# ECDNETOR 

Make Your Publications Visible.

A Service of 2BW

Leibniz-Informationszentrum Wirtschaft Leibniz Information Centre for Economics

## Kaiser, Karen

## Working Paper <br> Limited attention and choice

Working Papers, No. 2010-02

Provided in Cooperation with:<br>Bank of Mexico, Mexico City

Suggested Citation: Kaiser, Karen (2010) : Limited attention and choice, Working Papers, No. 2010-02, Banco de México, Ciudad de México

This Version is available at:
https://hdl.handle.net/10419/83759

## Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

## Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

# Banco de México <br> Documentos de Investigación 

Banco de México
Working Papers
$\mathrm{N}^{\circ}$ 2010-02

# Limited Attention and Choice 

Karen Kaiser

Banco de México

March 2010

La serie de Documentos de Investigación del Banco de México divulga resultados preliminares de trabajos de investigación económica realizados en el Banco de México con la finalidad de propiciar el intercambio y debate de ideas. El contenido de los Documentos de Investigación, así como las conclusiones que de ellos se derivan, son responsabilidad exclusiva de los autores y no reflejan necesariamente las del Banco de México.

The Working Papers series of Banco de México disseminates preliminary results of economic research conducted at Banco de México in order to promote the exchange and debate of ideas. The views and conclusions presented in the Working Papers are exclusively the responsibility of the authors and do not necessarily reflect those of Banco de México.

Documento de Investigación
2010-02

## Working Paper

2010-02

# Limited Attention and Choice* 

Karen Kaiser ${ }^{\dagger}$<br>Banco de México


#### Abstract

This paper analyzes a boundedly rational decision maker who is uncertain about his preference and faces the following trade-off: adding a good to the choice set has a positive option value but increases the complexity of the choice problem. The increased complexity is modeled as a reduction of the information available for each good. Because of this trade-off there is an optimal number of goods that the decision maker wants to analyze before making his final choice. The choice of the optimal set can be interpreted as the choice among stores. Stores maximize profits and choose a quality, an assortment, and a price. A lower cost of providing quality implies higher price and higher quality. Assortment will be small for very high levels of quality. Better quality of information implies greater variety and higher price. Greater variety combined with good consumer service can be a signal for high quality of the store.


Keywords: Decision making, bounded rationality, choice set, stores, quality.
JEL Classification: D01, D11, D21.

## Resumen

En este documento se analiza el comportamiento de un agente con racionalidad limitada, que no está seguro de sus preferencias y enfrenta el siguiente dilema: aumentar en un bien su conjunto de elección agrega valor pero incrementa la complejidad del proceso de elección. El aumento en la complejidad es modelado como una reducción en la información total que se tendrá de cada uno de los bienes a elegir. Debido a la disyuntiva que el agente enfrenta, va a existir un número óptimo de bienes que querrá analizar antes de escoger un producto. La preferencia por conjuntos de elección puede ser interpretada como la elección entre tiendas. Las tiendas son agentes maximizadores de ganancias que eligen la calidad de bienes que ofrecen, el tamaño del inventario y el precio. Un menor costo de proveer calidad implica que la tienda cobrará un mayor precio y ofrecerá mejor calidad. El tamaño del inventario será pequeño para valores muy altos de calidad y mientras mayor sea la calidad de la información que la tienda ofrezca ofrecerá una mayor variedad a un mayor precio. Mayor variedad combinada con un buen servicio al consumidor sirven como señal de alta calidad al consumidor.
Palabras Clave: Preferencia, racionalidad limitada, conjunto de elección, tiendas, calidad.

[^0]
## 1 Introduction

A boundedly rational decision maker (DM) that does not know his preference faces a two-stage decision problem. First, he has to choose a set of alternatives, and then he has to choose among those alternatives. The first stage of the process represents the idea of a consumer deciding to which store to go to, while the second refers to the consumer's behavior in the store.

DM does not know the goods ex-ante, so he does not know which good is preferred. Goods can be of two qualities: high or low, but with no information they are symmetric. Once he chooses a set, information about those goods will be revealed to him. DM will choose the good that has the highest probability of being of high quality.

Limited capacity to process information is modeled as an information trade-off: adding more goods to the set increases his option value but also decreases the amount of information he will learn about each good. The added option might be the best good in the set, but adding it entails the cost of losing information and not being able to recognize the best alternative.

Without bounded attention, adding more goods to the set adds option value and does not increase the complexity of the problem. In this scenario, DM will always be better off when choosing from larger sets. When there is a bounded capacity to process information adding more goods decreases the amount of information that he already has about other goods.

As the size of the set increases, the information trade-off causes the option value of a new good to decrease. In bigger sets the probability of having only bad goods is small and little is gained by adding one more good. As more goods are introduced into the set, the probability of ending in an uninformative information state increases, and the value of information increases.

The main result of the model states that as long as there is the possibility of losing some information when the size of the set increases, DM will find it optimal to restrict the number of elements in his choice set. There is a point where the expected costs of losing information becomes greater than the benefits of adding more options.

By choosing the number of objects DM wants to learn from, he allocates his capacity of attention optimally. If goods are classified according to how easy or difficult it is to learn about them, then the more complex learning about a good is the more attention it will require, and the optimal number of goods to be considered is going to be smaller than the consideration set for simpler goods.

One of the main contributions of this paper is that in contrast to previous models on consideration sets where the decision to include another brand is sequential, in this model DM has to make the decision at the beginning of the decision process. This allows a reinterpretation of the concept of consideration sets as stores or points of sale and creates a link between DM's bounded rationality and characteristics of the market structure, such as the relationships among variety, quality of information, and quality of a store.

In this framework, stores are interpreted as choice sets. A store is a profit maximizing agent that decides the average quality $q$ and the variety $n$ it will provide to its costumers. By fixing a price, the store is separating informed and uninformed consumers. An informed consumer is one whose expected utility of buying given the realization of information is greater than the price, but not necessarily someone who with
probability one will select a high quality good.
Higher average quality decreases the option value of adding one more good; as quality increases the probability of the added good to be the best in the set decreases. If DM recalls enough information, with very high probability, something of high quality will be found. Information becomes more valuable and the optimal assortment for the consumer is smaller.

As quality increases the consumer is willing to pay higher prices and the probability of buying increases, so the optimal assortment for the store will increase with quality, unless quality is so high that the increase in price is not enough to compensate for the decrease in the marginal benefits of that unit. In this case stores will be better offering smaller assortment.

Lower costs of quality imply higher quality. A store can increase its profits by increasing quality. For a given assortment price will increase and demand will too. This will increase store revenues: quantity demanded will be higher for a higher price.

On the other hand, for a fixed quality of store, the better the information quality provided to the consumer the bigger the assortment they will prefer. When quality of information is very high, the consumer needs fewer signals to detect a high-quality good, so the trade-off of information is smaller. If quality of information increases, the store will be better off by charging a price that requires consumers to be more informed in order to buy.

I also analyze the case where $q$ is unobservable. Stores can announce a price $p$, quality of information $\theta$, and variety $n$, and these three variables are observable to DM. If costs are increasing in $n, \theta$, and $q$, I find that price, assortment, and quality of information together will always reveal quality.

The incentive compatibility constraint for the firm not to deviate and provide low quality while charging high prices, is always satisfied. This is due to the fact the benefits of quality of information are greater when quality is greater. If the store charges the high price, and offers good quality of information, but offers low average quality then, the marginal benefits of good quality of information would be smaller than its marginal costs.

