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Exchange Rate Variability Inside and Outside the EMU

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ABSTRACT

An often heard view is that exchange rate variability will decrease for a country that joins the EMU. This is not necessarily true. Both real and nominal exchange rate variability increase under certain circumstances when asymmetric demand shocks occur inside or outside the union. These results are obtained theoretically within a standard international macro-model but they also remain valid in a numerical simulation of this model.

JEL Classification: F31; F33; F36; F41. **Key Words:** Exchange rate variability, EMU, asymmetric shocks.

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1. Introduction

There are at the moment three countries that have not yet decided whether they will participate fully in the European Economic and Monetary Union (the EMU): The United Kingdom, Sweden and Denmark. Some of the most central arguments regarding this important and probably irreversible decision are¹: A membership would reduce exchange rate variability and thus be advantageous to firms and others that are engaged in foreign economic activity². Furthermore, a membership would give microeconomic benefits in the form of lower costs of international transactions. The perhaps most important disadvantage, on the other hand, is the loss of independent monetary policy as a means to reduce the real impact of asymmetric shocks. This paper questions the EMU-positive argument above about reduced exchange rate variability. The nominal exchange rate variability between a possible entrant and the rest of the EMU will of course disappear as a result of a membership but the interesting thing is that it is not necessary that a membership in the EMU reduces total real (or nominal) exchange rate variability when variability to other currencies are included in the analysis.

The purpose of this paper is therefore to show how this possibly surprising result may arise, which is done by examining how total exchange rate variability due to macroeconomic shocks may change if a country joins the EMU. The paper will only deal with such exchange rate variability that has its origin in macroeconomic demand shocks and does not attempt to explain or discuss exchange rate fluctuations on a day to day basis.

The problem is analysed within an intertemporal framework with sticky prices for one period³. The shocks that create the need for exchange rate adjustments are permanent and asymmetric demand shocks which only affect one currency area directly. When a shock hits one country the exchange rates adjust immediately to restore equilibrium in the goods markets of the different countries. If a country joins the EMU, the value of the entrant's currency can no longer adjust and, because of that, output in the entering country will have to move away from its long run equilibrium level temporarily to clear the goods market. After one period prices become fully flexible and output returns to its long-run level. The exchange rates to countries outside the EMU are crucial to this analysis so the model has to consist of at least three countries: The EMU-entrant, the rest of the EMU and the rest of the world. In general, all exchange rates change due to a shock and the total effect is captured by an index that incorporates all the exchange rates that the entrant faces. Whether the possible entrant is a member of the union or not will affect the value of this index which means that one can

¹SOU 1996:158.

²For a presentation of some of the arguments in the literature regarding aversion to exchange rate variability, see *Section 3.1*.

³The realistic degree of nominal price rigidity is a controversial issue but what is needed to get the qualitative results in this model is only that goods prices and wages are more sticky than nominal exchange rates. Menu costs, for instance, may be a reason for the kind of price rigidity assumed. See Akerlof and Yellen (1985), Mankiw (1985) or Blanchard and Kiyotaki (1987).

calculate the value of the index under both regimes and compare the results to see which of the regimes that leads to most variability. Two exchange rate variability indices are used in this paper. Both incorporate all the exchange rates that the entrant faces but they take slightly different aspects into account.

The most central result is that total exchange rate variability may increase if a country joins the EMU. One gets less exchange rate variability with a flexible currency as well as less output variability when shocks occur outside the entering country. The reason is that a change in the value of the euro will change the entering country's competitiveness directly and thus affect the equilibrium of the goods market in that country. As long as nominal prices are not completely flexible, the entrant will have to accept that asymmetric demand shocks have real effects on the home economy. These effects could have been avoided, had the country stayed outside the monetary union.

The most important reference when constructing the model is without doubt Obstfeld's and Rogoff's (1995) influential paper on exchange rate dynamics. One of the most important differences compared to Obstfeld's and Rogoff's model is that their model in addition to consumption incorporate money in the utility function but also disutility of labour to allow for a somewhat richer supply side of the economy. Their model produces some rather nasty-looking expressions already in the case of two countries, though. By letting a traditional LM-equation handle the existence of money the model becomes much simpler which is necessary in order to describe as many as four countries simultaneously. This makes the model similar to a traditional static Mundell-Fleming model in the way described by Genberg (1989) but the model in this paper differs as it has a two-period reduced form coming from an underlying intertemporal macro model.

In section 2 the model is presented. It is followed by section 3 in which the consequences of macroeconomic demand shocks are examined analytically. Possible extensions are presented in section 4. In Section 5 numerical values are assigned to the analytical results in sections 3 and 4 through a simple simulation. Conclusions, finally, are presented in section 6.

2. The Model

For convenience, Sweden represents the entering country but the model is general enough to make it possible to study the effects of any country which considers entering a monetary union where the entrant is small relative to the rest of the union which in turn is small relative to the rest of the world. This means that the model will have a recursive structure as shocks only have a one way impact from large to small countries⁴. The common formal structure of the countries of the model makes it possible to derive the model in two steps: First one country is described in section 2.1 and then two more countries are incorporated in section 2.2.

2.1 An Open Economy

The goods market is the central market in the analysis but to be able to derive the reduced form for the goods market equilibrium condition some conditions from the international capital markets and from the money markets are needed.

First, the uncovered interest parity condition,

$$i^{j} - i^{k} = \frac{E_{e}^{j/k} - E^{j/k}}{E^{j/k}},$$
(1)

is assumed to hold to ensure equilibrium in the international capital markets. The term $(E_e^{j/k} - E^{j/k}) / E^{j/k}$ is the expected relative change in the exchange rate⁵. All shocks in the model are permanent and unexpected which means that all changes in nominal exchange rates are unexpected. This means that the right of (1) is zero at all times and that all nominal interest rates become equal to the world market interest rate *i*.

The exchange rate expectations above are rational as long as shocks are permanent. If no shocks occur then the world economy will stay in the initial equilibrium and there will be no adjustments. If a shock occurs, then the exchange rates will change once and for all to a new long run equilibrium level.

Second, suppose that a standard LM-equation is an appropriate description of the money market in each country. The expression

$$L^{j}(\mathbf{Y}^{j},i) = \frac{M^{j}}{P^{j}}, \ j = (\mathbf{S}, EMU)$$
(2a)

says that demand for real money is a positive function of aggregate output and a negative function of the nominal interest rate. When Sweden enters the monetary union, the expression

⁴In the numerical simulation this assumption is relaxed so that Sweden can affect the EMU economy.

⁵The exchange rate $E^{j'k}$ is is the exchange rate between countries j and k, expressed as the amount of currency j needed to buy one unit of currency k.

changes slightly as Sweden and the rest of the EMU establish a union (U) with a common money market:

$$L^{U}(Y^{S} + Y^{EMU}, i) = \frac{M^{U}}{P^{U}}.$$
 (2b)

The central banks are assumed to prefer more price and output stability to less. If all exchange rates are flexible, then there is no conflict between the targets of zero-inflation and output stability and the central banks will therefore keep the money supply constant and thus achieve both targets. It can be seen from (2a) that a constant money supply and a constant nominal interest rate will imply constant output as there is no need for prices to change since the once and for all adjustment in nominal exchange rates due to a shock will bring about the shift in international relative prices necessary to restore equilibrium immediately with no need for output or prices to adjust. Thus (2a) fixes output at its long run level and this equation can therefore be dropped. Equation (2b) must be kept, however, as it will determine short run output (which may be different from the long run equilibrium level) when Sweden is a part of the monetary union. It is optimal for the European Central Bank (ECB) to keep the supply of the euro constant as this policy stabilises the price and total output level of the union. Since Sweden is small⁶ compared to the rest of the European Union, the approximation that the weight addressed to Swedish output in equation (2b) approaches zero is used. This means that (2b) puts no restriction on Swedish output at all.

A very important presumption is that goods prices and wages are constant during one period and then completely flexible. The crucial point is that goods prices and wages must be more sticky than nominal exchange rates which seems very reasonable.

In order to derive the goods market equilibrium condition, start with the intertemporal maximisation problem of the consumers,

$$\max_{\substack{C_{St}^{S}, C_{EMUt}^{S}, C_{Wt}^{S} \in \mathcal{B}}} \sum_{t=1}^{\infty} \boldsymbol{b}^{t-1} \left[(C_{St}^{S})^{1-g} \left[(C_{EMUt}^{S})^{d} (C_{Wt}^{S})^{d} \right]_{F}^{f} \right]$$

s.t.
$$\sum_{t=1}^{\infty} \left[\frac{1}{1+i} \right]^{t-1} \left[P_{t}^{S} C_{St}^{S} + E_{t}^{S'EMU} P_{t}^{EMU} C_{EMUt}^{S} + E_{t}^{S'W} P_{t}^{W} C_{Wt}^{S} \right] = \sum_{t=1}^{\infty} P_{t}^{S} (Y_{t}^{S} - T_{t}^{S}) \left[\frac{1}{1+i} \right]^{t-1}$$
(3)

where C_{Wt}^{s} , for instance, denotes the Swedish individuals' period t consumption of the good produced in the rest of the world. The price of this good, P_{t}^{W} , must be multiplied by the nominal exchange rate, $E_{t}^{S/W}$, to be measured in kronor. The parameters β , β and α belong to the interval (0,1) in line with standard assumptions. Real income before taxes, Y_{1}^{S} , can vary in period one but the economy will immediately reach its steady state income level in period two

⁶Swedish GDP is approximately 3% of total output in the European Union.

when prices become fully flexible. Disposable income is equal to Y_t^s minus the lump-sum tax T_t^s . Preferences are assumed to be properly described by a standard CES utility function. In line with empirical results⁷ and as a convenient simplification, assume that the elasticity of imports with respect to the real exchange rate is equal to one. One way of entering this assumption into the model is by altering the utility function to the one in the maximisation problem (3) above. This particular utility function has Cobb-Douglas-utility between domestic goods and imports and CES-utility between imports from different countries. Through this specification expenditures on imports become a constant share of total expenditures which means that the elasticity of imports with respect to the real exchange rate is equal to one⁸. With one additional simplifying assumption about how the asymmetric shocks work the following reduced form is derived in Appendix A:

$$dY^{S} = f_{EMU}^{S} dE^{SEMU} + f_{W}^{S} dE^{SW} + du^{S},$$
(4)

where all f_j^i are positive. The right hand side of (4) states that a depreciation of the Swedish krona increases demand for the Swedish good. How large effect a depreciation of the krona against the euro, for instance, has on Swedish aggregate demand depends on the size of f_{EMU}^s which simply is a measure of how sensitive Swedish exports to the EMU-area are to changes in the exchange rate between the krona and the euro. Aggregate demand is also affected by the asymmetric demand shock, du^s , which, by definition, only has a direct effect on demand for the Swedish good. Equation (4) is the central relationship in the analysis below.

2.2 A Model with Three Countries

Using equation (4) and its counterpart in the EMU-area the following two equations will always hold regardless of the exchange rate regime:

$$dY^{S} = f_{EMU}^{S} dE^{S EMU} + f_{W}^{S} dE^{S W} + du^{S}$$

$$0 = f_{S}^{EMU} dE^{EMU/S} + f_{W}^{EMU} dE^{EMU/W} + du^{EMU}$$
(5b)

To be able to solve the model, expressions linking the different exchange rates are needed. The no arbitrage condition,

$$E^{j/k} = E^{i/k} / E^{i/j},$$

⁷Goldstein & Kahn (1985).

