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The Inefficiency of Price Quantity Bargaining*

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Abstract

In bargaining between a buyer and several sellers on prices and quantities, strategic inefficiencies arise. By reallocating quantities between agreements, the buyer can increase its share of the surplus. With two symmetric sellers producing substitutes, the quantities in the first agreement will be higher than the efficient, and lower than the efficient in the last, implying that sellers are strategically discriminated. In the asymmetric case, the buyer agrees first with the seller with lowest marginal cost, when the sellers produce substitutes. When goods are complements, both equilibrium quantities are lower than the efficient levels.

Keywords : Bargaining, inefficiencies, strategic discrimination.

JEL Classification : C72, C78, L10.

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1 Introduction

Negotiations are often interdependent. When a firm bargains with more than one worker, for example, it cannot reasonably be argued that the two negotiations are independent, since the surplus of hiring one worker usually depends on the characteristics of the other worker. Several models such as Horn & Wolinsky (1988) and Stole & Zweibel (1996a,b) extend Rubinstein (1982) to interdependent bargaining. One strong assumption in these models, however, is that the quantities agreed upon are fixed, i.e., there is only bargaining on how to split a given surplus. It seems reasonable that the payoffs also can be affected by varying quantities. In contrast to Binmore (1987) where there is only one seller and buyer and equilibrium quantities are efficient, we show below that inefficiencies are generic. As examples of bargaining over prices and quantities, we can think of one firm that bargains simultaneously with several workers or a downstream firm that bargains with several upstream firms.

In this paper, we analyze a bargaining model, when bargaining is on both prices and quantities. The main focus is on bargaining between one buyer and two sellers, but some results for the N seller case are also provided. We show that in equilibrium we have strategic inefficiencies, i.e. an outcome where equilibrium quantities are chosen different from the efficient in order to affect the distribution of surplus.

We show that equilibrium quantities generically are inefficient. Some intuition for this can be gained from the two seller case. When goods are substitutes, the buyer has a strategic incentive to increase the quantity in the first agreement, thereby decreasing the quantity and thus the price in the second agreement. Conjecture an agreement on the efficient quantities and consider a small increase in the first quantity. By the envelope theorem, the effect on the total surplus is small. The change in the quantity also decreases the payment in the last agreement, since the surplus in the last agreement decreases when the first quantity increases. Since the price effect is a first-order effect, while the effect on the total surplus is a second order effect, it is optimal for the buyer to increase the first quantity. When goods are complements, both quantities are inefficiently low.

Efficiency is affected by the relative bargaining power of the buyer and the sellers, in the sense of relative proposer probabilities. The outcome is efficient when the buyer makes all the bids. As all the surplus in both the first and the last agreement will be captured by the buyer, there is no strategic incentive to distort quantities. When sellers make all the bids, incentives to reallocate surplus to the first period is the largest, as all the surplus in the last agreement is captured by the seller. Reinterpreting the model in terms of two buyers and one seller, our model can be thought of as an auctions under full information with multiple objects and interdependent

valuations; see Krishna (2002). Our result is in contrast with Bulow & Klemperer (1996), who argue that auctions are more efficient than negotiations.

When sellers are completely symmetric, they will be treated symmetrically in the sense that the equilibrium will prescribe immediate agreement with either seller. The quantities agreed upon will be asymmetric, however. Small asymmetries between sellers are sufficient to determine equilibrium order of agreement, with the low cost seller agreeing first.

Under certain circumstances, the outcome will be efficient. If goods are independent the negotiations will also be independent, with efficiency as a result. Efficiency also holds if supply is completely inelastic, as the scope for using quantities strategically disappears. We also get efficiency when goods are perfect substitutes and sellers have constant marginal costs.

With linear marginal cost and quadratic revenues, we find that the degree of strategic inefficiencies increases, the flatter supply is relative to demand, the more homogeneous goods are and the more symmetric sellers are. Similarly, efficiency increases the steeper supply is relative to demand, the less homogeneous goods are and the more asymmetric sellers are.

The result that quantities are inefficient has implications for how to view e.g., union formation. Along the lines of Horn & Wolinsky (1988), in our model workers form a union if they are substitutes. When a union forms, the firm has only one counterpart. Then, from standard results in bargaining theory, quantities are efficient. Thus, forming a union leads to an increase in aggregate welfare.

Applying this model to intermediate goods markets, the more asymmetric the sellers are, the less inefficient is the outcome. We find a justification for the idea of countervailing power – high concentration on one side of the market can justify increased concentration on the other. Increasing the seller concentration reduces the ability of the buyer to strategically discriminate, with higher efficiency as a result.

Beyond potential applications of this mechanism, there are two reasons why the results are important. Firstly, it shows that the relationship between bargaining and efficiency can be more problematic than is commonly assumed. In the labor literature for instance, bargaining in wage and employment is often called “efficient bargaining”. Here, we show that efficient bargaining is inefficient when there is more than one worker. Secondly, there is a literature that implicitly assumes that this strategic possibility is not used. In Björnerstedt & Stennek (2004), the outcome is efficient because firms are assumed to have representatives negotiating, each bargaining with only one other firm. In contrast, our analysis is fully strategic.

In a series of papers, Segal (1999), Segal (2003) and Segal & Whinston (2003) analyze bilateral contracting with externalities. It is assumed that total surplus only depends on aggregate

quantities. We have a more general payoff structure as we allow for the distribution of trades to affect welfare. In the special case when welfare only depends on aggregate quantities, the allocation is efficient, in stark contrast with the result of these papers.

In section 2 the bargaining model is introduced. Section 3 analyzes equilibrium values and probabilities, section 4 analyzes the equilibrium quantities and finally section 5 concludes.

2 The Model

We will consider asynchronous simultaneous bargaining over both prices and quantities between a buyer A and two sellers 1 and 2. In section 4.1, a partial analysis is made of the n seller case.¹ One reason for restricting attention to the two buyer case is that when there are more than two sellers, the equilibrium structure becomes very complex.² As examples consider two workers who sell their labor to a firm or an upstream firm A selling a divisible intermediate good to downstream firms 1 and 2. Since the surplus in price quantity bargaining depends on the order of agreement, previous work such as Horn & Wolinsky (1988) is not applicable. Therefore, we extend the work of previous authors by analyzing a model where the total surplus depends on the order of agreement. The model is related to bargaining with externalities along the lines of Jehiel & Moldovanu (1995a and 1995b). For a complete discussion, see Björnerstedt & Westermark (2006a).

The buyer A bargains with both sellers 1 and 2 simultaneously. In each period, however, only one bid and response will be made, observed by all. Agreement is immediate and binding. We assume that bids alternate between $(A, 1)$ and $(A, 2)$, with the buyer A making the bid with probability λ , and the seller i with probability $1 - \lambda$. The buyer has the payoff function

$$R(q_1, q_2) - p_1 q_1 - p_2 q_2,$$

where p_i and q_i are the price and quantity sold by i to A . We say that q_1 and q_2 are *substitutes* when $\frac{\partial^2 R(q_1, q_2)}{\partial q_1 \partial q_2} < 0$, are *complements* if $\frac{\partial^2 R(q_1, q_2)}{\partial q_1 \partial q_2} > 0$ and are *independent* if $\frac{\partial^2 R(q_1, q_2)}{\partial q_1 \partial q_2} = 0$. The

¹Banerji (2002) has quantity choices in a bargaining setting with one buyer/firm bargaining with two sellers/unions. Their results, in particular Lemma 1 and the following corollary in that paper are incorrect; there exists a deviation from the proposed equilibrium profile where there is agreement on identical Cournot quantities with the two sellers. In the model, the sellers have zero marginal cost of supplying additional units. The firm then has the following profitable deviation. Offer, say, union 1 the same share of the surplus plus some small amount and a quantity equal to the sum of proposed equilibrium quantities. Since there is no marginal cost of supplying additional units, union 1 accepts the proposal. That the deviation is profitable can be seen from the fact that the firm only pays out half of the surplus it paid in the proposed equilibrium.

²This can be seen from Björnerstedt & Westermark (2006a) that analyzes bargaining over externalities in general where one seller sells an indivisible good to one of buyer n buyers.

payoff of seller i is given by:

$$p_i q_i - c_i(q_i)$$

where $c_i(q_i)$ represents the cost of supplying good q_i . We assume that R is concave and c_i convex. If R is strictly concave or c_i strictly convex, we say that production is *strictly convex*.

Quantities are restricted to be chosen from the sets Q_1 and Q_2 , respectively for seller 1 and 2. Most of the analysis focuses on the case when Q_i is continuous. As shorthand notation, if $(A, 1)$ agree first on the quantity q_1 , the surplus in this first agreement is given by $\Pi_1(q_1)$. Similarly, $\Pi_2(q_2)$ is the surplus when $(A, 2)$ agree first on q_2 . Given that there is agreement on q_1 , the additional surplus generated by $(A, 2)$ agreeing on q_2 is given by $\pi_2(q_2, q_1)$, with $\pi_1(q_1, q_2)$ similarly defined. In terms of the payoff functions $\Pi_1(q_1)$ and $\pi_2(q_2, q_1)$ are

$$\Pi_1(q_1) = R(q_1, 0) - c_1(q_1) \tag{1}$$

$$\pi_2(q_2, q_1) = R(q_1, q_2) - R(q_1, 0) - c_2(q_2). \tag{2}$$

Let V_i and W_i denote the value to A of bidding and receiving a bid from seller i , and v_i and w_i denote the value to seller i of bidding and receiving a bid. Let σ_{Ai} be the probability that A gives an acceptable bid to i when bidding and σ_{iA} the probability that i gives an acceptable bid. Consider a subgame following agreement on some quantity q_i . The values $V_j(q_i)$ and $W_j(q_i)$ are

$$V_j(q_i) = \max_{q_j} \pi_j(q_j, q_i) - w_j(q_i), \tag{3}$$

$$W_j(q_i) = \delta \lambda V_j(q_i) + \delta (1 - \lambda) W_j(q_i)$$

with $v_j(q_i)$ and $w_j(q_i)$ similarly defined. Let

$$q_j(q_i) = \arg \max_{q_j \in Q_j} \pi_j(q_j, q_i)$$

with $q_i(q_j)$ similarly defined. The solution to (3) gives $W_j(q_i) = \delta \lambda \pi_j(q_j(q_i), q_i)$ and $w_j(q_i) = \delta (1 - \lambda) \pi_j(q_j(q_i), q_i)$. Defining $\sigma_i = \lambda \sigma_{Ai} + (1 - \lambda) \sigma_{iA}$, the value equations in a subgame

without any agreement are

$$\begin{aligned}
V_i &= \max_{\{\sigma_{Ai}, q_i\}} (1 - \sigma_{Ai}) W_i + \sigma_{Ai} (\Pi_i(q_i) + \delta (\lambda V_j(q_i) + (1 - \lambda) W_j(q_i)) - w_i), \quad (4) \\
W_i &= \delta (\lambda V_j + (1 - \lambda) W_j), \\
v_i &= \max_{\{\sigma_{Ai}, q_i\}} (1 - \sigma_{iA}) w_i + \sigma_{iA} (\Pi_i(q_i) + \delta (\lambda V_j(q_i) + (1 - \lambda) W_j(q_i)) - W_i), \\
w_i &= \delta^2 ((1 - \lambda) \pi_i(q_i(q_j), q_j) \sigma_j + (1 - \sigma_j) ((1 - \lambda) v_i + \lambda w_i))
\end{aligned}$$

for $i = 1, 2$ with $j \neq i$. To understand (4), if A and i have come to agreement and A bargains with j , the firm gets $\delta (\lambda V_j(q_i) + (1 - \lambda) W_j(q_i))$. Thus in the first negotiation with i , the total amount at stake is, using the solution for $W_j(q_i)$, $\Pi_i(q_i) + \delta \lambda \pi_j(q_j(q_i), q_i)$. In giving an acceptable offer (with probability σ_{Ai}) it is sufficient to offer w_i to i . Since W_i is the continuation value conditional on disagreement, the value V_i in (4) follows. By similar reasoning v_i is determined. When rejecting a proposal by i , A gets V_j with probability λ and W_j with $1 - \lambda$, giving W_i in (4). When rejecting a proposal, i receives $(1 - \lambda) \pi_i(q_i(q_j), q_j)$ if A and j agree in the next period. With probability $1 - \sigma_j$ they do not, giving $(1 - \lambda) v_i + \lambda w_i$.

