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## Communicating Subjective Evaluations

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**Communicating  
Subjective Evaluations**

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September 2009, this version August 2013

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## Abstract

Should principals explain and justify their evaluations? In this paper the principal's evaluation is private information, but she can provide justification by sending a costly cheap-talk message. I show that the principal explains her evaluation to the agent if the evaluation turns out to be bad. The justification assures the agent that the principal has not distorted the evaluation downwards. In equilibrium, the wage increases in the agent's performance, when the principal justifies her evaluation. For good performance, however, the principal pays a constant high wage without justification. Furthermore, no payments to third parties are necessary if stochastic contracts are feasible.

**JEL classifications:** D82, D86, J41, M52

**Keywords:** Communication, Justification, Subjective evaluation, Stochastic contracts, Disclosure

This paper analyzes a principal-agent model in which the performance measure of the principal is nonverifiable by third parties and unobservable by the agent, but the principal has the possibility to communicate with the agent. Such subjective or nonverifiable measures of performance are widely used, as verifiable, i.e., objective, performance measures are often unavailable.<sup>1</sup> Examples for subjective measures of performance are the evaluations by supervisors, co-workers, and others. Their subjectivity, however, makes it the principal's choice whether to disclose and justify her evaluation of the agent's work. Hence, an endogenous hold-up problem arises. This problem explains some of the emphasis personnel policies place on feedback and communication.

In the model, an agent works for the principal who privately receives information about the agent's performance, like reports from colleagues, observations of the agent at work or of the agent's output. By random encounters or joint observations, the agent

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<sup>1</sup>The extensive use of subjective performance measures is confirmed by Dessler (2008, p. 339), Porter et al. (2008, p. 148), MacLeod and Parent (1999), and Murphy (1993). The reason is that agents can manipulate objective performance measures or multitask problems. Consequently, Gibbons (1998, p. 120) concludes that "objective performance measures typically cannot be used to create ideal incentives."

learns a very small fraction of the principal's information. These shared signals, however, are uninformative about the agent's evaluation by the principal. Then the principal has two options. Either she reports only the aggregated result of her evaluation or she justifies her evaluation by telling the agent about the information she collected.<sup>2</sup> Her message is not necessarily truthful, and providing justification is costly. The agent replies with a cheap-talk message about the shared signals. As the messages are the only third-party enforceable information, the contract just depends on these messages. The paper studies the resulting communication pattern: on the equilibrium path the principal justifies only bad evaluations. In this case, the wage is increasing in the evaluation. For good evaluations, the principal in equilibrium saves the hassle of explaining them and simply pays a high wage. This yields pooling and wage compression at the top.

The intuition for this communication pattern is the following. First, it is never optimal to justify all evaluations, because communication is costly. Second, the agent cannot verify the evaluation without justification. In this case the principal has an incentive to choose the evaluation yielding the lowest wage payment. Hence, no wage dispersion is feasible and there is pooling in the absence of justification. Third, giving a justification reduces the wage. Otherwise, abstaining from feedback would allow the principal to save on wage and communication costs. Hence, the highest wages lack justification. Finally, the monotone likelihood ratio property of the performance measure ensures that, with regard to justifications, a threshold strategy is optimal. Such a strategy is the most efficient way to give the agent incentives to exert effort. For bad performance, the principal has to bear the communication costs, but pays a lower wage. For good performance, on the other hand, she pays a higher wage instead of justifying her evaluation.<sup>3</sup>

The second contribution of this paper is to provide an explicit model of a justification. By justification, I refer to a message that transmits information previously unknown by the recipient and that is partially verifiable by the recipient. In the paper, the agent learns his evaluation by the principal when receiving the principal's message. In addition, the agent can to some extent verify the principal's message although the result of the evaluation and the agent's information are uncorrelated and stochastically independent. In contrast to previous literature I do not assume an exogenous verification technology, type-dependent messages spaces, or that messages are verifiable by a third party. All messages are cheap-talk. Hence, a third party cannot tell whether a message is truthful.

The mechanism uses the fact that the principal and the agent share some observations of the environment and the processes that lead to the evaluation. These shared observations are uninformative about the result of the evaluation and have mass zero with respect to the principal's information resulting in the evaluation. Nevertheless, the principal re-

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<sup>2</sup>Justifications of subjective evaluations are a common HR practice: "92% require a review and feedback session as part of the appraisal process." (Dessler, 2008, p. 366)

<sup>3</sup>Murphy (1993, p. 49) summarizes the reasoning as follows: Principals have "nonpecuniary costs [here, communication costs] associated with performance appraisal, which leads them to prefer to assign uniform ratings rather than to carefully distinguish employees by their performance."

calls all observations to justify the evaluation. If she were to distort the evaluation, she has to lie about some observations. No matter how she distorts the evaluation, there is a strictly positive probability that the agent becomes aware of the distortion. The reason is that the principal does not know which observations the agent learned.

