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Income redistribution going awry: The reversal power of the concern for relative deprivation

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Gerhard Sorger and Oded Stark

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Abstract

We demonstrate that a rank-preserving transfer from a richer individual to a poorer individual can exacerbate income inequality (when inequality is measured by the Gini coefficient). This happens when individuals' preferences depend negatively not only on work time (effort) but also on low relative income. It is rigorously shown that the set of preference profiles that gives rise to this perverse effect of a transfer on inequality is a non-empty open subset of all preference profiles. A robust example illustrates this result.

Keywords: A rank-preserving inequality-narrowing transfer; The Gini coefficient; Low relative income; Relative deprivation; Exacerbated inequality

JEL classification: D30; D31; D63

1. Motivation

There is good reason to expect that a rank-preserving transfer from a richer individual to a poorer individual will reduce inequality. This point was made a century ago by Pigou (1912), and nearly a century ago by Dalton (1920). For example, Dalton (1920, p. 351) wrote: "[I]f there are only two income-receivers, and a transfer of income takes place from the richer to the poorer, inequality is diminished. There is, indeed, an obvious limiting condition. For the transfer must not be so large, as more than to reverse the relative positions of the two income-receivers, and it will produce its maximum result, that is to say, create equality, when it is equal to half the difference between the two incomes." Over the years, this perceptive statement has been adapted to populations of any size and has assumed the status of an essential property for any admissible index of inequality (see, for example, Weymark, 2006). The modern literature that resorts to the Pigou-Dalton transfer principle in inequality measurement is extensive. To name but few of the leading studies: Atkinson (1970), Sen (1973), Kolm (1977), Blackorby and Donaldson (1978, 1980), Donaldson and Weymark (1980), Kakwani (1980), Weymark (1981), Ebert (1984, 1987), Foster (1983), Shorrocks (1980, 1984), Foster and Shorrocks (1988), Chakravarty (1988), Bossert (1990), Lambert and Aronson (1993), and Cowell (1995).

It is not an exaggeration to say that the Pigou-Dalton transfer principle is the cornerstone of inequality measurement theory. Yet, as we show in this paper, the widely accepted requirement to obey the principle may not hold if the individuals adjust their behavior in response to the transfer. In that case, there can be rank-preserving transfers from a richer individual to a poorer individual that increase the income disparity between them. This is a new problem and, because it questions the very basis of inequality measurement, it opens up a new research domain in the literature of inequality measurement.

To see this vividly, consider the Gini coefficient (Gini, 1912), which is arguably the most popular index of inequality. In population $N = \{1, 2, \ldots, n\}$, $n \geq 2$, let y_i be the income of individual i. Let the incomes be ordered: $y_1 \leq y_2 \leq \ldots \leq y_n$. The Gini coefficient of the population, G, is given by

$$G = \frac{\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (y_j - y_i)}{n \sum_{i=1}^{n} y_i}. \qquad (1)$$

Consider now a rank-preserving (small) transfer $\tau > 0$ from individual r (the rich) to individual p (the poor), where $p < r$, such that the post-transfer incomes $x_r = y_r - \tau$, $x_p = y_p + \tau$, and $x_i = y_i$ for all $i \in N \setminus \{p, r\}$ satisfy $x_1 \leq x_2 \leq \ldots \leq x_n$ (that is, the transfer preserves the ranking of the individuals according to their income). For the Gini coefficient of the post-transfer income distribution, it holds that[1]

$$G(\tau) = \frac{\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(x_j - x_i)}{n\sum_{i=1}^{n} x_i} = \frac{\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(y_j - y_i)}{n\sum_{i=1}^{n} y_i} - \frac{2(r-p)\tau}{n\sum_{i=1}^{n} y_i} < G(0).$$