It is easily seen from (4) that the optimal choice of q_i is independent of σ_{Ai} and, for any $\sigma_{Ai} > 0$, maximizes

$$\Pi_i(q_i) + \delta \lambda \pi_j(q_j(q_i), q_i) \quad (5)$$

Lemma 10 in the appendix shows that the expression above is strictly concave in q_i when production is strictly convex. Thus, whenever $\sigma_{Ai} > 0$ and $\sigma_{iA} > 0$, there is a unique solution for q_i to the maximization problem in V_i and v_i . Furthermore, quantity choices can be analyzed independently of equilibrium values and probabilities. Thus, letting $q_i^F(\delta)$ denote the maximizer in (4), we can define

$$\begin{aligned}
\pi_i^F(\delta) &= \Pi_i(q_i^F(\delta)) \\
\pi_j^L(\delta) &= \pi_j(q_j(q_i^F(\delta)), q_i^F(\delta))
\end{aligned}$$

Note that continuity and strict concavity ensures that $q_1^F(\delta)$ is a continuous function of δ . The last quantity q_2 only depends on δ indirectly through $q_1^F(\delta)$. By the maximum theorem, as the surplus in the last agreement is strictly concave in q_2 , q_2 is a continuous function of $q_1^F(\delta)$ and hence of δ . Hence, $\pi_i^F(\delta)$ and $\pi_i^L(\delta)$ are also continuous in δ . Also, we let $q_i^L(\delta) = q_i(q_j^F(\delta))$.

3 Equilibrium values and probabilities

Since quantities can be determined independently from equilibrium values and probabilities σ_i , we can analyze these quantity choices and equilibrium values, as defined in (4), and probabilities separately; see (4) and (5). We begin by analyzing equilibrium values and probabilities. The equilibrium outcome will be shown to depend upon on which order of agreement is more efficient and the degree of substitutability. To capture these notions we let, for $i = 1, 2$,

$$\varepsilon_i(\delta) = \delta (\pi_i^F(\delta) + \delta\pi_j^L(\delta)) - (\pi_j^F(\delta) + \delta\pi_i^L(\delta)) \quad (6)$$

$$\gamma_i(\delta) = \pi_i^F(\delta) - \delta^2(1 - \lambda)\pi_i^L(\delta) \quad (7)$$

The expression $\varepsilon_i(\delta)$ is interpreted as the difference in total surplus of waiting to agree with i in the next period, instead of agreeing with j today. Similarly $\gamma_i(\delta)$ is related to the degree of substitutability of i and j . Also, we let $\gamma_i = \gamma_i(1)$, $\pi_i^F = \pi_i^F(1)$, $\pi_i^L = \pi_i^L(1)$ and

$$\varepsilon = \pi_1^F + \pi_2^L - (\pi_2^F + \pi_1^L). \quad (8)$$

Note that $\varepsilon_1(\delta) \rightarrow \varepsilon$, $\varepsilon_2(\delta) \rightarrow -\varepsilon$ as $\delta \rightarrow 1$. When $\varepsilon > 0$, agreeing with 1 first gives a higher surplus than agreeing with 2 first. In the symmetric case ($\pi^F = \pi_1^F(1) = \pi_2^F(1)$ and $\pi^L = \pi_1^L(1) = \pi_2^L(1)$) when $\lambda = \frac{1}{2}$, equation (7) simplifies to $\gamma_1 = \pi^F - \frac{\pi^L}{2}$ (and $\varepsilon = 0$). Thus, γ_1 depends on the relative size of the gains of the first and second agreement, which is related to substitutability; see Proposition 5 below.

To distinguish (7) from the definition of substitutes for the R function, we say that 1 and 2 are *value substitutes* when $-\lambda\varepsilon_i \leq (1 - \lambda)\gamma_i$ for $i = 1, 2$.

One might think that the order of agreement do not matter for total surplus. The example below illustrates why $\varepsilon \neq 0$ is natural in applications.

Example 1 *Task allocation of workers in a firm. We assume that the firm can employ two workers for different tasks, t_x and t_y . The payoff of the firm is*

$$R(t_1, t_2) - p_1 - p_2$$

where p_i is the amount paid to i and R revenue to A . The payoff of seller i is given by:

$$p_i - c_i(t_i).$$

where the function c_i represents the cost of performing task t_i . When no agreement has been

reached, the proposed task when the buyer and seller 1 bargain is

$$t_1^F = \arg \max_{t_1 \in \{t_x, t_y\}} R(t_1, 0) - c_1(t_1) + \delta \lambda (R(t_1, t_2) - R(t_1, 0) - c_2(t_2)) \quad (9)$$

where $t_2 \neq t_1$. Then

$$\pi_1^F = R(t_1^F, 0) - c_1(t_1^F).$$

with π_2^F similarly defined. Letting t_i^L denote the i 's task in the last agreement, π_2^L is given by

$$\pi_2^L = R(t_1^F, t_2^L) - R(t_1^F, 0) - c_2(t_2^L)$$

with π_1^L similarly defined. If the solution to (9) has $t_1^F \neq t_2^F$, e.g. $t_1^F = t_x$ and $t_2^F = t_y$ the tasks are allocated in the same way, irrespectively if A agrees with 1 or 2 first. We get

$$\pi_1^F + \pi_2^L = R(t_x, t_y) - c_1(t_x) - c_2(t_y) = \pi_2^F + \pi_1^L$$

and hence $\varepsilon = 0$. However, if $t_1^F = t_2^F$ i.e., $t_2^F = t_x$ we get

$$\pi_2^F + \pi_1^L = R(t_y, t_x) - c_1(t_y) - c_2(t_x)$$

implying $\varepsilon \neq 0$, unless R is symmetric and c_1 and c_2 are identical.

This motivates the following definition. We say that an equilibrium is *order efficient* if, whenever $\varepsilon > 0$, the buyer agrees with seller 1 first, as $\delta \rightarrow 1$.

There are four types of equilibria, denoted I , P_i , M_i and H . There are equilibria with immediate agreement (denoted I), where A always agrees with the seller it meets. In equilibria P_i , A always agrees with seller i first. There are also mixed equilibria, denoted M_i , where A agrees with probability 1 when meeting i , and with probability $0 < \sigma_j < 1$ when meeting j , and mixed equilibria, denoted H , where $0 < \sigma_i < 1$ for $i = 1, 2$. We have the following result on order efficiency.

Proposition 2 *As $\delta \rightarrow 1$, when goods are value substitutes, any stationary subgame perfect equilibrium is order efficient.*

Assuming $\varepsilon > 0$, when $\lambda \geq \frac{1}{2}$, the equilibrium structure is as follows, using the first step in

the proof of Proposition 2 in the appendix when $\delta = 1$,

| | | |
|---|---|--|
| | $\gamma_2 < \frac{\lambda}{1-\lambda}\varepsilon$ | $\frac{\lambda}{1-\lambda}\varepsilon \leq \gamma_2$ |
| $\gamma_1 < -\frac{\lambda}{1-\lambda}\varepsilon$ | P_1, P_2, M_2 | P_2, M_1, M_2 |
| $-\frac{\lambda}{1-\lambda}\varepsilon \leq \gamma_1$ | P_1 | M_1 |

When $\lambda < \frac{1}{2}$, the equilibrium structure is slightly different, see the proof of the proposition in the appendix. The set of equilibria depend crucially on whether 1 and 2 are value substitutes or not. When 1 and 2 are value substitutes, then all equilibria are constrained efficient, in the sense that all equilibria are order efficient. However, in the case of value complements in (7), there are also equilibria that are order inefficient.

To understand why an inefficient order of agreement is an equilibrium can be seen from analyzing the pure strategy equilibria P_1 and P_2 with P_2 having an inefficient order of agreement. Both these exist if the degree of value complementarity is high. Assuming for expositional purposes that $\lambda = \frac{1}{2}$ and $\delta \rightarrow 1$, then if (A, i) agree last, then i gets $\frac{\pi_i^L}{2}$. Equilibrium payoffs can easily be derived by taking limits in Lemmas 11 - 14 in the appendix. Note that

$$\gamma_1 < -\varepsilon \iff \pi_1^F + \frac{\pi_2^L}{2} - \frac{\pi_1^L}{2} < \frac{1}{2} \left(\pi_2^F + \frac{\pi_1^L}{2} \right). \quad (10)$$

The P_2 equilibrium can be thought, in terms of A and 2 as a situation where A only bargains with 2 and the surplus consists of π_2^F and the net gain from hiring 1, given by $\frac{\pi_1^L}{2}$. Then both A and 2 get $\frac{1}{2} \left(\pi_2^F + \frac{\pi_1^L}{2} \right)$ each. In order for this to be an equilibrium, A should not want to agree with 1 first. In doing so, A would have to pay $\frac{\pi_1^L}{2}$ to 1, as this is the equilibrium payoff to 1. Conditional on acceptance, 2 is last, getting $\frac{\pi_2^L}{2}$. Then A gets $\pi_1^F + \frac{\pi_2^L}{2} - \frac{\pi_1^L}{2}$. In order for this deviation not to be profitable, condition (10) must hold. The net gain for the buyer when deviating and agreeing with 1 is depends on the relationship between the surpluses in the first and last agreement. Due to value complementarity π_1^L is large relative to π_1^F making it unprofitable to switch order. The existence of multiple equilibria when there are strong value complementarities can be seen as a coordination failure. If complementarities are strong enough, the gain in efficiency when switching to the efficient order of agreement cannot compensate A for paying 2 more.