⁸If the import elasticity is equal to one then an exchange rate depreciation by ten percent decreases the imported volume by ten percent which means that expenditures on imports stay the same since the volume imported decreases by ten percent at the same time as the price increases by ten percent. Import expenditures are thus a constant share of total expenditures.

can be used to rewrite the three different exchange rates above in terms of two exchange rates. Differentiate the no arbitrage condition and set the exchange rates equal to one in the initial equilibrium:

$$dE^{j/k} = \frac{E^{i/j} dE^{i/k} - E^{i/k} dE^{i/j}}{(E^{i/j})^2} = dE^{i/k} - dE^{i/j}.$$

It will turn out to be convenient to write all exchange rates in terms of $dE^{S/EMU}$ and $dE^{EMU/W}$ which gives:

$$dY^{s} = (f_{EMU}^{s} + f_{W}^{s})dE^{s'EMU} + f_{W}^{s}dE^{EMU/W} + du^{s}$$
(6a)

$$0 = -f_{S}^{EMU} dE^{S'EMU} + f_{W}^{EMU} dE^{EMU/W} + du^{EMU}$$
(6b)

This system consists of two equations and three unknown variables. When Sweden is outside the EMU the following additional condition applies:

$$dY^{\rm S} = 0, \tag{7}$$

since output in Sweden is determined by (2a). If, on the other hand, Sweden joins the union then the condition

$$dE^{S/EMU} = 0, \qquad (8)$$

must be added instead.

The next step to be carried out is to solve the model (6)-(8) while the effects of demand shocks on exchange rate variability will be examined in detail in section 3.

2.3 The Recursive Structure

It follows from the assumption that Sweden is a small country that a change in the exchange rate between Sweden and the EMU affects aggregate demand in the EMU to a very small extent. Thus the approximation

$$f_{\rm S}^{\rm EMU}=0$$

is used which makes the model recursive:

$$\begin{bmatrix} -1 & (f_{EMU}^{S} + f_{W}^{S}) & f_{W}^{S} \\ 0 & 0 & f_{W}^{EMU} \end{bmatrix} \begin{bmatrix} dY^{S} \\ dE^{S'EMU} \\ dE^{EMU/W} \end{bmatrix} = -\begin{bmatrix} du^{S} \\ du^{EMU} \end{bmatrix}.$$
(9a)
(9b)

The reason why the structure is recursive is simply that, whatever happens in Sweden, it is of no importance to the two other larger countries in the world.

The EMU The effects on the EMU are:

$$\frac{dE^{EMU/W}}{du^{S}} = 0; \qquad (10a)$$

$$\frac{dE^{EMU/W}}{du^{EMU}} = -\frac{1}{f_W^{EMU}} < 0.$$
(10b)

A shock in the small country Sweden does not affect the EMU so its exchange rate against the rest of the world does not need to adjust. When a positive demand shock hits the EMU the agents of that country demand more goods at the prevailing price level. Actual output in the EMU is fixed through the money market equilibrium condition (2a). Prices are fixed for one period so nominal exchange rates must adjust to clear the goods market. The relative price of the good produced in the EMU increases through the appreciation of the euro ($E^{EMU/W}$ decreases) which goes on until exports have decreased enough to balance the original increase in demand.

Sweden with a Flexible Exchange Rate Regime

Substitute the condition that aggregate income is fixed (7) into (9a) to obtain

$$(f_{EMU}^{S} + f_{W}^{S})dE^{S/EMU} + f_{W}^{S}dE^{EMU/W} = -du^{S}.$$
(11)

If the shock hits Sweden then the change in the exchange rate $E^{EMU/W}$ is equal to zero according to (10a) since a change in Swedish demand does not affect demand in the large countries. A new goods market equilibrium is reached through the appreciation of the krona which leads to reduced demand for the Swedish good through a decrease in exports.

The solution when the shock hits the EMU is

$$\frac{dE^{S/EMU}}{du^{EMU}} = \frac{f_{W}^{S}}{f_{W}^{EMU}(f_{EMU}^{S} + f_{W}^{S})} > 0.$$

In this case the euro appreciates against currency W as discussed above. As no shock has appeared in Sweden, a strengthening of the krona against one currency has to be offset by a weakening against another currency to prevent the goods market from being out of equilibrium. To preclude arbitrage profits in the foreign exchange market, the krona has to appreciate against currency W and depreciate against the euro, which can be seen from the no arbitrage condition

$$dE^{S/W} = dE^{S/EMU} + dE^{EMU/W}.$$

Sweden Inside the Monetary Union

Substitute condition (8) into (9a) and use (10a) and (10b) to solve for

$$\frac{dY^{s}}{du^{s}} = 1$$

and

$$\frac{dY^{S}}{du^{EMU}} = -\frac{f_{W}^{S}}{f_{W}^{EMU}}.$$

A shock in Sweden can no longer be met by exchange rate adjustments. Output therefore has to rise above the long run equilibrium level to clear the goods market. When a shock hits the EMU the euro appreciates which decreases Swedish exports to the rest of the world. The only way the Swedish goods market can remain in equilibrium is through a corresponding decline in output.

3 Theoretical Analysis

3.1 Variability of the Average Exchange Rate

It was mentioned in the introduction that firms are said to care about exchange rate variability. The question that naturally arises is: Why should they care about such variability? It seems reasonable to assume that what firms primarily care about is variation in relative prices or competitiveness. A change in the nominal exchange rate is no problem if wages and prices are fully flexible but when wages or prices are sticky the risk of prolonged mispricing emerges. Firms can of course protect themselves against fluctuations by using forward contracts but they increase costs. Hedging against a short-term international transaction is simple and straightforward, but hedging against long-term variability is likely to be very complicated, given never-ceasing changes in expectations about long-term conditions⁹. Small firms may lack necessary knowledge to know how and to what extent they should protect themselves against exchange rate variability.

The debate in newspapers and other media regarding the EMU reveals that many company managers speak strongly in favour of a Swedish membership in the EMU using the argument that exchange rate uncertainty reduces the level of aggregate investment. This alone is a reason to analyse the issues in this paper since the argument rests on the presumption that a Swedish membership in the EMU will reduce exchange rate variability. This is not necessarily the case, however, as will be seen later. One can go one step further, however, by asking if it is reasonable to assume that price variability really reduces investment. This is not a main question in this paper but it is still important for the relevance of the results. Three different approaches towards investments under uncertainty will be briefly discussed here: The theory of irreversible investments, the Arrow-Lind approach and an application of agency theory.

It is very easy to show that a single firm chooses to postpone irreversible investments if price variability increases¹⁰. Suppose that the price randomly takes on either a higher or a lower value next period. The investment project has a positive expected net present value but the company will not invest if it knows that the price goes down next period. This means that the value of the option to wait to invest until the price change is observed increases if the variance of the price movement increases as the upside potential payoff from the option increases while the downside payoff is left unchanged at zero since the option to invest will not be exercised if the price falls. Such a postponement can possibly extend over several years and lead to lower investment levels than those that would have been realised without uncertainty. However, the irreversible investment literature gives much less support to a reduction in *long run* investment levels in a general equilibrium context: "For aggregate or

⁹Friberg, R. & Vredin, A. (1997), p. 581.

¹⁰Dixit & Pindyck (1994) p. 39.

industry-wide uncertainty, the value of waiting for any one firm does drop to zero...¹¹. The results that remain are that irreversibility implies *slower* investment adjustments to price changes but not necessarily lower investment levels in the long run¹². Moreover, Ricardo J. Caballero states that the often heard argument that "more uncertainty implies less capital" is a misconception about the implications of the theory of irreversible investment¹³.

A very much used result when constructing models with a representative agent is the Arrow-Lind theorem. The theorem states that the manager of a firm owned by many small risk averse individuals should act as if the firm was owned by one risk-neutral agent. A crucial assumption is that the payoffs from the company's possible investment project and the owners' other incomes are uncorrelated. It seems reasonable, though, to suppose that project payoffs often are positively correlated with other incomes as both kinds of payoffs are likely to be correlated to the overall economic performance of the economy. In that case it is reasonable that managers will act as being risk averse in order to maximise the utility of the owners and that firms will require a risk premium despite the fact that projects are spread over many shareholders.¹⁴

Risk-averse managers is another issue. Managers have a large part of their personal capital tied up in their firms and they will therefore be concerned about total risk while shareholders only care about nondiversifiable risk with respect to a firm's different projects. Thus risk-averse managers are likely to choose safer projects with lower expected returns which is in conflict with the shareholders interests since they prefer riskier ventures as shareholders do not share a rise in profits with bondholders while the bondholders bear part of the risk of failure¹⁵.

Thus there may exist reasons for firms to act in a risk averse way when being exposed to uncertainty regarding the price of their products.

3.2 First Period Exchange Rate Variability

A simple and natural measure of aggregate variability in international relative prices, in the context of the model derived above and during the first period when goods prices are constant, is the Average Exchange Rate Index (*dAEI*):

¹¹Ibid. p. 248. This is a result of a general equilibrium where all firms have the potential to make the same innovations at the same time which implies that a single firm fears that other firms may have made the same innovation and that they also are planning to make the same investment. This makes waiting a meaningless alternative.

¹²Ibid. p. 261-267.

¹³"More uncertainty raises reluctance [to invest] precisely because it raises the need to reduce the extent of excessive capital during the now deeper recessions. Without raising reluctance, an increase in uncertainty would raise the average stock of capital in the presence of irreversibility constraints. This occurs because there would now be greater capital accumulation during extremely good times which would not be offset by large disinvestment during extremely bad times." (Caballero, R. J. (1997), p. 21.)

¹⁴Arrow, K. J. & Lind, R. (1970).

¹⁵Easterbrook, F. H. (1984), p. 653.

$$dAEI = \frac{f_{EMU}^{S} dE^{S/EMU} + f_{W}^{S} dE^{S/W}}{f_{EMU}^{S} + f_{W}^{S}}$$

The weights f_i^{s} simply measure the sensitivity of aggregate demand to changes in exchange rates, $dEX^{s}/dE^{s/i}$, and thus reflect how important foreign trade with a particular country is. Note that, since goods prices are constant in the first period, nominal and real exchange rate variability are equal. This index is appropriate to use if companies in Sweden, without additional costs, can redirect their exports from one country to another¹⁶. The effects when the krona appreciates against one currency and depreciates against another may therefore cancel and leave the value of the index unaffected. Exchange rate variability is in that case no burden to firms. The next step is to evaluate how the Average Exchange Rate Index is affected by different shocks.

Demand Shock in Sweden

Since Sweden is small the positive demand shock there has no effect on the EMU where $E^{EMU/W}$ is determined. As can be seen from (11) the appreciation of the krona completely absorbs the demand shock in the case where exchange rates are flexible and thus

$$\left.\frac{dAEI}{du^{s}}\right|_{F}=-\frac{1}{f_{EMU}^{s}+f_{W}^{s}}<0,$$

where F denotes the case of a flexible exchange rate between the krona and the euro. In the monetary union case, changes in exchange rates will be eliminated which means that

$$\left.\frac{dAEI}{du^{S}}\right|_{U}=0\,,$$

where U denotes the case of monetary union. The effect of the demand shock will instead be an increase in production. By joining the union one can eliminate exchange rate variability in the first period when shocks hit Sweden but at the cost of more output variation.