The experimental evidence suggests that the size of the choice set affects consumption behavior as well as levels of satisfaction. Iyengar and Lepper (2000) ran an experiment in which consumers could try a maximum of either 6 or 24 different exotic jam flavors, all of them from the same brand. They found that $30 \%$ of people who faced the limited-choice condition bought jam, while only $3 \%$ of the people who faced the extensive-choice condition did.

Gourville and Soman (2005) explain this phenomenon. Consumers suffer from information overload. They tested this hypothesis and found that when the cognitive process is simplified, the effect of more variety decreases. Boatwright and Nunes (2001) provided evidence of this phenomenon at the store level and analyzed the effects on profits for the store. As assortment was decreased, overall sales increased.

The results found in this paper are consistent with the experimental evidence. Consumers that face an extensive choice condition on average are less informed than consumers that faced smaller sets, and their reservation price is smaller. This explains why people bought less jelly in Iyengar and Lepper's experiment, and why stores might increase profits when reducing their assortment.

The paper is organized as follows: Section 1.1 analyzes relevant literature. Section 2 states the model and intuition about the optimal size of the set. In Section 3 the relation between quality of store and of information are related to the size of the set. Section 4 concludes. All proofs are in the Appendix.

### 1.1 Literature Review

Evidence of information overload was first gathered in an experiment by Jacoby, Speller and Berning (1974). They used 192 housewives as subjects to test if they could effectively use all the information provided to them to make a choice between different brands of rice and prepared dinners. In the different treatments there were either $4,8,12$, or 16 brands and either $4,8,12$, or 16 bits of information per product. They conclude that the amount of information that consumers are able to process is limited, and that providing too much information could result in poorer decisions.

Keller and Staelin (1987) separated the effects of providing the consumer with better quality information and more quantity of information. Different from Jacoby et al., they established a measure of the usefulness of each attribute to evaluate a product. They corroborated Jacoby et al.'s conclusion that there is information overload. The presence of too much information, even if it is of high quality, decreases information effectiveness.

Strong evidence has been collected about the existence of consideration sets. Hauser and Wernerfelt (1990) not only provide a survey of these studies, but also develop a model in which it is optimal for a utility maximizer to construct a consideration set before making the purchasing decision. The consumer faces some exogenous cost and benefit each time he has to decide whether to include some product in the set or not. In their model, the decision of incorporating one more element is sequential.

Previous literature in Bounded Rationality model a DM that can control which information to recall and which information to forget while the choice set remains fixed. Wilson (2006) models a DM with bounded memory as an automaton that has to choose between two alternatives. There is only a finite number of signals that can be recalled, so he has to choose the optimal memory rule to maximize the probability of choosing the correct state.

Gabaix, Laibson, Moloche, and Weinberg (2003) model limited attention. They develop and test a model in which cognition is costly, because it takes time. In the model DM allocates his thinking according to a cost-benefit analysis. The DM has to choose from a set of $N$ unknown goods, decision time is scarce, and he has to decide whether to ignore information or learn more. They found out that subjects think more when the marginal value of thinking is high. Their evidence suggests that subjects allocate attention following a cost-benefit analysis.

In contrast to this literature, this model does not allow DM to decide what information to recall. This model incorporates the evidence that people is distracted by other information or events when is performing a cognitive process.

## 2 Model

The decision maker (DM) is choosing a good $x$ from a set. Goods can be of two qualities: High or Low. The proportion of high quality goods in the population is $q$ and is known to the individual. However, ex-ante those goods that are of high quality cannot be distinguished from those that are of low quality.

At the beginning of the decision process, DM can select a choice set $B_{n}$ with cardinality $n$ and learn something about its elements before he makes a choice. The number of high quality goods $i \in\{0, \ldots, n\}$ that belong to set $B_{n}$ is distributed according to a Binomial Distribution $b(n, q)$ that is known to DM.

Once $B_{n}$ is chosen, the decision maker will receive some number of signals (determined by a learning technology) about each of the goods that belong to the set. Signals can be either good or bad (one/zero).

A learning technology is a function that will determine how many signals for each good DM will receive/learn. The number of signals $\ell$ is binomial with parameters $m_{n}$ and $\alpha_{n}$. Where $m$ is the maximum number of signals DM can receive, and $\alpha$ is the probability of a success.

In order to incorporate the limited capacity DM has to process information, I assume that both parameters are weakly decreasing in $n$, and at least one is strictly decreasing in $n$. By imposing these assumptions the amount of information that can be processed per good depends on the total number of goods in the set $n$, and has a decreasing relation to it.

The information for each good will be generated by sampling i.i.d signals from distribution $F_{H}$ or $F_{L}$ depending on the quality of the sampled good. I assume that high quality goods have a Binomial Distribution $b\left(\ell, \theta_{H}\right)$, where $\ell$ is the number of signals generated for a good and is determined by the learning technology. On the other hand, low quality goods can only generate bad signals $\theta_{L}=0$. Receiving one good signal is enough to distinguish a high quality good from a low one.

The way DM learns (learning technology), can be seen as a matrix whose number of columns are the number of goods and the number of rows $\ell$ is distributed according to a binomial distribution $b\left(m_{n}, \alpha_{n}\right)$. It can be thought as a matrix where each column is a good $\left\{x_{1}, x_{2}, x_{3}, \ldots\right\}$, and the number of attributes to be known, the length of the columns, $\left\{a_{1}, \ldots\right\}$ is determined by a binomial distribution $b\left(m_{n}, \alpha_{n}\right)$. For example: $b\left(m_{2}, \alpha_{2}\right)=b\left(3, \frac{1}{2}\right)$

$$
\begin{aligned}
& I_{4}=\left[\begin{array}{llll} 
& \mathbf{x}_{1} & \mathbf{x}_{2} & \mathbf{x}_{3} \\
\mathbf{a}_{1} & s_{11} & s_{12} & s_{13} \\
\mathbf{a}_{2} & s_{21} & s_{22} & s_{23} \\
\mathbf{a}_{3} & s_{31} & s_{32} & s_{33}
\end{array}\right], I_{2}=\left[\begin{array}{ccc} 
& \mathbf{x}_{1} & \mathbf{x}_{2} \\
\mathbf{a}_{1} & s_{11} & s_{12} \\
\mathbf{a}_{2} & s_{21} & s_{22}
\end{array}\right], \\
& I_{1}=\left[\begin{array}{lll}
\mathbf{x}_{1} & \mathbf{x}_{2} \\
\mathbf{a}_{1} & s_{11} & s_{12}
\end{array}\right], I_{0}=\mathbf{x}_{1} \mathbf{x}_{2}
\end{aligned}
$$

Each of this matrices will occur with probabilities: $\frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{8}$, respectively.
At the end of the information flow DM selects the good with the highest probability of being of high quality. Lets denote $I=\left\{I_{\ell}\left(x_{1}\right), . ., I_{\ell}\left(x_{n}\right)\right\}$ the matrix of information when DM receives $\ell$ signals.

With the learning technology that is imposed, at the beginning of the decision process, DM chooses the
size of a matrix and once he does that information will flow and fill it with signals. Once the matrix is filled DM chooses a product.

DM faces a constant trade off between increasing the amount of sampling ( $m$ ) and the number of goods $(n)$. The more goods he adds to the set the probability of having at least one high quality good, $p(i \geq 1)=$ $\left(1-(1-q)^{n}\right)$ increases, but the probability of distinguishing the high quality good from the low quality goods decreases, due to the fact that less information will be received about each good.