The case when sellers are symmetric, i.e., when $\varepsilon = 0$ the efficient order of agreement trivially prevails. When $\lambda \geq \frac{1}{2}$, using (34) in the appendix, the equilibria are characterized as follows;

| | | |
|-------------------|-------------------------|-------------------|
| | $\gamma_2 < 0$ | $0 \leq \gamma_2$ |
| $\gamma_1 < 0$ | I, P_1, P_2, M_1, M_2 | I, P_2, M_2 |
| $0 \leq \gamma_1$ | I, P_1, M_1 | I |

Although $\varepsilon = 0$ is non-generic, it is still interesting, as it covers some standard models, e.g., Horn & Wolinsky (1988). For example, a buyer bargaining with two sellers when quantities are fixed implies $\varepsilon = 0$. This can be seen from noting that the fixed quantity model can be analyzed in terms of Example 1 with $t_1^F = t_1^L$ and $t_2^F = t_2^L$. Also note that in the case of fixed quantities, the condition that $\gamma_1 \leq 0$ is identical to the condition in Horn & Wolinsky (1988).³

To understand why the immediate agreement equilibrium I is nongeneric, note that A can threaten both sellers with being last and can then force down the payoff of both sellers i to $\frac{\pi_i^L}{2}$. When agreeing with 1 first, A gets the remainder of the gains of trade: $\pi_1^F + \pi_2^L - \frac{\pi_1^L}{2} - \frac{\pi_2^L}{2} = \pi_1^F + \frac{\pi_2^L}{2} - \frac{\pi_1^L}{2}$. Similarly, when agreeing with 2 first, A gets $\pi_2^F + \frac{\pi_1^L}{2} - \frac{\pi_2^L}{2}$. To ensure that A does not have an incentive to disagree with either seller, the payoff for A when agreeing first with 1 or 2 have to be equal, implying $\varepsilon = 0$.

If $\varepsilon > 0$, no pure strategy equilibria exist when $\gamma_2 > \varepsilon$ and $\gamma_1 > -\varepsilon$, i.e., when the degree of value substitutability is high. The payoffs in the mixed strategy equilibria can best be understood in relation to bargaining with outside options. The share of the surplus of the agreement that A gets in negotiating with 1, depends on whether the equal split payoff $\frac{1}{2}(\pi_1^F + \frac{\pi_2^L}{2})$ is greater or less than the "outside option" agreeing with 2 first: $\pi_2^F + \frac{\pi_1^L}{2} - \frac{\pi_2^L}{2}$, i.e., condition (10) applied for seller 1. The condition thus shows that the outside option is not binding if there are strong value complementarities for 2, i.e., if $\gamma_2 < \varepsilon$. To understand why $\sigma_1 = 1$ and $0 < \sigma_2 < 1$ in the equilibrium M_1 , consider the cases where $\sigma_2 = 1$ or $\sigma_2 = 0$. From the discussion of the I equilibrium above, we know that if $\sigma_2 = 1$, if $\varepsilon > 0$, A will never want to agree with 2, as $\varepsilon > 0$ implies that $\pi_1^F + \frac{\pi_2^L}{2} - \frac{\pi_1^L}{2} > \pi_2^F + \frac{\pi_1^L}{2} - \frac{\pi_2^L}{2}$. Thus he gains by reducing the probability σ_2 . In the case where $\sigma_2 = 0$, the payoff to A is $\frac{1}{2}(\pi_1^F + \frac{\pi_2^L}{2})$ by a similar argument following (10). As (10) is violated, A gains by agreeing with 2 to obtain $\pi_2^F + \frac{\pi_1^L}{2} - \frac{\pi_2^L}{2}$. Thus to ensure that neither of these deviations are profitable, by continuity we have $0 < \sigma_2 < 1$.

Example 1 (continued). *It is clearly possible with inefficient equilibria. This is e.g. the case when λ is zero and*

$$R(0, t_x) - c_2(t_x) > R(0, t_y) - c_2(t_y)$$

Then inefficient equilibria can exist if complementarity is large enough, since we have $\varepsilon \neq 0$.

³To see this, note that we have, in the terminology of Horn & Wolinsky, $\pi^F = x$ and $\pi^L = y$. Then $\gamma_1 = \frac{x}{2} - \frac{y}{4} \leq 0$ or $x \leq 2y$ is identical to the cutoff condition in their Proposition 1.

4 Equilibrium Quantities

We now show that we can determine equilibrium quantities in both the first and last agreement, on and off the equilibrium path. From (4), for A and 1 the quantity q_1^F is chosen such that

$$q_1^F = \arg \max_{q_1} R(q_1, 0) - c_1(q_1) + \delta \lambda (R(q_1, q_2(q_1)) - R(q_1, 0) - c_2(q_2(q_1))). \quad (11)$$

Lemma 10 in the appendix shows that the maximand is strictly concave when production is strictly convex. Given that production is strictly convex, for any initial agreement on q_1 or q_2 there exists a unique q_2^L or q_1^L maximizing last period total surplus. The first-order condition to this problem is, using (2) and that the effects through $q_2(q_1)$ vanishes by the envelope theorem

$$\frac{\partial R(q_1, q_2(q_1))}{\partial q_1} - (1 - \lambda) \left(\frac{\partial R(q_1, q_2(q_1))}{\partial q_1} - \frac{\partial R(q_1, 0)}{\partial q_1} \right) - c_1'(q_1) \quad (12)$$

Let q_1^e and q_2^e denote the efficient quantities, i.e., the quantities that solve⁴

$$\max_{q_1, q_2} R(q_1, q_2) - c_1(q_1) - c_2(q_2).$$

The next proposition describes the relationship between equilibrium and efficient quantities.

Proposition 3 *Suppose production is strictly convex. For δ sufficiently close to 1, unless goods are independent or the buyer has all bargaining power, equilibrium quantities are inefficient. If goods are substitutes (complements), $q_1^F > q_1^e$ ($q_1^F < q_1^e$) and $q_2^L < q_2^e$ ($q_2^L > q_2^e$).*

Thus, if goods are substitutes, the seller that agrees first produces too much, and the last too little. The intuition is the following. The buyer uses the quantity in the first agreement to affect the price in the second agreement. Suppose the equilibrium agreement is on the efficient quantities and consider a small increase in the first equilibrium agreement q_1 as in (12). By efficiency, the first and last term in (12) vanishes. By substitutability,

$$\frac{\partial R(q_1, q_2)}{\partial q_1} - \frac{\partial R(q_1, 0)}{\partial q_1}$$

is negative and hence there is a gain from increasing q_1 . Note that this is the effect on the surplus in the last agreement which is closely related to the payment to buyer 2. Also, the amount paid to the first seller is unaffected by the deviation. Since the price effect is a first-order effect, while

⁴The analysis can be applied to a model where A sells goods to 1 and 2, where there is interdependence of buyer profits on the quantity on the other agreement. This could be due to the buyers being competitors in a final goods market. The payoff to A is then $p_1 q_1 + p_2 q_2 - c(q_1, q_2)$ and to i is $R(q_i, q_j) - p_i q_i$.

the effect on the total surplus is a second order effect, it is optimal for the buyer to increase q_1 . If goods are complements, a similar argument shows that both sellers produce too little. The buyer uses quantities strategically to increase profits.

The reason why there is efficiency when $\lambda = 1$ is that the maximand in (11) is equal to total surplus in the limit, implying that $q_1^F = q_1^e$.

In Proposition 3, the order of agreement is not determined. There is no simple relationship between value substitutability, i.e., condition (7), and the definition of substitutability for R . The following example shows that there are equilibria with an inefficient order of agreement when q_1 and q_2 are complements.

Example 4 *Symmetric perfect complements.* Assume $R(q_1, q_2) = \min\{q_1, q_2\}$. Then, for $q_1 > 1$, q_2^L solves,

$$\max_{q_2} R(q_1, q_2) - c(q_2)$$

implying that we either have $q_2 = q_1$ and $c'(q_2) \leq 1$ or $q_2 < q_1$ and $c'(q_2) = 1$. Then q_1 is chosen such that

$$q_1^F = \arg \max_{q_1} -c(q_1) + \delta\lambda (R(q_1, q_2(q_1)) - c(q_2(q_1))). \quad (13)$$

We cannot have a solution where $q_1^F > q_2^L$, since any increase of q_1 above q_2^L only increases costs through $c(q_1)$. Then $q_2(q_1) = q_1$ and the first-order condition is

$$-c'(q_1) + \delta\lambda (2 - c'(q_2(q_1))) \geq 0.$$

Suppose $c'(q_2) = 1$. Then we get $\delta\lambda \geq c'(q_1)$ a contradiction. Thus, $c'(q_2) = c'(q_1) < 1$ and hence quantities are inefficient. Also, it is easy to see that we have $\gamma_i < 0$ and $\varepsilon = 0$, implying that both order of agreements are equilibria. Introducing small asymmetries preserves this result and implies that there are equilibria with inefficient order of agreement.

When goods are substitutes, the following proposition shows that equilibria are order efficient, given that the sellers are not too asymmetric.⁵ Specifically, let $c_1(q_1) = \varphi c(q_1)$ and $c_2(q_2) = c(q_2)$ with $\varphi < 1$. Then

Proposition 5 *If goods are substitutes, $c_1(q_1) = \varphi c(q_1)$ with $\varphi < 1$ and $c_2(q_2) = c(q_2)$, then there exists a $\bar{\varphi} < 1$ such that all equilibria are order efficient for $\varphi > \bar{\varphi}$. If goods are sufficiently complementary, there are equilibria with an inefficient order of agreement for $\varphi > \bar{\varphi}$.*

⁵It will be shown in the next section that with linear marginal cost, the buyer always agrees first with the low cost seller.

Proposition 3 shows that quantities differ from the efficient outcome when production is strictly convex both when goods are substitutes and complements. If convexity is not strict, efficiency can be restored.

Proposition 6 *If goods are perfect substitutes, R strictly concave in aggregate quantities and sellers have constant marginal costs d_1 and d_2 , for δ sufficiently close to 1, the equilibrium outcome is efficient with positive quantities only in the first agreement. If $d_1 \leq d_2$, agreement is with 1 first, giving 1 a unit profit of at most $d_2 - d_1 \geq 0$.*

The proposition implies that if supply is perfectly elastic, we get a result similar to Bertrand, with two symmetric sellers. If asymmetric, the low cost seller receives positive profits. At most, the low cost seller can capture the entire difference in cost between the two sellers, as in Bertrand competition. When sellers are symmetric, both receive zero profit.

In Segal (1999) analyzing bilateral contracting with externalities, it is shown that aggregate quantities are too high. It is assumed that welfare depends only on aggregate quantities, implying that goods are perfect substitutes and marginal costs linear. As Proposition 6 shows, our results are in stark contrast to Segal (1999).

4.1 n workers

The case with more than two workers is complicated to analyze for several reasons. First, due to multiplicity of equilibria when goods are value complements as indicated by Proposition 2, the complements case is rather intractable to analyze. Second, even when goods are substitutes, there is a potential for multiplicity; see Björnerstedt & Westermark (2006a). In contrast to the case with two workers, the multiplicity has consequences for the quantity choices, since the values substituted into (4) depend on the equilibrium in the subgames. This is not the case in the two worker case, where the continuation values $V_i(q_i)$ etc. always are the same, irrespectively of the equilibrium analyzed. Another problem is that concavity of the maximand in the value functions cannot be guaranteed as in Lemma 10 below. However, generically, any equilibrium must be inefficient. To see this, suppose that $N - 2$ agreements has been concluded on the efficient quantities with all sellers except i and j . We fix these agreements and let V_i, W_i, v_i and w_i in (4) denote the value in the subgame following the $N - 2$ agreements, $V_j(q_i)$ and $W_j(q_i)$ the values when bargaining with j , given an additional agreement on q_i . Finally $\Pi_i(q_i)$ denote the surplus when the buyer agrees with i on q_i in addition to the $N - 2$ previous agreements and $\pi_i(q_i(q_j), q_j)$ denote the surplus when i bargains with the buyer, given agreement with j on q_j in addition to the $N - 2$ previous agreements. Appropriately redefining the R function gives us

the following corollary to Proposition 3.