Demand Shock in the EMU

A shock in the EMU, when Sweden is outside the union, leads to

$$\frac{dAEI}{du^{EMU}}\Big|_{F} = 0$$
(12a)

¹⁶If this is not the case, the Absolute Value Index used in section 4.5-4.7 may be a better measure of exchange rate variability.

through equation (11) as exchange rate adjustments will keep the Swedish goods market in equilibrium without any need for deviations in output. Equation (9b) will give the new equilibrium exchange rate $E^{EMU/W}$ while $E^{S/EMU}$ will respond just enough to keep Sweden in equilibrium.

Now consider the monetary union case where

$$\left. \frac{dAEI}{du^{EMU}} \right|_{U} = -\frac{f_{W}^{S}}{f_{W}^{EMU} \left(f_{EMU}^{S} + f_{W}^{S} \right)} < 0.$$
(12b)

This is a negative expression which means that the Swedish currency (the euro in this case) appreciates when the rest of the EMU is hit by a positive shock. The fact that (12b) is different from zero while (12a) is equal to zero means that Sweden faces larger total exchange rate variability when Sweden is inside the union than outside and the shock hits the rest of the EMU-area. The appreciation of the euro can only be met by a corresponding reduction in Swedish production. Thus exchange rate variability, as well as output variability, increases if Sweden joins the monetary union and shocks hit the rest of the EMU-area. From expression (12b) one can see that the larger Swedish trade with the rest of the world is, the more exchange rate variability increases. The reason for this is that the common currency, the euro, appreciates against currency W due to the shock in the EMU. This appreciation of the Swedish trade partner. With flexible rates the value of the krona will adjust to neutralise the effect on Sweden of the appreciation of the euro. This last mechanism will disappear if Sweden joins the EMU. Instead production must decrease to meet the weaker Swedish aggregate demand.

Results in Brief

When measuring exchange rate variability with the Average Exchange Rate Index and when shocks hit Sweden one gets the result that Sweden faces less variability being inside the EMU. This reduced exchange rate variability will however come at the cost of more variation in output. When shocks hit the EMU, though, Sweden will face more exchange rate and output variability inside the EMU. The results so far are illustrated in *Tables 1* and 2 on pages 15 and 16 (the first and last row of each table).

3.3 Second Period Price Variability

It is assumed that the kind of exchange rate variability that firms care about is changes in international competitiveness. Then all changes in relative prices because of a shock must be incorporated, not only first period changes in nominal exchange rates. In some cases, goods

prices have to adjust in the second period so that the Swedish economy can return to long run equilibrium.

Demand Shock in Sweden

If Sweden has a flexible exchange rate regime and a shock hits Sweden, then the nominal exchange rate immediately appreciates just enough to keep the Swedish goods market in long run equilibrium and thus there will be no need for any further relative price adjustments in the second period. Being inside the monetary union a relative price change, equal to the one in the case with a flexible regime, will take place but one period later through an increase in the price of the Swedish good¹⁷. This is illustrated in *Figure 1*, where P^{s}/P^{i} denotes the relative price of the Swedish good compared to an index of prices of international goods.

¹⁷It follows from (A16) in Appendix A that $Y \approx \overline{PY}$ if the export elasticity is approximately equal to 1. It can be shown easily that $\overline{Y}d\overline{P} = dY^{\varepsilon} = -dE^{\varepsilon_{EMU}}$ if $f_{_{EMU}}^{\varepsilon} + f_{_{W}}^{\varepsilon} = 1$ or, in other words, the export elasticity equals one. Using $d\overline{P} = -dE^{\varepsilon_{EMU}}$ to determine $d\overline{P}$ allows for export elasticities different from one.



Figure 1: Relative Price Changes and Output Changes due to a Swedish Demand Shock

Demand Shock in the EMU

When the krona is a flexible currency, adjustments in the value of the krona will always keep the Swedish goods market in long run equilibrium. Thus there is no need for price adjustments in the second period after the shock. The value of the krona will change in such a way that total terms of trade, or P^{s} / P^{*} , stay constant which keeps the Swedish economy in long run equilibrium which can be seen in *Figure 2a*.

In the case where Sweden is a member of the EMU, the Swedish currency (the euro) appreciates which brings the Swedish goods market out of long run equilibrium until a decrease in the price of the Swedish good can restore the initial equilibrium. The relative price changes in period one and two are of course equally large which is illustrated in *Figure 2b*.

Figure 2: Relative Price Changes and Output Changes due to a Demand Shock in the EMU



3.4 Total Real Exchange Rate Variability

The total real effect on competitiveness must incorporate the whole time-horizon. As *Figure 1* reveals, a shock in Sweden must lead to an international relative price change to restore long run equilibrium. There are two possible mechanisms through which these relative changes can take place; either through the nominal exchange rate immediately after the shock, or through the nominal price of the Swedish good one period later. Apart from the fact that the change in the price of the Swedish good comes with a one period lag, these changes in relative prices are completely identical. What really differs, though, is that one can avoid output variability by staying outside the EMU. *Table 1* sums these results.

	Flexible Regime	Monetary Union
Change in nominal exchange rates (period 1)	+	0
Change in goods prices (period 2)	0	+
Change in real exchange rates (period 1 and 2)	+	+
Change in output	0	+

Table 1: Effects of a Shock in Sweden

Sweden can get rid of all variation in relative prices as well as in output, when shocks hit the EMU, by staying outside the monetary union which can be seen in *Figure 2a*. The nature of the change in the real exchange rate, that takes place if Sweden is a member of the EMU, is different with this shock, though, as such a shock only gives temporary effects on the real exchange rate while a shock in Sweden results in a permanent change in the international relative price of the Swedish good. A shock in the EMU-area thus has no effect on Swedish competitiveness beyond the second period so the effect on total competitiveness would approach zero. Provided that prices and wages are more sticky than nominal exchange rates, there will remain some change in competitiveness that will result in a temporary change in output from its long run equilibrium level.

Table 2:	Effects	of a	Shock	in	the	EMU

	Flexible Regime	Monetary Union
Change in nominal exchange rates (period 1)	0	+
Change in goods prices (period 2)	0	+
Change in real exchange rates (period 1 and 2)	0	+
Change in output	0	+

The assumption of sticky prices is the single most important factor to explain the results in *Tables 1* and 2. If prices would be fully flexible then the real exchange rate could always adjust through nominal prices or wages and the existence of a flexible nominal exchange rate would be an unimportant issue. Faced with sticky prices the extreme flexibility of nominal exchange rates can play a very important role in keeping the economy close to long run equilibrium. What kind of exchange rate variability that counts is also important and, to allow

for an other approach towards exchange rate variability, an alternative index will be used in the next section.

4. Extensions

A natural extension is to allow for a fourth country. If this fourth country is a more important trade partner for Sweden than for the rest of the EMU, then it is reasonable to think that such asymmetries can affect how exchange rate variability changes if Sweden enters the monetary union. Another possible extension is to use an exchange rate variability index that builds on the assumption that all exchange rate variability matters, even if total competitiveness does not change. These two extensions will be dealt with in the next two sub-sections.

4.1 A Four-Country Model

A fourth country is added by dividing the rest of the world into two countries. Such a partition of the rest of the world may be important if the EMU and Sweden have asymmetric trade relations to the fourth country. Three possible candidate countries in the real world are the US, the UK and Norway. All these countries can be considered as important Swedish trade partners but whether they are more important to Sweden than to the EMU and to what extent such asymmetries really matter will be examined in the quantitative simulation in *Section 5*. The analysis that follows can be applied to all these countries but for convenience this new country will be labelled "the US". The two equations that describe the US are the following counterparts to (6a) and (2a):

$$dY^{US} = f_{S}^{US} dE^{US/S} + f_{EMU}^{US} dE^{US/EMU} + f_{W}^{US} dE^{US/W} + du^{US}$$
$$L^{US}(Y^{US}, i) = \frac{M^{US}}{P^{US}}$$

The US is assumed not to enter the monetary union which implies that output will always be constant in the US as a result of the money market equilibrium condition. The countries Sweden, the EMU and the US are still small compared to W while Sweden is also small compared to the EMU and the US. The following counterpart to the equation system (5a), (5b) will hold regardless of the exchange rate regime:

$$dY^{s} = f_{EMU}^{s} dE^{S/EMU} + f_{Us}^{s} dE^{S/US} + f_{W}^{s} dE^{S/W} + du^{s}$$
(13a)

$$0 = f_{S}^{EMU} dE^{EMU/S} + f_{US}^{EMU} dE^{EMU/US} + f_{W}^{EMU} dE^{EMU/W} + du^{EMU}$$
(13b)

$$0 = f_{S}^{US} dE^{US/S} + f_{EMU}^{US} dE^{US/EMU} + f_{W}^{US} dE^{US/W} + du^{US}.$$
 (13c)

To solve the model, arbitrage conditions of the kind used in the three-country model must be used. They make it possible to express all exchange rates in terms of $E^{S/EMU}$, $E^{EMU/US}$ and

 $E^{EMU/W}$. The resulting equation system will consist of three equations and four unknown variables so one also has to add condition (7) or (8) exactly as in the three-country model to make the model complete. Now make full use of the assumption that Sweden is small compared to the EMU and the US by setting both f_s^{EMU} and f_s^{US} to zero. This gives

$$\begin{bmatrix} -1 & (f_{EMU}^{S} + f_{US}^{S} + f_{W}^{S}) & f_{US}^{S} & f_{W}^{S} \\ 0 & 0 & f_{US}^{EMU} & f_{W}^{EMU} \\ 0 & 0 & (f_{US}^{VS} + f_{W}^{VS}) & f_{US}^{VS} \end{bmatrix} \begin{bmatrix} dY^{S} \\ dE^{S'EMU} \\ dE^{EMU/US} \end{bmatrix} = \begin{bmatrix} du^{S} \\ du^{EMU} \\ du^{EMU} \end{bmatrix}.$$
(14a)
(14b)
(14b)
(14c)

$$\begin{bmatrix} 0 & 0 & -(f_{EMU}^{US} + f_{W}^{US}) & f_{W}^{US} \end{bmatrix} \begin{bmatrix} dE^{EMU/U} \\ dE^{EMU/W} \end{bmatrix} \begin{bmatrix} du^{W} \end{bmatrix}$$
(14c)

Exchange Rate Solutions for the EMU and the US

Make use of the recursivity in system (14a)-(14c). Then solve the simplified simultaneous remainder of the model that holds regardless of the exchange rate regime:

$$\begin{bmatrix} f_{US}^{EMU} & f_{W}^{EMU} \\ -(f_{EMU}^{US} + f_{W}^{US}) & f_{W}^{US} \end{bmatrix} dE^{EMU/US} = -\begin{bmatrix} du^{EMU} \\ du^{US} \end{bmatrix}; \qquad |A| = f_{US}^{EMU} f_{W}^{US} + f_{W}^{EMU} (f_{EMU}^{US} + f_{W}^{US}) > 0,$$

where |A| is the determinant of the equation system. The solutions are:

$$dE^{EMU/US} = \frac{f_{W}^{EMU} du^{US} - f_{W}^{US} du^{EMU}}{|A|}; \qquad (15)$$

$$dE^{EMU/W} = -\frac{f_{US}^{EMU} du^{US} + (f_{EMU}^{US} + f_{W}^{US}) du^{EMU}}{|A|}.$$
 (16)

When a positive demand shock hits the EMU, the euro appreciates for the same reason as in the three-country model. The same mechanism also gives rise to an appreciation of the dollar $(dE^{EMU/US} > 0 \text{ and } dE^{US/W} = dE^{EMU/W} - dE^{EMU/US} < 0)$ when a demand shock occurs in the US.