The objective of DM is to reduce the probability of making a mistake. A mistake is made only when all information is $\operatorname{bad}, \max \left\{I_{f}\left(x_{\ell}\right)\right\}_{\ell=1}^{n}=0$, and there is at least one high quality good. In this case the probability of choosing a low quality good given that there are $i$ high quality goods is: $\left(1-\frac{i}{n}\right)$. The probability of receiving only bad signals for all the goods is $\left(1-\theta_{H}\right)^{m_{n} i}$.

If DM receives at least one good signal, his information matrix will be $I(1)$, and he will know for sure which good has high quality. If he only receives bad signals $I(0)$, he cannot distinguish among goods.

Denote $g(I)$ the expected utility of choosing a good given that $I$ is observed :

$$
g_{\ell}(I)=\left\{\begin{array}{cc}
1 & \max \left\{I_{\ell}\left(x_{k}\right)\right\}_{k=1}^{n} \geq 1 \\
\frac{q(1-\theta)^{\ell}}{q(1-\theta)^{\ell}+(1-q)} & \max \left\{I_{\ell}\left(x_{k}\right)\right\}_{k=1}^{n}=0
\end{array}\right.
$$

The expected utility given $\ell$ signals is:

$$
\begin{equation*}
V_{\ell, n}=\left(1-\beta_{\ell}^{n}\right)+\beta_{\ell}^{n} g_{\ell}(0) \tag{2.1}
\end{equation*}
$$

where $\beta_{\ell}^{n}=\left(q(1-\theta)^{\ell}+(1-q)\right)^{n}$
At the beginning of the decision process the expected utility of a set $B_{n}$ is :

$$
\begin{equation*}
U_{\left(m_{n}, \alpha_{n}\right)}\left(B_{n}\right)=E_{\ell}\left[V_{\ell, n}\right] \tag{2.2}
\end{equation*}
$$

and DM chooses $n$ in order to maximize it:

$$
\max _{n} U_{\left(m_{n}, \alpha_{n}\right)}\left(B_{n}\right)
$$

DM faces a constant trade off between increasing the amount of sampling ( $E[\ell]$ ) and the number of goods $(n)$. The cost of losing information if the number of objects in the set remains the same is: $C\left(n,\left(m_{n}, \alpha_{n}\right)\right)=$ $U_{\left(m_{n}, \alpha_{n}\right)}\left(B_{n}\right)-U_{\left(m_{n+1}, \alpha_{n+1}\right)}\left(B_{n}\right)$, while the benefits of increasing the number of objects by one if the amount of sampling remains the same is: $B\left(n,\left(m_{n}, \alpha_{n}\right)\right)=U_{\left(m_{n}, \alpha_{n}\right)}\left(B_{n+1}\right)-U_{\left(m_{n}, \alpha_{n}\right)}\left(B_{n}\right)$.

This gives:

$$
U_{\left(m, \alpha_{n+1}\right)}\left(B_{n+1}\right)-U_{\left(m, \alpha_{n}\right)}\left(B_{n}\right)=B\left(n,\left(m_{n+1}, \alpha_{n+1}\right)\right)-C\left(n,\left(m_{n}, \alpha_{n}\right)\right)
$$

Theorem 1 When the number of signals DM receives is binomial with parameters ( $m_{n}, \alpha_{n}$ ) with both parameters weakly decreasing in $n$, and at least one strictly decreasing in the number of goods, there is an $n^{*}$ such that for $n>n^{*}$ DM's payoff is strictly decreasing in $n$.

Proof. Is in Appendix 4.1.1
Theorem 1 states that as long as there is some probability of losing information by increasing the number of objects considered to make a choice, there is an optimal number of goods to be analyzed.

The costs of adding one more good will always be greater than zero. Different from the case $\alpha_{n}=\alpha$ where costs go to zero as the number of goods increases, in this case as $n$ increases the probability of receiving no information $\ell=0$ is always positive. As $n$ goes to infinity the probability of receiving no signals converges to $\lim _{n \rightarrow \infty}\left(1-\alpha_{n}\right)^{m}$.

On the other hand, for a fixed $n$ the costs of decreasing $\alpha$ are decreasing. Decreasing $\alpha$ when it is large is less costly than doing it when the probability of receiving information is already small.

When the size of the set is increased, for the same parameters $\left(m, \alpha_{n}\right)$, there are two effects: the first one is that the probability of receiving no information increases; and the second is that the probability of receiving only bad signals in states where there is information, decreases. For small $n$, the probability of receiving bad information is higher for any level of information $\ell$, and the difference between $V_{\ell}-V_{\ell+1}$ is greater; as $n$ increases the difference decreases but the second effect persists. So costs will be decreasing in $n$ but always be positive.

The benefit of adding one more good decreases as the size of the set increases. However, when $n$ is fixed benefits are increasing in $\alpha$. This means that the higher the probability of receiving information the more beneficial it is to add one more good.

Under the same conditions of quality and information, DM can be worse off when he has to select from bigger sets. The expected utility of a set starts to decrease once the optimal number is reached. It is important to notice that DM can not focus his attention on a subset once he has begun receiving information about the options he is considering. It can be argued that even if he could focus by tuning out information about some products, just the presence of these products distracts him, and causes, with some probability, a loss in information from the considered products.

This result can explain the experimental evidence found by Iyengar and Lepper (2000). DM will buy a product as long as his expected utility is at least as great as the price minus the discount coupon. Subjects that faced the extensive choice condition received less information per product, even if they tasted the same number of jams than subjects that faced only 6 . On average the expected utility of these consumers was lower than the average expected utility of the 6 -choice consumers, so fewer were willing to buy it. All subjects in the experiment faced the same quality of the store and of information, the only difference was in the variety.

Lemma 1 For a fixed quality of information, $\theta_{H}$, and learning technology, the greater $q$ the smaller the optimal set

Proof. is in Appendix 4.1.2

As expected quality increases, the probability of having only low quality goods is very small. If the set is small DM will be able to retain the necessary information to find at least one high quality good. If the set is too big, no matter that the expected quality is very high, his limited attention might make him choose wrongly. The better quality the store provides the consumer will be best satisfied with a smaller choice set. Is as if the store pre-selected the goods and consumers trusted the store to know their preferences with some noise.

When quality is low, the probability of having a consideration set with a very bad selection is very high. DM is willing to give up information in order to increase the probability of at least having one high quality good.

Lemma 2 For a fixed quality of the store, $q$, and learning technology, the greater $\theta_{H}$ the smaller the optimal set.

Proof. is in Appendix 4.1.3
The decrease in the probability of having bad signals for all the high quality goods that belong to the set, compensates for the increase in costs derived from losing more informative signals.

If quality is very high variety might harm the consumer, unless it is paired up with better information. Better attention and consumer service should go in hand with big high quality stores.

There is evidence that consideration sets can be as small as one. Lapersonne, et al., found that in France $22 \%$ of the people that are going to buy a new car considered only one brand before making their choice. But there is still no study that suggests if there is a relation between how complex learning about a good is and the size of its consideration set.

How difficult a task is, has an effect on divided attention performance. Baddeley (1986) asses that varying the difficulty of the tasks will affect dual-task performance. McDowd and Craik (1988) found out that the number of errors made by individuals increased as the complexity of the task was increased.

