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Persistency and Money Demand Distortions in a Stochastic DGE Model with Sticky Prices

Michael Gail*

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Abstract

Recently macroeconomists have intensified their efforts to develop models that are able to generate persistent reactions of real variables to monetary shocks in stochastic DGE models with nominal rigidities. This has proven to be quite difficult in models with price staggering only. Most papers show that output is above steady state only as long as prices are fixed for the firms. In this article particular attention is given to the role of money demand and to the form of the utility function. I consider cash-in-advance- (CIA) as well as money-in-the-utility-function- (MIU) models, with CRRA and GHH preferences, to evaluate their ability to generate persistence. Persistent reactions emerge only with a high value of the elasticity of labor supply with respect to the real wage *and* an interest rate sensitive money demand function. CIA-models generally create more persistence than MIU-models. In the CIA-setup a CRRA utility function generates more persistence than GHH preferences. The results highlight the importance of the way money is introduced in a New Neoclassical Synthesis model.

JEL classification: E52
Keywords: Monetary Policy, New Neoclassical Synthesis, Sticky Prices, Persistence, Transmission Mechanism

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1 Introduction

Can monetary shocks generate persistent responses of inflation and output? This question has been addressed in a battery of papers in the last few years. The most prominent paper is the one of [5, Chari/Kehoe/McGrattan (2000)] who conclude that standard models with staggered prices generate only a positive output reaction for the time of exogenous price stickiness. Several attempts have been made to challenge this result.

Recently [6, Christiano/Eichenbaum/Evans (2001)] developed a DGE model that is capable of generating the observed persistence of monetary shocks in US data. With an average duration of two to three quarters wage contracts are the critical nominal friction, not price contracts. If inertia in inflation and output persistence is the main goal to match then they show that variable capacity utilization is most important. To explain the reaction of all variables they include habit persistence in consumption as well as adjustment costs in investment. It should be noted that these authors use a limited information econometric strategy that is not yet common in the literature so that the results are difficult to compare to existing studies.

[9, Dotsey/King (2001)] stress the importance of variable capacity utilization as well. They demonstrate that persistence is possible even in a sticky price model that features labor supply variability through changes in employment and incorporates produced inputs as intermediate goods. All these three ingredients together produce a flat reaction of real marginal costs to a money growth shock. In turn this reduces the extent of price adjustments of the firms. Unfortunately this gradual adjustment of the price level is responsible for the rise in the nominal interest rate: the model does not display the liquidity effect.

[2, Bergin/Feenstra (2000)] use a modified DGE model with intermediate goods and so called translog preferences which is essentially a non-CES aggregator for intermediate goods that replaces the [8, Dixit/Stiglitz (1977)] aggregator. They show that intermediates in production are very important to generate persistent output responses but they also find a strengthening role for the translog preferences: The higher the share of intermediates in production the higher the persistence.

Intermediates also play an important role in the work of [17, Huang/Liu/Phaneuf (2001)]. They evaluate the performance of staggered wage models in relation to staggered price models. They show that only a model with intermediates, staggered price and staggered wage setting can explain persistent responses of output and, depending on the share of intermediates in production, a weak but slightly positive response of the real wage to a monetary shock, as is observed empirically in the postwar period.

In a model with a vertical input-output structure and only price staggering [15, Huang/Liu (2001a)] demonstrate that the higher the number of stages of production the more persistent the output response. With a sufficient number of stages the response can even be arbitrarily large, given that the share of intermediates is one at all stages of production.

In recent research [16, Huang/Liu (2001b)] demonstrate the importance of such an input-output structure in a two-country model to explain the significant cross-country correlations in aggregate output and the persistent deviations of real exchange rates from purchasing power parity.

[7, Dib/Phaneuf (2001)] discuss a model with price staggering instead of wage staggering. In a variant of the model with a nominal rigidity through costly price adjustment and a real rigidity through adjusting the labor input output, hours and real wages show a persistent reaction to a monetary shock. Moreover, the model can explain the decline in hours worked after a productivity shock, as observed in US postwar data.

In this paper special attention is given to the way money is introduced and to the form of the utility function to account for persistence. To do so CIA- as well as MIU-models are proposed. The importance of the way money demand is modeled in a DGE model has not yet been recognized by the papers summarized above. There is also no detailed analysis of the role played by the utility function. The results obtained here speak in favor of the setup. First, it turns out that the specific form of the utility function has important effects on the model outcomes. In the CIA-setup a CRRA utility function generates more persistence than GHH preferences. Second, persistent output and inflation responses depend only in part on the value of the elasticity of labor supply with respect to the real wage. Third, persistence depends also crucially upon the implied money demand function. Persistent output reactions emerge only in the MIU-model with GHH preferences and a high value for the elasticity of labor with respect to the real wage. In a CIA-model this result does not hold. Forth, CIA-models generally create more persistence than MIU-models. These results emerge from a model with price staggering only and with no other real or nominal rigidities, challenging results of [6, Christiano/Eichenbaum/Evans (2001)] or [9, Dotsey/King (2001)]. Neither variable capacity utilization nor labor supply variability through changes in employment nor wage staggering nor a vertical input-output structure are necessary to generate persistent output responses here. In addition the paper shows that [5, Chari/Kehoe/McGrattan's (2000)] contract multiplier has to be interpreted carefully as they only analyze a MIU-model. The multiplier seems to be different in a CIA-economy. To uncover the different reactions of labor inputs and firm's outputs I do not study a symmetric equilibrium. Instead, I look at firm specific labor inputs and outputs, as in [22, King/Wolman (1999)].

The paper is organized as follows: Section 2 describes in detail the different models and the calibration. In section 3 impulse responses are discussed for the CIA- and the MIU-model. Section 4 concludes and gives some suggestions for future research.

2 The Models

2.1 The Household

The representative household is assumed to have preferences over consumption (c_t) and leisure ($1 - n_t$). I consider two different sets of functions under two different setups. In the one setup, CIA-models are considered while in the other MIU-models are evaluated. Both will be calculated through for special utility functions. Since they differ for the setups they will be discussed separately below. The first momentary utility function considered under CIA is the one used by [22, King/Wolman (1999)] and is given by

$$u\left(c_t, n_t, a_t\right) = \frac{\left[c_t - \frac{a_t \theta}{1+\gamma} n_t^{1+\gamma}\right]^{1-\sigma} - 1}{1-\sigma} \tag{1}$$

Here a_t is a preference shock that also acts like a productivity shock. θ and γ are positive parameters, σ governs the degree of risk aversion. This function is familiar from the analysis of [14, Greenwood/Hercowitz/Huffman (1988)] and accordingly labeled GHH preferences. It has the special property that hours worked only depend upon the real wage and not upon consumption (no wealth effects).

The second utility function analyzed under CIA is the standard constant relative risk aversion function (CRRA) used in many Real Business Cycle models. ζ measures the relative weight of consumption for the representative agent.

$$u\left(c_t, n_t, a_t\right) = \frac{\left[a_t c_t^{\zeta}\left(1-n_t\right)^{1-\zeta}\right]^{1-\sigma}-1}{1-\sigma} \tag{2}$$

It should be noted that in contrast to the standard use of this utility function there is a disturbance a_t acting like a preference shock.[1]

Under a MIU-specification the corresponding GHH function to (1) is given by

$$u\left(c_t, \frac{M_t}{P_t}, n_t, a_t\right) = \frac{\left[\left(\eta c_t^{\nu}+(1-\eta)\left(\frac{M_t}{P_t}\right)^{\nu}\right)^{\frac{1}{\nu}}-\frac{a_t\theta}{1+\gamma}n_t^{1+\gamma}\right]^{1-\sigma}-1}{1-\sigma} \tag{3}$$

The MIU-specification was - among others - proposed by [24, Sidrauski (1967)]. Consumers are supposed to have preferences over real money balances M_t/P_t since they facilitate transactions. They are introduced using a CES function together with consumption. This expression replaces the consumption term in (1). η is a share parameter and ν will be shown to determine the interest elasticity of the implied money demand function. In case of CRRA preferences the specification in the CES form is embedded in a Cobb-Douglas structure with labor where ζ again acts as a weighting parameter.

$$u\left(c_t, \frac{M_t}{P_t}, n_t, a_t\right) = \frac{\left[a_t\left(\eta c_t^{\nu}+(1-\eta)\left(\frac{M_t}{P_t}\right)^{\nu}\right)^{\frac{\zeta}{\nu}}\left(1-n_t\right)^{1-\zeta}\right]^{1-\sigma}-1}{1-\sigma} \tag{4}$$

Note that for $\nu = \eta = 1$ both specifications collapse to their CIA-counterparts. The nonseparability allows to consider the influence of the money demand distortions on the dynamic evolution of consumption and labor because the variables will influence each other as cross derivatives will be non zero.

The intertemporal optimization problem for the household is to maximize lifetime utility subject to an intertemporal budget constraint. In the case of

[1] [22, King/Wolman (1999)] argue that it is necessary in (1) to have a_t affecting equally production and preferences in order to achieve balanced growth. This is doubtful because the model does not explicitly account for growth aspects as, e.g., in [18, King/Plosser/Rebelo (1988)].

utility function (1) and (2) it also faces a CIA-constraint. The household is assumed to have access to a bond market and to hold money. Its budget constraint is therefore given by

$$P_t c_t + M_t + B_t = P_t w_t n_t + M_{t-1} + (1 + R_{t-1}) B_{t-1} + M_t^s \tag{5}$$

The uses of wealth are nominal consumption $P_t c_t$, holdings of money balances M_t and bonds B_t. The household has several sources of its wealth. It earns money working in the market at the real wage rate w_t $(P_t w_t n_t)$ and can spend its money holdings carried over from the previous period (M_{t-1}). There are also previous period bond holdings including the interest on them $(1 + R_{t-1})(B_{t-1})$. Finally the household receives a monetary transfer M_t^s from the government or the monetary authority, respectively.[2] This transfer is equal to the change in money balances, i.e.

$$M_t^s = M_t - M_{t-1} \tag{6}$$

For utility functions (1) and (2) the household faces a CIA-constraint. It can consume only out of cash balances it has received before. This condition is therefore given by[3]

$$P_t c_t \leq M_{t-1} + M_t^s \tag{7}$$

The Lagrangian for the household in case of utility function (1) and (2) (index $H1$) (CIA-model) can then be written as follows:

$$\begin{aligned} L_{H1} &= E_0 \Bigg[\sum_{t=0}^{\infty} \beta^t u(c_t, n_t, a_t) \\ &+ \sum_{t=0}^{\infty} \beta^t \lambda_t \Bigg(w_t n_t + m_{t-1} \frac{P_{t-1}}{P_t} + m_t^s \\ &\quad + (1 + R_{t-1}) b_{t-1} \frac{P_{t-1}}{P_t} - c_t - m_t - b_t \Bigg) \\ &+ \sum_{t=0}^{\infty} \beta^t \Omega_t \left(m_{t-1} \frac{P_{t-1}}{P_t} + m_t^s - c_t \right) \Bigg] \end{aligned} \tag{8}$$

[2] The household also receives profits from the intermediate goods firms. Since these profits will be zero in the equilibrium they are not explicitly included in the budget constraint here.

[3] The formulation of the CIA-constraint, the monetary transfer and the intertemporal budget constraint is consistent with the timing in [26, Walsh (1998)], pp. 100-102.

