over the period from 1870 to 1914, Barsky and De Long (1991) argue that the deflation from 1879 to 1896 and the inflation from 1896 to 1913 were never built into inflation expectations. Their time-series evidence using ARIMA models suggests that univariate models could not predict inflation. Investors could have used more

<sup>5</sup> The survey data are available from the Federal Reserve Bank of Philadelphia from 1970. Before then the data were published twice a year in *Philadelphia Sunday Bulletin*.

<sup>6</sup> Inflation expectations are often measured as a moving average of past inflation. The problem with this approach is that it does not allow for unexpected inflation, which seems to have been a major force behind the low real interest rates in most of the post-war period as discussed above.

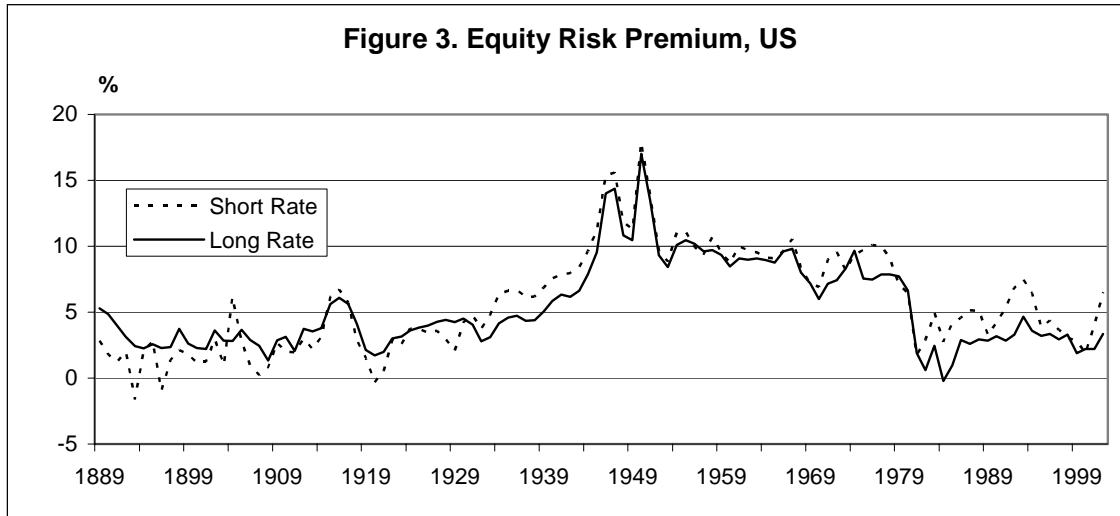
sophisticated models that allowed prices to be functions of world gold production since the US, and most of the rest of the world, was on the gold standard after 1879 and on a bimetallic standard before then. The world gold production would consequently be the most important determinant of the general price level. In fact Barsky and De Long (1991) find that world gold production is a statistically significant predictor of prices. However, studying the quantity-theoretical-leaning *Economist*, Barsky and De Long (1991) find no evidence that the gold boom over the period from 1895 to 1913 was ever embodied into inflation expectations. They find ‘no consensus of informed views and no explicit forecasts of positive inflation in the years before WWI’ (p 829). Based on this evidence they conclude that inflation expectations were close to zero in the period up to 1914.

Apart from the Great Depression period, very little research has been undertaken on inflation expectations over the period from 1914 to 1946. Whether inflation was expected during the Great Depression has been a highly controversial issue among economic historians and macroeconomists because inflation expectations play a key role in the identification of the causes of the Depression. Two important papers on inflation expectations during the Depression are Hamilton (1992) and Cecchetti (1992). Whereas Cecchetti (1992) argues that some of the deflation during the depression *could* have been forecast, Hamilton (1992) argues the deflation in the first years of the Depression could not have been predicted based on sophisticated econometric filtering of commodity prices in the futures market. Overall, the evidence from the Depression gives a mixed signal about whether the prices changes were expected. Most researchers of the Great Depression agree that only a fraction of the *ex post* price changes could have been predicted.