As an example consider a chef cooking for the principal. The health-conscious chef prepares the food using a number of ingredients, but without salt. Suppose the principal pretends not to like the food and justifies her assessment saying that the food was too salty. Although the chef is unaware of the principal's real evaluation of the food, he knows that the principal is lying. Nonetheless, the mechanism used here is not limited to employment relations. It applies more generally to moral hazard and hold-up settings whenever the contracting parties interact. The mechanism, however, requires that the informed party is able to provide justification.

Moreover, the optimal contract fits well with empirical observations that evaluations are lenient and wage dispersion for the best evaluations is low.<sup>4</sup> Those observations are typically referred to as leniency bias and centrality bias. This paper argues that this pattern can be understood as a feature of the optimal contract instead of biased behavior. In addition, many studies show that principals evaluating for developmental or feedback purposes are more likely to differentiate among subordinates than they are when the evaluation is used for administrative purposes, like merit increases or promotions.<sup>5</sup> In the latter case, evaluations are more compressed and show less variation between employees. The finding goes back to Taylor and Wherry (1951, p. 39) who compare ratings for different purposes. They find more lenient evaluations for administrative purposes “with considerably poorer discrimination at the top.” This observation is in line with the predictions of this paper. The principal must be given explicit incentives to report her evaluation truthfully. These incentives cause pooling of the best evaluations. If the evaluation is for developmental or feedback purposes, these incentives are unnecessary, as the preferences of the principal and the agent are likely to be better aligned. Managers at Merck, for example, experienced that “the salary link made discussions on performance improvement difficult.” (Murphy, 1993, p. 58) Psychological costs of supervisors to give bad evaluations to their subordinates yield no straightforward explanation of this pattern, since those costs apply to evaluations for all purposes similarly.

The final contribution of this paper is to prove that the optimal contract can be ex-post budget-balanced. Hence, the contract requires no payments to third parties in contrast

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<sup>4</sup>According to Bretz et al. (1992), usually 60–70% of all employees get an evaluation from the best or second-best category. Moreover, “Medoff and Abraham (1980) found in two companies that, among the 99% of employees in the same position who received the top three performance ratings, the difference in salary between the highest and lowest rated employees was about 5%.” (Gibbs, 1991, pp. 4-5) Similarly, Murphy (1993, p. 56) reports that the top 1% of employees at the pharmaceutical company Merck receive a pay raise just 3% higher than the median employee in 1985.

<sup>5</sup>This effect is found in Dessler (2008, p. 356), Milkovich et al. (2008, p. 351), a meta-study by Jawahar and Williams (1997), Jawahar and Stone (1997), Harris et al. (1995), McDaniel et al. (1994), Milkovich and Wigdor (1991, pp. 3, 72), and Landy and Farr (1980).

to previous models. Instead, stochastic contracts use differences in the risk preferences of the contracting parties to implement the required incentives. Compared to a common moral-hazard setting additional incentives are necessary: ex-ante the principal wants to explain her evaluation to the agent ex-post. Nevertheless, ex-post she might withhold this information to save on wage and communication costs. The principal has no commitment power other than the contract. Hence, she has to design contractual terms that make it ex-post incentive compatible for her to provide justification.

The remainder of the paper is organized as follows. Section 1 discusses the related literature. Section 2 explains the intuition using a simplified example. Section 3 sets up the model and characterizes the optimal contract. Section 4 makes the optimal contract ex-post budget-balanced. Then Section 5 points out a more familiar implementation of the optimal contract by an indirect mechanism. Section 6 examines the robustness of the model. Section 7 contains the concluding remarks. All proofs are relegated to the appendix.

## 1 Related Literature

As in Al-Najjar et al. (2006) and Anderlini and Felli (1994), I explicitly model certain features of a language. In their papers, restrictions of the contracting language make it ex-ante impossible to describe some events that are observable to all contracting parties ex-post. These restrictions make incomplete contracts optimal. In my paper agents can write any contract ex-ante. Yet, the state of the world is private information and needs to be communicated ex-post. This communication can be supplemented by justification that makes the principal's message partly verifiable. Although I use a similar representation of the states of the world as an infinite binary sequence, their approach is conceptually and technically different from what I do here. I illustrate my model of providing justification using a setting with subjective performance measures.