Here small variables indicate real quantities, i.e. for example $m_t = M_t/P_t$. Households optimize over c_t, n_t, m_t and b_t taking prices and the initial values of the price level P_0 as well as the outstanding stocks of money M_0 and bonds B_0 as given. The first order conditions for an interior solution are reported below.

$$\frac{\partial L_{H1}}{\partial c_t} = \beta^t \frac{\partial u(c_t, n_t, a_t)}{\partial c_t} - \beta^t \lambda_t - \beta^t \Omega_t = 0 \tag{9}$$

$$\frac{\partial L_{H1}}{\partial n_t} = \beta^t \frac{\partial u(c_t, n_t, a_t)}{\partial n_t} + \beta^t \lambda_t w_t = 0 \tag{10}$$

$$\frac{\partial L_{H1}}{\partial m_t} = -\beta^t \lambda_t + E_t \beta^{t+1} \lambda_{t+1} \frac{P_t}{P_{t+1}} + E_t \beta^{t+1} \Omega_{t+1} \frac{P_t}{P_{t+1}} = 0 \tag{11}$$

$$\frac{\partial L_{H1}}{\partial b_t} = -\beta^t \lambda_t + E_t \beta^{t+1} \lambda_{t+1} (1 + R_t) \frac{P_t}{P_{t+1}} = 0 \tag{12}$$

The derivatives with respect to λ_t and Ω_t are omitted since they are equal to the budget constraint and the CIA-constraint, respectively. It should be noted that these conditions result from the more general Kuhn-Tucker conditions assuming that all variables and multipliers are strictly positive. This implies especially that - given $\Omega_t > 0$ - the CIA-constraint is always binding and that the nominal interest rate R_t is positive. Otherwise (11) and (12) will not be compatible. In addition the household's optimal choices must also satisfy the transversality conditions:

$$\lim_{t \to \infty} \beta^t \lambda_t x_t = 0 \qquad \text{for } x = m, b \tag{13}$$

The familiar result that the first two efficiency conditions imply the equality of the marginal rate of substitution between consumption and labor and the real wage does not hold here because of the CIA-constraint. Instead one gets

$$w_t = -\frac{1}{\beta} E_t \left(\frac{\frac{\partial u(c_t, n_t, a_t)}{\partial n_t}}{\frac{\partial u(c_{t+1}, n_{t+1}, a_{t+1})}{\partial c_{t+1}}} \frac{P_{t+1}}{P_t} \right) \tag{14}$$

This equation can be derived by eliminating Ω_t in the efficiency condition for consumption using the efficiency condition for money. There is a different

timing of the marginal utility of consumption and labor which alters the dynamics of the real wage. In addition there is a direct influence of inflation. The marginal utility of consumption is given by $(1 + R_{t-1})\,\lambda_t$ so that the nominal interest rate acts like a tax on consumption.
The efficiency condition for bond holdings establishes a relation between the nominal interest rate and the price level. Rearranging terms yields

$$(1 + R_t) = E_t \left(\frac{\lambda_t}{\lambda_{t+1}} \frac{1}{\beta} \frac{P_{t+1}}{P_t} \right) \tag{15}$$

Supposed the Fisher equation is valid the real interest rate r_t is implicitly defined as

$$(1 + r_t) = E_t \left(\frac{\lambda_t}{\lambda_{t+1}} \frac{1}{\beta} \right) \tag{16}$$

because P_{t+1}/P_t equals one plus the rate of expected inflation which is approximated by the ex-post-inflation rate.

In case of the MIU-model the CIA-constraint is dropped since money demand will be determined endogenously through the derivative with respect to m_t. In this case m_t shows up in the utility function, of course. So the Lagrangian (index $H2$) will be given by

$$\begin{aligned} L_{H2} &= E_0 \Bigg[\sum_{t=0}^{\infty} \beta^t u\left(c_t, m_t, n_t, a_t\right) \\ &+ \sum_{t=0}^{\infty} \beta^t \lambda_t \Bigg(w_t n_t + m_{t-1} \frac{P_{t-1}}{P_t} + m_t^s \\ &+ \left(1 + R_{t-1}\right) b_{t-1} \frac{P_{t-1}}{P_t} - c_t - m_t - b_t \Bigg) \Bigg] \end{aligned} \tag{17}$$

In order to compare both setups the first order conditions are again reported.

$$\frac{\partial L_{H2}}{\partial c_t} = \beta^t \frac{\partial u\left(c_t, m_t, n_t, a_t\right)}{\partial c_t} - \beta^t \lambda_t = 0 \tag{18}$$

$$\frac{\partial L_{H2}}{\partial n_t} = \beta^t \frac{\partial u\left(c_t, m_t, n_t, a_t\right)}{\partial n_t} + \beta^t \lambda_t w_t = 0 \tag{19}$$

$$\frac{\partial L_{H2}}{\partial m_t} = \beta^t \frac{\partial u\left(c_t, m_t, n_t, a_t\right)}{\partial m_t} - \beta^t \lambda_t + E_t \beta^{t+1} \lambda_{t+1} \frac{P_t}{P_{t+1}} = 0 \tag{20}$$

$$\frac{\partial L_{H2}}{\partial b_t} = -\beta^t \lambda_t + E_t \beta^{t+1} \lambda_{t+1} \left(1 + R_t\right) \frac{P_t}{P_{t+1}} = 0 \tag{21}$$

The derivatives with respect to n_t and b_t are essentially the same as for $H1$. As before, P_0, M_0 and B_0 are given and the transversality conditions hold. In the consumption Euler equation the influence of the second Lagrange multiplier Ω_t disappears eliminating the dynamics present in the CIA-model. Now the marginal utility of consumption is just equal to the shadow price λ_t, there is no consumption tax working through the nominal interest rate. But in the efficiency condition for money the marginal utility of real balances has to be considered. This derivative determines the endogenous money demand function. Combining the optimum conditions for consumption, bonds and money yields the following equation:

$$\frac{\partial u\left(c_t, m_t, n_t, a_t\right)}{\partial m_t} = \frac{\partial u\left(c_t, m_t, n_t, a_t\right)}{\partial c_t} \frac{R_t}{1 + R_t} \tag{22}$$

This specification allows to estimate an empirical money demand function. A detailed description will be presented in the calibration section. The Taylor approximations are given in Appendix A.

Two important implications come out right here. First, the real wage rate will be determined by the usual marginal rate of substitution between consumption and labor, in contrast to the additional dynamics in the CIA-model (see (14)). Second, the implied money demand function is independent of the specific form of the monetary transfer M_t^s and, in addition, it depends directly upon the nominal interest rate (see (22)).

2.2 The Finished Goods Producing Firm

The firm producing the final good $c_t = y_t$ in the economy uses $c_{j,t}$ units of each intermediate good $j \in [0, 1]$ purchased at price $P_{j,t}$ to produce c_t units of the finished good. The production function is assumed to be a CES aggregator as in [8, Dixit/Stiglitz (1977)] with $\epsilon > 1$.

$$c_t = \left(\int_0^1 c_{j,t}^{(\epsilon-1)/\epsilon} dj \right)^{\epsilon/(\epsilon-1)} \tag{23}$$

The firm maximizes its profits over $c_{j,t}$ given the above production function and given the price P_t. So the problem can be written as

$$\max_{c_{j,t}} \left[P_t c_t - \int_0^1 P_{j,t} c_{j,t} dj \right] \text{ s.t. } \quad c_t = \left(\int_0^1 c_{j,t}^{(\epsilon-1)/\epsilon} dj \right)^{\epsilon/(\epsilon-1)} \tag{24}$$

The first order conditions for each good j imply

$$c_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\epsilon} c_t \tag{25}$$

where $-\epsilon$ measures the constant price elasticity of demand for each good j. Since the firm operates under perfect competition it does not make any profits. Inserting the demand function into the profit function and imposing the zero profit condition reveals that the only price P_t that is consistent with this requirement is given by

$$P_t = \left(\int_0^1 P_{j,t}^{(1-\epsilon)} dj \right)^{1/(1-\epsilon)} \tag{26}$$

In case that prices are fixed for just two periods and assuming that all price adjusting producers in a given period choose the same price the consumption aggregate can be written as

$$c_t = c\left(c_{0,t}, c_{1,t}\right) = \left(\frac{1}{2} c_{0,t}^{(\epsilon-1)/\epsilon} + \frac{1}{2} c_{1,t}^{(\epsilon-1)/\epsilon} \right)^{\epsilon/(\epsilon-1)} \tag{27}$$

where $c_{j,t}$ can then be interpreted as the quantity of a good consumed in period t whose price was set in period $t-j$. Similarly in the two period price setting case to be explored in detail in the next section the price equation simplifies. With prices set for two periods half of the firms adjust their price in period t and half do not. Moreover all adjusting firms choose the same price. Then $P_{j,t}$ is the nominal price at time t of any good whose price was set j periods ago and P_t is the price index at time t and is given by

$$P_t = \left(\frac{1}{2} P_{0,t}^{1-\epsilon} + \frac{1}{2} P_{1,t}^{1-\epsilon} \right)^{1/(1-\epsilon)} \tag{28}$$

2.3 The Intermediate Goods Producing Firm

Intermediate good firms produce with a technology that is linear in labor $n_{j,t}$ and subject to random productivity shocks a_t.

$$y_{j,t} = c_{j,t} = a_t n_{j,t} \tag{29}$$

Here $n_{j,t}$ is the labor input employed in period t by a firm who set the price in period $t - j$. Firms always meet the demand for their product, that is $y_{j,t} = c_{j,t}$. Those who do not adjust their prices in a given period can be interpreted as passive while those who do adjust do so optimally.

Firms set their prices to maximize the present discounted value of their profits.[4] Real marginal costs are given by $\psi_t = w_t / a_t$.[5] With a relative price defined by $p_{j,t} = P_{j,t}/P_t$ real profit $z_{j,t}$ for a firm of type j is equal to

$$z_{j,t} = p_{j,t} c_{j,t} - w_t n_{j,t} \tag{30}$$

Using the demand function for the intermediate goods $\left(c_{j,t} = p_{j,t}^{-\epsilon} c_t = a_t n_{j,t}\right)$ and the definition of real marginal costs given above the profit function can be rewritten as

$$z_{j,t} = z\left(p_{j,t}, c_t, \psi_t\right) = p_{j,t} c_{j,t} - \psi_t c_{j,t} = c_{j,t}\left(p_{j,t} - \psi_t\right) = p_{j,t}^{-\epsilon} c_t \left(p_{j,t} - \psi_t\right) \tag{31}$$

When prices are fixed for two periods the firm has to take into account the effect of the price chosen in period t on current and future profits. The price in period $t+1$ will be affected by the gross inflation rate Π_{t+1} between t and $t+1$ ($\Pi_{t+1} = P_{t+1}/P_t$).

$$p_{1,t+1} = \frac{p_{0,t}}{\Pi_{t+1}} \tag{32}$$

The optimal relative price has to balance the effects due to inflation between profits today and tomorrow. This intertemporal maximization problem is formally given by

$$\begin{aligned} &\max_{p_{0,t}} E_t \left[z\left(p_{0,t}, c_t, \psi_t\right) + \beta \frac{\lambda_{t+1}}{\lambda_t} z\left(p_{1,t+1}, c_{t+1}, \psi_{t+1}\right) \right] \\ &\text{s.t.} \quad p_{1,t+1} = \frac{p_{0,t}}{\Pi_{t+1}} \end{aligned} \tag{33}$$

[4] The model deviates in this respect from the standard textbook model in which profits are maximized over the quantity.

[5] It should be noticed that the wage is perfectly flexible in a competitive input market. So there is no index j for w_t and P_t which means that they are not firm-specific.