Sweden with a Flexible Exchange Rate Regime

A positive Swedish shock leads to an appreciation of the krona to compensate for the increase in demand just as in the three-country model. The other currencies will of course be unaffected as Sweden is a small country. This means that (14a) gives

$$\frac{dE^{S/EMU}}{du^{S}} = \frac{dE^{S/US}}{du^{S}} = \frac{dE^{S/W}}{du^{S}} = -\frac{1}{(f^{S}_{EMU} + f^{S}_{US} + f^{S}_{W})} < 0,$$

as the no arbitrage conditions state that

$$dE^{S/US} = dE^{S/EMU} + dE^{EMU/US}$$

and

$$dE^{S/W} = dE^{S/EMU} + dE^{EMU/W}$$

Now turn to shocks in the EMU and the US. Using the already obtained solutions for $dE^{EMU/US}$ (15) and $dE^{EMU/W}$ (16) and substituting them into equation (14a) one finds that:

$$\frac{dE^{S/EMU}}{du^{EMU}} = \frac{f_{US}^{S}f_{W}^{US} + f_{W}^{S}(f_{EMU}^{US} + f_{W}^{US})}{|A|(f_{EMU}^{S} + f_{US}^{S} + f_{W}^{S})} > 0;$$
(17)

$$\frac{dE^{S/EMU}}{du^{US}} = \frac{f_{US}^{EMU} f_{W}^{S} - f_{W}^{EMU} f_{US}^{S}}{|A|(f_{EMU}^{S} + f_{US}^{S} + f_{W}^{S})} \gtrsim 0.$$
(18)

When a positive shock hits the EMU the euro must appreciate to clear the goods market in the EMU-area. From equation $(14a)^{18}$ one can see that the krona will depreciate against the euro just enough to compensate for the decrease in the two exchange rates $E^{EMU/US}$ and $E^{EMU/W}$ so that Swedish total terms of trade are unchanged and the goods market of Sweden can remain in equilibrium.

A positive shock in the US leads to an appreciation of the dollar. How the exchange rate between the krona and the euro will be affected depends on the relations between Sweden and the rest of the EMU-countries. What determines the sign of (18) is whether

$$\frac{f_{US}^{EMU}}{f_{W}^{EMU}} < \frac{f_{US}^{S}}{f_{W}^{S}}.$$
(19)

The right-hand side of the expression is a measure of Swedish conditions and reflects the relation between the sensitivities of Swedish exports to changes in the dollar and currency W respectively. This can be interpreted as a measure of the relative economic distance from Sweden to the US and the rest of the world. Geographical and economic distance are related as a change in a currency of a nation close to Sweden in general is of greater importance than changes in an exchange rate to a more distant country. The sign of the expression above depends on the relative economic distances to the US and W from Sweden and the EMU-area respectively.

¹⁸Remember that Swedish output is constant when all exchange rates are flexible.

Just to enable an intuitive interpretation of expression (19), make the temporary assumption that

$$f_W^S = f_W^{EMU}$$

and examine this special case where Sweden and the EMU-area are equally sensitive to changes in their respective exchange rate versus the rest of the world. Suppose, in addition, that a change in the value of the dollar affects demand for the Swedish good more than demand for the EMU-good $f_{US}^{S} > f_{US}^{EMU}$. Then the appreciation of the dollar (because of the shock in the US) against the krona and the euro leads to a larger increase in Swedish exports to the US than in EMU-exports to the US. This asymmetric stimuli of Sweden compared to the EMU has to be compensated for by an appreciation of the krona against the euro to stimulate the economy of the EMU relative to Sweden so that Sweden and the EMU both can remain in equilibrium.

Sweden Inside the Monetary Union

Whether Sweden joins the EMU or not will of course have no effect on the other countries but, exactly as in the three-country model, changes in the value of the krona can no longer restore equilibrium. Instead the output level must be above or below its long run level until a change in the price of the Swedish good can bring the Swedish economy back to long run equilibrium.

4.2 First Period Exchange Rate Variability

The Average Exchange Rate Index now includes variation to the US and will because of that appear as follows:

$$dAEI = \frac{f_{EMU}^{S} dE^{S/EMU} + f_{US}^{S} dE^{S/US} + f_{W}^{S} dE^{S/W}}{f_{EMU}^{S} + f_{US}^{S} + f_{W}^{S}}.$$

Demand Shock in Sweden

The qualitative results and interpretations are identical to those given in section 3.2.

Demand Shock in the EMU A shock in the EMU-area leads to

$$\left. \frac{dAEI}{du^{EMU}} \right|_F = 0$$

and

$$\frac{dAEI}{du^{EMU}}\Big|_{U} = -\frac{f_{US}^{S}f_{W}^{US} + f_{W}^{S}(f_{EMU}^{US} + f_{W}^{US})}{|A|(f_{EMU}^{S} + f_{US}^{S} + f_{W}^{S})} < 0$$
(20)

through the same mechanisms as in section 3.2. Expression (20) is negative for the same reason as the corresponding expression is negative in the three-country model. The fact that it is different from zero means that Sweden faces larger total exchange rate variability when being inside the union and shocks hit the rest of the EMU-area.

Demand Shock in the US

When Sweden has its own flexible currency, there is no difference between the EMU and the US from the Swedish point of view. Thus one gets the analogous result that

$$\frac{dAEI}{du^{US}}\Big|_{F} = 0.$$

If, on the other hand, Sweden and the EMU are joined in a monetary union, then

$$\left. \frac{dAEI}{du^{US}} \right|_{U} = \frac{f_{US}^{S} f_{W}^{EMU} - f_{W}^{S} f_{US}^{EMU}}{|A|(f_{EMU}^{S} + f_{US}^{S} + f_{W}^{S})} \gtrsim 0.$$

This expression will in general be different from zero meaning that exchange rate variability increases if Sweden joins the monetary union. It may be equal to zero, however, in the interesting special case when

$$\frac{f_{US}^{S}}{f_{W}^{S}} = \frac{f_{US}^{EMU}}{f_{W}^{EMU}} \,.$$

In that case the relative importance of the US and the rest of the world is the same for Sweden and the EMU-area. Then Sweden and the EMU are so similar that they in this regard constitute an optimum currency area and that is why Sweden can join the EMU without any increase in exchange rate variability. Membership in the EMU generally means more variability, though, when shocks hit the US.

Results in Brief

The only important difference when a fourth country is added arises when shocks hit the US. The effect on Sweden of such a shock resembles that of a shock in the rest of the EMU-area as it in general leads to an increase in both exchange rate and output variability as a result of membership in the EMU. There is one exception, though: If Sweden and the EMU are

symmetric in their relations to their international environment, then they constitute an optimum currency area and exchange rate and output variability will not increase with membership in the EMU.

4.3 Second Period Price Variability

The results of shocks in the US are analogous to those that follow from shocks in the EMU in section 3.3. Shocks in Sweden and the EMU have the same qualitative effects on Sweden as in the analysis with three countries.

4.4 Total Real Exchange Rate Variability

The total real effects of shocks in the EMU and the US are qualitatively equivalent to those shown in *Table 2*. The effects of shocks in Sweden completely correspond to those in *Table 1*.

4.5 Using the Absolute Value Index

For many large and multinational companies it seems reasonable that decreasing exports to Germany may be compensated for by increasing exports to the USA and thus the previous analysis using the Average Exchange Rate Index can be justified. One can, however, imagine a situation where a firm has a sales organisation in only one of the countries mentioned above or that other factors make it costly to redirect exports from one country to another. If such redirectional costs exist then all changes in exchange rates are costly even if total terms of trade for Sweden remain unchanged after a shock. If this description is more adequate, then the Absolute Value Index (dAVI) below may be a better measure of variability.

Absolute Value Index (*dAVI*):
$$dAVI = \frac{f_{EMU}^{S} |dE^{S/EMU}| + f_{W}^{S} |dE^{S/W}|}{f_{EMU}^{S} + f_{W}^{S}}.$$

The number of countries in the model will now be reduced to three to facilitate an easier way of interpreting the changes in the Absolute Value Index ¹⁹.

$$\frac{dAVI}{du^{\text{EMU}}}\bigg|_{U} - \frac{dAVI}{du^{\text{EMU}}}\bigg|_{F}$$

¹⁹In a four-country setting the use of the Absolute Value Index gives rise to some complicating technicalities which make it harder to focus on the fundamental difference between the two indices used in this paper. For instance, conditions on relative economic distances to different countries in the sense explained on pages 19 and 20 will play an important role in determining the sign of the central expression

It is by no means impossible to handle these complications but they give rise to quite a few different cases necessary to study. Conclusions founded on empirical evidence about which cases are the most relevant to study are in some cases sensitive to such things as whether the United Kingdom enters the EMU. These issues are not irrelevant but analysing them here would be too far-reaching.

4.6 First Period Absolute Value Exchange Rate Variability

Demand shock in Sweden

The results are identical to those when the Average Exchange Rate Index is used since the krona appreciates against all other currencies which means that all exchange rate changes have the same sign and then the choice of index does not matter. Just as before variation in output replaces variation in the value of the krona if Sweden joins the monetary union.

Demand shock in the EMU

The change in the Absolute Value Index caused by an EMU-shock is

$$\frac{dAVI}{du^{EMU}} = \frac{1}{f_{EMU}^{S} + f_{W}^{S}} \left[f_{EMU}^{S} \left| \frac{dE^{S'EMU}}{du^{EMU}} \right| + f_{W}^{S} \left| \frac{dE^{S'W}}{du^{EMU}} \right| \right]$$

As discussed earlier $dE^{S/EMU}/du^{EMU} > 0$ since the euro appreciates against all exchange rates including the krona while $dE^{S/W}/du^{EMU} < 0$ as the krona will follow the euro in its appreciation against currency W. Then comparing the two exchange rate regimes by taking the difference between the values of the index under the two regimes gives:

$$\frac{dAVI}{du^{EMU}}\Big|_{U} - \frac{dAVI}{du^{EMU}}\Big|_{F} = \frac{f_{W}^{S}(f_{W}^{S} - f_{EMU}^{S})}{f_{W}^{EMU}(f_{EMU}^{S} + f_{W}^{S})^{2}}$$

The sign of this expression depends on the importance of different trade partners. If the importance, seen from the Swedish point of view, of the the rest of the world is greater than the importance of the EMU, then this expression is positive meaning that Sweden faces larger exchange rate variability being inside the monetary union. To see why, consider the following no-arbitrage condition:

The size of the change in $E^{S/W}$ increases if Sweden decides to enter the EMU as variation in $dE^{EMU/W}$ stays unaffected while variation in $E^{S/EMU}$ disappears completely. If the rest of the world is the most important trade partner then variability increases as the effect from (21) dominates over the direct effect of the elimination of variation in $E^{S/EMU}$.

4.7 Second Period Price Variability

When shocks hit Sweden the appreciation of the krona, through the once and for all change in the nominal exchange rate, completely absorbs the effect of a shock if Sweden stays outside

the EMU. If Sweden enters the monetary union the same change in international relative prices takes place one period later through a change in the price of the Swedish good.