If the learning technology is interpreted as a function that measures how much attention the realization of a task (analysis of a good) requires. For a fixed capacity, we can relate how complex a good is (how difficult it is to learn about it) with the optimal number of goods in the set. As expected, if the good is simple the optimal set will be greater.

Lemma 3 If $b\left(m_{n}, \alpha_{n}\right)$ and $\widehat{b}\left(\widehat{m}_{n}, \widehat{\alpha}_{n}\right)$ are two learning technologies such that at least one parameter of technology $b$ is always greater than the corresponding parameter in technology $\widehat{b}$, and the other is no smaller, then $n^{*} \geq \widehat{n}^{*}$.

Proof. is in Appendix 4.1.4
Note that by endowing DM with a learning technology he loses control over which information to recall/forget. Other learning technologies that are not included in this paper, allow the decision maker to select what information to recall or learn. In this case, how the decision maker selects the information becomes a problem to be studied by itself. These kinds of technologies would completely change the structure of the model. Wilson (2006) and Gabaix, Laibson (2003) are examples of frameworks used to analyze this "selection" behavior.

Another assumption that I impose is that the amount of information received for each good is the same. Due to the fact that ex-ante all goods are indistinguishable, that there are no labels, and that the amount of information received of a good does not convey information about its quality, this assumption should not make a difference in the conclusions. If the flow of information were coming from advertising, and if high quality goods were expected to be more/less advertised the amount of information would be informative, and this would change the results.

The learning technology that is assumed covers a wide variety of possible learning technologies. Technologies that constrain the total amount of bits of information are special cases. Some technologies where there can be some complementarities in the learning process are also included. This would be the case where by adding one more good the total number of signals that can be learned increases, even though the number of signals per good decreases.

When $\alpha_{n}=1$ for all $n$, and $m_{n}$ is decreasing in $n$. Technologies in which the total amount of information increases as $n$ increases are included, as long as the total number of signals per good always decreases. A special case is technologies that constrain the total amount of information $\sum_{\ell=1}^{n} m_{n}$. The analysis of this case helps to understand the mechanisms behind the information trade-off.

The other extreme case is when there is a maximum number of signals DM can receive per good $m$ and the only thing that changes is the distribution over $m, m$ is fixed for all $n$ and $\alpha_{n}$ is strictly decreasing in $n$.

If a good at most has $m$ attributes, there is no way of receiving more than $m$ signals, so the fact that there is a maximum does not seem to be an imposition. On the other hand, the more goods there are, distractors will increase and attention will decrease so the probability of retaining many signals decreases.

### 2.1 Stores

In this section I analyze how preferences of consumers over sets affect the decisions of stores. Stores are sets of goods that provide to costumers a certain number of options $n$, a quality level $q$, and information quality $\theta$. A consumer, that is going to buy a product, has to consider first to which store to go, and then which product to buy.

A consumer will buy a product only if the expected probability of choosing a high quality good, given the information, is greater than or equal to the price: $g_{\ell}(I) \geq p$. If he receives a good signal $I(1)$ then his utility of buying a good is $1-p$. If he only receives $\ell$ bad signals his utility of buying a good is $g_{\ell}(I(0))-p$ . If his expected utility is not greater than the price he leaves empty-handed.

A store is looking to maximize its profits:

$$
\Pi(q, n, p)=p D(q, n, p)-c(n)-g(q)
$$

$c(n)$ and $g(q)$ are the cost of variety and quality and both are increasing. $D_{k}(q, n, p)$ is the expected demand: the probability of the consumer buying a good once he is in the store:

$$
D(q, n, p)=p\left(V_{\ell} \geq p \mid q, n\right)
$$

where $V_{\ell}=\left(1-\beta_{\ell}^{n}\right)+\beta_{\ell}^{n} g_{\ell}(0)$, and $\ell$ is distributed according to the learning technology $b\left(m_{n}, \alpha_{n}\right)$.
A store will choose a price from the set $p \in\left\{1,\left\{g_{\ell}(I(0))\right\}_{\ell=0}^{m}\right\}$. If the price is 1 , only costumers that receive a good signal will buy. If the price is $g_{m}(I(0))=p_{m}$ every costumer that walks in the store will buy and information in the store becomes irrelevant. When the firm maximizes profits and fixes a price it is establishing how informed a consumer must be in order to buy a product.

As quality $q$ increases, for the same realization of information $(\ell, I)$ the consumer is willing to pay a higher price. If the consumer only receives bad signals, the greater $\ell$ the worst informed he is.

For a fixed quality $q$, the firm will choose the assortment $n$, up to the point where marginal revenues of adding one more good to the set equals the marginal cost. When it is optimal for the firm to charge a price smaller than one, the store will sell to at least two level of informed consumers, and she is indifferent between selling the product to any of them. This creates an incentive to the store to increase its assortment making consumers worse off.

Lemma 4 lower marginal costs of quality imply higher quality $q$, higher price, and smaller assortment $n$ if quality is high enough.

Proof. is in Appendix 4.2.1
As expected quality increases, the probability of having only low quality goods is smaller. If the set is small, DM will be able to retain the necessary information to find at least one high quality good. If the set is too big, no matter that the expected quality is very high, his limited attention might make him choose incorrectly. So for higher quality $q$ the consumer prefers smaller assortment $n$.

However, as quality increases the store can increase its profits by increasing the number of objects. When the store sets a price smaller than one, demand is increasing in $n$. This happens because as $n$ increases the probability of being in a state with no signals increases, but those costumers are already part of its demand, so demand increases.

When quality is high enough it is optimal for the store to decrease assortment. As quality increases the marginal benefit of adding one more good is very small and decrease even further as quality increases, making the marginal revenue of that last unit smaller. The store will be better if it decreases its assortment.

When quality is high, the store is willing to set a price in order to attract more uninformed consumers. This still increases its profits due to the fact that for high quality levels, the expected utility of consumers, even when they are not well informed, is also high. On the other hand, as quality decreases expected utility for each level of information decreases more, so the store might be better selecting consumers that are more informed, and therefore willing to pay more.

Lemma 5 Better quality of information $\theta_{H}$ imply bigger assortment $n$.
Proof. is in Appendix 4.2.2
When quality of information $\theta_{H}$ increases, each signal becomes more informative so having a bigger assortment becomes less costly for the consumer. Demand increases, because consumers will be more informed, and the store will be better if it sets a price such that better informed consumers buy. The
marginal benefit of adding a good to the store for a level of assortment increases, with high probability if it is of high quality it will be recognized by the consumer.

When stores are interpreted as consideration sets, there is the assumption that the consumer cannot focus on a subset of the products that are offered in that store. This assumption is supported by the evidence that the ability to remain focused on goal oriented-tasks is affected by distractors.

Lavie (2005) finds evidence that "distractor processing depends critically not only on the type of load involved in the processing of goal-relevant information." When the task requires cognitive control functions, the more attention is required to perform the task the more interference is caused by the distractors. Having more goods on display will generate more interference.

### 2.2 Unobservable Quality

Evidence about the relationship between price and quality of a product is not conclusive. Some studies had found that there is positive correlation between price and quality but usually is not significant, meaning that price is a poor signal for quality (for a list of relation between price and quality per product see Gerstner (1985)). Rao and Monroe (1989) perform a meta-analysis of previous studies and find out that the positive correlation becomes stronger when the subject faces multiple responses to different prices.