The term λ_{t+1}/λ_t is equal to the ratio of future to current marginal utility of labor and the respective real wage ratio (derived in the household's optimization problem) and considered to be - in conjunction with β - the appropriate discount factor for real profits. This is a consequence of the assumption that households own the production factor labor and rent it to the firms. They also own a diversified portfolio of claims to the profits earned by the firms. Although there will be no asset accumulation in equilibrium λ_t can be used to determine the present value of profits.[6] Solving the efficiency condition for the optimal price to be set in period t yields a forward-looking form of the price equation and is in that respect similar to the one in [25, Taylor (1980)].

$$p_{0,t} = \frac{\epsilon}{\epsilon - 1} \frac{\lambda_t c_t \psi_t + \beta E_t \lambda_{t+1} \left(P_{t+1}/P_t \right)^{\epsilon} c_{t+1} \psi_{t+1}}{\lambda_t c_t + \beta E_t \lambda_{t+1} \left(P_{t+1}/P_t \right)^{\epsilon - 1} c_{t+1}} \tag{34}$$

The optimal relative price $p_{0,t}$ depends upon the current and future real marginal costs, the gross inflation rate, current and future consumption as well as today's and tomorrow's interest rates (through the influence of the λ-terms). (34) can be manipulated in a way that yields a form which is exactly equal to the one studied in [26, Walsh (1998)], p. 197, when using (15) for the interest rate factor. To derive the Taylor approximation in the Appendix it is useful to write (34) as

$$P_{0,t} = \frac{\epsilon}{\epsilon - 1} \frac{\lambda_t P_t^{\epsilon} c_t \psi_t + \beta E_t \lambda_{t+1} P_{t+1}^{\epsilon} c_{t+1} \psi_{t+1}}{\lambda_t P_t^{\epsilon - 1} c_t + \beta E_t \lambda_{t+1} P_{t+1}^{\epsilon - 1} c_{t+1}} \tag{35}$$

Finally, aggregate labor demand must be equal to the aggregate labor supply of the household.[7]

$$n_t = \frac{1}{2} n_{0,t} + \frac{1}{2} n_{1,t} \tag{36}$$

2.4 Market Clearing Conditions and Other Equations

It is well known that models like the one at hand imply multiple equilibria and sunspots because bonds are not determined. To escape this problem the household budget constraint is dropped and bonds are set to zero: $b_t =$

[6]More details on this can be found in [11, Dotsey/King/Wolman (1999)], p. 659-665 as well as in [10, Dotsey/King/Wolman (1997)], p. 9-13.

[7]The factor 0.5 shows up because $n_{j,t}$ is labor hired per j-type firm and half the firms are of each type.

0 for all t.[8] Note that due to Walras' law the intertemporal budget constraint will also hold in equilibrium.
In the CIA-model the implicit money demand function is derived by substituting M_t^s in the CIA-constraint - holding with equality. This implies:

$$M_t = P_t c_t \tag{37}$$

It is essentially a quantity theoretic type of money demand. It is important to stress that it depends crucially upon the form of the monetary transfer M_t^s. [3, Carlstrom/Fuerst (2001)] include bond holdings in their CIA-constraint. Using this specification, including bond holdings also in M_t^s, leads to multiple equilibria.
In the MIU-model the efficiency condition for money determines the money demand function, of course (see the discussion of (22)).

The markup μ_t is just the reciprocal of real marginal cost so that

$$\mu_t = \frac{1}{\psi_t} \tag{38}$$

2.5 The Monetary Authority

The model is closed by adding a monetary policy rule. Therefore an exogenous process for the money growth rate is considered. To achieve persistent but non permanent effects the level of money follows an AR(2)-process. Assume that money grows at a factor g_t:

$$M_t = g_t M_{t-1} \tag{39}$$

If $\widehat{g}_t$ follows an AR(1)-process $\widehat{g}_t = \rho_g \widehat{g}_{t-1} + \epsilon_{g_t}$ then money will follow an AR(2)-process.[9] Note that inflation is zero at the steady state so also money growth is zero there ($g = 1$).

There is another shock in the model, namely the productivity shock a_t. As is clear from the utility functions this shock can also act as a taste shock. So one can easily analyze the model's impulse responses to this productivity/ taste shock. Under these circumstances $\widehat{a}_t$ follows an AR(1)-process

$$\widehat{a}_t = \rho_a \widehat{a}_{t-1} + \epsilon_{a_t} \tag{40}$$

with ϵ_{a_t} white noise and $0 < \rho_a < 1$.

[8] See [12, Flodén (2000)], p. 1413. He argues that bonds are introduced to determine the nominal interest rate.

[9] A hat ($\widehat{\ }$) represents the relative deviation of the respective variable from its steady state (see the Appendix). ρ_g lies between 0 and 1 and ϵ_{g_t} is white noise.

2.6 Calibration

To compute impulse responses the parameters of the model have to be calibrated. Some parameters depend upon the specific utility function used so it is useful to look at first at the parameters which are independent of these.

It is possible to either specify β or r exogenously. Here β will be set to 0.99 implying a value of r of about 0.0101 per quarter which is in line with other values used for the real interest rate in the literature. ψ and μ can be determined by fixing a value for the elasticity of the demand functions for the differentiated products. This elasticity being equal to 6 causes the static markup $\mu = \epsilon/(\epsilon - 1)$ to be 1.2 which is the mean value found in the study of [23, Linnemann (1999)] about average markups. In order to determine the steady state real wage w the productivity shock a has to be specified. As there is no information available about that parameter it is arbitrarily set at 10.[10] Either n or c have to be set exogenously to calculate $c = an$. Because more information is available about hours worked, n is specified to be equal to 0.25 implying that agents work 25 % of their non-sleeping time.

In the benchmark case, σ, the parameter governing the degree of risk aversion, is set to 2 in all models. For GHH preferences γ has to be specified. To make results comparable to the CRRA utility function γ is set to $1.\bar{3}$ which implies the same static steady state elasticity of labor supply with respect to the real wage. In the sensitivity analysis the value will be changed to 0.1. The implied value of θ under CIA is 5.2384.

Using the CRRA preference specification under CIA the parameter ζ can be calculated using equation (14) which implies $\zeta = 0.2878$, a value that is reasonably in line with other studies.

In the MIU-model, both for CRRA and GHH preferences, the parameters ν and η are calibrated by estimating an empirical money demand function the form of which is implied by the efficiency conditions of the household. This functional form is obtained by solving (22) for m_t and taking logarithms:

$$\ln m_t = \frac{1}{\nu - 1} \ln \frac{\eta}{1 - \eta} + \frac{1}{\nu - 1} \ln \left(\frac{R_t}{1 + R_t} \right) + \ln c_t \tag{41}$$

Estimates of [5, Chari/Kehoe/McGrattan (2000)] reveal that $\eta = 0.94$ and $\nu = -1.56$. They use US data from Citibase covering 1960:1-1995:4 regressing

[10] In contrast to the well known basic neoclassical model of [18, King/Plosser/Rebelo (1988)] there is no escape from specifying parameters such as a at the steady state. The system cannot be reduced until only deep parameters remain to be calibrated.

the log of consumption velocity on the log of the interest rate variable $R_t/(1+R_t)$. Since the focus is on the qualitative results of the model the money demand function is not estimated for specific German or other data. For CRRA utility the implied value of ζ changes slightly to 0.2899 while m/c is equal to 2.06. Under GHH preferences $\theta = 5.3240$.

For the exogenous money growth process $\rho_g = 0.5$ is used. As the focus of the paper is on persistence of money shocks productivity shocks will not be considered. But they can be used to check whether the model displays reasonable impulse responses to technology shocks.

3 Impulse Response Functions

The solution is conducted using an extended version of the algorithm of [19, King/Plosser/Rebelo (2002)] which allows for singularities in the system matrix of the reduced model. The theoretical background of this algorithm is developed in [20, King/Watson (1999)] whereas computational aspects and the implementation are discussed in [21, King/Watson (2002)].

3.1 CIA-Model

Because results differ it is useful to subdivide this subsection in two further sections containing results for the GHH preferences and for the standard CRRA utility function.

3.1.1 GHH Preferences

Here the impulse responses of the model variables to a 1% shock to the money growth rate will be discussed. Figures 1-2 display the reaction of selected variables to this shock in the benchmark calibration. The reaction of $\widehat{c}_{0,t}$ and of the prices are the most persistent ones of the variables under observation. Real marginal costs as well as consumption of non-adjusting firms show a cyclical reaction which is counterfactual. Aggregate consumption rises on impact and falls immediately below the steady state in the next period. There is some persistence after the initial positive impact, beginning in the second quarter. Unfortunately the persistence consists of a tendency of aggregate consumption to remain below its steady state level for several successive periods. This is a feature not empirically observed either. Real marginal

costs display a strong increase which amounts to a quite strong rise in the price firms set when they are allowed to do so. But it takes some 7 or 8 periods for the price level to reach the new equilibrium value so one can conclude that prices show at least some persistence. Inflation shows a hump as it does empirically. The decline in the real interest rate is more than three times the rise in the nominal rate. As for many DGE models with sticky prices also this one fails to generate the liquidity effect (a falling nominal interest rate). But the nominal rate reacts quite persistently.
In the literature several authors argue in favor of models generating flat marginal cost curves because then there is little incentive for firms to raise prices. In turn money growth shocks can have persistent effects on output. In case of the GHH utility function the static steady state elasticity of real marginal cost with respect to output is constant and equal to γ.

$$\frac{\partial\psi}{\partial c}\frac{c}{\psi} = \gamma \tag{42}$$

In the benchmark case γ was calibrated to be $1.\bar{3}$. Changing this value to 0.1 would considerably reduce this elasticity and would probably enhance the persistence effects of money growth shocks in the model. But a low value for this elasticity implies at the same time a high static steady state elasticity of labor supply with respect to the real wage. Formally this elasticity is given by

$$\frac{\partial n}{\partial w}\frac{w}{n} = \frac{1}{\gamma} \tag{43}$$

and it is equal to 10 here. In light of empirical estimates of the labor supply elasticity this value must be regarded as too high. But in spite of this implication there is not much more persistence in the aggregate consumption reaction (see Figures 3-4). There is a smoother reaction but again consumption is cyclical approaching the new steady state from below.[11] $\widehat{c}_{0,t}$ displays considerably more persistence than before. This is also true for real marginal costs $\widehat{\psi}_t$ but they react stronger than 0.1% as could have been expected due to the low output elasticity. Note that the price level overshoots its new

[11]Although there is now persistence in the optimal price of the firms ($\widehat{P}_{0,t} = 0.7870\widehat{P}_{0,t-1} + 1.5060\widehat{M}_t - 1.2930\widehat{M}_{t-1}$) the response of the nominal interest rate does not change by much. This seems to be at the heart of a solution to the problem.

equilibrium value of 2 quite strongly, contrasting the result in Figure 2 for a higher value of γ.

The reason why even the variant of the model with a low elasticity of real marginal costs with respect to output fails to generate a persistent output reaction seems to be the implied money demand function, which is essentially of a quantity theoretic type here. Real marginal costs are more reactive to a money growth shock because of the additional dynamics in the model which work through Ω_t, the shadow price of the monetary transfer, in the efficiency condition for consumption. As ψ_t is proportional to the real wage w_t one can evaluate the reaction of real marginal costs using w_t. Rewriting (14) using (15) yields

$$w_t = -E_t \left[\frac{\frac{\partial u(c_t, n_t, a_t)}{\partial n_t}}{\frac{\partial u(c_{t+1}, n_{t+1}, a_{t+1})}{\partial c_{t+1}}} \frac{\lambda_{t+1}}{\lambda_t} (1 + R_t) \right] \tag{44}$$

An intuition for the result could be the following.[12] An expansionary money growth shock leads to an increase in real aggregate demand since prices are sticky so that firms have to hire more workers, so that n goes up, resulting in a decrease in $\partial u/\partial n$. Consumption rises leading to a fall in $\partial u/\partial c$. But this decline gets only effective in the next period (see the timing of this marginal utility above). Overall this causes w and ψ to rise.[13] As the nominal interest rate also rises the wage rate will even rise more. But λ_t and λ_{t+1} will fall dampening the other effects. In sum, ψ_t will react quite strong so that the effect of the declining shadow price of wealth λ will be dominated by the other factors.