That shocks outside Sweden have no effects in the second period if the krona is flexible is also easy to understand as the period one adjustments in nominal exchange rates are sufficient to restore long run equilibrium immediately in all countries of the world. *Figure 3a* illustrates the effects of an EMU-shock, where the P^{EMU} / P^{W} -line shows the immediate and permanent appreciation of the euro. The krona appreciates against currency W and depreciates against the euro, which the P^{S} / P^{EMU} - and P^{S} / P^{W} -lines illustrate. The P^{S} / P^{i} line is an average of Swedish relative prices against the EMU and the rest of the world and shows changes in total Swedish terms of trade. This just illustrates the earlier result that the krona adjusts to shocks just enough to keep total terms of trade constant which keeps the Swedish goods market in long-run equilibrium at all times.

Figure 3: Swedish Relative Prices



A shock in the EMU, when Sweden is a part of the monetary union, also leads to an immediate and permanent appreciation of the euro (P^{EMU} / P^{W} increases) which can be seen in *Figure 3b*. This appreciation affects the Swedish relative price against the rest of the world (P^{S} / P^{W}) in exactly the same way during the first period since Sweden and the EMU have a common currency. The relative price between Sweden and the EMU stays constant in period one, of course, which the P^{S} / P^{EMU} -line illustrates. The model in this paper has a unique

long-run solution so the relative prices must move to the same levels as in *Figure 3a* as soon as prices become flexible in the second period. This explains the new levels of P^S / P^W and P^S / P^{EMU} from the second period and onwards. The Absolute Value Index measures the size of the distance "a" in *Figure 3b*. The total change in relative prices in period two (corresponding to the change in the Absolute Value Index in the first period) is given by the sum of the absolute values of the price changes "b" and "c" in the figure. It can be shown that the second period price change is equal to the absolute value of

$$\frac{dE^{S/EMU}}{du^{EMU}}\bigg|_{F},$$

which is the change in the price of the Swedish good that would have taken place through the nominal exchange rate if this exchange rate would have been flexible. This means that the, in fact, inevitable appreciation of the krona will take place through an equally large rise in the price of the Swedish good in period two. The P^{S}/P' -line illustrates that total terms of trade depart from its long-run level during the first period causing a temporary change in output. In the second period both total terms of trade and output return to their original levels.

4.8 Total Real Exchange Rate Variability

Adding second period price variability to the results of section 4.6 gives

$$\frac{dAVI}{du^{EMU}}\Big|_{U} - \frac{dAVI}{du^{EMU}}\Big|_{F} + ABS\left[\frac{dE^{S/EMU}}{du^{EMU}}\Big|_{F}\right] > 0,$$

which is a measure of the increase in total real exchange rate variability when second period effects have been taken into account. This expression states that total real exchange rate variability increases if Sweden joins the EMU and shocks appear outside Sweden. The justification of the addition of the relative price changes in the two periods is that firms have to take the full cost of adjustments to the new situation in the first period after a shock and then make another costly adjustment when goods prices and wages change one period later.

Tables 3 and 4 below sum the results when the Absolute Value Index is used. *Tables 1* and 3 look the same which illustrates the fact that the choice of index does not affect the results when shocks hit Sweden. Table 4 differs from table 2 as the AVI assigns a positive value to a change in exchange rates even if those changes leave total terms of trade unaffected.

Table 3: Effects of a Shock in Sweden

	Flexible Regime	Monetary Union
Change in nominal exchange rates (period 1)	+	0
Change in goods prices (period 2)	0	+
Change in real exchange rates (period 1 and 2)	+	+
Change in output	0	+

Table 4: Effects of a Shock in the EMU

	Flexible Regime	Monetary Union
Change in nominal exchange rates (period 1)	+	+
Change in goods prices (period 2)	0	+
Change in real exchange rates (period 1 and 2)	+	++
Change in output	0	+

5 Numerical Simulation

5.1 Method

The purpose of the simulation in this section is to calculate numerical measures of the effects of shocks in different countries. Two possible EMU-entrants will be considered: Sweden and the UK. The US, Norway and the UK are important Swedish trade partners and shocks in these countries may potentially have asymmetric effects on Sweden and the EMU. From the British point of view, asymmetric relations to the US seem to be the most important as shocks in the small countries Sweden and Norway have limited effects on the UK. To examine the potential asymmetries mentioned above, data from all EU-countries, Norway and the US are used in the simulation.

Based upon the model derived in Appendix A a variant of the reduced form (A17) is used to simulate the theoretical model. Start from the goods market equilibrium condition (A16a), disregard from the intertemporal effects of changes in the Swedish real interest rate between the first and second periods, change notation by swapping EX_{EMU}^{s} for C_{s}^{EMU} (and EX_{w}^{s} for C_{s}^{w})²⁰, Then substitute (A10) for C_{s}^{s} and differentiate (A16a) to get²¹

²⁰Swedish exports to the EMU are of course equivalent to the EMU citizens' consumption of the Swedish good.

$$\frac{dY^{S}}{Y^{S}} = \frac{g(1+i)}{1+gi} \frac{dG^{S}}{Y^{S}} + \frac{1+i}{1+gi} \frac{\P E X_{EMU}^{S}}{E X_{EMU}^{S}} \frac{E X_{EMU}^{S}}{Y^{S}} dE^{S/EMU} + \frac{1+i}{1+gi} \frac{\P E X_{W}^{S}}{E X_{W}^{S}} \frac{E X_{W}^{S}}{Y^{S}} dE^{S/W}.$$
(23)

The elasticity of exports with respect to changes in exchange rates is set to be equal to 2 as there is empirical evidence suggesting that this elasticity is roughly equal to two²². The impact of exchange rate changes on aggregate demand is less than what the value of the export elasticity first seems to suggest, however, as exports to some extent consist of imported goods. To take this into account the elasticity is multiplied by (1 - g). The implicit assumption behind this is that exports have the same degree of import content as total output in a particular country²³. Equation (23) can thus be expressed in terms of g, the world market interest rate, GDP, exports and elasticities of exports with respect to the exchange rate. Let e/ denote the export elasticity, add a fourth country and restate the counterpart to (14) that will be used in this section:

$$\frac{(1-g)el}{g^{Y^{S}}} \begin{bmatrix} -\frac{1+g}{(1+i)} \frac{Y^{S}}{(1-g)el} & EX_{E}^{S} + EX_{US}^{S} + EX_{W}^{S} & EX_{US}^{S} & EX_{W}^{S} \\ \frac{(1-g_{EMU})el}{g_{EMU}} \end{bmatrix}^{eIMU} = 0 & -EX_{S}^{E} & EX_{US}^{E} & EX_{W}^{E} \\ \frac{(1-g_{US})el}{g_{US}} \frac{(1-g_{US})el}{q_{US}} \end{bmatrix} = 0 & -EX_{S}^{US} & -(EX_{S}^{US} + EX_{US}^{US} + EX_{W}^{US}) & EX_{W}^{US} \end{bmatrix} = \left[\frac{dG^{e}}{Y^{S}}}{\frac{dE^{eMU}US}{Y^{US}}} \right].$$

$$= -\left[\frac{dG^{e}}{Y^{S}}}{\frac{dG^{US}}{Y^{US}}} \right].$$

$$(24a)-(24c)$$

The world market real interest rate is set to 5.5%²⁴. All data refer to the year of 1995 and all shocks are Government demand shocks that create a direct increase in aggregate demand of a size equivalent to one percent of GDP.

 $^{^{21}}$ To simplify the expressions for the elasticities of exports with respect to exchange rates, all variables possible to express in euro are expressed in that currency which means that all exchange rates become equal to one in the initial equilibrium.

 $^{^{22}}$ The Goldstein & Kahn (1985) survey shows that there is little reason to allow for differentiated export elasticities as the ranking of individual countries regarding their export elasticities differs between the different studies quoted in Goldstein & Kahn.

²³Remember that g is the ratio of (Swedish) imports to GDP.

²⁴About one half of the financial savings of Swedish households are in common stocks or in funds that invest in common stocks and the other half in interest-bearing assets. As Brealey & Myers (1991) state on page 131, the

5.2 Results and Interpretations for Sweden

The results from the numerical simulation are presented in *Tables 5a-5c*. These tables show results from the case where the US is the fourth country of the model and both the UK and Denmark remain outside the EMU. The other cases have very similar interpretations so to give a more focused presentation, the results of these cases can be found in *Table 8* in Appendix B.

First consider an example that explains how the mechanisms work. Let aggregate demand increase by 1% in Sweden. When this happens the krona appreciates to keep the Swedish goods market in equilibrium. One can see from *Table 5a* that

$$\frac{dAEI}{du^{s}}\Big|_{F} = -1.9852\%$$

which means that the krona appreciates approximately 2% as a result of the demand shock. Since the elasticity of exports with respect to exchange rates is assumed to be equal to 2, this appreciation results in a 4% decrease in exports. The import content in Swedish exports is about 27% so only 73% of the decrease in exports affect Swedish aggregate demand. Exports are roughly one third of aggregate demand so aggregate demand is reduced by $0.73 \cdot 1/3 \cdot 4\% \approx 1.0\%$. Adjustments in the value of the krona thus ensure that the Swedish goods market remains in long-run equilibrium.

Being a member of the monetary union, this shock will have another impact on the Swedish economy. The direct increase in aggregate demand is still 1%. The value of the krona can no longer adjust to compensate for the shock so output must jump to a higher level for one period until prices can adjust. This temporary increase is 1% which can be seen in *Table 5a*.

Table 5a: Effects on Sweden of a Shock in Sweden

average annual real returns on government bons and corporate bonds between 1926 and 1988 are 1.7% and 2.4%, respectively. The corresponding average annual rate of return on common stocks is 8.8%. The average (5.5% annually) of a normal risk-free return (2%) and a normal return on common stocks (9%) is therefore used as an estimate of the appropriate discount factor for the households in their intertemporal maximisation problem. Changes in the interest rate only have small effects on the numerical results and thus the value of this parameter is not particularly critical.

	dA	EI	dAVI		
Change in	Flexible Regime	Monetary Union	Flexible Regime	Monetary Union	
nominal exchange rates	-1.9852%	0%	1.9852%	0%	
goods prices	0%	-1.9852%	0%	1.9852%	
real exchange rates	-1.9852%	-1.9852%	1.9852%	1.9852%	
output	0%	1.0000%	0%	1.0000%	

Table 5b: Effects on Sweden of a Shock in the EMU

	dA	EI	dAVI		
Change in	Flexible Regime	Monetary Union	Flexible Regime	Monetary Union	
nominal exchange rates	0%	-3.1393%	2.6549%	3.1393%	
goods prices	0%	3.1393%	0%	3.0412%	
real exchange rates	0%	-3.1393%*	2.6549%	6.1805%	
output	0%	-1.5814%	0%	-1.5814%	

Table 5c: Effects on Sweden of a Shock in the US

	dAEI		dA	VI
Change in	Flexible Regime	Monetary Union	Flexible Regime	Monetary Union
nominal exchange rates	0%	-0.1442%	0.9194%	0.9199%
goods prices	0%	0.1442%	0%	0.1397%
real exchange rates	0%	-0.1442%*	0.9194%	1.0596%
output	0%	-0.0726%	0%	-0.0726%

Variability Measured by the Average Exchange Rate Index

^{*} The total terms of trade return to their original value after one period which motivates 0% as an alternative entry. The entry given in the table can be motivated by the fact that a change in nominal exchange rates in the first period also is a temporary change in *real* exchange rates as prices are constant.