Corollary 7 *When there are N workers, unless goods are independent or the buyer has all bargaining power, equilibrium quantities are inefficient.*

To get more specific results on the relationship between equilibrium quantities, we proceed by analyzing the case when goods are perfect substitutes and sellers are symmetric. The reason is that this case is tractable since the state variable in bargaining is the aggregate quantity already agreed upon.⁶ Let N denote the set of sellers. A seller i is selected to bargain with the buyer according to some given order. For simplicity we renumber sellers so that seller 1 meets first with the buyer, seller 2 meets next and so on. When the buyer has agreed with the sellers in the set B , let $Q^B = \sum_{j \in B} q_j$ denote the aggregate quantity. Let $V_i(Q^B)$ denote the value of the buyer when proposing to i , given agreement on Q^B . Similarly, let $v_i(Q^B)$ be the value of seller i when being proposer and let $W_i(Q^B)$ be the value of the buyer when being respondent. Finally, let $w_i^j(Q^B)$ be the value to seller i when the buyer bargains with j . When $j = i$ this is the respondent payoff for i . For an immediate agreement equilibrium. the value functions are, when the seller bargains with i ,

$$\begin{aligned} V_i(Q^B) &= \max_{q_i} (\Pi_i(Q^B + q_i) + \delta (\lambda V_{i+1}(Q^{B \cup i}) + (1 - \lambda) W_{i+1}(Q^{B \cup i})) - w_i^i(Q^B)), \quad (14) \\ W_i(Q^B) &= \delta (\lambda V_{i+1}(Q^B) + (1 - \lambda) W_{i+1}(Q^B)), \\ v_i(Q^B) &= \max_{q_i} (\Pi_i(Q^B + q_i) + \delta (\lambda V_{i+1}(Q^{B \cup i}) + (1 - \lambda) W_{i+1}(Q^{B \cup i})) - W_i(Q^B)), \\ w_i^i(Q^B) &= \delta w_i^j(Q^B) \end{aligned}$$

where j is the next seller to bargain with the buyer and

$$\Pi_i(Q^B + q_i) = R(Q^B + q_i) - R(Q^B) - c(q_i).$$

where $R: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is strictly concave and c is strictly convex. One thing worth noting here is that, when goods are imperfect substitutes, the state variable above would be the entire vector of quantities, instead of just Q^B , greatly complicating the analysis. Lemma 15 in the appendix shows that an immediate agreement equilibrium exists. We have the following results, showing that the agreement quantities is a decreasing sequence, partially generalizing Proposition 3.

Proposition 8 *Suppose goods are perfect substitutes and sellers symmetric. In equilibrium, if i agrees before j then $q_i > q_j$.*

⁶With imperfect substitutes, the state variable is the entire vector of previous agreements.

Also, it is easy to see that early agreement quantities are larger than the efficient quantities and late agreement quantities smaller. To see this, for the last agreement the first-order condition with respect to q_n is

$$R'(Q^N) - c'(q_n) = 0. \quad (15)$$

The efficient quantities are determined by

$$R'(nq_n^e) - c'(q_n^e) = 0.$$

If all quantities are larger than the efficient we have $Q^N > nq_n^e$ and $q_n > q_n^e$, contradicting (15). Similarly, if all quantities are smaller than the efficient, we have $Q^N < nq_n^e$ and $q_n < q_n^e$, again contradicting (15).

4.2 Strategic Discrimination and Inefficiency

The propositions above can be used to derive some qualitative comparative statics results. More specifically, we want to study how quantities, profits and efficiency varies with substitutability, relative slope of supply and demand and the degree of asymmetry between sellers.

In Proposition 6, we have efficiency with completely elastic supply. Also, efficiency holds if supply is completely inelastic, as there is no scope for strategic discrimination through reallocating purchases from one seller to the other. By Proposition 2, we see that for intermediate values however, we do not get efficiency.

To see that efficiency is not monotonic in the degree of substitutability in general, consider constant marginal cost of production, Proposition 6 shows that the outcome is efficient with perfect substitutes. Also, we have efficiency when goods are independent. If goods are not perfect substitutes, however, inefficiency arises.

Elasticity of supply affects the distribution of surplus. When goods are perfect substitutes and supply is completely elastic, Proposition 6 shows that the firm obtains all the gains of trade (with symmetric sellers). If supply is completely inelastic, however, the standard results of Horn & Wolinsky hold: sellers are treated equally with both obtaining a positive share of the surplus.

To obtain results on how efficiency and unequal treatment varies with the degree of substitutability, seller asymmetries and relative slopes, we have to make more specific assumption on functional forms. In section 4.2, such specific assumptions are made.

The qualitative results of the previous section can be strengthened by more specific assumptions on revenue and cost functions. In the special case of linear marginal cost functions, comparative statics of the determinants of strategic discrimination can be analyzed.

We wish to study how strategic discrimination depends on exogenous parameters, such as marginal costs, substitutability and asymmetries between sellers. As relative bargaining power will not be the focus in the following, we will assume that $\lambda = \frac{1}{2}$. In light of the definition in section 4, we say that strategic discrimination increases in a parameter ξ if $q_i^F(\xi)/q_i^e(\xi)$ is increasing in ξ and $q_i^L(\xi)/q_i^e(\xi)$ is decreasing in ξ .

To study how equilibrium surplus depends on parameters is slightly more complicated than just analyzing the effect on the equilibrium surplus $\pi_1^F + \pi_2^L$, as changes in parameters affect the efficient surplus as well. Therefore, we look how the ratio of equilibrium to efficient surplus

$$\rho_e = \frac{\pi_1^F + \pi_2^L}{\pi_1^{F,e} + \pi_2^{L,e}}$$

varies as we change parameters. Let revenue and cost functions be given by

$$\begin{aligned} R(q_1, q_2) &= r(q_1 + q_2) - \frac{1}{2}(q_1^2 + 2sq_1q_2 + q_2^2) \\ c_1(q_1) &= \frac{c}{1-\theta}q_1^2 \\ c_2(q_2) &= \frac{c}{\theta}q_2^2. \end{aligned}$$

As $\theta \rightarrow 0$, only seller 1 is selling, with $\theta = 1/2$, sellers are symmetric. The relative slope of supply and demand is given by c . Solving for equilibrium quantities gives, when 1 is first,

$$\begin{aligned} q_1^F &= r \frac{(2c + (2-s)\theta)(1-\theta)}{2c + 2c^2 + \theta(2-s^2)(1-\theta)} \\ q_2^L &= 2r \frac{(c + (1-s)(1-\theta))\theta}{2c + 2c^2 + \theta(2-s^2)(1-\theta)}. \end{aligned} \tag{16}$$

The next proposition shows that, with linear marginal cost, we can say more than in Proposition 3. Agreement will be first with the seller with lower marginal cost and the quantity in the first agreement is higher than the efficient and the last quantity smaller. Strategic discrimination is high if supply is elastic, goods are easily substitutable and sellers relatively symmetric.

Proposition 9 *If goods are substitutes and $\theta < \frac{1}{2}$ any equilibrium is order efficient. Also:*

1. *Strategic discrimination increases*

- (a) *as the slope of supply relative to demand c , decreases.*
- (b) *as the homogeneity of goods s increases for $c > 1$.*
- (c) *as the symmetry of sellers θ increases.*

2. Efficiency increases

- (a) *as the slope of supply relative to demand c increases, for $c > 1$.*
- (b) *as the homogeneity of goods s decreases, for $c > 1$.*
- (c) *as the symmetry of sellers θ decreases.*

In the appendix, only the result on order efficiency is proved. As the proof of the rest of the proposition involves tedious algebra, it is available separately in Björnerstedt & Westermark (2006b).

The degree of strategic discrimination decreases with the slope of supply relative to demand. The slope of supply relative to demand can be interpreted as the disutility of effort for the sellers. Reallocation from one seller to the other costs more. With linear marginal costs, for sufficiently inelastic supply, efficiency increases in the slope of supply relative to demand.

Strategic discrimination increases and efficiency decreases in the degree of substitutability if the slope of supply relative to demand is high enough. The higher the degree of substitutability, the lower the cost of using quantities strategically, implying more strategic discrimination. Also, strategic discrimination increases and efficiency decreases in the symmetry of sellers. If $\theta = 0$ there is just one seller and standard results shows, e.g. Binmore (1987) that we get efficiency. If sellers are symmetric, we get inefficiencies. Also, the relationship is monotonic.

5 Concluding remarks

With simultaneous negotiations, we show that the buyer can obtain higher payoffs by using quantities strategically in order to reduce payment to sellers. This is the case even when sellers are symmetric and generically leads to inefficiencies.

The possibility of arbitrage might affect the results when goods are perfect substitutes, since then sellers could costlessly trade goods between themselves without being observed by the buyer. Then, the buyer potentially cannot use quantities to reduce the surplus in the last agreement. In the case of imperfect substitutes, it is costly to shift production of one good to the other seller, diminishing the problem with arbitrage.

It should be noted that the buyer will not have an incentive to renegotiate the contract in the first agreement to reduce the quantity agreed upon. Although quantities would be more efficient, in bargaining with seller 1 again, the buyer has to split surplus equally. This will not be beneficial for the buyer. The equilibrium is thus renegotiation proof in some sense.

In Björnerstedt & Westermark (2006c), we show that the model is robust to changes in how proposals are made. In such a model, equilibrium values and probabilities cannot be determined independently of equilibrium quantities, greatly increasing the complexity of the analysis. In the case when sellers are symmetric, results similar to those presented here can be obtained.

The model has a potential to explain some of the differences in work hours and pay between men and women, assuming that men prefer to work a little more than women; see Björnerstedt & Westermark (2006c). In this setting, employees are the sellers and the firm is the buyer. Introducing small asymmetries in a symmetric setup leads to potentially large asymmetries in treatment of workers. When workers are substitutes, the worker that is more/less inclined to work increases/decreases work hours more than what is motivated by efficiency considerations. Thus, the large difference in work hours between men and women could at least partially be caused by strategic effects. An implication of the model when the workers are sufficiently symmetric, is that, *given* the hourly wages, worker 1 will want to work less. Worker 2 will want to work more or less, depending on the relative slope of aggregate demand. This is in line with empirical evidence in Bell & Freeman (2000). Evidence from German GSOEP data indicates that workers feel that the actual working time is larger than the desired working time, taking the effect of reduced pay into account.

A Proofs

The following lemma is used to show that (11) is strictly concave when production is strictly convex.

Lemma 10 *The maximand of (11)*

$$(1 - \delta\lambda) R(q_1, 0) - c_1(q_1) + \delta\lambda(R(q_1, q_2(q_1)) - c_2(q_2(q_1))) \quad (17)$$

is strictly concave.