There is a long literature on subjective performance measures. Usually, it is assumed that evaluations are observable and relationships are long-term. This yields implicit contracts, like for example in Compte (1998), Kandori and Matsushima (1998), Baker et al. (1994), MacLeod and Malcomson (1989), and Bull (1987). Then reputation effects created by the continuation value for both contracting parties allow subjective performance measures to gain credibility and to be used as the basis for the agent's incentives. Levin (2003) drops the assumption that the subjective performance measure is perfectly observable by both contracting parties. In this case optimal contracts often have a termination form, i.e., the contract ends after observing bad performance. In contrast to these repeated interactions, subjective evaluations are also used in static settings.

MacLeod (2003) was the first to implement subjective performance measures in a static setting. He assumes that the agent has a signal that is correlated with the principal's evaluation and introduces a message game. Each party reports their information by

sending a public message. This enables the parties to condition their contract on these messages, which essentially solves the credibility problem. As the information structure is exogenously given, the principal cannot decide, depending on the performance measure, whether to justify her evaluation. Thus, the results correspond to two special cases of my model. If the agent's and the principal's signal are correlated, MacLeod (2003) achieves the common second-best solution. This corresponds to obligatory or costless justification in my model as in Lemma 2. If the signals are uncorrelated, the optimal contract in MacLeod (2003) resembles the case of prohibitively expensive justification in my model. The case of imperfect correlation in combination with a binding upper limit on wage payments shares some features with the optimal contract here, but the reasoning and the proofs are different. First, I do not assume an upper limit on payments. Second, the agent receives no private signals telling him that he received no information. Instead, it is the principal's incentive – resulting from the contract and the communication costs – to withhold and distort her evaluation that yields the compression at the top result. Economically, the main difference between this paper and MacLeod (2003) is that I consider the principal's decision whether to justify her evaluation.

In the current paper, I follow a static approach. Some justification can be found in Fuchs (2007) who considers a finitely repeated principal-agent model. He shows that it is optimal for the principal to announce her subjective evaluation only once at the end of the interaction. In this case, the agent does not learn whether a good performance has already occurred. Hence, it is sufficient to penalize only the worst outcome, while paying a constant wage following all other terminal histories. Brown and Heywood (2005) and Addison and Belfield (2008) provide additional justification for a static approach. They show empirically that performance evaluations are more likely to be used for employees with shorter expected tenure.

This paper also relates to the literature on endogenous contracts, like Kvaløy and Olsen (2009). Yet, I do not assume any cost for writing specific contractual arrangements. The contract can be any functions of the messages, but communication is costly. As justification allows verifying the performance measure, there is a parallel to the literature on costly state verification, like Hart and Moore (1998), Gale and Hellwig (1985), and Townsend (1979). These models allow an investor to verify the firm's performance by a costly audit. They show the optimality of debt contracts, which are similar to the optimal contract in my paper, as there are no audits for high payments. In this literature, however, the firm learns its performance, while the investor chooses whether to perform an audit. In my model, due to the nature of a justification it is the better-informed party that makes the communication decision. This is also the reason why mixed strategies with respect to the communication strategy are not optimal in my setting. In addition, the communication need not be truthful and cannot be verified directly by one of the contracting parties, while the result of an audit is truthful and verifiable.



In Rahman (2012), the principal instructs the agent to shirk sometimes. These instructions create shared observations between the principal and the monitor. The shared observations allow the principal to verify the monitor's report if the probability of shirking is strictly positive. In my model of justification, it is sufficient that there are some shared observations, but they can be uninformative and have mass zero.

Following truthful communication, the performance measure becomes observable, but unverifiable – similarly to a hold-up setting. Aghion et al. (2012), Hart and Moore (1988), and Grossman and Hart (1986) discuss solutions to this problem. In my model, preferences are independent of the evaluation, while in the hold-up setting the preferences depend on the types or the effort of the parties. Therefore I cannot replicate the solutions of these models. In contrast to the literature on informed principals, the principal's information arises during the principal-agent relationship and is unavailable at the contracting stage.

Furthermore, credibility of promised incentives is sometimes discussed under the notion of fairness and trust. According to Bernardin and Orban (1990, p.197) “trust in appraisal accounted for a significant proportion of variance in performance ratings.” In my model, justification establish this trust. In Giebe and Gürtler (2012), Al-Najjar and Casadesus-Masanell (2001), and Rotemberg and Saloner (1993), this trust is created by the extent to which the principal's preferences incorporate the agent's well-being.