Before exploring this preference specification in the MIU-model let's turn to the CRRA utility function first.

3.1.2 CRRA Preferences

Figures 5-6 summarize the impulse responses in the model with CRRA preferences (see(2)). At first glance these graphs seem to be very similar to

[12] It must be kept in mind that the argument made here is very tentative since it refers to (44) using some kind of comparative static analysis. Since the efficiency conditions are linearized around the steady state there are many more complicated mechanisms at work. Remember that agents are optimizing taking care of all state and exogenous variables. A correct analysis would look at the model solution which is given in linearized form around the steady state (see the previous footnote for an example).

[13] Note that $\partial u/\partial n$ is negative.

those under GHH preferences. But there are some small interesting differences. First, there is a reduced cyclicality of the real interest rate and real marginal costs. Nevertheless aggregate consumption rises only on impact and approaches the steady state from below. Second, the reaction of $\widehat{c}_{0,t}$ is smoother showing no kink as under GHH utility. The same holds for prices and inflation (compare Figures 5 and 1 as well as 6 and 2).

This is an interesting result pointing out the role played by the utility function. For the CRRA utility function the static steady state elasticity of labor supply with respect to the real wage rate depends only on the value of hours worked at the steady state, n.[14]

$$\frac{\partial n}{\partial w}\frac{w}{n} = 1 - n \tag{45}$$

This implies a value of 0.75 which is the same as in case of benchmark GHH preferences. Similarly the elasticity of real marginal cost with respect to output can be shown to be given by

$$\frac{\partial \psi}{\partial c}\frac{c}{\psi} = \frac{1}{1-n} \tag{46}$$

which is equal to $1.\bar{3}$ in the stationary equilibrium and equal to γ under GHH. Now in the CIA-setup this leads to overall a bit more persistent reactions under CRRA preferences than under GHH utility. Obviously it makes a difference which type of utility function is used in DGE models with sticky prices. Preferences thus are at least partly responsible for the degree of persistence.[15]

3.2 MIU-Model

Similar to the CIA-case results differ in the MIU-model so there will be two subsections to treat each utility function separately.

3.2.1 GHH Preferences

Figures 7-8 visualize the impulse responses for the MIU-model with GHH preferences in the benchmark case. A first inspection of the impulses reveals

[14] It is important to consider this elasticity at the steady state where $c = an$.

[15] Looking at the model solution reveals that for CRRA preferences there is a negative impact of firms' optimal price on the nominal interest rate while it is positive under GHH preferences.

that now all variables but the nominal interest behave cyclical: a positive (negative) reaction is followed by an immediate negative (positive) one which reverts to positive (negative) behavior again. This is certainly counterfactual and not observed empirically. A second important result is the complete absence of persistence in the reactions of the variables, with the exception of the nominal interest rate which rises persistently. Third, price adjusting firms react very strongly in the first period so that the price level overshoots considerably. Even the behavior of prices shows no persistence at all. Forth, real money balances decline on impact and then approach the steady state from below, a reaction which is also not observed empirically. A very low value of the risk aversion parameter σ creates extremely cyclical impulse responses with humps and dips for several periods. On the other hand high values of σ dampen the peaks and troughs.[16]

Obviously it makes a big difference how money is introduced in DGE models. Since the benchmark models have been calibrated the same way the absence of persistence must be due to the implied money demand function. So it can be concluded that in a MIU-model where money demand is interest rate sensitive persistent reactions to money growth shocks cannot be explained. An implied quantity theoretic type of money demand seems to be a more appropriate formulation if the aim is to achieve persistent output reactions in a sticky price model. Is there some intuition for this in the efficiency condition for the real wage? The corresponding equation to (44) can be written as follows:

$$w_t = -\frac{\frac{\partial u(c_t,m_t,n_t,a_t)}{\partial n_t}}{\frac{\partial u(c_t,m_t,n_t,a_t)}{\partial c_t}} = -\frac{\frac{\partial u(c_t,m_t,n_t,a_t)}{\partial n_t}}{\frac{\partial u(c_t,m_t,n_t,a_t)}{\partial m_t}} \frac{R_t}{1+R_t} \tag{47}$$

The second equality takes into account the combined efficiency conditions for consumption, bonds and money (see (22)) which implies the interest rate sensitive money demand function in the MIU-model. As there is now no delayed effect of the rise of consumption on the fall in the marginal utility of consumption the marginal rate of substitution between consumption an labor rises immediately. This rise is strenghtened by the rise in the nominal interest rate R_t. Moreover there is no dampening effect due to λ_t. So probably the rise in w_t and thus in ψ_t is much stronger than in the CIA-setup which leads price adjusting firms to increase their price $P_{0,t}$ very strongly. Maybe this

[16]This is not shown in the Figures. Results are available from the author upon request.

can explain the overshooting behavior one can see in Figure 8.[17]

But can GHH preferences with a low value for the elasticity of real marginal costs with respect to output generate more persistent reactions than in a CIA-setup? The results of the experiment are shown in Figures 9-10. Surprisingly, now all variables display very strong persistence after a money growth shock. Results are completely different to the CIA-outcome. Intermediate as well as aggregate consumption react strongly and stay above (or below) the steady state value for more than 8 quarters after the shock. Real marginal costs are flat, showing only a 0.12% deviation from the equilibrium value. Real money balances rise all the time, due to the smooth and moderate price level increase. Intermediate goods firms raise their prices accordingly very slowly. Inflation displays a hump as observed empirically. Unfortunately the nominal interest rate counterfactually rises again. Thus, just changing from a CIA-setup to a MIU-model leads to completely different model outcomes. A low marginal cost elasticity is obviously not enough to generate persistence in output. It must be combined with an interest rate sensitive money demand function which is implied by a MIU-model.
This result is very surprising since one would expect from the discussion above that the results would even be worse compared to the CIA-model. There are obviously some other mechanisms at work which make firms' real marginal costs rise only slightly. Note that in Figure 9 the nominal rate rises very little so that this effect is indeed very small in generating upward pressure upon the wage rate. However it is difficult to reveal the reasons for this behavior. All that can be concluded is that is has something to do with the money demand function that depends on the nominal interest rate here.[18]

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[17]A look at the model solution reveals that the main difference between the MIU- and the CIA-setup for GHH preferences is the process for the nominal interest rate. In the MIU-model there is a negative impact of the firms' optimal price while it is positive in the CIA-setup. This causes the kink in the impulse response of $\widehat{R}_t$. So obviously the behavior of the interest not only depends on the utility function but also on the way money is introduced (see footnote 15).

[18]Technically the most important difference between these two versions is the reaction of price adjusting firms' consumption (output) $\widehat{c}_{0,t}$ to their optimal price: it is positive in the MIU-model while negative in the CIA-setup. In addition, and maybe most important, the nominal interest rate reacts very weakly both to the optimal price and to the money growth shock.

3.2.2 CRRA Preferences

Finally Figures 11-12 show the results for the MIU-model with CRRA preferences. Compared to the GHH version the outcome does not differ very much. But as in the CIA-setup there are some small differences. First, the reactions are all weaker than under GHH preferences. Second, the strength of the cyclical behavior is less, i.e. the dips and humps are smaller in size. Lowering the value of σ leads to more pronounced dips and humps whereas a higher risk aversion makes them smaller.[19]

Again, the MIU-model version generates considerably less persistent reactions than the CIA-setup. This is especially the case for $\widehat{c}_{0,t}$ as well as the prices. As the models are again calibrated the same way the loss of persistence is due to the different implied money demand functions.[20] This leads to the conclusion that two conditions have to be fulfilled in order to enable a DGE model with sticky prices alone to generate persistent output and inflation responses: first, the static steady state elasticity of labor supply with respect to the real wage must be high, and second, the money demand function has to be interest rate sensitive. Only one of these ingredients is not enough to generate persistence. This refines results in the literature, for example of [1, Ascari (2003)] who only looks at MIU-specifications and concludes that a high labor supply elasticity plays the most important role for persistent output reactions in a price staggering model. Similarly [5, Chari/Kehoe/McGrattan (2000)] study a MIU-model and investigate a similar utility function to the GHH specification in their sensitivity analysis. They also point out only the role of a high labor supply elasticity for a persistent output reaction.

4 Conclusions

In light of the main question of the paper it must be concluded that persistent reactions of output and inflation to money growth shocks can only be explained in a MIU-model with GHH preferences and a high labor supply elasticity. All other economies considered fall short of reaching persistence.

An interesting future direction of research is to look at models that are

[19] These Figures are again not shown. Results are available from the author upon request.

[20] Surprisingly now there is a negative response of $\widehat{R}_t$ on the firms' optimal price under GHH, just the reverse of the result in a CIA-setup (see footnote 15).

generalized to include capital accumulation considerations. Results of [5, Chari/Kehoe/McGrattan (2000)] are very discouraging. They find almost no persistence in models with capital. It would be interesting to see how the results change in a CIA-model.

Another promising line of research is to analyze open economy models. Recently [13, Ghironi (2002)] has shown that once openness is taken into account a sticky price model can generate endogenous output persistence.[21] This depends crucially on incomplete asset markets. It would be interesting to generalize the model at hand to such a framework.

A Appendix

A.1 Household's Equations: CIA-Model

The efficiency condition for aggregate consumption results in

$$\begin{aligned} & -D_1 u(c,n,a)\widehat{P}_{t+1} + nD_{12}u(c,n,a)\widehat{n}_{t+1} + cD_{11}u(c,n,a)\widehat{c}_{t+1} \\ = \; & D_1 u(c,n,a)\widehat{\lambda}_t - D_1 u(c,n,a)\widehat{P}_t - aD_{13}u(c,n,a)\widehat{a}_{t+1} \end{aligned} \tag{48}$$

using Ω_t from the derivative with respect to m_t.