Different from previous empirical studies, using this model we can analyze the consequences of bounded rationality and its effect on the relationship between price and quality of the store. Do we expect better stores to be more expensive, even if the quality of their best products is the same as the quality of the best products on a low quality store? In order to study the relationship between price and quality in this section, the choice of quality and variety are endogenous.

The store will select the quality of the store $q$, quality can only be either high or low $\left\{q_{H}, q_{L}\right\}$, the assortment or number of goods $n$, price $p$, and quality of information $\theta$. Consumers will go only once to the store.

Once the consumer is in the store he can only observe the assortment $n$, the price $p$, and quality of information $\theta$. Using this information and the information that is revealed to him about the goods, he has to choose whether to buy or not. The consumer knows the potential qualities $\left\{q_{H}, q_{L}\right\}$ so associated to them there is an $\left(p_{H}, n_{H}, \theta_{H}\right)$ and $\left(p_{L}, n_{L}, \theta_{L}\right)$ that he also knows. For every possible combination of $(p, n, \theta)$ the consumer forms beliefs about $q$.

The store chooses $\phi=\left(q_{j}, n_{r}, p_{s}, \theta_{s}\right)$. The consumer will buy after information is realized if given the realization of information and his beliefs about $q$, his expected utility of buying is greater than the price. The expected demand for the store is:

$$
D\left(q_{j}, \phi \mid \widehat{q_{j}}\right)=p\left(V_{\ell} \geq p_{s} \mid\left(\widehat{q_{j}}, \phi\right)\right)
$$

where $\widehat{q_{j}}$ denotes the beliefs about $q_{j}$ given $\left(n_{r}, p_{s}, \theta_{j}\right) . V_{\ell}=\left(1-\beta_{\ell}^{n}\right)+\beta_{\ell}^{n} g_{\ell}(0)$, and $\ell$ is distributed according to the learning technology $b\left(m, \alpha_{n}\right)$.

The store chooses $\phi$ in order to maximize its profits:

$$
\Pi\left(q_{j}, \phi \mid \widehat{q_{j}}\right)=p_{s} D\left(\widehat{q_{j}}, \phi\right)-c\left(n_{i}\right)-g\left(q_{j}\right)-c_{1}\left(\theta_{r}\right)
$$

$c\left(n_{r}\right), g\left(q_{j}\right)$, and $c_{1}\left(\theta_{r}\right)$ are the costs of variety, quality, and quality of information; all costs are increasing.

A store will be truthful if for some $j=H, L$ :

$$
\Pi\left(q_{j}, \phi_{j} \mid \widehat{q_{j}}\right)>\Pi\left(q_{k}, \phi_{j} \mid \widehat{q_{j}}\right)
$$

for $k \neq j$.

Proposition 1 Price, assortment, and quality of information together are a signal for quality of the store iff $p_{H}>p_{L}$, and $p_{H}$ is such that for some realizations of information consumers won't buy.

Proof. is in Appendix 4.2.3
Proposition 1 says that if quality of information is endogenous then price, assortment, and quality of information will signal quality. If quality of information were fixed then price and variety alone are not enough to avoid the store deviating and offer low quality while charging high price. This is due to the fact that quality of information affects directly the marginal benefit of an assortment unit.

A store that cheats and offers low quality while charging high price will incur in very high costs due to the fact that she also needs to provide the bigger assortment and the greater quality of information that high quality will imply. This causes that at the optimal high quality assortment the marginal costs are smaller than the benefits.

An extreme case is when quality is so high that every consumer that walks in the store buys, no matter what was the realization of information. In this case, the store will provide a small assortment even if quality is high; an example of this are high-end restaurants that offer one fixed menu. If consumers belief that stores are being truthful then stores will have an incentive to deviate.

## 3 Conclusions

In this paper I present a model that helps to understand better how information overload can affect market structure. As regards to individual behavior the main insight of the model is that it studies the problem of information overload in a two-stage framework that allows to interpret the choice in the first stage as the choice of a store.

One important characteristic of the model is that it incorporates the psychological evidence of information overload and limited attention in a learning technology that has a trade-off between being better informed about each good and being informed about more goods.

The main result of the model is that a boundedly rational decision maker will find it optimal to restrict the number of objects he wants to learn from before making a choice.

This result is consistent and gives a better understanding of the experimental evidence about people deferring choice when there are too many options. There is extensive experimental evidence that supports the hypothesis that information overload creates conflict when making a choice. In this paper a better understanding of how this phenomena affects and is affected by other variables, such as quality of the store, complexity of the goods, quality of information, and pricing mechanisms is gained.

Another interesting finding is that size of the optimal assortment depends on how complex/easy is to learn about a good. If the process of learning about a good is very costly in terms of the attention needed to learn about it, fewer options will be considered.

This model shows that there are important consequences for stores that derived from limited attention in consumers. Stores, are the first step in the choice process, and the choice conditions of each store will determine whether a consumer will go there or not. I find that there is a negative relationship between the optimal number of choices DM wants and the quality of the store. However, unless the store has very high quality, it will increase its profits from increasing assortment, decreasing consumers utility.

The next step of the model is to see how preferences over stores affect market structure, when there is a competitive environment. The positive relation between quality of information on variety happens even when there is only one firm, but it seems that at least two firms are necessary to have the negative relation between assortment and average quality for any value of quality, and not only for high values.

On the other hand, there is a positive relation between quality of information and optimal number of options. Both, the consumer and the store are better off by increasing assortment when quality of information increases.

Whether consumers prefer small or large sets is mostly a consequence of how well informed they will be to choose the product. Not even high quality by itself would make consumers want more variety. Good consumer service and accessibility to information, seem to be essential for a consumer to choose more variety.

When quality is not observable, and the optimal price when quality is high is higher than the price when quality is low, high quality of information or better consumer service help the store to signal its true quality to the consumer. Only a high price is not enough for consumers to think that the store if of high quality, but a high price with good consumer service are enough. This is because the benefits per unit of a store that provides good consumer service but offers low quality are not as high as for the store that offers also high quality.

So far experimental evidence has been focused on the effects of larger sets on choice and on whether or not DM choose better in extensive choice conditions or not. But it has not been tested whether decision makers will internalize their bounded capacity optimally in their decision process and would prefer to restrain their choice set.

## 4 Appendix

### 4.1 Proofs to Section 2

### 4.1.1 Proof of Theorem 1

$$
\begin{aligned}
& E_{\ell}\left(V_{\ell}\right)=E_{\ell}\left[1-\beta_{\ell}^{n-1}(1-q)\right] \\
& f(\ell, n)=1-\beta_{\ell}^{n-1}(1-q) \text { is concave in } \ell: \\
& \frac{\partial\left(1-\beta_{\ell}^{n-1}(1-q)\right)}{\partial \ell}>0 \\
& \frac{\partial^{2}\left(1-\beta_{\ell}^{n-1}(1-q)\right)}{\partial \ell^{2}}=q \ln (1-\theta)^{2}(1-\theta)^{\ell}(n-1) \beta_{\ell}^{n-3}(1-q)\left(-(n-2) q(1-\theta)^{\ell}-\beta_{\ell}\right)<0
\end{aligned}
$$