This is what conventional wisdom and the received literature have led us to expect: a rank-preserving transfer from the rich to the poor reduces inequality. However, a possible perverse outcome might have been overlooked: once the *reason* for the initial income gap $y_r - y_p > 0$ is factored in and the adjustment of behavior in the wake of the transfer is taken into account, it could as well be the case that income inequality will be exacerbated rather than diminished. To demonstrate this possibility, we resort to distaste for low relative income and the associated concepts of relative deprivation and reference groups. We use a measure of relative deprivation based on the seminal work of Runciman (1966), and proposed by Yitzhaki (1979). We note that since the 1960s, a considerable body of research has demonstrated empirically that interpersonal comparisons of income (that is, comparisons of the income of an individual with the incomes of higher income members of his reference group) bear significantly on the perception of well-being, and on behavior.[2]

The relative deprivation of individual $i \in N$, whose (post-transfer) income is x_i, is defined as

$$RD_i \equiv \begin{cases} (1/n)\sum_{j=i+1}^{n}(x_j - x_i) & \text{if } i < n, \\ 0 & \text{if } i = n, \end{cases} \tag{2}$$

and the aggregate relative deprivation of the population is defined as

[1] Note that by the very nature of the transfer, $\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i$.

[2] For a review see Clark et al. (2008). Additional references are provided in Section 2 below as well as in Appendix 1.

$$TRD_N \equiv \sum_{i=1}^{n-1} RD_i = (1/n)\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(x_j - x_i).$$

From the expression of the Gini coefficient in (1) we get that[3]

$$G(\tau) = \frac{TRD_N}{\sum_{i=1}^{n} x_i}. \tag{3}$$

The explanation for the unexpected outcome of a rank-preserving transfer from the rich to the poor that we present in this paper is that the poor seek income for two reasons: to obtain income "for its own sake," and to obtain income in order to hold at bay relative deprivation. When income is taken away from the rich, the relative deprivation sensed by the poor is reduced, and his incentive to work in order to maintain a "bearable level" of relative deprivation is correspondingly weakened. Add to this the additional reduction in relative deprivation for the poor from receiving that very income that is taken away from the rich. To take an example: when the population N consists of just two individuals with post-transfer incomes x_1 and x_2 such that $x_2 \geq x_1$, we have that $TRD_N = (x_2 - x_1)/2$, and from (3) it follows that

$$G(\tau) = \frac{x_2 - x_1}{2(x_2 + x_1)}. \tag{4}$$

In this simple case of two individuals, it is obvious that the transfer reduces total relative deprivation, that is, the numerator in (4) is smaller than the pre-transfer value $y_2 - y_1$. But what happens to the denominator? It is reasonable to suppose that the rich individual adjusts his working time (effort) such that he will not be subjected to as great a reduction in income as has been taken away from him yet that the adjustment falls short of neutralizing the (negative) transfer. The poor individual will surely scale back his working time. In combination, the two individuals working less than before implies that the sum of their incomes (the denominator of the Gini coefficient in (4)) is smaller than the corresponding pre-transfer value $2(y_2 + y_1)$. If the reduction in total income in the denominator dominates the reduction in total deprivation in the numerator, income inequality as measured by the Gini coefficient will worsen.

[3] In equation (1) we presented the Gini coefficient as a function of the pre-transfer incomes y_i, $i \in N$. Here, in equation (3), we already adjust for the transfers and, hence, express the Gini coefficient as a function of the post-transfer incomes x_i, $i \in N$.

2. A robust example

To obtain rigorously the possibility illustrated heuristically above, we consider an economy in which the individuals produce a single consumption good. Specifically, we assume that individual $i \in N$ transforms labor (costly effort) into output of the consumption good at the rate of one-to-one. Individual i has preferences that are described by the utility function $U^i(c_i, RD_i, y_i)$, where c_i denotes the consumption of individual i, RD_i is the relative deprivation of individual i given in (2), and y_i is the effort exerted by individual i (which is equal to his output). Suppose that there is a transfer of size $\tau \geq 0$ from a rich individual r to a poor individual p, where $1 \leq p < r \leq n$. The post-transfer income of individual i is defined by

$$x_i \equiv \begin{cases} y_i + \tau & \text{for } i = p, \\ y_i - \tau & \text{for } i = r, \\ y_i & \text{for } i \notin \{p, r\}. \end{cases} \tag{5}$$