A hat ($\widehat{\ }$) represents the relative deviation of the respective variable from its steady state ($\widehat{a}_t = (a_t - a)/a$). $D_i u(\cdot)$ denotes the first partial derivative of the u-function with respect to the i-th argument. Similarly $D_{ij}u(\cdot)$ denotes the partial derivative of $D_i u(\cdot)$ with respect to the j-th argument, all evaluated at the steady state. For aggregate labor one gets

$$\begin{aligned} 0 = \; & nD_{22}u(c,n,a)\widehat{n}_t + cD_{21}u(c,n,a)\widehat{c}_t \\ & -D_2 u(c,n,a)\widehat{\lambda}_t - D_2 u(c,n,a)\widehat{w}_t + aD_{23}u(c,n,a)\widehat{a}_t \end{aligned} \tag{49}$$

The cyclical behavior of money demand can be deduced from (37).

$$\widehat{M}_t = \widehat{c}_t + \widehat{P}_t \tag{50}$$

The nominal interest rate follows, according to (15),

$$-\widehat{P}_{t+1} + \widehat{\lambda}_{t+1} = -\widehat{P}_t - \frac{R}{1+R}\widehat{R}_t + \widehat{\lambda}_t \tag{51}$$

[21] See also [4, Cavallo/Ghironi (2002)].

in the approximated form, with R (respective r for the real rate) as the steady state values. The real rate r_t was deduced via the Fisher equation (see (16)) so that the approximated equation is given by

$$\widehat{\lambda}_{t+1} = -\frac{r}{1+r}\widehat{r}_t + \widehat{\lambda}_t \tag{52}$$

A.2 Household's Equations: MIU-Model

In the MIU-model the following three equations replace the first three in Appendix A.1.

$$\begin{aligned} 0 = \quad & -mD_{12}u\left(c,m,n,a\right)\widehat{P}_t + nD_{13}u\left(c,m,n,a\right)\widehat{n}_t \\ & +cD_{11}u\left(c,m,n,a\right)\widehat{c}_t - D_1u\left(c,m,n,a\right)\widehat{\lambda}_t \\ & +mD_{12}u\left(c,m,n,a\right)\widehat{M}_t + aD_{14}u\left(c,m,n,a\right)\widehat{a}_t \end{aligned} \tag{53}$$

Optimal labor is determined by

$$\begin{aligned} 0 = \quad & nD_{33}u\left(c,m,n,a\right)\widehat{n}_t + cD_{31}u\left(c,m,n,a\right)\widehat{c}_t \\ & -D_3u\left(c,m,n,a\right)\widehat{\lambda}_t - D_3u\left(c,m,n,a\right)\widehat{w}_t \\ & +mD_{32}u\left(c,m,n,a\right)\widehat{M}_t + aD_{34}u\left(c,m,n,a\right)\widehat{a}_t \\ & -mD_{32}u\left(c,m,n,a\right)\widehat{P}_t \end{aligned} \tag{54}$$

The efficiency condition for money now determines the respective money demand function. So one gets

$$\begin{aligned} & \beta D_1u\left(c,m,n,a\right)\widehat{P}_{t+1} - \beta D_1u\left(c,m,n,a\right)\widehat{\lambda}_{t+1} \\ = \quad & cD_{21}u\left(c,m,n,a\right)\widehat{c}_t + mD_{22}u\left(c,m,n,a\right)\widehat{M}_t \\ & +nD_{23}u\left(c,m,n,a\right)\widehat{n}_t - D_1u\left(c,m,n,a\right)\widehat{\lambda}_t \\ & +\left[\beta D_1u\left(c,m,n,a\right) - mD_{22}u\left(c,m,n,a\right)\right]\widehat{P}_t \\ & +aD_{24}u\left(c,m,n,a\right)\widehat{a}_t \end{aligned} \tag{55}$$

The equations for the nominal and real interest rate stay the same.

A.3 Finished Goods Firm's Equations

It is possible to combine the demand functions for the differentiated products c_0 and c_1 (see (25)) to arrive at

$$\widehat{P}_{0,t} = -\frac{1}{\epsilon}\widehat{c}_{0,t} + \frac{1}{\epsilon}\widehat{c}_{1,t} + \widehat{P}_{1,t} \tag{56}$$

The consumption aggregator (27) implies

$$0 = \frac{1}{2}\widehat{c}_{0,t} + \frac{1}{2}\widehat{c}_{1,t} - \widehat{c}_t \quad (57)$$

The price level is uniquely determined since $P_{1,t}$ is predetermined and $P_{0,t}$ is given by (56). Using (28) one gets

$$0 = \frac{1}{2}\widehat{P}_{0,t} + \frac{1}{2}\widehat{P}_{1,t} - \widehat{P}_t \quad (58)$$

A.4 Intermediate Goods Firm's Equations

In contrast to the household's conditions the equations of the firms to not change under different utility functions. The production functions for the differentiated goods must obey

$$0 = \widehat{n}_{0,t} - \widehat{c}_{0,t} + \widehat{a}_t \quad (59)$$
$$0 = \widehat{n}_{1,t} - \widehat{c}_{1,t} + \widehat{a}_t \quad (60)$$

As discussed earlier firms are unable to change their prices for two periods so $P_{0,t-1} = P_{1,t}$. The Taylor approximation for this condition is given by

$$0 = -\widehat{P}_{0,t-1} + \widehat{P}_{1,t} \quad (61)$$

The condition for optimal two period pricing is given in (34). Its Taylor approximation can be written as

$$\begin{aligned} &\beta\left[\epsilon\psi - (\epsilon-1)\right]\widehat{\lambda}_{t+1} + \beta\left[\epsilon^2\psi - (\epsilon-1)^2\right]\widehat{P}_{t+1} + \beta\left[\epsilon\psi - (\epsilon-1)\right]\widehat{c}_{t+1} \\ &+\beta\epsilon\psi\widehat{\psi}_{t+1} = (\epsilon-1)(1+\beta)\widehat{P}_{0,t} + \left[(\epsilon-1) - \epsilon\psi\right]\widehat{\lambda}_t \\ &+\left[(\epsilon-1)^2 - \epsilon^2\psi\right]\widehat{P}_t + \left[(\epsilon-1) - \epsilon\psi\right]\widehat{c}_t - \epsilon\psi\widehat{\psi}_t \end{aligned} \quad (62)$$

Real marginal cost ψ_t is given by the ratio of the real wage w_t over the productivity shock a_t. Since the markup μ_t is determined by the ratio of price over nominal marginal cost ($\mu = P/(P\psi)$ and as there is no inflation it follows that $\mu_t = a_t/w_t$. So the Taylor approximations can be written as

$$0 = \widehat{\mu}_t + \widehat{w}_t - \widehat{a}_t \quad (63)$$
$$0 = \widehat{\mu}_t + \widehat{\psi}_t \quad (64)$$

The Taylor approximation of the labor market clearing condition amounts to

$$0 = \widehat{n}_t - \frac{1}{2}\widehat{n}_{0,t} - \frac{1}{2}\widehat{n}_{1,t} \quad (65)$$

A.5 Monetary Authority's and Other Equations

To close the model one needs to assume some exogenous process for the money supply. Here it will be assumed that money $\widehat{M_t}$ follows an AR(2)-process (see the discussion in the main text). This implies that the growth rate of $\widehat{M_t}$ follows an AR(1)-process. In order to model this properly one has to add the equation

$$0 = \widehat{M_t} - \widehat{g}_{M_t} \tag{66}$$

where $\widehat{g}_{M_t}$ is the exogenous stochastic process that will have the same characteristics as $\widehat{M_t}$, that is, follows the same AR(2)-process.
As it is interesting to study the implications for the inflation rate Π this equation is further added to the system:

$$0 = -\widehat{\Pi}_t + \widehat{P}_t - \widehat{P}_{t-1} \tag{67}$$

There are now 19 variables
$\widehat{c}_{0,t}, \widehat{c}_{1,t}, \widehat{c}_t, \widehat{\lambda}_t, \widehat{n}_{0,t}, \widehat{n}_{1,t}, \widehat{n}_t, \widehat{w}_t, \widehat{\mu}_t, \widehat{\psi}_t, \widehat{r}_t, \widehat{R}_t, \widehat{P}_t, \widehat{P}_{t-1}, \widehat{P}_{0,t}, \widehat{P}_{0,t-1}, \widehat{P}_{1,t}, \widehat{\Pi}_t, \widehat{M_t}$
but only 17 equations so two tautologies must be added to the model. These are

$$\widehat{P}_{0,t} = \widehat{P}_{0,t} \tag{68}$$

$$\widehat{P}_t = \widehat{P}_t \tag{69}$$

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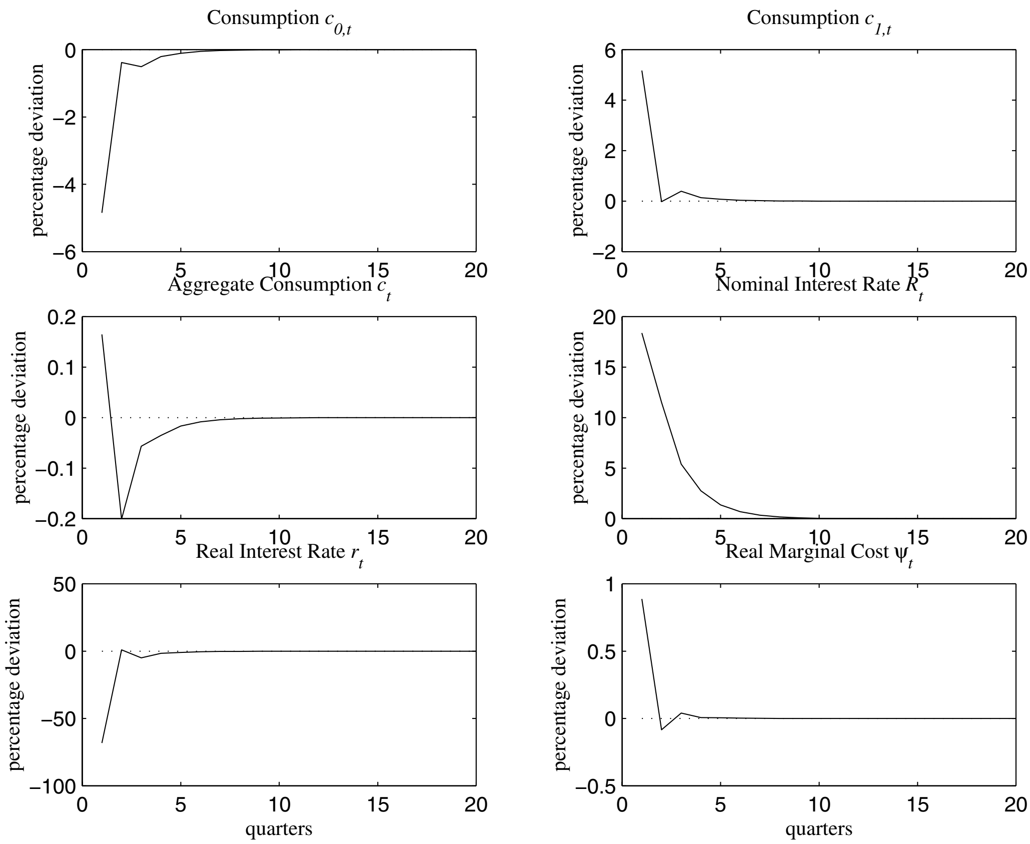

Figure 1: Impulse Response Functions for $\widehat{c}_{0,t}, \widehat{c}_{1,t}, \widehat{c}_t, \widehat{R}_t, \widehat{r}_t, \widehat{\psi}_t$, CIA-model, GHH preferences