The shock considered above gives rise to a 1.9852% appreciation of the nominal exchange rate when all exchange rates are flexible. Being inside the union an equally large change in the value of the krona takes place one period later through a change in the price of the Swedish good but the output level will in this case be away from long-run equilibrium until goods prices adjust.

If a shock hits the EMU-area instead, then the euro appreciates against all currencies including the krona. To keep the Swedish goods market in equilibrium, the krona appreciates against the dollar and currency W, leading to

$$\left.\frac{dAEI}{du^{EMU}}\right|_{F} = 0$$

which is a theoretical result already discussed. If Sweden becomes a member of the EMU does not affect the appreciation of the euro but the appreciation of the euro will contract Swedish aggregate demand and to clear the goods market Swedish output now has to decrease by 1.5814%. Sweden will also experience an increase in real exchange rate variability due to a temporary appreciation of terms of trade (-3.1393%). This happens because the krona cannot depreciate to compensate for the appreciation of the euro.

A shock in the US works in much the same way as the shock in the EMU above. One difference, though, is that the magnitude of the effects is much smaller. To compensate for the appreciation of the dollar against the euro, the euro has to appreciate against currency W to leave aggregate demand unaffected in the EMU-area. Since Sweden and the EMU have similar relations to the US, these exchange rate changes that keeps aggregate demand unchanged in the EMU, leave Sweden almost unaffected too. There will only remain a small need for the krona to depreciate against the euro²⁵ (by 0.1397%) or, alternatively, a need for a small decline in output (by 0.0726%).

As the change in real exchange rates reveals, shocks in the EMU affect Swedish competitiveness much more (-0.1637%) than shocks in the US (-0.0075%) which suggests that the existence of asymmetries between Sweden and the EMU are much more important than asymmetries in trade relations with the US²⁶.

²⁵See Table 6a in Appendix B for details.

 $^{^{26}}$ This result still holds when the UK or Norway is used as the fourth country of the model. The reason for this result is that the EMU is by far the most important trade partner to Sweden.

Variability Measured by the Absolute Value Index

When using the Absolute Value Index, Swedish shocks give exactly the same results as above so the interesting cases are those when shocks hit the EMU or the US. The Average Exchange Rate Index states clearly that variability increases if Sweden joins the EMU and shocks hit the EMU or the US. For the Absolute Value Index, on the other hand, a theoretical analysis gives ambiguous results regarding variability in the first period and therefore it is interesting to examine the numerical results. Tables 5b and 5c show that Sweden will have to face more variability in the first period if it enters the monetary union but the differences are quite small compared to the differences in variability presented above when using the other index. The reason for the large differences when the Average Exchange Rate Index is used is that all exchange rate changes cancel when the krona is flexible, while the appreciation of the euro, if Sweden is a part of the EMU, affects the index to a large extent. The changes in Swedish exchange rates, when the krona is flexible, are added in the Absolute Value Index, instead of cancelling out, assigning positive values to the change in the value of the index under both exchange rate regimes. When Norway is the fourth country and the UK and Denmark have joined "Euroland", then variability decreases if Sweden enters the EMU so there is no definite answer to the question about what exchange rate regime is preferable when focus is on nominal exchange rate variability. What can be seen is that, no matter which countries are studied, shocks in the EMU result in more exchange rate variability than shocks in the US, the UK or Norway. One can also see that, when changes in goods prices in the second period are taken into account, real exchange rate variability increases when Sweden becomes a part of the EMU and shocks hit the EMU or the US, but that is, of course, a theoretical result already discussed.

Sensitivity Analysis

The length of the period with fixed prices has been assumed to be one year in this section. Time enters the model via the interest rate so by cutting the interest rate to one half of its original value one can study both the effects of a lower interest rate or of a six months period with fixed prices. All results are only affected to a very small extent by different interest rates in the range from 2% to 9% annually.

Changes in the elasticity of exports with respect to exchange rates affect all changes in exchange rates proportionally so by increasing the export elasticity from 2 to 4 all values of the indices become half as large.

One can also let an equally large shock hit Sweden and the EMU simultaneously. This reduces the need for adjustments in the exchange rate between the krona and the euro as well as in output but the variability does not disappear since Sweden and the EMU have different trading patterns and degrees of openness.

Weighting shocks in different countries

It seems reasonable that the probability of the event that a shock of the size of one percent of GDP hits EMU is smaller than the probability that a correspondingly large shock hits Sweden. The reason is that the EMU consists of many countries whose GDP changes are not perfectly correlated. Therefore shocks in different EMU-countries are likely to cancel to some extent. In order to examine this, standard deviations of the relative changes in GDP for the EMU-countries, Sweden and the US are calculated. The data used are real annual GDP from 1975 to 1995. This standard deviation does not measure the standard deviation of the demand shocks discussed in this paper but, given the assumption that demand shocks have the same importance in explaining GDP variability in all these countries, it should give some indication of the relative sizes of normal shocks²⁷.

Let Sweden be hit by a normal-sized 2.3 percent shock and let the other countries be hit by shocks, adjusted in size according to the differences in standard deviations of the relative GDP-changes. Then one can compare the effects of normal-sized shocks in the different countries. The results of this can be seen in *Table 6*. The unambiguous result is that there is no sign of a reduction in neither total real exchange rate variability nor output variability as a result of a Swedish membership in the monetary union, no matter what index is used.

	A	AEI AVI Y		(
Shock in	F	U	F	U	F	U
Sweden	4.6%	4.6%	4.6%	4.6%	0%	2.3%
the EMU	0%	-5.3%	4.5%	10.5%	0%	-2.7%
the US	0%	-0.36%	2.3%	2.6%	0%	-0.2%

Table 6: Effects on the Real Exchange Rate and on Output

Results in Brief

First consider the Average Exchange Rate Index. With a flexible exchange rate regime the krona has to appreciate approximately twice as much as the percentage change in aggregate demand caused by a Swedish demand shock. A 1% increase in aggregate demand in the EMU gives rise to a 3.1% appreciation of the Average Exchange Rate Index if Sweden is a member of the monetary union compared to a 0.14% appreciation if a corresponding shock occurs in

²⁷The standard deviations are: the EMU: 1,7%, Sweden: 2,3% and the US: 2,5%. The large standard deviation in the US can to a large extent be explained by one outlier: the large decrease in GDP between 1981 and 1982.

the US. The much larger impact of EMU-shocks is a general result and holds regardless of which country is used as the fourth country.

Then consider the results when the Absolute Value Index is used. Variability increases as a consequence of a Swedish membership if variation in goods prices in the second period is taken into account. When focus is on nominal exchange rate variability, instead, and shocks hit the fourth country or the EMU the picture is rather dispersed. Variability can both increase and decrease if Sweden enters the monetary union depending on which country is chosen and also depending on what decision Denmark and the UK take regarding their membership in the EMU.

5.3 Results and Interpretations for the United Kingdom

The simulation can naturally be done from a British point of view as well. The results are only reported for the most interesting case where the US is the fourth country.²⁸. The most important conclusion is that the United Kingdom will get similar-sized effects of an entry to the EMU which can be seen in *Tables 7a-7c*. To clear the British goods market after a British shock the pound has to appreciate by approximately 3%. This appreciation is larger than the corresponding Swedish appreciation reflecting the fact that the United Kingdom is a slightly more closed economy. If the UK enters the EMU and a shock hits the EMU, then British

	dA	EI	dAVI		
Change in	Flexible Regime	Monetary Union	Flexible Regime	Monetary Union	
nominal exchange rates	-2.8919%	0%	2.8919%	0%	
goods prices	0%	-2.8919%	0%	2.8919%	
real exchange rates	-2.8919%	-2.8919%	2.8919%	2.8919%	
output	0%	1.0000%	0%	1.0000%	

Table 7a: Effects on the UK of a Shock in the UK

Table 7b: Effects on the UK of a Shock in the EMU

 $^{^{28}}$ Sweden and Norway affect the British economy to a much smaller extent than the US as trade with Sweden or Norway is less than one fourth of the trade with the US.

	dA	EI	dAVI		
Change in	Flexible Regime	Monetary Union	Flexible Regime	Monetary Union	
nominal exchange rates	0%	-2.0306%	2.1976%	2.0306%	
goods prices	0%	2.0306%	0%	1.8547%	
real exchange rates	0%	-2.0306%*	2.1976%	3.8853%	
output	0%	-0.7022%	0%	-0.7022%	

Table 7c: Effects on the UK of a Shock in the US

	dA	EI	dAVI		
Change in	Flexible Regime	Monetary Union	Flexible Regime	Monetary Union	
nominal exchange rates	0%	-0.2594%	1.1004%	0.9018%	
goods prices	0%	0.2594%	0%	0.2369%	
real exchange rates	0%	-0.2594%*	1.1004%	1.1387%	
output	0%	-0.0897%	0%	-0.0897%	

competitiveness deteriorates, of course, but the deterioration is smaller (2.0306%) than the corresponding Swedish deterioration when Sweden is studied (3.1393%). The main reason for this is that the share of UK exports going to the EU (60%) is larger than the corresponding share for Sweden (44%)²⁹. This means that the exchange rate between the pound and the euro has more weight in the exchange rate index for the UK. The great importance of the elimination of variation in this exchange rate, as a result of an entry into the EMU, reduces the total increase in the Average Exchange Rate Index. Shocks in the US affect the UK more than Sweden as the US is more important to the UK than to Sweden³⁰ but such shocks still affect the UK much less than shocks emanating from the EMU.

²⁹This figure is valid when Denmark and the UK stay outside the EMU.

 $^{^{30}}$ Exports from the UK to the US constitute 11% of total British exports while the corresponding figure for Sweden is 7%.

6. Conclusions

The most important result is that a Swedish and British membership in the EMU in many cases leads to *more* exchange rate variability and also to more output variability. The perhaps most crucial assumption to generate these results is that prices are more sticky than nominal exchange rates. There is no doubt that goods prices and wages are more sticky than nominal exchange rates but, the question of how important such stickiness is, has no clear answer.

Two measures on total exchange rate variability have been used: the Average Exchange Rate Index (AEI) and the Absolute Value Index (AVI). The difference between the indices is how the sign of an exchange rate change is handled. All changes are added when the Absolute Value Index is used while exchange rates moving in opposite directions may cancel out when the Average Exchange Rate Index is used.

Start with the simplest shock to analyse: the domestic shock. When shocks hit the entrant, nominal exchange rate variability can be eliminated by joining the EMU. If the nominal exchange rate is not allowed to change, then the output level of the entering country temporarily moves away from its long run equilibrium level. An equally large change in goods prices in the second period will, however, replace the change in nominal exchange rates that can be eliminated by a membership so the conclusion for domestic shocks is that a decision to enter the monetary union will only postpone an inevitable change in the real exchange rate but it will lead to more output variability.

Shocks in countries outside the entrant unambiguously give the result that real exchange rate variability increases as a result of an entry into the monetary union. The reason is that the Swedish or British currency (the euro in this case) appreciates to keep the EMU-economy in long-run equilibrium. If the entrant stayed outside the EMU, then the value of its currency could adjust to leave total terms of trade unaffected, meaning less variability of both exchange rates and output³¹.