Individual i maximizes

$$U^i(c_i, RD_i, y_i)$$

with respect to c_i, y_i, and x_i, subject to (2), (5), and the budget constraint

$$c_i \leq x_i. \tag{6}$$

With regard to the utility functions, we assume that U^i maps $(0, +\infty) \times \mathbf{R}_+^2$ to $\mathbf{R}$, is twice continuously differentiable, concave, strictly increasing in c_i, and strictly decreasing in RD_i and in y_i. Let U be the set of all N-tuples $\mathbf{U} = (U^1, U^2, \ldots, U^n)$ of functions with these properties and endow U with the metric

$$\mathbf{d}(\mathbf{U}, \tilde{\mathbf{U}}) = \max\{d_k(U^i, \tilde{U}^i) \mid i \in N, k \in \{0,1,2\}\},$$

where $d_k(f, g)$ is the supremum norm of the k-th derivative of $f - g$. In words, two profiles $\mathbf{U}$ and $\tilde{\mathbf{U}}$ are close to each other if all their n components as well as the corresponding first and second partial derivatives are close in the supremum norm. We can now state our main result.

Theorem 1. There exists an open set $\mathrm{V} \subset \mathrm{U}$ and a number $\bar{\tau} > 0$ such that the Gini coefficient $G(\tau)$ is strictly increasing with respect to the size of the transfer τ for all $\tau \in [0, \bar{\tau})$ and all $\mathbf{U} \in \mathrm{V}$.

The proof of this Theorem, which is presented in Appendix 2, is a constructive one. Specifically, we present a robust example of an economy in which the Gini coefficient depends *positively* on the size of the transfer. It is clear from the reasoning in Section 1 that Theorem 1 requires dependence of the individuals' preferences on relative deprivation. The following example illustrates our main result. It is derived by the very same constructive approach that underlies the proof of Theorem 1, and it indicates that the effect of relative deprivation on the overall utility of an individual need not be excessively large. Specifically, when in this example we compute the marginal rate of substitution (MRS) between consumption and relative deprivation, and the MRS between effort and relative deprivation, we find that all the marginal rates of substitution are larger than one (in absolute value), such that the individuals would be willing to trade a reduction of relative deprivation by one unit against an increase of consumption by less than one unit, or against a reduction of effort by less than one unit. In other words, the individuals in this example value relative deprivation strictly less than either consumption or effort.

Example. Consider the case of $n = 5$ individuals, and suppose that their preferences are described by the utility functions

$$U_i(c_i, RD_i, y_i) = \begin{cases} (17/25)c_1 - (1/2)(RD_1)^2 - y_1 & \text{if } i = 1, \\ (107/125)c_2 - (1/2)(RD_2)^2 - y_2 & \text{if } i = 2, \\ (99/25)\ln(c_3) - RD_3 - (1/2)y_3^2 & \text{if } i = 3, \\ (124/125)c_4 - (1/2)(RD_4)^2 - y_4 & \text{if } i = 4, \\ (13/5)c_5 - (1/2)(RD_5)^2 - (1/2)y_5^2 & \text{if } i = 5. \end{cases}$$

If there is no transfer, the unique equilibrium results in income $x_i = (8+i)/5$ for every $i \in \{1, 2, \ldots, 5\}$, and the Gini coefficient is $G(0) = 4/55$.

Now assume that there is a transfer from the rich individual $r = 3$ to a poor individual $p \in \{1, 2\}$ (which of the two poor individuals receives the transfer is irrelevant in this particular example). Figure 1 shows the income levels of all five individuals for all transfers $\tau \in [0, 1/2)$, starting with x_1 at the bottom and ending with x_5 at the top. Figure 2 shows the

corresponding values of the Gini coefficient. It can be seen that the Gini coefficient is strictly increasing for the entire displayed parameter range.