Furthermore, based on the model of Barsky and De Long (1991), the price changes in the period leading up to the Depression could not have been predicted. World gold production increased gradually by 2.2% annually on average over the period from 1914 to 1931 (see Table 11 in Warren and Pearson, 1933), which is close to the growth in real world GDP over the same period (Maddison, 1995). It is, therefore, unlikely that inflation expectations were significantly different from zero before the US abandoned the gold standard in 1933.

There is very little research, if any, on inflation expectations over the period from 1934 to 1946. There were no formal rules about gold backup ratios and gold reserve requirements in this period. The rate of inflation fluctuated around a slightly positive number that was not significantly different from zero, on average, over the period from 1934 to 1940. It is therefore unlikely that the new monetary regime, in the wake of the break-down of the gold standard, gave investors any reason to change their inflation expectations significantly from zero. Over the period from 1941 to 1946 consumer price inflation was, on average, 5.7%. It is questionable, however, that inflation expectations were positively affected by the higher inflation, because there were no reasons to expect the inflation over that period to be permanent. The Livingston surveys on inflation expectations in

1947 and 1948 showed negative inflation expectations despite the fact that the rate of consumer price inflation exceeded 10% on average in these two years. More interestingly, long nominal interest rates fell below 3% in 1934 and remained in the 2-3% band up to 1955 (Homer and Sylla, 1991). It is, therefore, unlikely that inflation expectations were more positive in the *post*-1934 period than they were in the *pre*-1934 period.



The path of the equity risk premiums over the past 133 years are displayed in Figure 3. The average equity premium over the whole period is 5.7% using short bond rates and 5.3% when long bonds are used. This is close to the *ex post* arithmetic real returns, which makes sense since the required share returns were close to each other in the end periods. The hump-shaped profile identified for required share returns is also visible for the equity risk premium. The figure gives several insights into the movements in the equity risk premium.

The equity risk premium before WWII was largely independent of monetary policies. The increasing equity premium from 1920 to WWII probably reflected increasing macroeconomic instability, such as the inflation and deflation following WWI, and the depressions in the beginning of the 1920s and the beginning of the 1930s, as highlighted, by for instance, Siegel (2002a, 2002b). The high risk premium from WWII to the beginning of the 1980s was predominantly a result of high required share returns, but it has also been influenced by monetary policies and the financing needs of governments. The real interest rate decreased after WWII as the government debt-GDP ratio gradually declined from its WWII high and the expansive monetary policies in the 1970s contributed to a continuation of the decline.

The period from WWII to 1980 stands out in modern history as a unique period of excess share returns. The arithmetic average *ex post* excess returns relative to a long bond was 7.9% over the period from 1942 to 1980, which is three percentage points above the 4.9% arithmetic average in

the periods 1871-1941 and 1981-2002. The historically high excess returns over the period from 1942 to 1980 was a result of a combination of high required returns and low real interest rates, where the low real interest rates, to a large extent, reflected loose monetary policies.

The equity risk premium has stabilised at the 4-6% mark over the past two decades, which is slightly below the consensus estimate among finance professors (Welch, 2000), but close to the assessment of fund managers. Surveys of fund managers find that the equity risk premium was 4.1% in 1989 and 5.7% in 1997 (Thaler, 2002, p 4).

## 5 Empirical evidence

An apparently paradoxical implication of the model is that the share price is a negative function of earnings per unit of capital and the  $Y/K$  ratio when the influence of temporary demand and supply shocks are controlled for. This is because the model implies that cyclically adjusted earnings per unit of capital and the  $Y/K$  ratio are only affected by innovations in the required returns. This implication is tested in this section.