When evaluating which index is the most appropriate to use, the crucial point is, as mentioned earlier, whether firms can redirect their exports to other countries easily or not. Large multinational companies can probably redirect their exports more easily as they have already existing international networks of sales organisations and as they have easier access to the human capital needed to enter new foreign markets. The major part of international trade goes via large companies which suggests that the Average Exchange Rate Index is more appropriate. Small firms may be much more exposed to one or a few countries, however, which gives support to the use of the Absolute Value Index. Moreover, redirecting exports is probably costly not only for small firms but also for large firms which gives additional

³¹Variability of the nominal exchange rate can both increase and decrease when variability is measured by the AVI depending on where shocks occur (in the EMU, the US, the UK or Norway) and also on the decisions of the UK and Denmark regarding their possible future membership in the EMU but this ambiguity disappears if the price changes in the second period are taken into account. In that case a membership in the EMU definitely leads to an increase in exchange rate variability.

support to the Absolute Value Index. The economy as a whole is likely to be characterised partly by flexible export streams and partly by redirectional costs.

If one lets an equally big shock hit the EMU and the entrant simultaneously one gets a symmetric shock. Such shocks will reduce the magnitude of the results in this paper but the results will not disappear as long as the entrant and the EMU have different trading patterns. The relevance of the results in this paper thus partly depends on the degree of asymmetry between the entering country and the EMU. Empirical measures of the degree of asymmetry are sensitive to the model and time period chosen³² when studying this issue but it can be stated that "country-specific asymmetric shocks largely explain the short-term macroeconomic development in Sweden"³³. Another important issue is whether real or financial shocks dominate. The results of an analysis by Alun Thomas (1997) suggest that real shocks play an important role in real exchange rate movements in Sweden and thus giving support to the approach with real shocks used in this paper. Financial disturbances exist, however, and it would therefore in some cases be optimal to have a fixed exchange rate to insulate the real economy from such shocks but several studies indicate that financial disturbances only explain a small part of variation in output but a large part of variation in inflation³⁴. The possibly large temporary deviations from the long term value of the exchange rate caused by lack of credibility for fiscal policy will probably be smaller for an EMUcountry than for a small outsider as a crisis in Sweden, for instance, will affect the value of the krona much more than the value of the euro.

All the results above are derived under the assumption that prices are sticky and shocks permanent. Regardless of which of the two possible exchange rate regimes that is chosen there will be a need for permanent adjustments in international relative prices to restore equilibrium if shocks are permanent. Therefore a membership in the monetary union will have no effect on changes in terms of trade in the long-run. The real exchange rate will change in any case but, by staying outside the monetary union, Sweden or the United Kingdom can obtain shorter periods away from long-run equilibrium.

7. Appendices

7.1 Appendix A: Derivation of a Reduced Form for the Goods Market

The reduced form for aggregate demand used in the main text will be derived from microeconomic behaviour in this appendix. Just as in the main text, the world consists of three countries: Sweden (S), the EMU and the rest of the world (W). There are three different goods available to consumers; one produced in each country.

³²Jansson, P. (1997).

³³Ibid.

³⁴Ibid.

The Government

By assumption, the government only demands domestic goods and services. Shocks in government expenditures are nominal and permanent and contribute additively to individual utility. Since Ricardian equivalence holds in this model, nothing is lost by simply assuming that the government budget is always balanced:

$$G_t = T_t, \forall t$$
.

A permanent positive shock in government expenditures that hits the EMU thus has two effects on the Swedish economy:

1) The increase in aggregate demand in the EMU is an asymmetric stimuli of demand for the EMU-good relative to the Swedish good.

2) A rise in government expenditures in the EMU-area means that the resulting increase in taxes takes disposable income away from consumers living in the EMU. Faced with less income the EMU-citizens cut consumption both of domestic and foreign goods. This results in a direct decrease of demand for the Swedish good through a decrease in Swedish exports.

To facilitate the derivation of a simple reduced form for the goods market³⁵, suppose the second effect above does not exist. This means that the shocks henceforth will be somewhat less asymmetric. The effects caused by asymmetries in the simplified model will thus appear smaller which should be remembered when the results are interpreted.

The Firms

Suppose that production is characterised by constant returns to scale and that nominal wages and goods prices remain constant until one period of time has elapsed after the appearance of a shock. Production is then completely demand-determined during the period with constant prices. After this period, nominal wages and goods prices adjust to immediately bring output back to its long run equilibrium level. Goods prices are set in domestic currency.

The Consumers

Preferences are assumed to be properly described by a standard CES utility function. In line with empirical results³⁶ and as a convenient simplification assume that the elasticity of imports with respect to the real exchange rate is equal to one. One way of entering this

³⁵Demand for the Swedish good in expression (A17) will then depend positively on total output abroad instead of disposable income.

³⁶Goldstein & Kahn (1985).

assumption into the model is by altering the utility function to the one in the maximisation problem (A4) below. This particular utility function has Cobb-Douglas-utility between domestic goods and imports and CES-utility between imports from different countries. Through this specification expenditures on imports become a constant share of total expenditures which is equivalent to the elasticity of imports with respect to the real exchange rate being equal to one.

Consumers are assumed to hold no assets initially to avoid the possibly important effects of exchange rate changes on permanent income via changes in the size of foreign net assets³⁷. large negative net foreign assets may result in Privately held assets are assumed to be zero initially. It is straightforward to allow for assets but they

The optimisation problem of the individuals in Sweden can be written:

$$\max_{\substack{C_{St}^{S}, C_{EMUt}^{S}, C_{Wt}^{S} \\ t=1}} \sum_{t=1}^{\infty} b^{t-1} U_{t} \left[C_{St}^{S}, C_{EMUt}^{S}, C_{Wt}^{S} \right]$$
s.t.
$$\sum_{t=1}^{\infty} \left[\frac{1}{1+i} \right]^{t-1} \left[P_{t}^{S} C_{St}^{S} + E_{t}^{S'EMU} P_{t}^{EMU} C_{EMUt}^{S} + E_{t}^{S'W} P_{t}^{W} C_{Wt}^{S} \right] = \sum_{t=1}^{\infty} P_{t}^{S} (Y_{t}^{S} - T_{t}^{S}) \left[\frac{1}{1+i} \right]^{t-1}$$
(A1)

where C_{Wt}^{s} , for instance, denotes the Swedish individuals' period t consumption of the good produced in the rest of the world. The price of this good, P_{t}^{W} , must be multiplied by the nominal exchange rate, $E_{t}^{S/W}$, to be measured in kronor. The subjective discount factor \mathcal{B} belongs to the interval (0,1) in line with standard assumptions. Real income before taxes, Y_{1}^{S} , can vary in period one but the economy will immediately reach its steady state income level in period two when prices become fully flexible. Disposable income is equal to Y_{t}^{S} minus the lump-sum tax T_{t}^{S} .

Solving the optimisation problem gives standard Euler conditions:

$$\frac{\iint J_{t}}{\iint C_{St}^{S}} = \beta(1+i)\frac{P_{t}^{S}}{P_{t+1}^{S}}\frac{\iint J_{t+1}}{\iint C_{St+1}^{S}},$$

$$\frac{\iint J_{t}}{\iint C_{EMUt}^{S}} = \beta(1+i)\frac{E_{t}^{S'EMU}}{E_{t+1}^{S'EMU}}\frac{P_{EMU}}{P_{t+1}^{EMU}}\frac{\oiint J_{t+1}}{\oiint C_{EMUt+1}^{S}} \text{ and}$$

$$\frac{\oiint J_{t}}{\iint C_{Wt}^{S}} = \beta(1+i)\frac{E_{t}^{S'W}}{E_{t+1}^{S'W}}\frac{P_{t}^{W}}{P_{t+1}^{W}}\frac{\oiint J_{t+1}}{\oiint C_{Wt+1}^{S}}.$$
(A2)

It can be seen from (A2) that steady state from period two and onwards demands that

³⁷It is, in principle, straightforward to allow for non-zero assets but the effects of changes in the value of such assets are not a main issue of this paper.

$$\boldsymbol{b} = \frac{1}{1+i} \frac{P_{t+1}^{S}}{P_{t}^{S}} = \frac{1}{1+i} \frac{E_{t+1}^{S/EMU} P_{t+1}^{EMU}}{E_{t}^{S/EMU} P_{t}^{EMU}} = \frac{1}{1+i} \frac{E_{t+1}^{S/W} P_{t+1}^{W}}{E_{t}^{S/W} P_{t}^{W}} = \frac{1}{1+i},$$
(A3)

since inflation in steady state is assumed to be zero and nominal exchange rates only change instantly due to a shock. Use the conditions for this steady state and the utility function mentioned above to rewrite³⁸ (A1):

$$\max \ln \left\{ (C_{S}^{s})^{1-g} \left[(C_{EMU}^{s})^{d} + (C_{W}^{s})^{d} \right]^{g} \right\} + \frac{1}{i} \ln \left\{ (\overline{C}_{S}^{s})^{1-g} \left[(\overline{C}_{EMU}^{s})^{d} + (\overline{C}_{W}^{s})^{d} \right]^{g} \right\}$$

s.t. $C_{S}^{s} + E^{S'EMU} C_{EMU}^{s} + E^{S'W} C_{W}^{s} + \frac{1}{i} \left[\overline{P} \overline{C}_{S}^{s} + \overline{E}^{S'EMU} \overline{C}_{EMU}^{s} + \overline{E}^{S'W} \overline{C}_{W}^{s} \right] = YD^{s} + \frac{\overline{P}}{i} \overline{YD}^{s}$ (A4)

where overbars indicate steady state values of the variables, and disposable income YD^{s} is equal to $Y^{s} - T^{s}$. To simplify notation further all nominal prices, except the price of the Swedish good in steady state, are normalised to one. The reformulated problem (A4) has the following first order conditions:

$$C_{\rm S}^{\rm S} = \overline{P}^{\rm S} \overline{C}_{\rm S}^{\rm S} \tag{A5}$$

$$\frac{1-g(C_{EMU}^{s})^{d}+(C_{W}^{s})^{d}}{gC_{s}^{s}(C_{EMU}^{s})^{d+1}} = \frac{1}{E^{s/EMU}}$$
(A6)

$$\frac{1-g(C_{EMU}^{s})^{d}+(C_{W}^{s})^{d}}{gC_{s}^{s}(C_{W}^{s})^{d-1}} = \frac{1}{E^{s/W}}$$
(A7)

$$\frac{1-g}{g} \frac{(\overline{C}_{EMU}^{S})^{d} + (\overline{C}_{W}^{S})^{d}}{\overline{C}_{S}^{S} (\overline{C}_{EMU}^{S})^{d-1}} = \frac{\overline{P}^{S}}{\overline{E}^{S/EMU}}$$
(A8)

$$\frac{1-g(\overline{C}_{EMU}^{S})^{d}+(\overline{C}_{W}^{S})^{d}}{\overline{C}_{S}^{S}(\overline{C}_{W}^{S})^{d-1}} = \frac{\overline{P}^{S}}{\overline{E}^{S/W}}$$
(A9)

³⁸Make use of the fact that $\sum_{t=2}^{\infty} a^{t-1}$ is equal to $\frac{a}{1-a}$ to show that the assumption of steady state from the

second period onwards means that the problem can be handled as a two period problem with one sticky prices period and one steady state period.