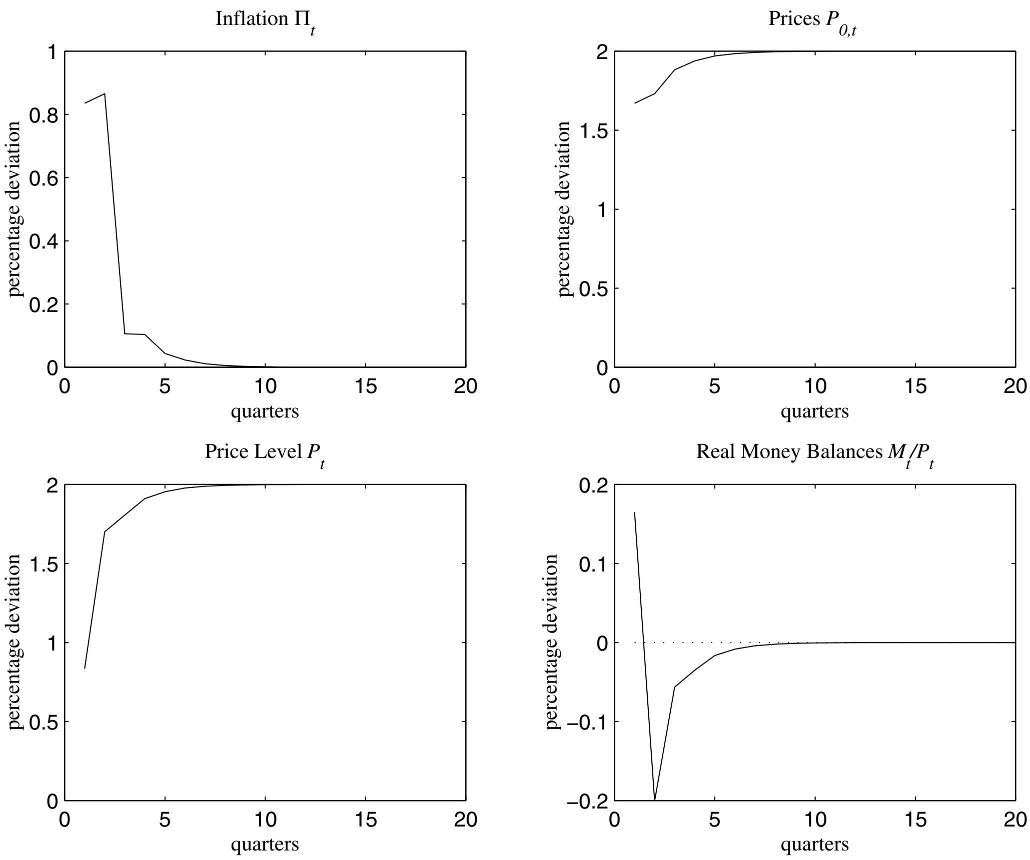

Figure 2: Impulse Response Functions for $\widehat{\Pi}_t, \widehat{P}_{0,t}, \widehat{P}_t, \widehat{M_t} - \widehat{P}_t$, CIA-model, GHH preferences

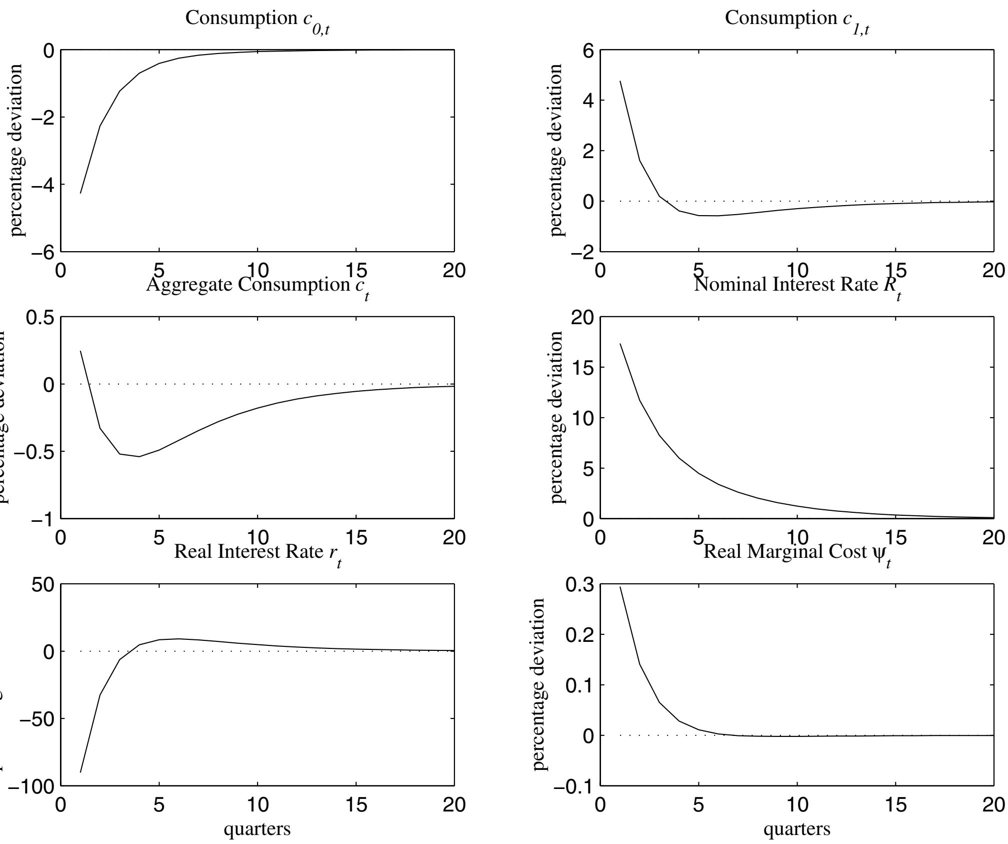

Figure 3: Impulse Response Functions for $\widehat{c}_{0,t}, \widehat{c}_{1,t}, \widehat{c}_t, \widehat{R}_t, \widehat{r}_t, \widehat{\psi}_t$, CIA-model, GHH preferences, high labor supply elasticity

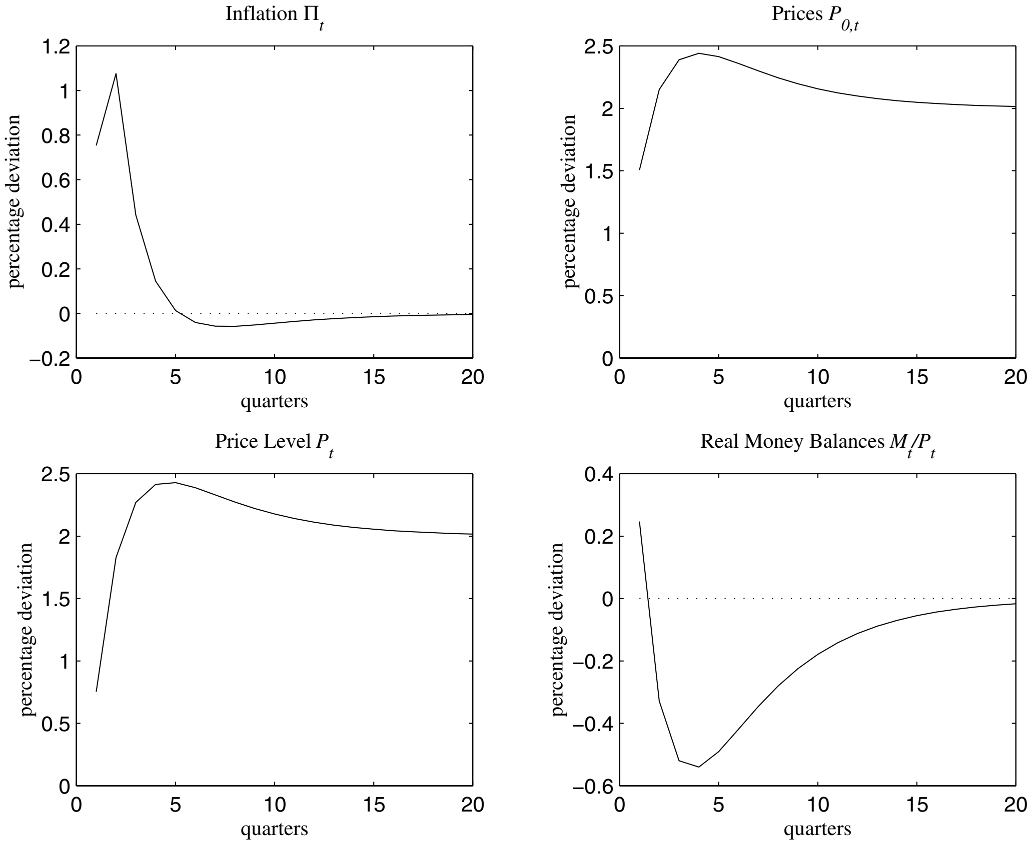

Figure 4: Impulse Response Functions for $\widehat{\Pi}_t, \widehat{P}_{0,t}, \widehat{P}_t, \widehat{M}_t - \widehat{P}_t$, CIA-model, GHH preferences, high labor supply elasticity

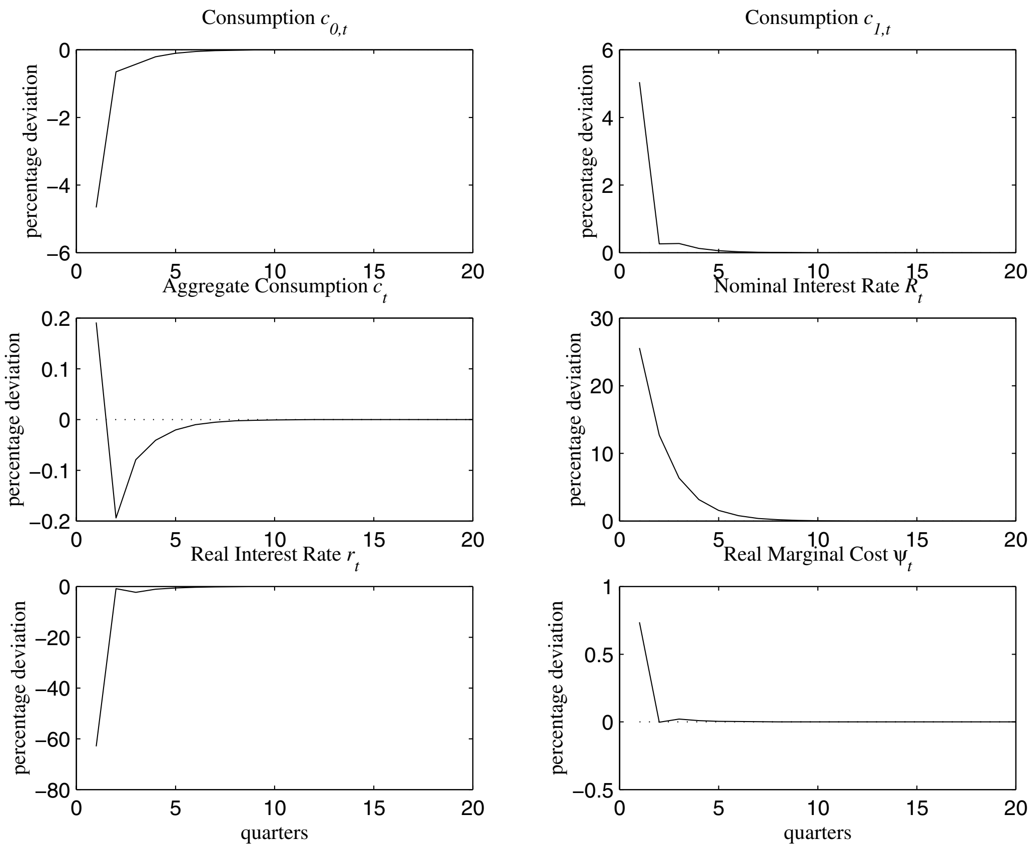

Figure 5: Impulse Response Functions for $\widehat{c}_{0,t}, \widehat{c}_{1,t}, \widehat{c}_t, \widehat{R}_t, \widehat{r}_t, \widehat{\psi}_t$, CIA-model, CRRA preferences