Proof: We have a sum of concave functions of the last term in (17) is concave in q_1 . Using the Envelope theorem, the second derivative of of the last term in (17) is

$$R_{11}(q_1, q_2(q_1)) + R_{12}(q_1, q_2(q_1)) \frac{dq_2}{dq_1} \quad (18)$$

Using

$$\frac{dq_2}{dq_1} = -\frac{R_{12}(q_1, q_2(q_1))}{R_{22}(q_1, q_2(q_1)) - c_2''(q_2)} \quad (19)$$

and that, by strict convexity of production, the denominator of (19) is negative, we have

$$-R_{11}(q_1, q_2(q_1))c_2''(q_2) + R_{11}(q_1, q_2(q_1))R_{22}(q_1, q_2(q_1)) - R_{12}^2(q_1, q_2(q_1)) \geq 0 \quad (20)$$

The first term is nonnegative by the concavity of R and convexity of c_2 . The latter two terms are nonnegative as the Hessian of R is nonnegative. Thus the last term in (17) is concave in q_1 . Thus, (17) is strictly concave, as one of first two the terms in (17) is strictly concave. ■

A.1 Pure equilibrium values and probabilities

To show Proposition 2, we first characterize the different equilibrium types in Lemmas 11 - 14. The proposition then collects the conditions of these Lemmas, letting $\delta \rightarrow 1$.

We first analyze pure strategy equilibria and then turn to mixed strategy equilibria. We focus on stationary subgame perfect equilibria (SSPE). Let

$$\varepsilon_i(\delta) = \delta(\pi_i^F(\delta) + \delta\pi_j^L(\delta)) - (\pi_j^F(\delta) + \delta\pi_i^L(\delta)) \quad (21)$$

$$\gamma_i(\delta) = \pi_i^F(\delta) - \delta^2(1 - \lambda)\pi_i^L(\delta) \quad (22)$$

Note first that there is no equilibrium where $\sigma_1 = 0$ and $\sigma_2 = 0$. Using this in the value equations implies that $V_i = W_i = v_i = w_i = 0$ for $i = 1, 2$. Then any proposer has a profitable deviation by offering slightly more than zero. Similarly, there is no equilibrium with $0 < \sigma_i < 1$ and $\sigma_j = 0$. Using this in the value equations for i gives $V_i = W_i = v_i = w_i = 0$. Indifference when i proposes gives $w_i = \pi_i^F(\delta) + \delta\lambda\pi_j^L(\delta) - W_i = \pi_i^F + \delta\lambda\pi_j^L(\delta)$, contradicting $w_i = 0$. Let

$$A_i(\delta) = \frac{\delta\lambda(\varepsilon_i(\delta) - \delta(1 - \delta^2)(1 - \lambda)\pi_i^L(\delta))}{1 - \delta^2(1 - \lambda)^2}$$

$$a_i(\delta) = \frac{\delta\lambda\varepsilon_i(\delta) + (1 - \delta^2)\left(\left(\pi_i^F(\delta) + \delta\lambda\pi_j^L(\delta)\right) - \delta^2\lambda(1 - \lambda)\pi_i^L(\delta)\right)}{1 - \delta^2(1 - \lambda)^2}$$

Note that, when $\varepsilon = 0$, we have $\lim_{\delta \rightarrow 1} A_i(\delta) = \lim_{\delta \rightarrow 1} a_i(\delta) = 0$ as $\lim_{\delta \rightarrow 1} \varepsilon_i(\delta) = 0$.

Lemma 11 *There is an equilibrium with immediate agreement between A and sellers 1 and 2 in*

any subgame if $(1 - \delta^2) (1 - \lambda) \gamma_1 (\delta) \geq \lambda \varepsilon_2 (\delta)$ and $(1 - \delta^2) (1 - \lambda) \gamma_2 (\delta) \geq \lambda \varepsilon_1 (\delta)$. We have

$$\begin{aligned}
V_i &= \pi_i^F (\delta) + \delta \lambda \pi_j^L (\delta) - \delta^2 (1 - \lambda) \pi_i^L (\delta), \\
W_i &= \frac{\delta^2 \lambda (2 - \lambda)}{1 - \delta^2 + \delta^2 \lambda (2 - \lambda)} (\pi_i^F (\delta) + \delta \lambda \pi_j^L (\delta) - \delta^2 (1 - \lambda) \pi_i^L (\delta)) - A_i (\delta), \\
v_i &= \frac{\delta^2 \lambda (2 - \lambda)}{1 - \delta^2 + \delta^2 \lambda (2 - \lambda)} \delta^2 (1 - \lambda) \pi_i^L (\delta) + a_i (\delta), \\
w_i &= \delta^2 (1 - \lambda) \pi_i^L (\delta)
\end{aligned} \tag{23}$$

Proof: If an agreement is reached in any meeting between A and the sellers then we have $\sigma_{Ai} = \sigma_{iA} = \sigma_i = 1$. Using this in the value equations (4) and some algebra gives (23).

In order for it to be an equilibrium, no player should have an incentive to set the probability σ_{Ai} or σ_{iA} less than 1. This is true iff $W_i \leq V_i$ and $w_i \leq v_i$ for $i = 1, 2$ or, since $v_i - w_i = V_i - W_i$,

$$\begin{aligned}
V_i &\geq \frac{\delta \lambda}{1 - \delta^2 (1 - \lambda)} V_j \\
V_j &\geq \frac{\delta \lambda}{1 - \delta^2 (1 - \lambda)} V_i
\end{aligned}$$

Using the solution for V_i and V_j gives

$$(1 - \delta^2) (\pi_i^F (\delta) - \delta^2 (1 - \lambda) \pi_i^L (\delta)) \geq \frac{\lambda}{1 - \lambda} (\delta (\pi_j^F (\delta) + \delta \pi_i^L (\delta)) - (\pi_i^F (\delta) + \delta \pi_j^L (\delta)))$$

This condition holds if $(1 - \delta^2) (1 - \lambda) \gamma_i (\delta) \geq \lambda \varepsilon_j (\delta)$ for $i = 1, 2$. ■

Lemma 12 *There is an equilibrium where an agreement is reached first between the buyer and seller i if $\lambda \varepsilon_i (\delta) \geq (1 - \lambda) \gamma_j (\delta)$. We have*

$$\begin{aligned}
V_i &= (1 - \delta^2 (1 - \lambda)) (\pi_i^F (\delta) + \delta \lambda \pi_j^L (\delta)) \\
W_i &= \delta^2 \lambda (\pi_i^F (\delta) + \delta \lambda \pi_j^L (\delta)) \\
v_i &= (1 - \delta^2 \lambda) (\pi_i^F (\delta) + \delta \lambda \pi_j^L (\delta)) \\
w_i &= \delta^2 (1 - \lambda) (\pi_i^F (\delta) + \delta \lambda \pi_j^L (\delta)) \\
v_j &= w_j = \delta^2 (1 - \lambda) \pi_j^L (\delta).
\end{aligned} \tag{24}$$

Proof: Using the value equations, setting $\sigma_i = 1$ and $\sigma_j = 0$ gives (24). Since $\frac{\delta^2 \lambda}{1 - \delta^2 (1 - \lambda)} < 1$ and $\frac{\delta^2 (1 - \lambda)}{1 - \delta^2 \lambda} < 1$ we have $V_i \geq W_i$ and $v_i \geq w_i$. Thus, both the buyer and seller i find it profitable to make acceptable offers. In order for A not to want to bid to j , the value to A has to be less

than waiting, i.e. $\pi_j^F(\delta) + \delta\lambda\pi_i^L(\delta) - w_j \leq W_j$ or, using that $W_j = \delta\lambda(\pi_i^F(\delta) + \delta\lambda\pi_j^L(\delta))$:

$$\delta\lambda(\pi_i^F(\delta) + \delta\lambda\pi_j^L(\delta)) - \lambda(\pi_j^F(\delta) + \delta\lambda\pi_i^L(\delta)) \geq (1-\lambda)(\pi_j^F(\delta) - \delta^2(1-\lambda)\pi_j^L(\delta)).$$

This condition holds if $\lambda\varepsilon_i(\delta) \geq (1-\lambda)\gamma_j(\delta)$. As the deviation condition when j is the proposer is identical, it is satisfied in this case also. ■

A.2 Mixed equilibria

The following lemmas state conditions for existence of mixed equilibria. Note that $\lim_{\delta \rightarrow 1} \sigma_i = 0$.

Lemma 13 *There is a SSPE with $0 < \sigma_1 < 1$ and $0 < \sigma_2 < 1$ if*

$$(1-\delta^2)(1-\lambda)\pi_i^L(\delta) - \delta\lambda\pi_j^L(\delta) > \gamma_i(\delta)$$

for $i = 1, 2$. We have

$$\sigma_i = (1-\delta^2) \frac{\pi_j^F(\delta) + \delta\lambda\pi_i^L(\delta)}{\delta^2 \left((1-\lambda)\pi_j^L(\delta) - (\pi_j^F(\delta) + \delta\lambda\pi_i^L(\delta)) \right)}. \quad (25)$$

and

$$\begin{aligned} V_i &= W_i = 0 \\ v_i &= w_i = \pi_i^F + \delta\lambda\pi_j^L. \end{aligned} \quad (26)$$

Proof: Both sellers are indifferent between making an acceptable offer or not, and thus

$$\begin{aligned} w_1 &= \pi_1^F(\delta) + \delta\lambda\pi_2^L(\delta) - W_1 \\ w_2 &= \pi_2^F(\delta) + \delta\lambda\pi_1^L(\delta) - W_2. \end{aligned}$$

Solving the value equations (4) for V_i and W_i ,

$$\begin{aligned} V_1 &= W_1 = \delta(\lambda V_2 + (1-\lambda)W_2), \\ V_2 &= W_2 = \delta(\lambda V_1 + (1-\lambda)W_1), \end{aligned}$$

which implies that $V_i = \delta^2 V_i$, which cannot hold, unless $V_i = 0$. Using this and the other equations in (4) gives (25) and (26). If the denominator in the last equation is positive, i.e.

$$(1-\delta^2)(1-\lambda)\pi_i^L(\delta) - \delta\lambda\pi_j^L(\delta) > \pi_i^F(\delta) - \delta^2(1-\lambda)\pi_i^L(\delta) = \gamma_i(\delta)$$

for $i = 1, 2$ and $j \neq i$ then a mixed equilibrium exists. ■

Lemma 14 *If*

$$\begin{aligned} (1 - \delta^2) \gamma_j(\delta) < \frac{\lambda}{1 - \lambda} \varepsilon_i(\delta) \quad \text{and} \quad \gamma_j(\delta) > \frac{\lambda}{1 - \lambda} \varepsilon_i(\delta) > 0 \quad \text{or} \\ (1 - \delta^2) \gamma_j(\delta) > \frac{\lambda}{1 - \lambda} \varepsilon_i(\delta) \quad \text{and} \quad -\delta \lambda \pi_i^L(\delta) \leq \gamma_j(\delta) < \frac{\lambda}{1 - \lambda} \varepsilon_i(\delta) < 0 \end{aligned} \quad (27)$$

there is a mixed SSPE where the buyer agrees with probability one with seller i and with probability $\sigma_j \in (0, 1)$ with seller j where

$$\sigma_j = \frac{1 - \delta^2}{\delta^2} \frac{(1 - \lambda) \gamma_j(\delta) - \lambda \varepsilon_i(\delta)}{\lambda \varepsilon_i(\delta)}$$