The model implies that *ex post* share returns are a negative function of the change in the required returns, a positive function of the cyclical demand shocks and a negative function of the share of income going to labour.<sup>7</sup> The last two variables account for the temporary influence on earnings of demand and supply shocks. The following two equations are estimated:

$$SR_t = \alpha_0 + \alpha_1 \Delta \ln \hat{\rho}_t^{c,Y/K} + \alpha_2 \Delta \ln Y_{t+1}^e + \alpha_3 \Delta \ln S_t^L + \varepsilon_1, \quad (11)$$

and

$$SR_t = \beta_0 + \beta_1 \Delta \hat{\rho}_t^{c,E} + \beta_2 \Delta \ln Y_{t+1}^e + \beta_3 \Delta \ln S_t^L + \varepsilon_2, \quad (12)$$

where  $SR$  is real share returns,  $Y_{t+1}^e$  is expected per capita income at period  $t+1$ ,  $S^L$  is the share of income going to labour in manufacturing,  $\varepsilon$  is a white noise error term, and  $\rho^{c,Y/K}$  and  $\rho^{c,E}$  are estimated using Equations (9) and (10). The real share returns are calculated as the log first-difference of the accumulated share index deflated by consumer prices and  $\rho^{c,E}$  is not measured in logs because it is negative during the Great Depression. Expected income growth is measured as the instrumented income growth at period  $t+1$ . The equations were first estimated with all regressors unlagged and lagged one period and variables with insignificant coefficient estimates, at the 10% level, were deleted using the general-to-specific model reduction procedure.

The results of estimating the restricted versions of Equations (11) and (12) are shown in Table 1. The estimated coefficients have their expected sign. Both expected income growth and

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<sup>7</sup> The influence of income shares on share returns are not explicitly incorporated into the model to keep it as simple as possible. Varying income shares can be incorporated into the model by allowing the elasticity of substitution between capital and labour to vary on medium term frequencies or by assuming imperfect competition, and, therefore, varying mark-ups on medium term frequencies.

labour's income share are important determinants of share returns. Since  $S^L$  fluctuates by 20-30% on medium term frequencies it is an important factor behind medium-term movements in share prices. Labour's income share in US manufacturing has decreased by 35% since 1980; thus explaining 50-60% of the increase in share prices over the same period. Income growth is also a significant contributor to share price fluctuations on business cycle frequencies.

**Table 1. Parameter estimates of Equations (11) and (12).**

$\hat{SR}_t = 0.01 - 0.31 \Delta \ln \rho_{t-1}^{c,Y/K} + 2.68 \Delta \ln Y_{t+1}^e - 1.74 \Delta \ln S_t^L$ <small>(0.50) (2.04) (2.39) (2.49)</small>	$R^2 = 0.13$	$DW = 2.22$
$\hat{SR}_t = 0.00 - 0.02 \Delta \ln \rho_{t-1}^{c,E} + 3.64 \Delta \ln Y_{t+1}^e - 1.43 \Delta \ln S_t^L$ <small>(0.05) (1.87) (2.66) (2.01)</small>	$R^2 = 0.14$	$DW = 2.11$

**Notes.** The numbers in parenthesis are absolute  $t$ -statistics. Estimation period: 1892-2001 (Equation (11)) and 1912-2001 (Equation (12)). The following instruments are used for  $\Delta \ln Y_t^e$ :  $\Delta \ln S_t^L$ ,  $\Delta \ln S_{t-1}^L$ ,  $\Delta \ln Y_t$ , and  $\Delta \ln Y_{t-1}$ .

The estimated coefficients of the required returns are both negative and statistically significant. In a dividend discount model with no dividend growth we would expect an estimated coefficient of the log of the required returns to be -1, but lower in dynamic dividend-discount models and in models with positive expected dividend growth, because  $\ln(\rho)$  is a biased measure of  $\ln(\rho - g)$ ; thus, the estimated coefficient of  $\ln(\rho)$  will be biased downward. The estimated coefficients of required returns of about -0.3, therefore, do not seem unreasonable (note that the coefficient estimate of  $\rho^{c,E}$  is not in logs).

The estimates of the simple model in this section highlight the dynamic movements in share prices on various frequencies: Output growth is a key determinant of *ex post* share returns on business cycle frequencies, factor shares are key determinants of *ex post* share returns on medium term frequencies and required share returns are the key determinants of *ex post* share returns in the long run.