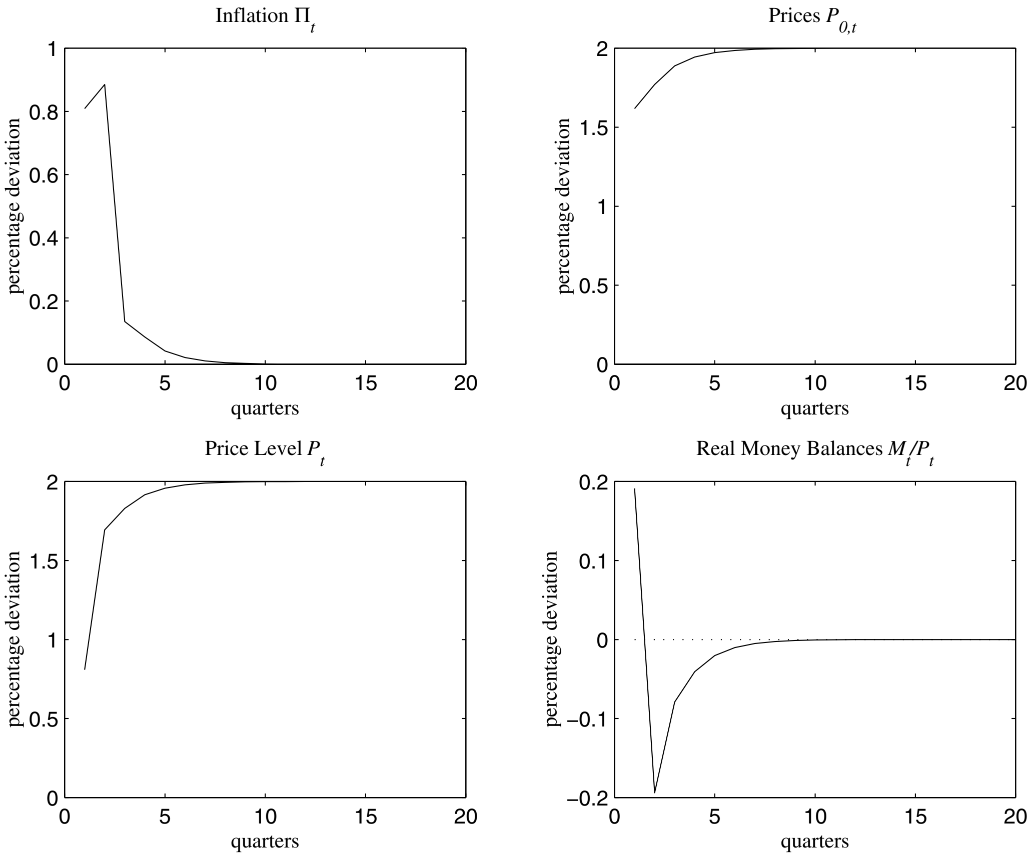

Figure 6: Impulse Response Functions for $\widehat{\Pi}_t, \widehat{P}_{0,t}, \widehat{P}_t, \widehat{M}_t - \widehat{P}_t$, CIA-model, CRRA preferences

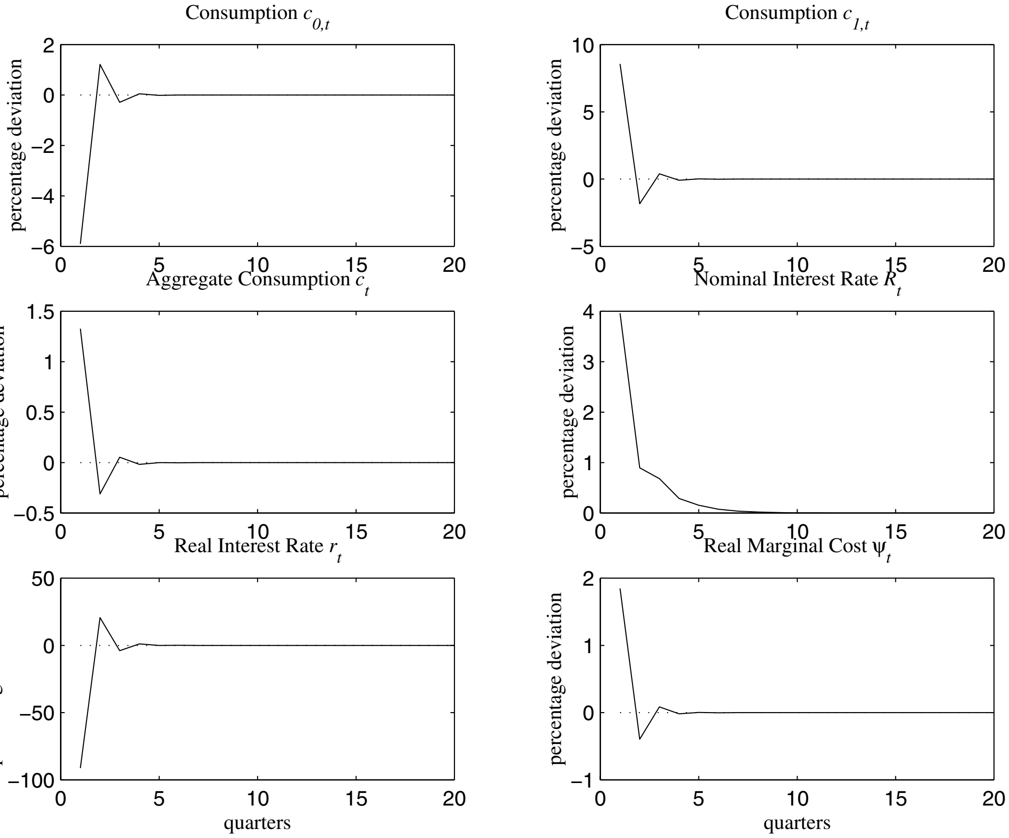

Figure 7: Impulse Response Functions for $\widehat{c}_{0,t}, \widehat{c}_{1,t}, \widehat{c}_t, \widehat{R}_t, \widehat{r}_t, \widehat{\psi}_t$, MIU-Model, GHH preferences

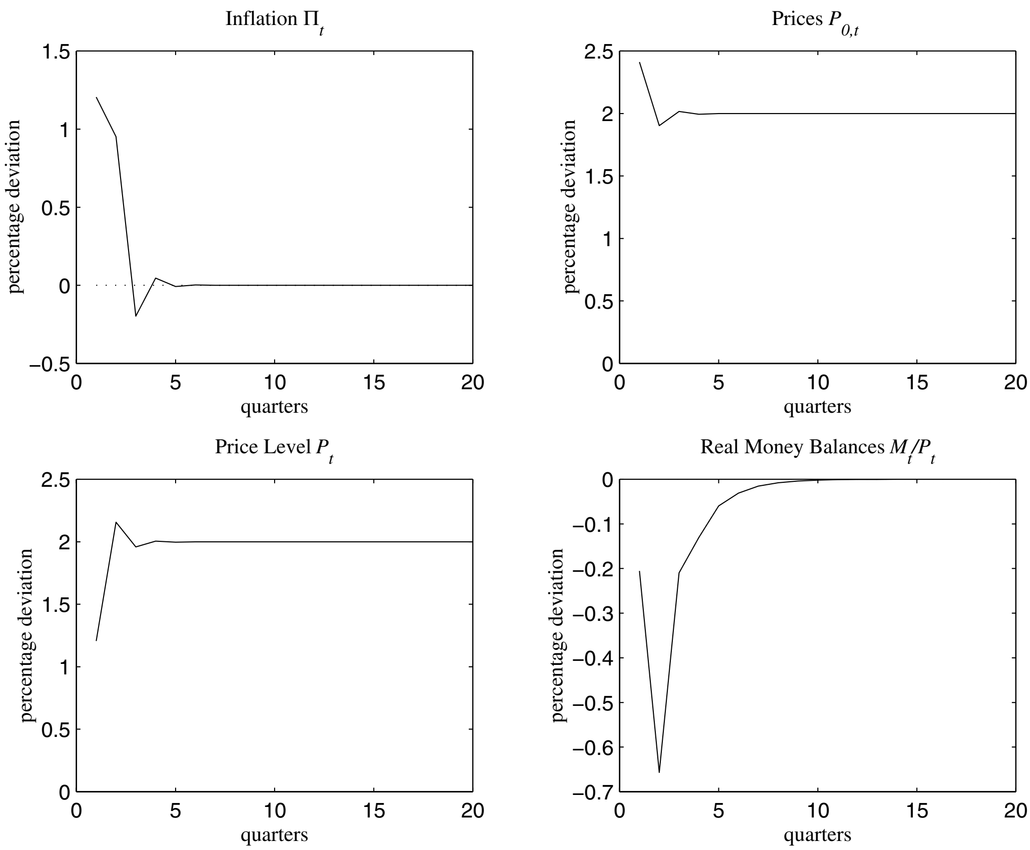

Figure 8: Impulse Response Functions for $\widehat{\Pi}_t, \widehat{P}_{0,t}, \widehat{P}_t, \widehat{M}_t - \widehat{P}_t$, MIU-Model, GHH preferences

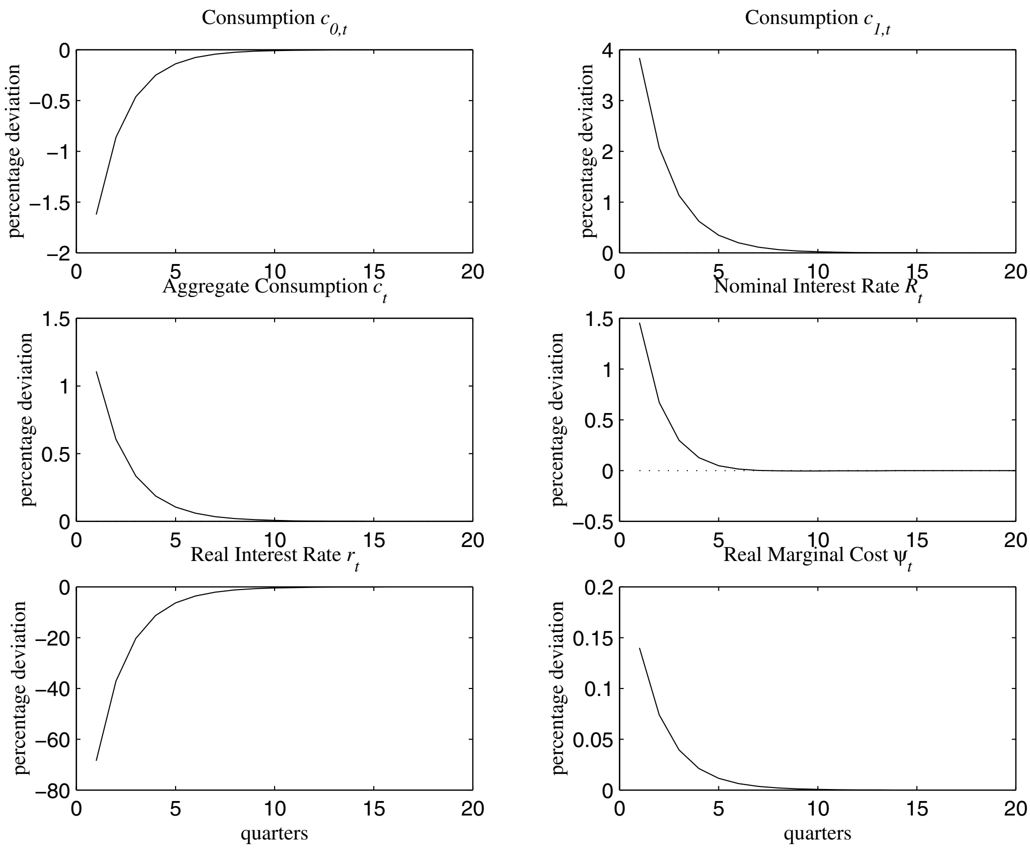

Figure 9: Impulse Response Functions for $\widehat{c}_{0,t}, \widehat{c}_{1,t}, \widehat{c}_t, \widehat{R}_t, \widehat{r}_t, \widehat{\psi}_t$, MIU-Model, GHH preferences, high labor supply elasticity