If $\gamma_j(\delta) = \varepsilon_i(\delta) = 0$ for any probability $\sigma_j \in (0, 1)$ there is a mixed SSPE where the buyer agrees with probability one with seller i and with σ_j with seller j . We have

$$\begin{aligned} V_i &= W_i + \frac{1 - \delta^2}{\delta^2 \lambda} W_i, \\ W_i &= \delta V_j = \delta W_j = \delta (\pi_j^F(\delta) + \delta \lambda \pi_i^L(\delta) - \delta^2 (1 - \lambda) \pi_j^L(\delta)), \\ v_i &= \pi_i^F(\delta) + \delta \lambda \pi_j^L(\delta) - W_i, \\ w_i &= \pi_i^F(\delta) + \delta \lambda \pi_j^L(\delta) - W_i - \frac{1 - \delta^2}{\delta^2 \lambda} W_i, \\ v_j &= w_j = \delta^2 (1 - \lambda) \pi_j^L(\delta). \end{aligned} \quad (28)$$

Proof: In order for a A to mix with j , we must have

$$V_j = W_j = \pi_j^F(\delta) + \delta \lambda \pi_i^L(\delta) - w_j$$

Similarly we have $v_j = w_j$. Using this in the expression for W_i gives

$$W_i = \delta (\lambda V_j + (1 - \lambda) W_j) = \delta W_j = \delta (\pi_j^F(\delta) + \delta \lambda \pi_i^L(\delta) - w_j)$$

Then, using $\sigma_i = 1$ and solving for the values in (4), we have (28). Note that

$$((1 - \lambda) \pi_i^L(\delta) - ((1 - \lambda) v_i + \lambda w_i)) = -\frac{1}{\delta} \varepsilon_i(\delta) \quad (29)$$

$$\delta^2 (((1 - \lambda) v_i + \lambda w_i) - w_i) = (1 - \delta^2) \frac{1}{\delta \lambda} ((1 - \lambda) \gamma_j(\delta) - \lambda (\varepsilon_i(\delta))) \quad (30)$$

We can solve for σ_j , using the expression for w_i in (4). If $\varepsilon_i(\delta) \neq 0$ we have

$$\sigma_j = \frac{1 - \delta^2}{\delta^2 \lambda} \frac{(1 - \lambda) \gamma_j(\delta) - \lambda \varepsilon_i(\delta)}{\varepsilon_i(\delta)}$$

If $\varepsilon_i(\delta) > 0$, to ensure $\sigma_j > 0$ we require $(1 - \lambda) \gamma_j(\delta) > \lambda \varepsilon_i(\delta)$. Also, for $\sigma_j < 1$, we must have

$$(1 - \delta^2) (1 - \lambda) \gamma_j(\delta) < \lambda \varepsilon_i(\delta) \quad (31)$$

A similar argument takes care of the case when $\varepsilon_i(\delta) < 0$. If $\varepsilon_i(\delta) = 0$, then, using (4) and (29),

$$w_i = \delta^2 ((1 - \lambda) v_i + \lambda w_i).$$

Then any $\sigma_j \in (0, 1)$ satisfies the above expression. Also, from (30) we have $\gamma_j(\delta) = 0$.

Acceptable offers between A and i will be preferred, since $\frac{1 - \delta^2(1 - \lambda)}{\delta^2 \lambda} > 1$ implies $V_i > W_i$ and $v_i > w_i$ whenever $W_i > 0$ and $w_i > 0$ from (28). Also from (28) we have

$$\frac{W_i}{\delta} = \gamma_j(\delta) + \delta \lambda \pi_i^L(\delta), \quad (32)$$

and $W_i \geq 0$ iff

$$\gamma_j(\delta) + \delta \lambda \pi_i^L(\delta) \geq 0$$

If $\varepsilon_i(\delta) \geq 0$, then from (27) $\gamma_j(\delta) \geq 0$, and if $\varepsilon_i(\delta) < 0$, then $\gamma_j(\delta) \geq -\delta \lambda \pi_i^L(\delta)$ by assumption.

To see that $w_i > 0$, consider

$$\begin{aligned} w_i &= \frac{\delta}{\delta^2 \lambda} (\lambda \varepsilon_i(\delta) - (1 - \lambda) (\gamma_j(\delta))) + \frac{\delta^2 (1 - \lambda)}{\delta^2 \lambda} (\delta (\gamma_j(\delta) + \delta \lambda \pi_i^L(\delta))) \\ &= \frac{\delta}{\delta^2 \lambda} (\lambda \varepsilon_i(\delta) - (1 - \delta^2) (1 - \lambda) \gamma_j(\delta)) + \delta^2 (1 - \lambda) \pi_i^L(\delta). \end{aligned}$$

If $\varepsilon_i(\delta) < 0$ then $(1 - \lambda) \gamma_j(\delta) < \lambda \varepsilon_i(\delta)$ and hence $w_i > 0$ if $W_i \geq 0$. If $\varepsilon_i(\delta) \geq 0$ then $(1 - \delta^2) (1 - \lambda) \gamma_j(\delta) \leq \lambda \varepsilon_i(\delta)$ and hence $w_i > 0$. Thus there are no incentives to deviate. ■

When $\lambda \geq \frac{1}{2}$ the condition $-\delta \lambda \pi_i^L(\delta) \leq \gamma_j(\delta)$ in the statement of the Lemma is redundant, as

$$\gamma_j(\delta) + \delta \lambda \pi_i^L(\delta) = -\lambda \varepsilon_i(\delta) - \delta^2 (1 - 2\lambda) \pi_j^L(\delta) + (1 - \lambda) \pi_j^F(\delta) + \lambda \delta \pi_i^F(\delta)$$

To simplify expressions, let $\Delta = \frac{1}{1 - \delta^2}$ and $\Lambda = \frac{\lambda}{1 - \lambda}$.

Proof of Proposition 2.

Step 1: Collecting results Lemmas 11 - 14. Since $\varepsilon_i(\delta)$ is continuous in δ and $\varepsilon_1(1) = \varepsilon$, $\varepsilon_2(1) = -\varepsilon$ there is some $\hat{\delta} < 1$ such that $\varepsilon_1(\delta) > 0$ and $\varepsilon_2(\delta) < 0$ for all $\delta > \hat{\delta}$. Let

$$\begin{aligned}
a_i &= -\delta\lambda\pi_i^L(\delta) \\
b_i &= a_i + (1 - \delta^2)(1 - \lambda)\pi_j^L(\delta) \\
c_i &= \Lambda\varepsilon_i(\delta) \\
d_i &= \Delta c_i
\end{aligned} \tag{33}$$

Case 1: Suppose $\lambda \geq \frac{1}{2}$.

We can summarize the equilibria in Lemmas 11 - 14 in terms of the following table, using that $\varepsilon_i(\delta) > 0$ implies $\Delta\varepsilon_i(\delta) > \varepsilon_i(\delta)$ and $\varepsilon_i(\delta) < 0$ implies $\Delta\varepsilon_i(\delta) < \varepsilon_i(\delta)$,

| | | | |
|---------------------------------------|---------------------------------------|-----------------------------------|--------------------------------------|
| | $\gamma_2(\delta) \in (-\infty, c_1]$ | $\gamma_2(\delta) \in (c_1, d_1)$ | $\gamma_2(\delta) \in [d_1, \infty)$ |
| $\gamma_1(\delta) \in (-\infty, d_2)$ | P_1, P_2 | P_2, M_1 | P_2 |
| $\gamma_1(\delta) \in [d_2, c_2]$ | P_1, P_2, M_2 | P_2, M_1, M_2 | I, M_2, P_2 |
| $\gamma_1(\delta) \in (c_2, \infty)$ | P_1 | M_1 | I |

Case 2: Suppose $\lambda < \frac{1}{2}$.

Since $\varepsilon > 0$, there exists a $\hat{\delta} < 1$ such that

$$-\delta\lambda\pi_1^L(\delta) + (1 - \delta^2)(1 - \lambda)\pi_2^L(\delta) < \Lambda\varepsilon_1(\delta) < \Delta\Lambda\varepsilon_1(\delta)$$

and, if $-\delta\lambda\pi_2^L(\delta) < \Lambda\varepsilon_2(\delta)$ then

$$\Delta\Lambda\varepsilon_2(\delta) < -\delta\lambda\pi_2^L(\delta) < -\delta\lambda\pi_2^L(\delta) + (1 - \delta^2)(1 - \lambda)\pi_1^L(\delta) < \Lambda\varepsilon_2(\delta)$$

and, if $-\delta\lambda\pi_2^L(\delta) \geq \Lambda\varepsilon_2(\delta)$ then

$$\Delta\Lambda\varepsilon_2(\delta) < \Lambda\varepsilon_2(\delta) \leq -\delta\lambda\pi_2^L(\delta) + (1 - \delta^2)(1 - \lambda)\pi_1^L(\delta).$$

We can summarize the equilibria in Lemmas 11 - 14 in terms of the following tables

If $-\delta\lambda\pi_2^L < \Lambda\varepsilon_2(\delta)$

| | $\gamma_2(\delta) \in (-\infty, b_1)$ | $\gamma_2(\delta) \in [b_1, c_1]$ | $\gamma_2(\delta) \in (c_1, d_1)$ | $\gamma_2(\delta) \in [d_1, \infty)$ |
|---------------------------------------|---------------------------------------|-----------------------------------|-----------------------------------|--------------------------------------|
| $\gamma_1(\delta) \in (-\infty, d_2)$ | P_1, P_2, H | P_1, P_2 | P_2, M_1 | P_2 |
| $\gamma_1(\delta) \in [d_2, a_2)$ | P_1, P_2, H | P_1, P_2 | P_2, M_1 | I, P_2 |
| $\gamma_1(\delta) \in [a_2, b_2)$ | P_1, P_2, M_2, H | P_1, P_2, M_2 | P_2, M_1, M_2 | I, P_2, M_2 |
| $\gamma_1(\delta) \in [b_2, c_2]$ | P_1, P_2, M_2 | P_1, P_2, M_2 | P_2, M_1, M_2 | I, M_2, P_2 |
| $\gamma_1(\delta) \in (c_2, \infty)$ | P_1 | P_1 | M_1 | I |

If $-\delta\lambda\pi_2^L \geq \Lambda\varepsilon_2(\delta)$

| | $\gamma_2(\delta) \in (-\infty, b_1)$ | $\gamma_2(\delta) \in [b_1, c_1]$ | $\gamma_2(\delta) \in (c_1, d_1)$ | $\gamma_2(\delta) \in [d_1, \infty)$ |
|---------------------------------------|---------------------------------------|-----------------------------------|-----------------------------------|--------------------------------------|
| $\gamma_1(\delta) \in (-\infty, d_2)$ | P_1, P_2, H | P_1, P_2 | P_2, M_1 | P_2 |
| $\gamma_1(\delta) \in [d_2, c_2]$ | P_1, P_2, H | P_1, P_2 | P_2, M_1 | I, P_2 |
| $\gamma_1(\delta) \in (c_2, b_2)$ | P_1, H | P_1 | M_1 | I |
| $\gamma_1(\delta) \in [b_2, \infty)$ | P_1 | P_1 | M_1 | I |

Step 2: Taking limits. When $\varepsilon > 0$, as $\delta \rightarrow 1$ we have $\Delta \rightarrow \infty$, and hence $d_1 = \Delta\Lambda\varepsilon_1(\delta) \rightarrow \infty$, $d_2 = \Delta\Lambda\varepsilon_2(\delta) \rightarrow -\infty$ and $a_2 \rightarrow b_2$, eliminating some intervals.