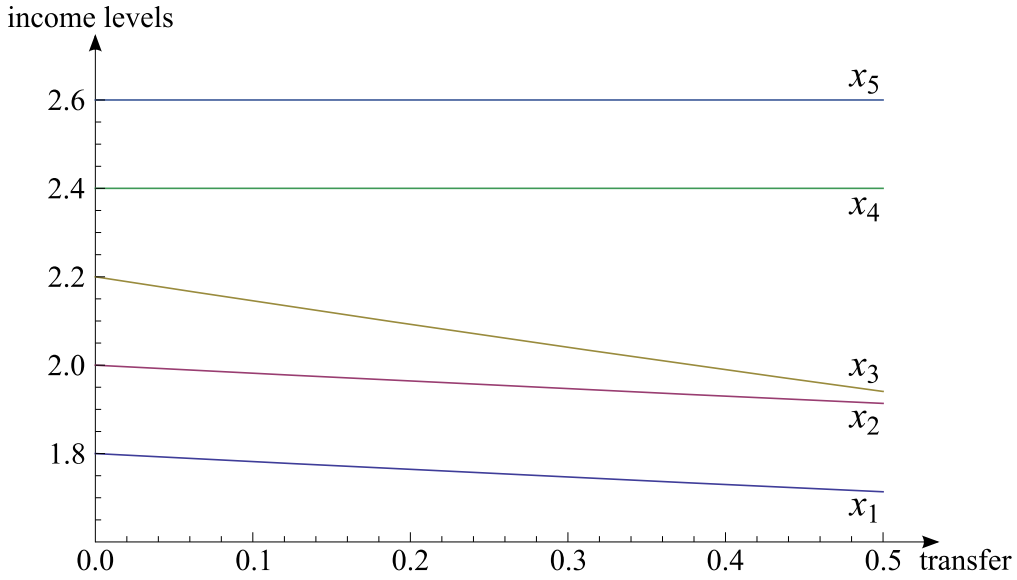

Figure 1: Income levels as functions of the transfer.

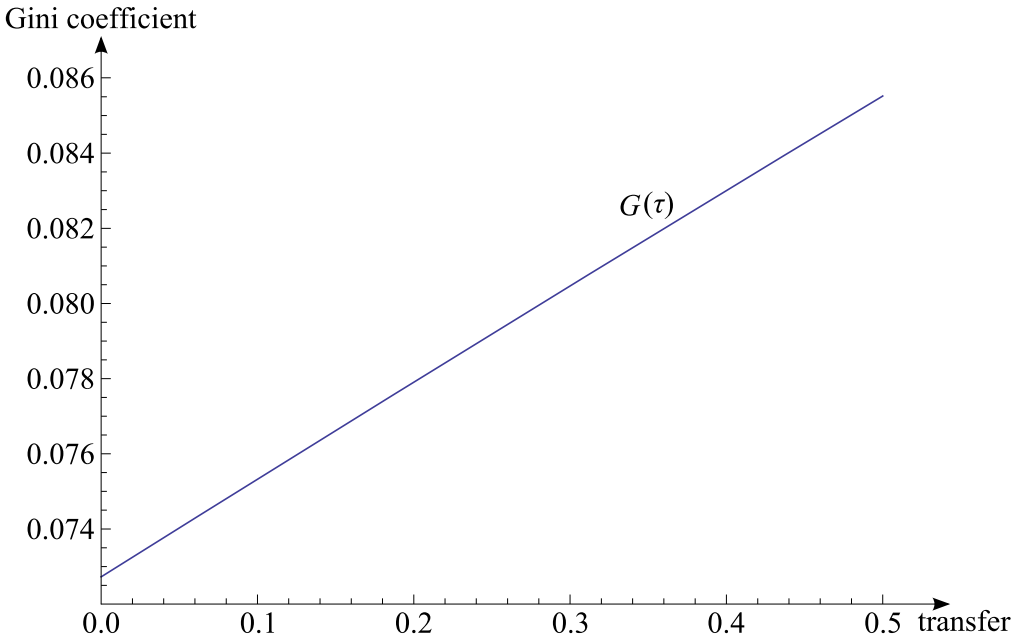

Figure 2: The Gini coefficient as a function of the transfer.