Together with the budget constraint in (A4) the first order conditions (A5) to (A9) form an equation system in the six unknown variables C_s^s , C_{EMU}^s , \overline{C}_w^s , \overline{C}_{EMU}^s and \overline{C}_w^s . Solve for the different components of Swedish period one consumption:

$$C_{s}^{s} = (1 - g \frac{iYD^{s} + \overline{P}^{s} \overline{YD}^{s}}{1 + i}$$
(A10)

$$E^{S/EMU}C^{S}_{EMU} = g^{\underline{i}YD^{S}} + \overline{P}^{S}\overline{YD}^{S} \left[1 + \left(\frac{E^{S/W}}{E^{S/EMU}}\right)^{\frac{d}{d-1}} \right]^{1}$$
(A11)

$$E^{S/W}C_{W}^{S} = g \frac{iYD^{S} + \overline{P}^{S}\overline{YD}^{S}}{1+i} \left[1 + \left(\frac{E^{S/EMU}}{E^{S/W}}\right)^{\frac{d}{d-1}} \right]^{-1}$$
(A12)

The Consumption Decision in the EMU and the Rest of the World

The optimisation problem of the individuals in the EMU has a completely analogous structure:

$$\max \ln \left\{ \left(C_{EMU}^{EMU} \right)^{1-g_{EMU}} \left[\left(C_{S}^{EMU} \right)^{d_{EMU}} + \left(C_{W}^{EMU} \right)^{d_{EMU}} \right]^{\frac{g_{EMU}}{f_{MU}}} \right\} + \frac{1}{i} \ln \left\{ \left(\overline{C}_{EMU}^{EMU} \right)^{1-g_{EMU}} \left[\left(\overline{C}_{S}^{EMU} \right)^{d_{EMU}} + \left(\overline{C}_{W}^{EMU} \right)^{d_{EMU}} \right]^{\frac{g_{EMU}}{f_{MU}}} \right\} \right\}$$

$$\operatorname{s.t.} C_{EMU}^{EMU} + \frac{1}{E^{S/EMU}} C_{S}^{EMU} + \frac{E^{S/W}}{E^{S/EMU}} C_{W}^{EMU} + \frac{1}{i} \left[\overline{C}_{EMU}^{EMU} + \frac{1}{\overline{E}} C_{S}^{EMU} + \frac{\overline{E}_{S}^{S/W}}{\overline{E}} C_{W}^{EMU} \right] = YD^{EMU} + \frac{1}{i} \overline{YD}^{EMU}$$
(A13)

This analogy means that one can immediately state that

$$\frac{1}{E^{S/EMU}}C_{S}^{EMU} = \mathbf{g}_{EMU}\frac{iYD^{EMU} + \overline{YD}^{EMU}}{1+i} \left[1 + \left(\frac{1}{E^{S/W}}\right)^{\frac{d_{EMU}}{d_{EMU}-1}}\right]^{1}.$$
(A14)

Define the optimisation in an analogous way in the rest of the world and solve for the world's import of Swedish goods:

$$\frac{1}{E^{S'W}} C_{S}^{W} = g_{V} \frac{iYD^{W} + \overline{YD}^{W}}{1+i} \left[1 + \left(\frac{1}{E^{S'EMU}}\right)^{\frac{d_{W}}{d_{W}-1}} \right]^{-1}.$$
(A15)

Goods Market Equilibrium in Sweden

Equilibrium in the Swedish goods market demands that

$$Y^{S} = C_{S}^{S} + C_{S}^{EMU} + C_{S}^{W} + G^{S}$$
$$\overline{P}^{S}\overline{Y}^{S} = \overline{P}^{S}\overline{C}_{S}^{S} + \overline{P}^{S}\overline{C}_{S}^{EMU} + \overline{P}^{S}\overline{C}_{S}^{W} + G^{S}$$

which can be rewritten as

$$Y^{s} = C_{s}^{s} + C_{s}^{EMU} + C_{s}^{W} + G^{s}$$
(A16a)

$$\overline{P}^{s}\overline{Y}^{s} = C_{s}^{s} + C_{s}^{EMU} + C_{s}^{W} + G^{s}, \qquad (A16b)$$

if the elasticity of exports with respect to the real exchange rate is close to one. The equations (A16a) and (A16b) imply

$$\overline{P}^{s} = \frac{Y^{s}}{\overline{Y}^{s}},$$

which can be substituted into (A16) to give

$$Y^{S} = F^{S}(E^{S/EMU}, E^{SW}, G^{S}, \overline{Y}^{S}, Y^{EMU}, \overline{Y}^{EMU}, Y^{W}, \overline{Y}^{W}, i, \Gamma), \qquad (A17)$$

where Γ is the parameter vector (gg_{EMU} , g_V , dd_{MU} , d_V). The important property used in the main text, that aggregate demand depends positively on the exchange rates, holds in (A17) too, which can be shown easily. In this way government expenditures shocks can be analysed but one can see from (A17) that the results in the main text can be interpreted as emanating from other shocks such as shocks in foreign income or Swedish permanent income, in the world market interest rate or in preferences. Differentiating gives

$$dY^{s} = f_{EMU}^{s} dE^{s/EMU} + f_{W}^{s} dE^{s/W} + du^{s},$$
(A18)

where $du^{s} = f_{G}^{s} dG^{s}$. Expression (A18) is equivalent to the central equation (4) in the main text.

7.2 Appendix B: Numerical Results for Sweden

This appendix provides tables showing the effects on output and on all exchange rates due to shocks in different countries. The tables also show changes in the value of the exchange rate indices. The columns dAEIu-dAEIf and dAVIu-dAVIf measure the difference of the absolute values of the corresponding indices. A negative value in such a column thus indicates that variability would decrease if Sweden would enter the EMU.

Table 8a: Numerical results

4th country: the US (DK and the UK outside the EMU)

Flexible Reg	gime						
Shock in	dEse	dEeu	dEew	dEsu	dEsw		
Sweden	-1.9231%	-0.0813%	-0.1145%	-2.0044%	-2.0376%		
EMU	3.0412%	-4.5841%	-5.5215%	-1.5428%	-2.4803%		
US	0.1397%	5.1839%	-1.0806%	5.3236%	-0.9409%		
Monetary U	J nion						
Shock in	dYs	dEeu	dEew				
Sweden	1.0000%	0.0000%	0.0000%				
EMU	-1.5814%	-4.7126%	-5.7025%				
US	-0.0726%	5.1780%	-1.0889%				
Indices (firs	st period)						
Shock in	dAEIf	dAEIu	dAEIu-dAEIf		dAVIf	dAVIu	dAVIu- dAVIf
Sweden	-1.9852%	0.0000%	-1.9852%		1.9852%	0.0000%	-1.9852%
EMU	0.0000%	-3.1393%	3.1393%		2.6549%	3.1393%	0.4844%
US	0.0000%	-0.1442%	0.1442%		0.9194%	0.9199%	0.0004%

Table 8b: Numerical results

4th country: Norway (DK and the UK outside the EMU)

Flexible Regime

Shock in	dEse	dEen	dEew	dEsn	dEsw
Sweden	-1.9389%	0.1433%	-0.1147%	-1.7956%	-2.0536%
EMU	2.8572%	-2.6221%	-5.4246%	0.2351%	-2.5674%
Norway	-0.1283%	2.1539%	-0.0510%	2.0256%	-0.1793%

Monetary Union

Shock in	dYs	dEen	dEew
Sweden	1.0000%	0.0000%	0.0000%
EMU	-1.4736%	-2.4109%	-5.5937%
Norway	0.0662%	2.1444%	-0.0434%

Indices (first period)

Shock in	dAEIf	dAEIu	dAEIu-dAEIf	dAVIf	dAVIu	dAVIu-
						dAVIf
Sweden	-1.9852%	0.0000%	-1.9852%	1.9852%	0.0000%	-1.9852%
EMU	0.0000%	-2.9254%	2.9254%	2.5278%	2.9254%	0.3976%
Norway	0.0000%	0.1314%	0.1314%	0.2886%	0.1741%	-0.1144%

Table 8c: Numerical results

4th country: the UK (DK and the UK outside the EMU)

Flexible Regime								
Shock in	dEse	dEeu	dEew	dEsu	dEsw			
Sweden	-1.9161%	-0.0106%	-0.1423%	-1.9268%	-2.0584%			
EMU	3.2197%	-2.8395%	-6.2230%	0.3801%	-3.0033%			
UK	0.1970%	2.4900%	-0.8528%	2.6870%	-0.6559%			
Monetary Union								
Shock in	dVs	dEeu	dEew					

Shock in	ars	dEeu	dEew
Sweden	1.0000%	0.0000%	0.0000%
EMU	-1.6803%	-2.8574%	-6.4622%
UK	-0.1028%	2.4889%	-0.8675%

Indices (first period)

Shock in	dAEIf	dAEIu	dAEIu-dAEIf	dAVIf	dAVIu	dAVIu-
Sweden	-1.9852%	0.0000%	-1.9852%	1.9852%	0.0000%	dAVIf -1.9852%
EMU	0.0000%	-3.3357%	3.3357%	2.8752%	3.3357%	0.4605%
UK	0.0000%	-0.2041%	0.2041%	0.6279%	0.6264%	-0.0015%

Table 8d: Numerical results

4th country: the US (DK and the UK inside the EMU)

Flexible Regime

Shock in	dEse	dEeu	dEew	dEsu	dEsw
Sweden	-1.9213%	-0.1140%	-0.1664%	-2.0353%	-2.0877%
EMU	2.7263%	-5.3703%	-6.9930%	-2.6440%	-4.2667%
US	0.1231%	4.9415%	-1.4839%	5.0645%	-1.3608%

Monetary Union

Shock in	dYs	dEeu	dEew
Sweden	1.0000%	0.0000%	0.0000%
EMU	-1.4190%	-5.5320%	-7.2290%
US	-0.0640%	4.9342%	-1.4946%

Indices (first period)

Shock in	dAEIf	dAEIu	dAEIu-dAEIf	dAVIf	dAVIu	dAVIu-
						dAVIf
Sweden	-1.9852%	0.0000%	4.1131%	1.9852%	0.0000%	0.0000%
EMU	0.0000%	-2.8169%	4.8701%	3.2321%	2.8169%	0.0000%
US	0.0000%	-0.1271%	0.0000%	0.9045%	0.8663%	0.0000%

Table 8e: Numerical results

4th country: Norway (DK and the UK inside the EMU)

Flexible Regime	
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Shock in	dEse	dEen	dEew	dEsn	dEsw
Sweden	-1.9408%	0.1588%	-0.1657%	-1.7820%	-2.1065%
EMU	2.4015%	-1.6400%	-6.7995%	0.7615%	-4.3979%
Norway	-0.1274%	2.1596%	-0.0786%	2.0322%	-0.2060%

Monetary Union

Shock in	dYs	dEen	dEew
Sweden	1.0000%	0.0000%	0.0000%
EMU	-1.2374%	-1.4435%	-7.0045%
Norway	0.0656%	2.1492%	-0.0678%

Indices (first period)

Shock in	dAEIf	dAEIu	dAEIu-dAEIf	dAVIf	dAVIu	dAVIu- dAVIf
Sweden	-1.9852%	0.0000%	-1.9852%	1.9852%	0.0000%	-1.9852%
EMU	0.0000%	-2.4564%	2.4564%	2.9555%	2.4564%	-0.4991%
Norway	0.0000%	0.1303%	0.1303%	0.2895%	0.1759%	-0.1137%

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