We need to find what we converge to if $\lambda\varepsilon = (1 - \lambda)\gamma_2$ or $\lambda\varepsilon = -(1 - \lambda)\gamma_1$ above, i.e., the boundary cases in proposition 2. If $\lambda\varepsilon = (1 - \lambda)\gamma_2$ then $(1 - \lambda)\gamma_2(\delta) > \lambda\varepsilon_1(\delta)$ for all $\delta < 1$ as

$$(1 - \lambda)\gamma_2(\delta) - \lambda\varepsilon_1(\delta) = (1 - \lambda)\left((1 - \delta^2)(1 - \lambda)\pi_2^L(\delta)\right) + \lambda(1 - \delta)\left(\pi_1^F(\delta) + \delta\pi_2^L(\delta)\right) > 0.$$

By the same argument, $(1 - \lambda)\gamma_1(\delta) > \lambda\varepsilon_2(\delta)$ for all $\delta < 1$. Also, when $-\lambda\pi_1^L = \gamma_2$ we get

$$\gamma_2(\delta) - (-\delta\lambda\pi_1^L + (1 - \delta^2)(1 - \lambda)\pi_2^L) = \pi_2^F - (1 - \lambda)\pi_2^L - ((1 - \delta)\lambda\pi_1^L - \lambda\pi_1^L) = -(1 - \delta)\lambda\pi_1^L < 0$$

for all $\delta < 1$. A similar argument establishes the corresponding inequality when $-\lambda\pi_2^L(\delta) = \gamma_1$.

When $\lambda \geq \frac{1}{2}$ we have $-\delta\lambda\pi_2^L(\delta) \leq \gamma_1(\delta)$ since, using $\varepsilon_2(\delta) < 0$, we have

$$\gamma_1(\delta) + \delta\lambda\pi_2^L(\delta) = -\lambda\varepsilon_2(\delta) - \delta^2(1 - 2\lambda)\pi_1^L(\delta) + (1 - \lambda)\pi_1^F(\delta) + \lambda\delta\pi_2^F(\delta) > 0$$

Step 3: Efficiency. When goods are value substitutes, the result follows from the two cases in Step 1 and the limit argument in Step 2. ■

When $\varepsilon = 0$ the Lemmas 11 - 14 can be used for a characterization. It is complicated to summarize the results in tables, since when $\lambda < \frac{1}{2}$ there are four different cases, as compared

to the two in Proposition 2. The complexity when $\lambda < \frac{1}{2}$ is due to the fact that we can have $\Delta\Lambda\varepsilon_i(\delta) \leq -\delta\lambda\pi_i^L(\delta)$ for $i = 1, 2$. This leads to four possible interval configurations. The case when $\lambda \geq \frac{1}{2}$ can be more conveniently illustrated. Noting that $\frac{\partial\varepsilon_i(1)}{\partial\delta} > 0$ there is some $\hat{\delta}$ such that $\varepsilon_i(\delta) < 0$ for $\hat{\delta} < \delta < 1$ and hence $\Delta\varepsilon_i(\delta) < \varepsilon_i(\delta)$ when $\hat{\delta} < \delta < 1$ for $i = 1, 2$. Also, when $\varepsilon = 0$, $\lim_{\delta \rightarrow 1} \Delta\varepsilon_i(\delta) = -\frac{\pi_j^F + \pi_j^L}{2}$ and $\gamma_i(\delta) > \Delta\Lambda\varepsilon_j(\delta)$ for δ close to 1, as

$$\lim_{\delta \rightarrow 1} \gamma_i(\delta) - \Delta\Lambda\varepsilon_j(\delta) = \frac{1}{1-\lambda} \left(\left(1 - \frac{\lambda}{2}\right) \pi_i^F + \left(\frac{\lambda}{2} - (1-\lambda)^2\right) \pi_i^L \right) > 0.$$

We can summarize the equilibria in Lemmas 11 - 14 as follows, using (33)

| | $\gamma_2(\delta) \in (-\infty, d_1]$ | $\gamma_2(\delta) \in (d_1, c_1)$ | $\gamma_2(\delta) \in [c_1, \infty)$ |
|---------------------------------------|---------------------------------------|-----------------------------------|--------------------------------------|
| $\gamma_1(\delta) \in (-\infty, d_2]$ | P_1, P_2 | P_1, P_2, M_1 | P_2 |
| $\gamma_1(\delta) \in (d_2, c_2)$ | P_1, P_2, M_2 | I, P_1, P_2, M_1, M_2 | I, P_2, M_2 |
| $\gamma_1(\delta) \in [c_2, \infty)$ | P_1 | I, P_1, M_1 | I |

(34)

A.3 Equilibrium quantities

Proof of Proposition 3: Step 1. Assume that the buyer and seller 1 have come to agreement on q_1 . The first-order condition to the problem in (2) is

$$\frac{\partial R(q_1, q_2)}{\partial q_2} - c'_2(q_2) = 0 \tag{35}$$

Step 2. Now, let us analyze bargaining between the buyer and seller 1.

Suppose $\lambda < 1$. Evaluating (12) at q_1^e , we have $q_2(q_1^e) = q_2^e$ and, using that $\frac{\partial R(q_1^e, q_2^e)}{\partial q_1} = c'_1(q_1^e)$,

$$\begin{aligned} & \frac{\partial R(q_1^e, q_2^e)}{\partial q_1} - (1-\lambda) \left(\frac{\partial R(q_1^e, q_2^e)}{\partial q_1} - \frac{\partial R(q_1^e, 0)}{\partial q_1} \right) - c'_1(q_1^e) \\ &= - (1-\lambda) \left(\frac{\partial R(q_1^e, q_2^e)}{\partial q_1} - \frac{\partial R(q_1^e, 0)}{\partial q_1} \right) > 0 \end{aligned}$$

when goods are substitutes. When evaluated at q_1^F and q_2^L , expression (12) is zero. As (11) is strictly concave in q_1 , (12) is decreasing in q_1 . Thus $q_1^F > q_1^e$. Since $q_1^F > q_1^e$ and $q'_2(q_1) < 0$, we have $q_2^L < q_2^e$. A similar argument establishes the results when goods are complements and independent, respectively.

When $\lambda = 1$ it is easy too see from (12) that the efficient quantities solve the first-order condition. Also, the objective (11) is equal to social surplus.

Step 3. Monotonicity in λ .

Consider $\lambda, \hat{\lambda}$ with $\lambda < \hat{\lambda}$ and let (q_1^F, q_2^L) and $(\hat{q}_1^F, \hat{q}_2^L)$ be the corresponding equilibrium

quantities. Define

$$\Theta(q_1^F, \hat{\lambda}) = \frac{\partial R(q_1^F, q_2(q_1^F))}{\partial q_1} - (1 - \hat{\lambda}) \left(\frac{\partial R(q_1^F, q_2(q_1^F))}{\partial q_1} - \frac{\partial R(q_1^F, 0)}{\partial q_1} \right) - c_1'(q_1^F)$$

We have $\Theta(q_1^F, \lambda) = 0$ from the first-order condition. When $\frac{\partial^2 R(q_1, q_2)}{\partial q_1 \partial q_2} < 0$ then $\Theta(q_1^F, \hat{\lambda})$ is decreasing in $\hat{\lambda}$ and we have $\Theta(q_1^F, \hat{\lambda}) < 0$ and when $\frac{\partial^2 R(q_1, q_2)}{\partial q_1 \partial q_2} > 0$ then $\Theta(q_1^F, \hat{\lambda})$ is increasing in $\hat{\lambda}$ and we have $\Theta(q_1^F, \hat{\lambda}) > 0$. Since $\Theta(q_1^F, \hat{\lambda})$ is decreasing in q_1^F it follows that $q_1^F > \hat{q}_1^F$ when $\frac{\partial^2 R(q_1, q_2)}{\partial q_1 \partial q_2} < 0$ and $q_1^F < \hat{q}_1^F$ when $\frac{\partial^2 R(q_1, q_2)}{\partial q_1 \partial q_2} > 0$. Also, since we have $\frac{dq_2}{dq_1} < 0$ when $\frac{\partial^2 R(q_1, q_2)}{\partial q_1 \partial q_2} < 0$ then $q_2^L < \hat{q}_2^L$ and since we have $\frac{dq_2}{dq_1} > 0$ when $\frac{\partial^2 R(q_1, q_2)}{\partial q_1 \partial q_2} > 0$ then $q_2^L > \hat{q}_2^L$. The relationship with efficient quantities follows from Proposition 3. ■

Proof of Proposition 5:

Case 1. Substitutes.

Step 1. Show that $\varepsilon > 0$.

$$\begin{aligned} \frac{d\varepsilon}{d\varphi} &= \frac{d(\pi_1^F + \pi_2^L)}{d\varphi} - \frac{d(\pi_2^F + \pi_1^L)}{d\varphi} = \left(\frac{\partial R(q_1^F, q_2(q_1^F))}{\partial q_1} - \varphi c'(q_1^F) \right) \frac{dq_1}{d\varphi} - c(q_1^F) \\ &\quad - \left(\frac{\partial R(q_1(q_2^F), q_2^F)}{\partial q_2} - c'(q_2^F) \right) \frac{dq_2}{d\varphi} + c(q_1(q_2^F)) \end{aligned}$$

Evaluating at $\varphi = 1$, by symmetry, we have, using the first-order conditions

$$\frac{d\varepsilon}{d\varphi} = -c(q_1^F) + c(q_1(q_2^F))$$

As workers are substitutes, by Proposition 3, we have $q_1^F > q_1(q_2^F)$ and thus $\frac{d\varepsilon}{d\varphi} \Big|_{\varphi=1} < 0$. For $\varphi = 1$ we have $\varepsilon = 0$. As the derivative is continuous in φ , we have $\varepsilon > 0$ for φ sufficiently close to 1, and since ε is continuous in φ , there is a $\bar{\varphi}$ such that for all $\varphi > \bar{\varphi}$ we have $\varepsilon > 0$.

Step 2. We need to show that

$$\begin{aligned} (1 - \lambda) \gamma_1 &> -\lambda \varepsilon \\ \gamma_1 &> -\lambda \pi_2^L \end{aligned} \tag{36}$$

We have

$$\gamma_1 = \pi_1^F - (1 - \lambda) \pi_1^L = R(q_1^F, 0) - \varphi c(q_1^F) - (1 - \lambda) (R(q_1^L, q_2^F) - R(0, q_2^F) - \varphi c(q_1^L)). \tag{37}$$

The first-order condition for q_1^F can be rewritten as

$$\lambda \frac{\partial R(q_1^F, q_2^L)}{\partial q_1} + (1 - \lambda) \frac{\partial R(q_1^F, 0)}{\partial q_1} - \varphi c'(q_1^F) = 0.$$

When workers are substitutes, we have $\frac{\partial R(q_1^F, q_2^L)}{\partial q_1} < \frac{\partial R(q_1^F, 0)}{\partial q_1}$ and hence we have

$$\frac{\partial R(q_1^F, 0)}{\partial q_1} - \varphi c'(q_1^F) > 0. \quad (38)$$

Note that, by using that, since $\frac{\partial^2 R(q_1, q_2)}{\partial q_1 \partial q_2} < 0$ we have

$$R(q_1, 0) > R(q_1, q_2) - R(0, q_2)$$

and hence (37) is

$$\pi_1^F - (1 - \lambda) \pi_1^L > R(q_1^F, 0) - \varphi c(q_1^F) - (1 - \lambda) (R(q_1^L, 0) - \varphi c(q_1^L)).$$

Also, (38) implies that $R(q_1^F, 0) - \varphi c(q_1^F) > R(q_1^L, 0) - \varphi c(q_1^L)$ and hence $\gamma_1 > 0$. For $\varphi > \bar{\varphi}$ we have, by continuity of ε , π_1^F and π_1^L that (36) holds.