Theorem 1 not only states that there exist examples like the one shown above, but that these examples are robust. As a consequence, the property of a strictly increasing Gini coefficient holds for all economies in which the utility functions are small perturbations of those in the example.

3. Conclusion

The result obtained in this paper is of considerable significance in the spheres of inequality measurement and social welfare. The prospect that a Pigou-Dalton transfer - a rank-preserving marginal transfer from a richer individual to a poorer individual - exacerbates (rather than reduces) inequality will require social planners and policy makers to evaluate closely the preferences of individuals (and the individuals' expected behavioral responses) before subjecting the individuals to policy measures (tax and transfer) which could lead to outcomes that are orthogonal to the conventionally expected ones. If a Pigou-Dalton transfer ends up increasing inequality, then, upon such a transfer, an equality-favoring social welfare function will record a loss. The very choice of the social welfare function could then be affected: although a Pigou-Dalton transfer can entail a social welfare loss when the Gini coefficient registers an increase and social welfare is "standard" egalitarian, if social welfare is utilitarian and incorporates the individuals' distaste for low relative income, social welfare need not decline. Furthermore, demonstrating that a Pigou-Dalton transfer fails to decrease income inequality could imply that a more stringent transfer principle will be needed to secure reduced inequality.

Appendix 1: A brief foray into relative deprivation

Our analysis is based on the sociological-psychological concepts of relative deprivation and reference groups, which are fitting tools for studying comparisons that affect an individual's behavior, in this case comparisons with related individuals whose incomes are higher than his own income (cf. the large literature spanning from Duesenberry, 1949, up to, for example, Clark et al., 2008). An individual has an unpleasant sense of relative deprivation when he lacks a desired good and perceives that others in his reference group possess that good; see Runciman (1966).[4] Given the income distribution of the individual's reference group, the individual's relative deprivation is the sum of the deprivation caused by every income unit that he lacks (Yitzhaki, 1979; Hey and Lambert, 1980; Ebert and Moyes, 2000; Bossert and D'Ambrosio, 2006; and Stark and Hyll, 2011).

The pioneering study in modern times that opened the flood-gate to research on relative deprivation and primary (reference) groups is the two-volume set of Stouffer et al.

[4] In Runciman's (1966) theory of *RD*, an individual's reference group is the group of the individuals with whom the individual compares himself; cf. Singer (1981).

(1949a, 1949b). That work documented the dissatisfaction caused not by a given low military rank and weak prospects of promotion (military police) but rather by the pace of promotion of others (air force), and the lesser dissatisfaction of black soldiers stationed in the South who compared themselves to the local South black civilians than the dissatisfaction of their counterparts stationed in the North who compared themselves to the local North black civilians. Stouffer's research was followed by a large social-psychological literature. Economics has caught up relatively late, and only somewhat. This is rather surprising because eminent economists in the past understood well that people compare themselves to others around them, and that social comparisons are of paramount importance for individuals' happiness, motivations, and actions. Even Adam Smith (1776) pointed to the social aspects of the necessities of life, and stressed the *relative* nature of poverty: "A linen shirt, for example, is, strictly speaking, not a necessary of life. The Greeks and Romans lived, I suppose, very comfortably, though they had no linen. But in the present times, through the greater part of Europe, a creditable day-laborer would be ashamed to appear in public without a linen shirt, the want of which would be supposed to denote that disgraceful degree of poverty […]." (p. 465). Karl Marx's (1849) observations that "Our wants and pleasures have their origin in the society; [… and] they are of a relative nature" (p. 33) emphasize the social nature of utility, and the impact of an individual's relative position on his satisfaction. Inter alia, Marx (1849) wrote: "A house may be large or small; as long as the surrounding houses are equally small, it satisfies all social demands for a dwelling. But if a palace arises beside the little house, the house shrinks into a hut" (p. 33). Paul Samuelson (1973), one of the founders of modern neoclassical economics, pointed out that an individual's utility does not depend only on what he consumes in *absolute* terms: "Because man is a social animal, what he regards as 'necessary comforts of life' depends on what he sees others consuming." (p. 218).