## 6 How much has the declining required return contributed to the share price increase in the post-war period?

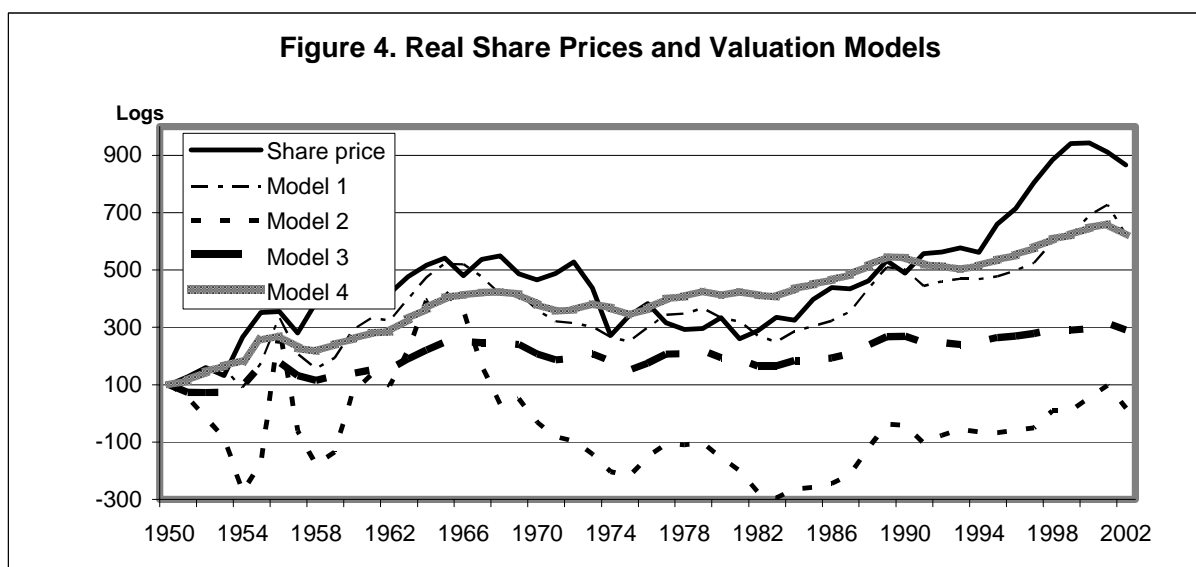
The pronounced increase in real share prices in the post-war period has been either a result of an unexpected increase in expected earnings growth or an unexpected reduction in the required share returns. This section makes a rough decomposition of the post-war increase in real share prices into earnings-growth-induced and required-returns-induced increases. The period starts in 1950 following Fama and French (2002).

The simplest possible framework is used for this decomposition, namely the following Gordon growth model (Barsky and De Long, 1993):



$$Q_t = \frac{Div_t}{\hat{\rho}_t^{Y/K} - g_t}, \quad (13)$$

where  $g$  is the growth in real dividends per share,  $Q$  is share prices deflated by consumer prices, and  $Div$  is dividends per share deflated by consumer prices. This model assumes that the required share returns and the expected growth in dividends vary over time, but, that their values at time  $t$ , are expected to persist to infinity. This is, of course, a simplification and the evidence in this section should only be considered as an illustration. The expected growth rate in real dividends is arbitrarily set to the average geometric growth rate in real dividends over the previous 18 years. Using a 10-year growth rate instead does not affect the results.



**Notes.** Share prices and dividends are deflated by consumer prices and the growth rate in dividends is an 18-year moving average. The valuation models are based on Equation (13). Model 1:  $\rho$  is based on  $Y/K$  and  $g$  is allowed to vary. Model 2:  $\rho = 0.06$  and  $g$  is allowed to vary. Model 3:  $\rho$  and  $g$  are constants. Model 4.  $(\rho - g)$  is set to  $0.5\rho$ , where  $\rho$  is based on  $Y/K$ .