By Jensen's inequality $E_{\ell}(f(\ell, n))$ is bounded above by a concave function in $n$ :
$E_{\ell}(f(\ell, n)) \leq f\left(E_{\ell}(\ell), n\right)=1-\left(q(1-\theta)^{m_{n} \alpha_{n}}+1-q\right)^{n-1}(1-q)$
$f\left(m_{n} \alpha_{n}, n\right)$ is concave in $n$.
$f\left(m_{n} \alpha_{n}, n\right)$ is bounded below by $q$.
To prove there is $N>0 \in \mathbb{N}$, such that for every $n>N, E_{\ell}(f(\ell, n))$ is decreasing and $E_{\ell}(f(\ell, n))<q+\varepsilon$ :
For a fix $m \in \mathbb{N}$;
$\frac{\partial E_{\ell}(f(\ell, n))}{\partial n}=-\binom{E_{l}\left[\beta_{\ell}^{n-1} \ln \beta_{\ell}\right]}{+E_{l}\left[\left(l \alpha_{n}^{-1} \frac{\partial \alpha_{n}}{\partial n}+\left(-\frac{\partial \alpha_{n}}{\partial n}\right)(m-l)\left(1-\alpha_{n}\right)^{-1}\right) \beta_{\ell}^{n-1}\right]}$
$\frac{\partial E_{\ell}(f(\ell, n))}{\partial n}$ is continuous in $\alpha_{n}$ and $\left.\frac{\partial E_{\ell}(f(\ell, n))}{\partial n}\right|_{\alpha_{n}=1}>0$ and $\left.\frac{\partial E_{\ell}(f(\ell, n))}{\partial n}\right|_{\alpha_{n}=0}<0$
Then for every strictly decreasing function $\alpha_{n}$ with $\lim _{n \rightarrow \infty} \alpha_{n} \rightarrow 0$
there is an $\bar{\alpha}$ such that $\left.\frac{\partial E_{\ell}(f(\ell, n))}{\partial n}\right|_{\alpha_{n}<\bar{\alpha}}<0$
also if $m<\left.m^{\prime} \Rightarrow \frac{\partial E_{\ell}(f(\ell, n))}{\partial n}\right|_{m}<\left.\frac{\partial E_{\ell}(f(\ell, n))}{\partial n}\right|_{m}$,
then, if $n^{\prime}>n$ and $\left.\frac{\partial E_{\ell}(f(\ell, n))}{\partial n}\right|_{n}<0$ then $\left.\frac{\partial E_{\ell}(f(\ell, n))}{\partial n}\right|_{n^{\prime}}<0$.
Therefore there is an $n$ that maximizes $E_{\ell}(f(\ell, n))$.

### 4.1.2 Proof of Lemma 1

To prove: if $q^{\prime}>q$ then $\left.\frac{\partial E_{\ell}(f(\ell, n))}{\partial n}\right|_{q} \geq\left. 0 \Rightarrow \frac{\partial E_{\ell}(f(\ell, n))}{\partial n}\right|_{q^{\prime}} \geq 0$
$\frac{\partial E_{l}\left[\beta_{\ell}^{n-1} \ln \beta_{\ell}\right]}{\partial q}=E_{l}\left[(n-1) \beta_{\ell}^{n-2}\left((1-\theta)^{\ell}-1\right) \ln \beta_{\ell}+\beta_{\ell}^{n-2}\left((1-\theta)^{\ell}-1\right)\right]<0$
then $0>E_{l}\left[\beta_{\ell}^{n-1} \ln \beta_{\ell}\right]_{q}>E_{l}\left[\beta_{\ell}^{n-1} \ln \beta_{\ell}\right]_{q^{\prime}}$
$\frac{\partial \frac{\partial m_{n}}{\partial n} E_{l}\left[\left(\sum_{k=0}^{l-1} \frac{1}{m_{n}-k}\right) \beta_{\ell}^{n-1}\right]}{\partial q}>0$

$$
\begin{aligned}
& 0>\frac{\partial m_{n}}{\partial n} E_{l}\left[\left(\sum_{k=0}^{l-1} \frac{1}{m_{n}-k}\right) \beta_{\ell}^{n-1}\right]_{q^{\prime}}>\frac{\partial m_{n}}{\partial n} E_{l}\left[\left(\sum_{k=0}^{l-1} \frac{1}{m_{n}-k}\right) \beta_{\ell}^{n-1}\right]_{q} \\
& \frac{\partial E_{l}\left[\left(l \alpha_{n}^{-1} \frac{\partial \alpha_{n}}{\partial n}+\left(\ln \left(1-\alpha_{n}\right)+\left(-\frac{\partial \alpha_{n}}{\partial n}\right)(n-l)\left(1-\alpha_{n}\right)^{-1}\right)\right) \beta_{\ell}^{n-1}\right]}{\partial q}<0 \\
& \left.\frac{\partial E_{\ell}(f(\ell, n))}{\partial n}\right|_{q} \geq 0 \Rightarrow \\
& \frac{\partial m_{n}}{\partial n} E_{l}\left[\left(\sum_{k=0}^{l-1} \frac{1}{m_{n}-k}\right) \beta_{\ell}^{n-1}\right]+E_{l}\left[\beta_{\ell}^{n-1} \ln \beta_{\ell}\right] \leq \\
& E_{l}\left[\left(l \alpha_{n}^{-1} \frac{\partial \alpha_{n}}{\partial n}+\left(\ln \left(1-\alpha_{n}\right)+\left(-\frac{\partial \alpha_{n}}{\partial n}\right)(n-l)\left(1-\alpha_{n}\right)^{-1}\right)\right) \beta_{\ell}^{n-1}\right] \\
& \frac{\partial m_{n}}{\partial n} E_{l}\left[\left(\sum_{k=0}^{l-1} \frac{1}{m_{n}-k}\right) \beta_{\ell}^{n-1}\right]<0 \text { and } E_{l}\left[\beta_{\ell}^{n-1} \ln \beta_{\ell}\right]<0
\end{aligned}
$$

therefore:

$$
\left.\frac{\partial E_{\ell}(f(\ell, n))}{\partial n}\right|_{q^{\prime}} \geq 0
$$

### 4.1.3 Proof of Lemma 2

To prove if $\theta^{\prime}>\theta$ then $\left.\frac{\partial E_{\ell}(f(\ell, n))}{\partial n}\right|_{\theta} \geq\left. 0 \Rightarrow \frac{\partial E_{\ell}(f(\ell, n))}{\partial n}\right|_{\theta^{\prime}} \geq 0$
$0>E_{l}\left[\beta_{\ell}^{n-1} \ln \beta_{\ell}\right]_{\theta}>E_{l}\left[\beta_{\ell}^{n-1} \ln \beta_{\ell}\right]_{\theta^{\prime}}$
$0>\frac{\partial m_{n}}{\partial n} E_{l}\left[\left(\sum_{k=0}^{l-1} \frac{1}{m_{n}-k}\right) \beta_{\ell}^{n-1}\right]_{\theta}>\frac{\partial m_{n}}{\partial n} E_{l}\left[\left(\sum_{k=0}^{l-1} \frac{1}{m_{n}-k}\right) \beta_{\ell}^{n-1}\right]_{\theta^{\prime}}$
and

$$
\begin{aligned}
0 & >E_{l}\left[\left(l \alpha_{n}^{-1} \frac{\partial \alpha_{n}}{\partial n}+\left(\ln \left(1-\alpha_{n}\right)+\left(-\frac{\partial \alpha_{n}}{\partial n}\right)(n-l)\left(1-\alpha_{n}\right)^{-1}\right)\right) \beta_{\ell}^{n-1}\right]_{\theta^{\prime}} \\
& >E_{l}\left[\left(l \alpha_{n}^{-1} \frac{\partial \alpha_{n}}{\partial n}+\left(\ln \left(1-\alpha_{n}\right)+\left(-\frac{\partial \alpha_{n}}{\partial n}\right)(n-l)\left(1-\alpha_{n}\right)^{-1}\right)\right) \beta_{\ell}^{n-1}\right]_{\theta}
\end{aligned}
$$

therefore:

$$
\left.\frac{\partial E_{\ell}(f(\ell, n))}{\partial n}\right|_{q^{\prime}} \geq 0
$$