The relative income hypothesis, formulated by Duesenberry (1949), posits an asymmetry in the comparisons of income which affect the individual's behavior: the individual looks upward when making comparisons.[5] Thorstein Veblen's (1899) concept of *pecuniary emulation* explains why the behavior of an individual can be influenced by comparisons with the incomes of those who are richer. Because income determines the level of consumption, higher income levels may be the focus for emulation. Thus, an individual's

[5] The empirical findings support the relative income hypothesis. Duesenberry (1949) found that individuals' savings rates depend on their positions in the income distribution, and that the incomes of the richer people affect the behavior of the poorer ones (but not *vice versa*). Schor (1998) showed that, keeping annual and permanent income constant, the individuals whose incomes are lower than the incomes of others in their community saved significantly less than those who are relatively better off in their community.

income aspirations (to obtain the income levels of other individuals whose incomes are higher than his own) are shaped by the perceived consumption standards of the richer. In that way, invidious comparisons affect behavior, that is, behavior which leads to "the achievement of a favourable comparison with other men [...]"; see Veblen (1899, p. 33).

Modern day evidence from econometric studies, experimental economics, social psychology, and neuroscience indicates that humans routinely compare themselves with other individuals who constitute their "comparison" or "reference" group, and that the outcome of that engagement impinges on their sense of well-being. People are dissatisfied when their consumption, income, or social standing fall below those of others with whom they naturally compare themselves (those who constitute their "reference group"). Consequently, economic processes are impacted, and economic realizations differ from what they would have been had comparisons with others not mattered. Clark and Senik (2010) reviewed data collected in 2006-2007 as part of Wave 3 of the European Social Survey. Their analysis of a usable sample of around 19,000 observations for 18 countries reveals that income comparisons are acknowledged as at least somewhat important by a majority of Europeans; are mostly upward; and are associated with lower levels of happiness. In principle, there are (at least) five basic responses to the sensing of discontent or dismay from having an income that is lower than the incomes of others with whom comparisons are made: exerting more effort, exiting (migrating), acquiring better skills (enhancing productivity), demanding transfers by means of political redistribution, and sabotaging (the performance of) others. Examples of such responses are in Stark and Taylor (1991), Zizzo and Oswald (2001), Luttmer (2005), Fliessbach et al. (2007), Blanchflower and Oswald (2008), Takahashi et al. (2009), Stark and Fan (2011), Stark and Hyll (2011), Fan and Stark (2011), and Stark et al. (2012). The specific response depends on individual perceptions, preferences, and capabilities; on the nature of the economic and social environment; on the set of opportunities; on the time frame; and on the social and cultural norms. In our definition of relative deprivation we resort to income-based comparisons, namely, an individual feels relatively deprived when others in his comparison group earn more than he does.

The theoretical possibility that behavior is modulated by individuals deriving satisfaction from looking down, rather than only by them experiencing deprivation from looking up, does not appear to have much of a basis. Andolfatto (2002) argues that while the utility of an individual rises in his own consumption, it declines in the consumption of any of his neighbors if that consumption falls below some minimal level; individuals are adversely

affected by the material well-being of others in their reference group when this well-being is sufficiently lower than theirs. Already a decade ago, Frey and Stutzer (2002) and Walker and Smith (2002) marshaled a large body of evidence that overwhelmingly supports the ‘‘upward comparison’’ view.