Restricted and unrestricted versions of Equation (13) and real share prices are displayed in Figure 4. Real share prices have increased by approximately 900% over the period from 1950 to the turn of this century. Consider the model where  $\rho$  and  $g$  are assumed to be constant and real share values are consequently driven solely by real dividends (Model 3). This model predicts an increase in real share prices of approximately 300% over the whole period. Allowing  $g$  to vary but keeping  $\rho$  constant would lead to unaltered share prices over the entire period (Model 2). However, Model 1, where  $\rho$  and  $g$  are allowed to vary, predicts a 400% increase in share prices since 1982.

When  $\rho$  is allowed to vary the valuation model tracks share prices much closer compared to the models with fixed  $\rho$ . In Model 1 where  $\rho$  and  $g$  are allowed to vary, the warranted share values track almost all movements in share prices and the model predicts a 700% increase in share prices

over the 50-year period considered. Model 4, however, only predicts half of the 400% increase in real share prices in the 1990s, which suggests that share prices over that period have been, to a large extent, driven by high earnings expectations that have so far not been rewarded. Restricting the growth rate in  $g$  to  $\kappa\rho$ , as predicted by the model in Section 2, where  $\kappa$  is set to a constant of 0.5, yields Model 4. This model has predicts most of the increase in real share prices in the post-war period, but has a smoother path than Model 1 and real share prices. The question is whether the warranted share prices generated by this model are too smooth. The evidence from Figure 4 suggests not. In the post-war period real share prices have gravitated to the level that is predicted by this model.

The warranted share prices generated by the models where  $\rho$  is allowed to vary indicate that a large proportion of the post-war increase in share prices is explained by a reduction in the required returns. The reduction in the required returns has resulted in a 550% increase in share prices over the whole period, which reinforces the findings in Section 4 that the high *ex post* share returns over the period from 1942 to 1980 defines a unique period for share returns in modern history.

## 7 Concluding remarks

This paper has presented a simple method to calculate the time-varying required share returns based on Tobin's  $q$  model. The idea is that the cyclically adjusted output-capital ratio and earnings per unit of capital will only change in response to changes in the required share returns. Endogenous adjustment of the capital stock means that earnings shocks have only temporary effects and will, therefore, not affect the steady state values of the output capital ratio and earnings per unit of capital. Earnings shocks will lead to an endogenous response in the capital stock, which will continue to adjust until the *pre*-shock earnings per unit of capital and the output capital ratios are re-established. A change in the required returns, however, will lead to an endogenous adjustment in the capital stock until the earnings per unit of capital and the output capital ratio equal the new required share returns. It follows that the cyclically adjusted earnings per unit of capital and the output capital ratio are excellent indicators of the required share returns.

Empirical evidence on the time-profile of the required share returns and the equity premium confirms the suspicion in the literature that the equity premium and the required share returns increased sharply following the Great Depression and WWII and first reached their *pre*-WWII levels in the 1980s and the 1990s. In fact, the period from WWII to 1980 stands out as a unique period in the modern economic history of the equity risk premium and the required share returns and has contributed substantially to an upward bias in *ex post* required share returns and the equity premium. The arithmetic average of the equity premium was pushed to eight percent in that period, which is three percentage points above of the historical returns in the surrounding periods. Furthermore, the

post-war reduction in the required share returns explains more than half of the nine-fold increase in share prices over the same period and, therefore, suggests that the high post-war share returns are not going to be repeated in the foreseeable future. The equity risk premium has stabilised around the 4-6% mark over the past two decades, which is close to the pre-Depression equilibrium.

The implication of the model that adjusted to earnings per unit of capital and the output capital ratio reflect the required returns result in the apparently paradoxical result that share prices are negatively affected by the earnings per unit of capital and the output capital ratio. Controlling for the influences of the business cycle, the regression results showed a significant negative correlation between share prices, on the one hand, and earnings per unit of capital and the output capital ratio, on the other hand, as predicted by the model.