### 4.1.4 Proof of Lemma 3

$b\left(m_{n}, \alpha_{n}\right)$ and $\widehat{b}\left(\widehat{m}_{n}, \widehat{\alpha}_{n}\right)$
$\left(m_{n}, \alpha_{n}\right) \geq\left(\widehat{m}_{n}, \widehat{\alpha}_{n}\right)$
$\left.\frac{\partial E_{\ell}(f(\ell, n))}{\partial n}\right|_{\left(m_{n}, \alpha_{n}\right)}<\left.0 \Rightarrow \frac{\partial E_{\ell}(f(\ell, n))}{\partial n}\right|_{\left(\widehat{m}_{n}, \widehat{\alpha}_{n}\right)}<0$
therefore $n \geq \widehat{n}$

### 4.2 Store Decisions:

Stores:
Profits can be rewritten as:
$\Pi(q, n, p)=p_{\ell}\left(1-\sum_{j=\ell+1}^{m}\binom{m}{j} \alpha_{n}^{j}\left(1-\alpha_{n}\right)^{m-j} \beta_{j}^{n}\right)-c(n)-g(q)$

As long as price is smaller than one, $\left(1-\sum_{j=\ell+1}^{m}\binom{m}{j} \alpha_{n}^{j}\left(1-\alpha_{n}\right)^{m-j} \beta_{j}^{n}\right)$ is increasing in $n$ Marginal benefit of adding one more element:
$p_{\ell}\left(\sum_{j=1}^{m}\binom{m}{j} \beta_{j}^{n}\left(\alpha_{n}^{j}\left(1-\alpha_{n}\right)^{m-j}-\alpha_{n+1}^{j}\left(1-\alpha_{n+1}\right)^{m-j} \beta_{j}\right)\right)$
marginal cost of adding one more element: $c^{\prime}(n)$.

Lemma 6 Higher quality imply bigger/smaller optimal assortment for the store (if demand is smaller than one/one).

Proof. If price is such that demand is less than one:
As quality increases, profits increase by two channels:

1. $\beta_{j}^{n}$ decreases for every $j$.The greater $j$ the bigger the decrease.
2. Price increases

This means that the marginal benefit of each $n$ is increasing in $q$.
The condition to set the optimal $n$ is:
$p_{\ell} D_{n}=c^{\prime}(n)$
when $q$ changes:
$\frac{\partial p_{\ell}}{\partial q} D_{n}+p_{\ell} \frac{\partial D_{n}}{\partial q}$
$\frac{\partial p_{\ell}}{\partial q} D_{n}$ positive and decreases as quality increases
$p_{\ell} \frac{\partial D_{n}}{\partial q}$ is positive and decreases as quality increases
Therefore if demand is less than one: as $q$ increases optimal assortment is at least as big
If quality is high enough such that demand is already one: optimal assortment decreases.

### 4.2.1 Proof of Lemma 4

$\Pi(q, n, p)=p_{\ell}\left(1-\sum_{j=\ell+1}^{m}\binom{m}{j} \alpha_{n}^{j}\left(1-\alpha_{n}\right)^{m-j} \beta_{j}^{n}\right)-c(n)-g(q)$
To prove lower marginal costs imply higher quality:
if $g_{1}(q)<g_{2}(q)$ for every $q$ and $g_{1}^{\prime}(q)<g_{2}^{\prime}(q)$
optimal decision for the firm to invest in quality is:
$p_{\ell} D_{q}+p_{\ell q} D-c^{\prime}(n) \frac{\partial n}{\partial q}=g^{\prime}(q)$
In the optimum when costs are high: $\left(n_{2}, q_{2}, p_{2}\right)$
$p_{\ell} D_{q}+p_{\ell q} D-c^{\prime}(n) \frac{\partial n}{\partial q}>g_{1}^{\prime}(q)$
$\frac{\partial\left(p_{\ell} D_{q}+p_{\ell q} D\right)}{\partial q}$ is decreasing in $q$, and $c^{\prime}(n) \frac{\partial n}{\partial q}$ is increasing in $q$. So increasing $q$ is optimal.
To prove higher quality implies higher price:
$p_{\ell}$ is increasing in $q$.If for the new quality it is still optimal to price $\ell$ we are done.
If for $q$ low, the optimal price is $p_{\ell}^{L}$ and for $q_{H}$ it is optimal to price $p_{\ell+k}^{H}$ and $p_{\ell+k}^{H}<p_{\ell}^{L}$
If instead of charging $p_{\ell}^{L}$ the store charges $p_{\ell+1}^{H}$
revenues will
a) decrease by: $\left(p_{\ell}^{L}-p_{\ell+k}^{H}\right) D_{\ell}^{L}<\left(p_{\ell}^{H}-p_{\ell+k}^{H}\right) D_{\ell}^{H}$
b) increase by: $p_{\ell+k}^{H} x_{L}>p_{\ell+k}^{H} x_{H}$

Where $x_{j}=\binom{m}{k+1} \alpha_{n}^{k+1}\left(1-\alpha_{n}\right)^{m-k-1} \beta_{k+1, j}^{n}$
This implies that it would not be profit maximizing either to charge $p_{\ell+1}^{H}$ when quality is higher.

### 4.2.2 Proof of Lemma 5

The marginal revenue of one more product $n$ when $\theta$ increases is:
$\frac{\partial p_{\ell}}{\partial \theta} D_{n}+p_{\ell} \frac{\partial D_{n}}{\partial \theta}$
$p_{\ell}=q$ and quality is fixed.

$$
\frac{\partial D_{n}}{\partial \theta}>0
$$

therefore the marginal revenue of $n$ is increasing in $\theta$ and the optimal assortment when $\theta$ increases is greater.