Appendix 2: Proof of Theorem 1

The general idea of the proof is to construct an example and to show that this example is robust with respect to small perturbations of the preference profile $\mathbf{U}$. In what follows, we proceed under the assumption (to be verified later) that the incomes are always ordered as $x_1 < x_2 < \ldots < x_n$. The assumption means that in the income hierarchy, the transfer τ does not alter the ranking of the individuals by their incomes. We only elaborate on the case where $r < n$. The alternative case $r = n$ can be dealt with in a similar way.

Consider a preference profile given by

$$U_i(c_i, RD_i, y_i) = \begin{cases} A_n c_n - (1/2)(RD_n)^2 - (1/2)y_n^2 & \text{if } i = n, \\ A_r \ln(c_r) - RD_r - (1/2)y_r^2 & \text{if } i = r, \\ A_i c_i - (1/2)(RD_i)^2 - y_i & \text{otherwise,} \end{cases}$$

where A_1, A_2, ..., A_n are positive real numbers to be determined later. Obviously, it holds that $\mathbf{U} \in \mathrm{U}$. Using conditions (2) and (5), and noting that the budget constraint (6) must hold as an equality, we can eliminate the variables c_i, y_i, and RD_i. This implies, for example, that individual n chooses x_n in order to maximize

$$A_n x_n - (1/2)x_n^2.$$

This problem has the unique solution

$$x_n = A_n. \tag{7}$$

Analogously, individual r chooses x_r so as to maximize

$$A_r \ln(x_r) - (1/n)\sum_{j=r+1}^{n}(x_j - x_r) - (1/2)(x_r + \tau)^2.$$

The first-order condition for this problem is

$$A_r / x_r + (n-r)/n - x_r - \tau = 0$$

and has the unique positive solution

$$x_r = (1/2)[(n-r)/n - \tau] + \sqrt{[(n-r)/n - \tau]^2/4 + A_r}. \quad (8)$$

Finally, individual $i \in N \setminus \{r, n\}$ chooses x_i in order to maximize

$$A_i x_i - (1/2)\left[(1/n)\sum_{j=i+1}^{n}(x_j - x_i)\right]^2 - x_i + \delta_{ip}\tau,$$

where $\delta_{ip} = 1$ if $i = p$, and $\delta_{ip} = 0$ otherwise. The first-order conditions for these problems are

$$A_i + \left[(n-i)/n^2\right]\sum_{j=i+1}^{n}(x_j - x_i) - 1 = 0. \quad (9)$$

We continue with a series of lemmas.

Lemma 1. For all $i \in N \setminus \{r\}$ it holds that RD_i is independent of τ.

Proof. For $i = n$ the claim holds trivially because $RD_n = 0$. Now consider $i \in N \setminus \{r, n\}$. For any such i condition (9) must hold and it follows therefore that

$$RD_i = (1/n)\sum_{j=i+1}^{n}(x_j - x_i) = n(1 - A_i)/(n-i),$$

which is independent of τ. □

Lemma 2. For all $i \in \{r+1, r+2, \ldots, n\}$ it holds that x_i is independent of τ.

Proof. For all $i \in \{r+1, r+2, \ldots, n-1\}$ condition (9) must hold. Conditions (7) and (9) for $i \in \{r+1, r+2, \ldots, n-1\}$ form a system of $n-r$ linear equations in the $n-r$ unknowns $\{x_{r+1}, x_{r+2}, \ldots, x_n\}$. Since these equations are linearly independent and also independent of τ, it follows that the values x_{r+1}, x_{r+2}, ..., x_n are uniquely determined and independent of τ. □

Lemma 3. For all $i \in \{1, 2, \ldots, r\}$ it holds that x_i is strictly decreasing with respect to τ.

Proof. For $i = r$ the statement follows easily from (8). We proceed by induction. Suppose we have already shown that x_j is strictly decreasing with respect to τ for all $j \in \{k, k+1, \ldots, r\}$. Consider $i = k-1$ and note that for this value of i, equation (9) holds. Solving this equation for $x_i = x_{k-1}$ we obtain

$$x_{k-1} = \left[(n-k+1)\sum_{j=k}^{n} x_j - n^2(1-A_{k-1}) \right] / (n-k+1)^2.$$

Since we have already proved that x_j is strictly decreasing with respect to τ for all $j \in \{k, k+1, \ldots, r\}$ and since x_j is independent of τ for all $j \in \{r+1, r+2, \ldots, n\}$, it follows from the displayed equation that x_{k-1} must be strictly decreasing with respect to τ. □

Lemma 4. RD_r is strictly increasing with respect to τ.