## DATA APPENDIX

The  $Y/K$ -based data on required returns are constructed as follows. Tangible capital stock is real private non-farm and non-residential capital stock from John W Kendrick, 1961, *Productivity Trends in the United States*, NBER, Princeton: Princeton University Press. The data are updated using the perpetual inventory method and 8% depreciation rate for gross real private non-farm and non-residential investment from the *Economic Report of the President*. The intangible capital stock is based on industry R&D expenditures from the National Science Foundation using the inventory perpetual method and 20% depreciation rate. The capital stock is first constructed at fixed prices using the economy-wide GDP deflator from Bureau of Economic Analysis (BEA), and then converted to current prices. The initial R&D capital stock was computed from the equation  $K_0 = I_0 / (\delta + g)$ , where  $I_0$  is the gross investment at period 0,  $\delta$  is the rate of depreciation, and  $g$  is the arithmetic average annual growth rate in investment over the whole data period. The data are backdated using patents granted to the corporate sector from Department of Commerce, 1975, *Historical Statistics of the United States: Colonial Times to 1970*, Bureau of the Census: Washington DC. Over the period from 1901 to 1921, in which data are only available on 5-year intervals, the ratio of patents to the corporate sector and total patents is interpolated by multiplying with total patents. The ratio between patents to the corporate sector and the total number of patents granted in 1901 multiplied by the total number of patents granted is used over the period from 1870 to 1901. The patent capital stock is adjusted to the R&D capital stock in 1953. GNP in the non-farm business sector is used over the period from 1929 to 2002 from *Historical Statistics* (F 42) and *Report of the President*. GDP in the non-farm sector is used over the period from 1919 to 1929 from *Historical Statistics* (F 128) and industrial production before then from B. R. Mitchell, 1983, *International Historical Statistics: Americas and Australasia*, Macmillan: London. The overall capital depreciation rate is estimated as a weighted average of the depreciation rate for tangibles (8%) and the depreciation rate for intangibles (20%). The earnings-based data on required returns are constructed as follows. Both the numerator and the denominator in earnings per unit of capital are in nominal terms are from the private corporate sector. The tangible capital stock is from Arnold J Katz and Shelby W Herman, 1997, "Improved estimates of fixed Reproducible Tangible Wealth, 1929-95," BEA, [www.bea.doc.gov/bea/an/0597niw/maintext.htm](http://www.bea.doc.gov/bea/an/0597niw/maintext.htm) and is updated by data from OECD, *National Accounts*. The real capital stock data before 1929 are from Kendrick, *op cit*, and is converted to nominal values using the GDP deflator from *Historical Statistics*. The intangible capital stock is based on industry R&D expenditures from the National Science Foundation and the

principles in the  $Y/K$ -based estimates are applied. The capital stock is first constructed from at fixed prices using the economy-wide GDP deflator from BEA, and then converted to current prices. The patent capital stock is converted to nominal values using consumer prices from *Historical Statistics*. Corporate earnings are from *Historical Statistics* Series Y 384 up to 1970 and from BEA from 1970 to 2002. The nominal R&D expenditures that are used to add to accounting earnings are directly available from 1953 to 2002. Over the period from 1929 to 1953 the industrial patents granted multiplied by consumer prices are used and scaled by the ratio of nominal R&D expenditures and the index of nominal patents in 1953. Labour's income share. Is computed as total labour costs divided by nominal value-added income in manufacturing from Liesner, *op cit*, Table US. 6 and updated using OECD, *National Accounts*. Real GDP and population are from Maddison (1995). The figures are updated using OECD, *National Accounts*, and ILO, *Year-Book*. Share returns. 1870-1999: Jack W Wilson and Charles P Jones, 2002, "An Analysis of the S&P 500 Index and Cowles's Extensions: Price Indexes and Stock Returns, 1870-1999," *Journal of Business*, 75, 505-533, and updated using the data from Shiller's homepage. Long interest rates. Federal Govn bonds 1889-1900 and 1919-1949, corporate and municipal long-term bond yields 1900-1919 (Homer and Sylla, 1991), and long-term government bond rate, 1950-2002, IMF, *International Financial Statistics*. Short interest rates. Stock exchange call loans (renewals) 1889-1918, Prime bankers 90-days acceptance (1919-57) (Homer and Sylla, 1991), and 3-month Treasury bill rate, IMF, *International Financial Statistics* (1958-2002).

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