Step 3. Since $\varepsilon > 0$, $(1 - \lambda) \gamma_1 > -\lambda \varepsilon$ and $\gamma_1 > -\lambda \pi_2^L$ then, by Proposition 1, the first agreement is with seller 1 with probability 1 as $\delta \rightarrow 1$.

Case 2. Complements.

Follows from Example 4. ■

Proof of Proposition 6: When goods 1 and 2 are perfect substitutes and marginal costs are constant, then R is a function of aggregate quantity only, i.e., we can write

$$R(q_1 + q_2)$$

and

$$c_i(q_i) = d_i q_i.$$

Without loss of generality we assume that $d_1 \leq d_2$. In order for the problem to be non-trivial, we also assume that $R'(0) > d_2$.

Step 1. The efficient quantities.

When $d_1 < d_2$, it is easy to see that the efficient quantities satisfy

$$R'(q_1^e) - d_1 = 0 \quad (39)$$

and $q_2^e = 0$. When $d_1 = d_2$, we have

$$R'(q_1^e + q_2^e) - d_1 = 0. \quad (40)$$

There is a multiplicity of solutions where all q_1 and q_2 satisfying $R'(q_1 + q_2) = d_1$ are solutions.

Step 2. The equilibrium quantities.

Suppose the firm agrees first with seller 1. The first-order condition in the last bargain is

$$R'(q_1 + q_2) - d_2 \leq 0. \quad (41)$$

In the first bargain, the first-order condition is

$$R'(q_1 + q_2) - (1 - \lambda)(R'(q_1 + q_2) - R'(q_1)) - d_1 \leq 0. \quad (42)$$

If $q_2 > 0$, then (41) holds with equality, and (42) is

$$R'(q_1) \leq \frac{d_1 - \lambda d_2}{1 + \lambda}.$$

Then $R'(q_1) \leq R'(q_1 + q_2)$, a contradiction. We thus have q_1^F solving $R'(q_1^F) = d_1$ and $q_2^L = 0$.

Suppose the firm agrees first with seller 2. Similar arguments as above establishes that if $d_1 < d_2$ we have $q_2^F > 0$ and $q_1^L > 0$. The aggregate quantity solves $R'(q_2^F + q_1^L) = d_1$. If $d_1 = d_2$ we have q_2^F solving $R'(q_2^F) = d_2$ and $q_1^L = 0$.

If $d_1 < d_2$ agreeing with 1 first gives a total surplus of

$$\pi_1^F + \pi_2^L = R(q_1^F) - d_1 q_1^F \quad (43)$$

and agreeing with 2 first gives

$$\pi_2^F + \pi_1^L = R(q_2^F + q_1^L) - d_1 q_1^L - d_2 q_2^F \quad (44)$$

$$\gamma_1 = \pi_1^F - (1 - \lambda)\pi_1^L$$

Note that the aggregate quantities are the same in the two cases. Hence, we have $\varepsilon > 0$. Also, when agreeing with 1 first we have $\pi_2^L = 0$ ensuring that $\pi_1^F - \pi_1^L > \pi_2^F > 0$ and hence $(1 - \lambda)\gamma_1 > -\lambda\varepsilon$. Using Proposition 2 establishes that all equilibria prescribe agreement with 1 first. Since $R'(q_1^F) = d_1$ we have, from expression (39) that $q_1^F = q_1^e$ and $q_2^L = q_2^e = 0$.

If $d_1 = d_2$ and the firm agrees first with 1, we have q_1^F solving $R'(q_1^F) = d_1$ and $q_2^L = 0$. If

the firm agrees first with 2, we have q_2^F solving $R'(q_2^F) = d_2$ and $q_1^L = 0$. Clearly, both of these candidates satisfy expression (40).

Step 3. Equilibrium payoffs.

When agreement is with 2 first, we have:

$$R'(q_2^F) = \frac{d_2 - \lambda d_1}{1 + \lambda}. \quad (45)$$

The payoff to 1 is $(1 - \lambda) \pi_1^L + \varepsilon$. Using (43) and (44) we have $\varepsilon = (d_2 - d_1) q_2^F$, the efficiency loss of agreeing with 2 first. Since $R'(q) \leq d_2$ from (45) and R' is decreasing,

$$\pi_1^L = R(q_2^F + q_1^L) - R(q_2^F) - d_1 q_1^L = \int_{q_2^F}^{q_2^F + q_1^L} R'(q) dq - d_1 q_1^L < (d_2 - d_1) q_1^L.$$

Then $(1 - \lambda) \pi_1^L + \varepsilon$ is at most $(d_2 - d_1) (q_2^F + (1 - \lambda) q_1^L) < (d_2 - d_1) q_1^F$. ■

Lemma 15 *There exists an immediate agreement equilibrium.*

Proof: For existence, note that, by symmetry, we have $V_i(Q^B) = V_{i+1}(Q^B)$ and $W_i(Q^B) = W_{i+1}(Q^B)$. From the second value equation, we then have

$$W_i(Q^B) = \frac{\delta \lambda}{1 - \delta(1 - \lambda)} V_i(Q^B)$$

Since $\frac{\delta \lambda}{1 - \delta(1 - \lambda)} < 1$ we have $W_i(Q^B) < V_i(Q^B)$ whenever $V_i(Q^B) > 0$. Also, from (14), we have

$$v_i(Q^B) - w_i^i(Q^B) = V_i(Q^B) - W_i(Q^B).$$

We proceed by induction to show that $V_i(Q^B) > 0$. Note that $V_i(Q^{N \setminus i}) > 0$. Suppose $V_{i+1}(Q^{B \cup i}) > 0$. Note that, since the seller always can threaten buyers with being last, the value for buyers is in the limit,

$$w_i^i(Q^B) = (1 - \lambda) R(Q^{n-1}, q_n(Q^{n-1})) - R(Q^{n-1}, 0) - c(q_n(Q^{n-1}))$$

By substitutability, we have, when $q_i = q_n(Q^{n-1})$,

$$\Pi_i(Q^B + q_i) > w_i^i(Q^B)$$

Since $q_i = q_n(Q^{n-1})$ is not necessarily optimal in the problem in $V_i(Q^B)$, it follows that $V_i(Q^B) > 0$. By continuity, we have $V_i(Q^B) > 0$ for δ close to one, establishing existence. ■

Proof of Proposition 8.

The first-order condition to the value function problem is

$$R'(Q^B, q_i, 0) - c'(q_i) + \delta W'(Q^{B \cup i}) = 0$$

We also have

$$\frac{dq_{i-1}}{dq_{i-2}} = -\frac{R''(Q^B, 0) + W''(Q^B)}{R''(Q^B, 0) - c''(q_{i-1}) + W''(Q^B)}$$

and hence $\frac{dq_{i-1}}{dq_{i-2}} > -1$.

The first-order condition is, using

$$W'(Q^B) = R'(Q^{B \cup i}, 0) - R'(Q^B, 0) + W'(Q^{B \cup i}) - w_{iQ^B}^i(Q^B),$$

in the limit

$$c'(q_i) - c'(q_{i-1}) - w_{iQ^B}^i(Q^B) = 0 \tag{46}$$

Consider $w_{iQ^B}^i(Q^B)$. We show that $w_{iQ^B}^i(Q^B) < 0$ by induction. First, note that $w_n^n(Q^{n-1})$ is decreasing and suppose $w_{i+1}^{i+1}(Q^{B \cup i})$ is decreasing. In the limit, a seller is indifferent between agreeing and waiting for another buyer to agree, implying that

$$w_i^i(Q^B) = w_{i+1}^{i+1}(Q^{B \cup i}) = w_{i+1}^{i+1}(Q^B + q_{i+1}(Q^B))$$

Then

$$w_{iQ^B}^i(Q^B) = w_{i+1Q^{B \cup i}}^{i+1}(Q^B + q_{i+1}(Q^B)) \left(1 + \frac{dq_{i-1}}{dq_{i-2}}\right)$$

Since $\frac{dq_{i-1}}{dq_{i-2}} > -1$ we have $w_i^i(Q^B)$ decreasing.

Since $w_{iQ^B}^i(Q^B) < 0$ we have $c'(q_i) < c'(q_{i-1})$ in (46) and hence $q_{i-1} > q_i$. ■

A.4 Strategic Discrimination and Inefficiency

Let $u_1 = \frac{c}{1-\theta}$ and $u_2 = \frac{c}{\theta}$. Using the equilibrium quantities we get

$$\begin{aligned} \pi_1^F &= \frac{r^2 (2 - s + 2u_2) (2 + s(1 - 2s) + 2u_2 + u_1(2 + s + 2u_2))}{2(2 - s^2 + 2u_2 + 2u_1(1 + u_2))^2} \\ \pi_2^L &= \frac{2r^2 (1 - s + u_1)^2 (1 + u_2)}{(2 - s^2 + 2u_2 + 2u_1(1 + u_2))^2} \end{aligned} \tag{47}$$

Again, when 2 is first π_2^F and π_1^L are similar with indices interchanged.

Proof of first part of Proposition 9: From Proposition 2, if $\varepsilon > 0$ and $\gamma_1 > -\varepsilon$ and

$\gamma_2 > \varepsilon$, then the unique equilibrium is M_1 . Using (47) gives

$$\varepsilon = \frac{r^2 s^2 (u_2 - u_1)}{2 (2 - s^2 + 2u_2 + 2u_1 (1 + u_2))^2}$$

Thus $\varepsilon > 0$ for $u_1 < u_2$. Again using (47) gives

$$\gamma_1 = \frac{r^2 (2 + 4s - 7s^2 + 2s^3 + 4(1 + s - s^2)u_2 + 2u_2^2 + u_1(2 + 4s - 3s^2 + 4(1 + s)u_2 + 2u_2^2))}{2 (2 - s^2 + 2u_2 + 2u_1 (1 + u_2))^2}$$

$$\gamma_2 = \frac{r^2 (2 + 4s - 7s^2 + 2s^3 + (2 + 4s - 3s^2)u_2 + 2u_1^2(1 + u_2) + 4u_1(1 + s - s^2 + (1 + s)u_2))}{2 (2 - s^2 + 2u_2 + 2u_1 (1 + u_2))^2}$$

Some tedious algebra shows that we have $\gamma_1 > -\varepsilon$ and $\gamma_2 > \varepsilon$ for all u_1 and u_2 . ■

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