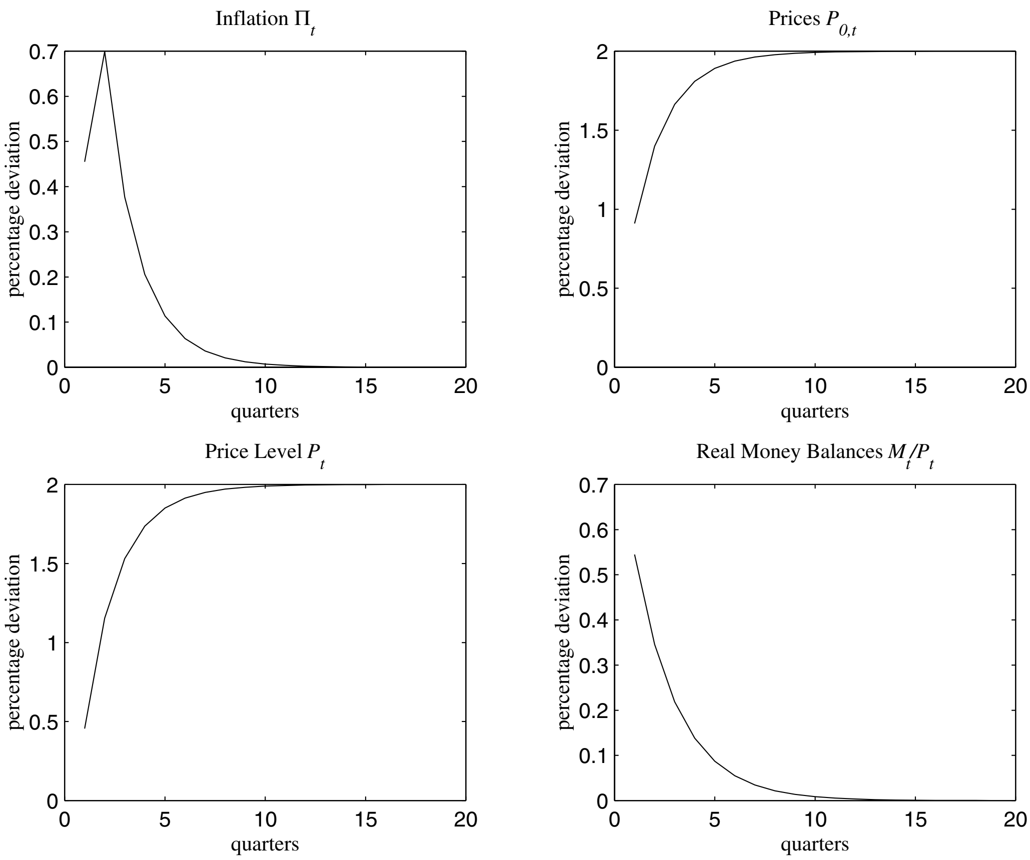

Figure 10: Impulse Response Functions for $\widehat{\Pi}_t$, $\widehat{P}_{0,t}$, $\widehat{P}_t$, $\widehat{M_t} - \widehat{P}_t$, MIU-Model, GHH preferences, high labor supply elasticity

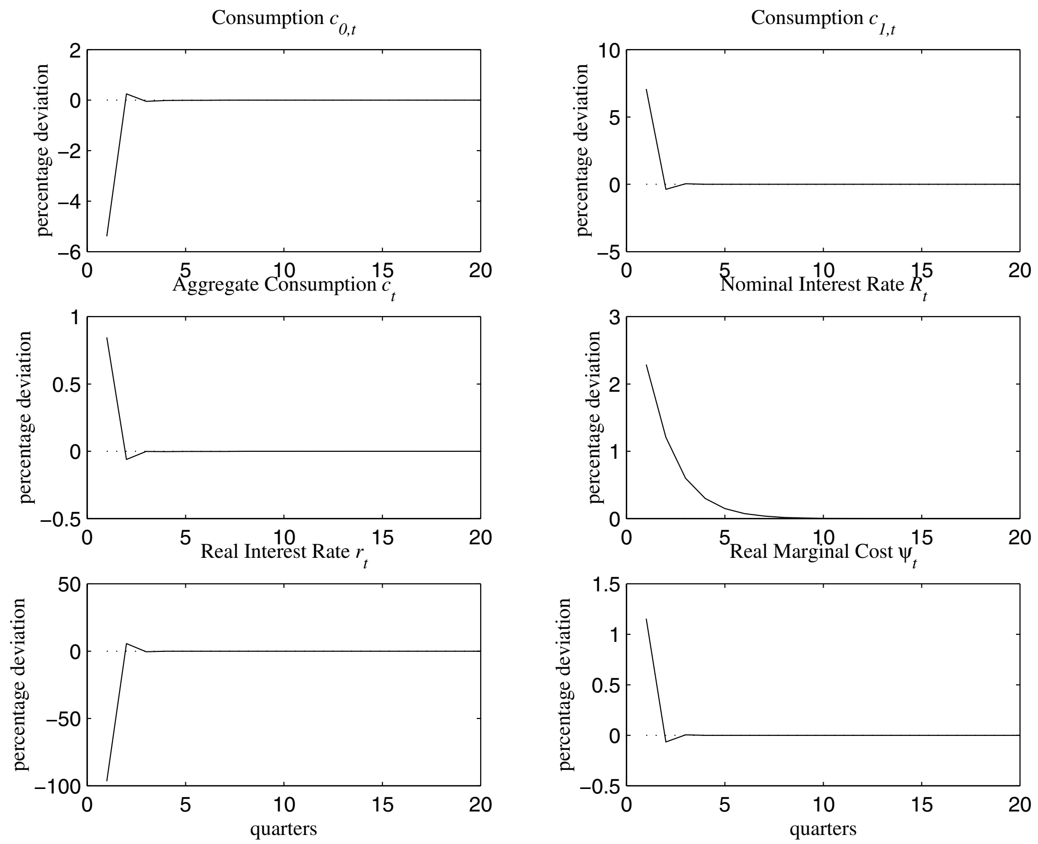

Figure 11: Impulse Response Functions for $\widehat{c}_{0,t}, \widehat{c}_{1,t}, \widehat{c}_t, \widehat{R}_t, \widehat{r}_t, \widehat{\psi}_t$, MIU-Model, CRRA preferences

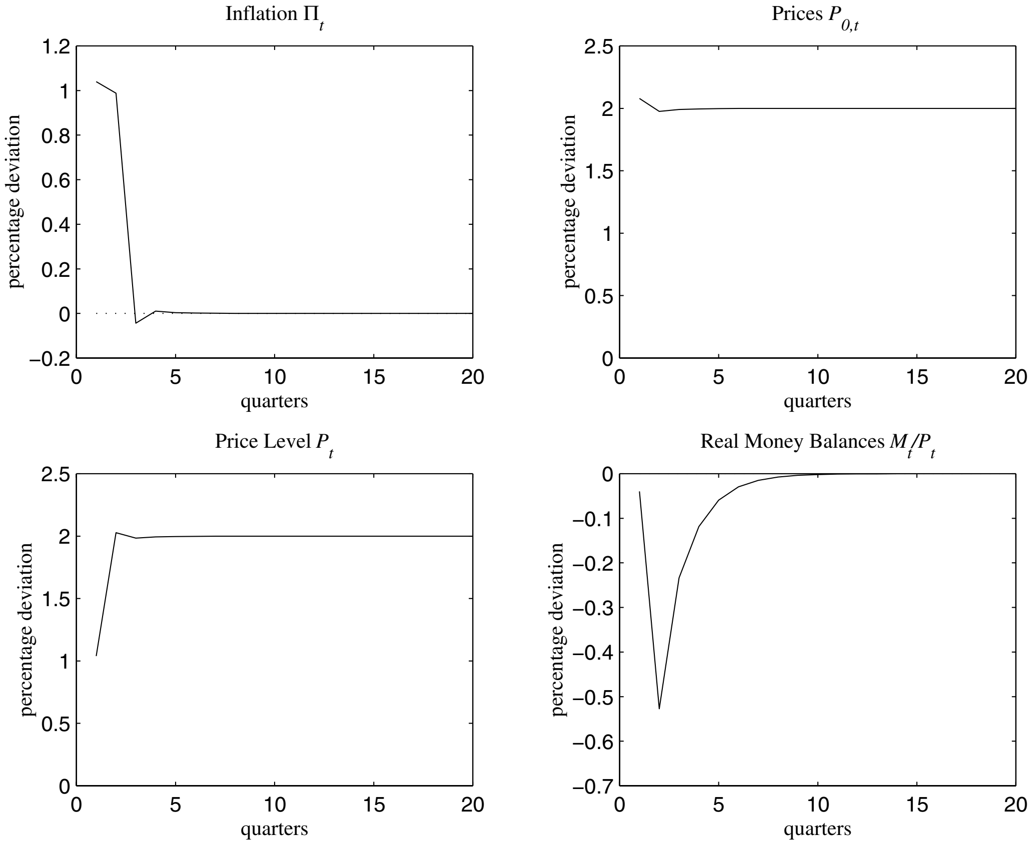

Figure 12: Impulse Response Functions for $\widehat{\Pi}_t$, $\widehat{P}_{0,t}$, $\widehat{P}_t$, $\widehat{M_t} - \widehat{P}_t$, MIU-Model, CRRA preferences

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32-92 **Günter Knieps** and **Rüdiger Pethig**, Uncertainty, Capacity Costs and Competition in the Electric Power Industry

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34-92 **Walter Buhr**, Regional Economic Growth by Policy-Induced Capital Flows: II. Policy Simulation Results

35-92 **Andreas Pfingsten** and **Reiner Wolff**, Endowment Changes in Economic Equilibrium: The Dutch Disease Revisited

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40-93 **Axel A. Weber**, Monetary Policy in Europe: Towards a European Central Bank and One European Currency

41-93 **Axel A. Weber**, Exchange Rates, Target Zones and International Trade: The Importance of the Policy Making Framework

42-93 **Klaus Schöler** und **Matthias Schlemper**, Oligopolistisches Marktverhalten der Banken

43-93 **Andreas Pfingsten** and **Reiner Wolff**, Specific Input in Competitive Equilibria with Decreasing Returns to Scale

44-93 **Andreas Pfingsten** and **Reiner Wolff**, Adverse Rybczynski Effects Generated from Scale Diseconomies

45-93 **Rüdiger Pethig**, TV-Monopoly, Advertising and Program Quality

46-93 **Axel A. Weber**, Testing Long-Run Neutrality: Empirical Evidence for G7-Countries with Special Emphasis on Germany

47-94 **Rüdiger Pethig**, Efficient Management of Water Quality

48-94 **Klaus Fiedler**, Naturwissenschaftliche Grundlagen natürlicher Selbstreinigungsprozesse in Wasserressourcen

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52-95 **Gerhard Brinkmann**, Über öffentliche Güter und über Güter, um deren Gebrauch man nicht rivalisieren kann

53-95 **Marlies Klemisch-Ahlert**, International Environmental Negotiations with Compensation or Redistribution

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83-99 **Thomas Eichner** and **Rüdiger Pethig**, Recycling, Producer Responsibility and Centralized Waste Management

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94-01 **Thomas Christiaans**, Economic Growth, the Mathematical Pendulum, and a Golden Rule of Thumb

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