If for some $\ell: \frac{\partial p_{\ell}}{\partial \theta} D_{n}+p_{\ell} \frac{\partial D_{n}}{\partial \theta}<0$ then the optimal would be to reduce $n:\left(q, \theta, n_{L}, p_{\ell}\right)$, but there exists an $\ell^{\prime}<\ell$ such that $\frac{\partial p_{\ell^{\prime}}}{\partial \theta} D_{n}+p_{\ell^{\prime}} \frac{\partial D_{n}}{\partial \theta}>0$, and profits are higher:
the optimal would be to increase $n:\left(q, \theta, n_{h}, p_{\ell-k}\right)$
$\Pi\left(q, \theta, n_{h}, p_{\ell-k}\right)>\Pi\left(q, \theta, n_{L}, p_{\ell}\right)$
$D\left(q, \theta, n_{h}, p\right)>D\left(q, \theta, n_{L}, p\right)$ for every price $p$
$D\left(q, \theta, n_{h}, p_{\ell-k}\right)>\Pi\left(q, \theta, n, p_{\ell-k}\right)>\Pi\left(q, \theta, n, p_{\ell}\right)$
$\Pi\left(q, \theta, n, p_{\ell-k}\right)=p_{\ell-k} D\left(q, \theta, n, p_{\ell-k}\right)-c(n)$
$\frac{\partial p_{\ell^{\prime}}}{\partial \theta} D_{n}+p_{\ell^{\prime}} \frac{\partial D_{n}}{\partial \theta}>0$ and
For every $n<n_{L}: p_{\ell-k} D_{n}>p_{\ell} D_{n}$ so
$\Pi\left(q, \theta, n_{h}, p_{\ell-k}\right)>\Pi\left(q, \theta, n_{L}, p_{\ell}\right)$

### 4.2.3 Proof of Proposition 1

Let $\phi_{j}^{*}=\left(p_{j}, n_{j}, \theta_{j}\right)=\arg \max _{\phi} \Pi\left(q_{j}, \phi\right) \quad$ and $p_{H}>p_{L}$
To prove there is an equilibrium such that:
i) $\widehat{q}\left(\phi_{j}^{*}\right)=q_{j}$
ii) $\phi \in\left\{\phi_{H}^{*}, \phi_{L}^{*}\right\}$

First we prove that for any $p_{H}>p_{L}$ the firm won't have incentives to deviate and cheat:
Case 1: If $p_{H}=q_{H}$ and $p_{L}=q_{L}$ :
If the firm chooses $q_{L}$, and $p_{H}, \theta_{H}, n_{H}$ :
$p_{H}\left(1-\sum_{j=1}^{m}\binom{m}{j} \alpha_{n_{H}}^{j}\left(1-\alpha_{n_{H}}\right)^{m-j}\left(q_{L}\left(1-\theta_{H}\right)^{j}+1-q_{L}\right)^{n_{H}}\right)-c\left(n_{H}\right)-g\left(q_{L}\right)-c_{1}\left(\theta_{H}\right)<$
$p_{H}\left(1-\sum_{j=1}^{m}\binom{m}{j} \alpha_{n_{H}}^{j}\left(1-\alpha_{n_{H}}\right)^{m-j}\left(q_{H}\left(1-\theta_{H}\right)^{j}+1-q_{H}\right)^{n_{H}}\right)-c\left(n_{H}\right)-g\left(q_{H}\right)-c_{1}\left(\theta_{H}\right)$
This happens because $n_{H}>n_{L}$ and the marginal benefit for each unit is smaller when he cheats. Due to the fact that they have to offer $n_{H}$ so that they can trick the consumers, then they have to offer assortment up to a point where marginal costs exceeds the marginal benefits.

$$
\begin{aligned}
& p_{H} D_{n}^{H, L}<c^{\prime}\left(n_{H}\right) \\
& p_{H} D_{\theta}^{H, L}<c_{1}^{\prime}\left(\theta_{H}\right) \\
& p_{H} D_{q}^{H, L}>g^{\prime}\left(q_{L}\right)
\end{aligned}
$$

Quality has to increase up to $q_{H}$ to get to the maximization of profits.
Case 2: If $p_{H}=p_{\ell}$ and $p_{L}=p_{\ell+1}$, and Case 3: If $p_{H}=1$ and $p_{L}=p_{\ell+1}$ are similar.
If $p_{H}$ is such that any consumer that enters the store buys: $p_{H}=p_{m}$ and $p_{L}=p_{\ell}$
This case only happens when $q_{H}$ is so high that it is optimal for the store to charge the lowest price and have a demand of one. Note that even though the high quality price is such that every consumer buys, still $p_{H}>p_{L}$. In this case $n_{H}<n_{L}$

$$
p_{H}-c\left(n_{H}\right)-g\left(q_{L}\right)-c_{1}\left(\theta_{H}\right)>p_{H}-c\left(n_{H}\right)-g\left(q_{H}\right)-c_{1}\left(\theta_{H}\right)
$$

So the store will have incentives to deviate.

## References

[1] Baddeley, A., 1986. Working Memory. Clarendon Press, Oxford.
[2] Boatwright, P., Nunes, J., 2001. Reducing assortment: An attribute-based approach. J. Marketing 65 (july) 50-63.
[3] Gabaix, X., Laibson, D., 2003. Bounded rationality and directed cognition. Unpublished working paper. Massachusetts Institute of Technology and Harvard University, Cambridge, MA.
[4] Gabaix, X., Laibson, D., Moloche, G., Weinberg, S., 2003. The allocation of attention: theory and evidence. Unpublished working paper. Massachusetts Institute of Technology and Harvard University, Cambridge, MA.
[5] Gerstner, E. 1985. Do Higher Prices Signal Higher Quality? Journal of Marketing Research, 22 (2), 209-215.
[6] Gourville, J., Soman, Dilip., 2005. Overchoice and Assortment Type: When and Why Variety Backfires. Marketing Science 24 (3), 382-395.
[7] Hauser, J. and Wernerfelt, B., 1990. An Evaluation Cost Model of Consideration Sets. Journal of Consumer Research 16, 393-408.
[8] Iyengar, S. and Lepper, M., 2000. When Choice is Demotivating: Can One Desire Too Much of a Good Thing? Journal of Personality and Social Psychology, 79 (6), 995-1006.
[9] Jacoby, J., Speller, D., Berning,C., 1974. Brand Choice Behavior as a Function of Information Load: Replication and Extension. The Journal of Consumer Research 1 (1), 33-42.
[10] Keller, K., Staelin, R., 1987. Effects of Quality and Quantity of Information on Decision Effectivenenss. The Journal of Consumer Research 14 (2), 200-213.
[11] Lapersonne, E., Laurent, G., Le Goff, J., 1995. Consideration sets of size one: An empirical investigation of automobile purchases. Inern. J. of Research in Marketing 12, 55-66.
[12] Lavie, Nilli., 2005. Distracted and confused?: Selective attention under load. TRENDS in Cognitive Science 9 (2), 75-82.
[13] McDowd, J. and Craik, F., 1988. Effects of Aging and Task Difficulty on Divided Attention Performance. Journal of Experimental Psychology: Human Perception and Performance 14 (2), 267-280
[14] Pashler, H., Johnston, J., 1998. Attentional limitations in dual-task performance. In: Pashler, H. (Ed.), Attention. Psychology Press, Hove, UK.
[15] Rao, A., Monroe, K. 1989. The Effect of Price, Brand Name, and Store Name on Buyers' Perceptions of Product Quality: An Integrative Review. Journal of Marketing Research, 26 (3), 351-367.
[16] Rohrer, D., Pashler, H., 2003. Concurrent task effects on memory retrieval. Psychonomic Bulletin \& Review 10 (1), 96-103.
[17] Shapiro, J. A "Memory Jamming" Theory of advertising, University of Chicago, May 2006.
[18] Wilson, A. "Bounded Memory and Biases in Information Processing." Harvard University, Mimeograph, April 2006.About This Shell


[^0]:    ${ }^{*}$ I am extremely grateful to Wolfgang Pesendorfer for his guidance. I thank Marco Battaglini and Faruk Gul for their useful advice. I thank Julio Cacho, Sylvain Chassang, David Dillenberger,Wiola Dziuda, Sambuddha Ghosh, Rainer Schwabe, and Satoru Takahashi, for their support and comments.
    ${ }^{\dagger}$ Dirección General de Investigación Económica. Email: kkaiser@banxico.org.mx.