Proof. We have that

$$RD_r = (1/n) \sum_{j=r+1}^{n} (x_j - x_r).$$

In Lemma 2 we have shown that x_j is independent of τ whenever $j \in \{r+1, r+2, \ldots, n\}$, and from Lemma 3 it follows that x_r is strictly decreasing with respect to τ. Combining these results it follows that RD_r is strictly increasing with respect to τ. □

From equation (3) it follows that

$$G(\tau) = \frac{\sum_{i=1}^{n} RD_i}{\sum_{i=1}^{n} x_i} = \frac{RD_r + \sum_{i \in N \setminus \{r\}} RD_i}{\sum_{i=1}^{r} x_i + \sum_{i=r+1}^{n} x_i}.$$

From Lemmas 1 and 4 it follows that the numerator on the right-hand side is strictly increasing with respect to τ, and from Lemmas 2 and 3 we obtain that the denominator is strictly decreasing with respect to τ. Together this implies that the Gini coefficient $G(\tau)$ is a strictly increasing function of τ.

Up to now we have assumed that $x_1 < x_2 < \ldots < x_n$ holds, and that all incomes x_i are strictly positive. We next show that we can indeed choose the parameters A_1, A_2, ..., A_n in such a way that these properties hold at least for all sufficiently small positive values of τ. By continuity, it suffices to show that we can find parameters A_i, $i \in N$, such that for $\tau = 0$ it holds that $x_1 > 0$ and

$$x_{i+1} - x_i = 1/n. \tag{10}$$

Condition (10) and $x_1 > 0$ imply that

$$x_r = x_1 + (r-1)/n > (r-1)/n.$$

Together with (8) and $\tau = 0$, this implies that

$$(n-r)/(2n) + \sqrt{(n-r)^2/(4n^2) + A_r} > (r-1)/n.$$

It is obvious that we can choose A_r in such a way that this inequality holds. We next note that we must have that $x_n - x_r = (n-r)/n$. Taking into account (7) and (8) for $\tau = 0$, this translates into

$$A_n - x_r = A_n - (n-r)/(2n) - \sqrt{(n-r)^2/(4n^2) + A_r} = (n-r)/n$$

which is equivalent to

$$A_n = 3(n-r)/(2n) + \sqrt{(n-r)^2/(4n^2) + A_r}.$$

Since A_r has already been chosen, this equation determines A_n. Finally, we note that $x_{i+1} - x_i = 1/n$ implies that $x_j - x_i = (j-i)/n$ holds whenever $j > i$ and it follows from (9) that for all $i \in N \setminus \{r, n\}$

$$A_i = 1 - \left[(n-i)/n^3\right] \sum_{j=i+1}^{n} (j-i) = 1 - \left[(n-i)/n^3\right] \sum_{k=1}^{n-i} k = 1 - \frac{(n-i)^2(n-i+1)}{2n^3} > 0.$$

Thus, we have determined all parameters A_i, $i \in N$, in such a way that the maintained assumption $0 < x_1 < x_2 < \ldots < x_n$ is satisfied for all sufficiently small positive transfers.

Finally, we have to show that the example is a robust one. But this follows immediately from the fact that all our results were derived from the first-order conditions, which involve only first-order partial derivatives of the utility functions. If we perturb the utility functions in such a way that the values of their first-order and second-order derivatives evaluated at the solution corresponding to $\tau = 0$ remain close to the corresponding values of the example, then the positive dependence of the Gini coefficient on the size of the transfer will still obtain, at least locally at $\tau = 0$. This concludes the proof of Theorem 1.

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