Finally, the present article concerns stochastic contracts and ex-post budget balance. Previous literature, like MacLeod (2003) or Fuchs (2007), requires payments to third parties. This allows the contracting parties to renegotiate in order to avoid paying money to an outsider – as already discussed by Hart and Moore (1988). If stochastic contracts are possible, I show how to establish ex-post budget balance. Maskin and Tirole (1999) use a similar mechanism to implement incomplete contracts in an investment setting. Rasmusen (1987) shows that stochastic wage payments ensure ex-post budget balance in a team-production setting. He does not consider differences in risk aversion between the principal and the agent, as the principal's payment is complete deterministic, only the sharing rule between the agents is stochastic. In my model, the principal's payment has to be stochastic to guarantee budget balance.

## 2 Subjective Evaluations at Work

As an example, consider the performance evaluation at Arrow Electronics, a Fortune 500 company.<sup>6</sup> Employees are evaluated in seven performance areas, capturing, for example, costumer satisfaction, their business judgment, and skills as a team worker. In each area, they receive a grade on a scale from one to five. The average grade across the seven performance areas yields the result of the evaluation that is used for compensation.

Suppose that the principal receives a report for each performance area from a differ-

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<sup>6</sup>Hall and Madigan (2000) describe the details of evaluations at Arrow Electronics.

ent source. Hence, she listens to costumers complaining or praising the employee. Then, she talks to the agent's colleagues to learn about his skills as a team worker. Finally, she observes the agent at work or the agent's output. This closely captures a practical evaluation process, as "an appraiser would use evidence from direct observation of the employee, or by reports from others, to make judgment about the appraisee's performance." (Porter et al., 2008, p. 149) These reports of the different sources are subjective and private information of the principal. The agent, however, sometimes gets direct feedback from customers or is told by colleagues about their reports. Hence, he also observes a small number of these reports.

Finally, Arrow Electronics requires managers to communicate the evaluation. For this purpose, the principal can choose either to tell the agent only the result of the evaluation or to justify the evaluation. A justification tells the agent all the reports of all sources. Providing justification is costly, as it requires the principal to spend additional time on the evaluation. The agent replies to the principal's message and receives his wage.<sup>7</sup>

Analyzing the problem requires more structure. Consider a risk-averse agent working for a risk-neutral principal. The principal proposes a contract that specifies the agent's wage  $W$ . After signing such a contract, the agent chooses his work effort  $e \in [0, 1)$ , which is unobservable by the principal. For simplification, consider only two performance areas and a binary scale for each area. Then the principal receives the information  $I(t) \in \{0, 1\}$  from two sources  $t \in T = \{1, 2\}$ , namely, colleagues and customers. With probability  $e$  both colleagues and customers report a positive signal, 1. Otherwise, either only the colleagues or only the customers report a positive signal with probability  $\frac{1-e}{2}$  each. Define the result of the subjective evaluation as the average  $\mu = \frac{1}{2} \sum_{t \in T} I(t)$ . Hence, the result  $\mu$  of the evaluation equals 1 with probability  $e$  and  $1/2$  with probability  $1 - e$ . The agent learns the report from one positive source  $S$  by coincidence. If both colleagues and customers report positively,  $S$  is drawn randomly with probability  $1/2$  for each source.  $S$  is private information of the agent. Notice that the result  $\mu$  of the subjective evaluation and the agent's information  $I(S)$  are stochastically independent.

Now, turn to the communication of the evaluation. A justification requires communication effort  $\kappa > 0$  and allows the principal to send a message  $m_P \in \mathcal{I} = \{0, 1\}^T = \{0, 1\}^2$ . Then the principal can tell the agent all information  $I = (I(1), I(2))$  upon which her evaluation is based. If the principal decides not to provide justification, she can tell the agent only the result of the evaluation  $\mu$ , here 1 or  $1/2$ . Hence, her message space is restricted to  $R = \{(1, 1), (1, 0)\}$ . Independent of the principal's choice, the agent replies with a cheap-talk message  $m_A \in \mathcal{I}$ . Both parties can lie and send any message from the corresponding messages sets. The wage depends on these two messages. Figure 1 summarizes the timing.

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<sup>7</sup>Assume that the agent quits his job at Arrow Electronics afterwards. Indeed, turnover rates at Arrow Electronics could reach 20%-25%.

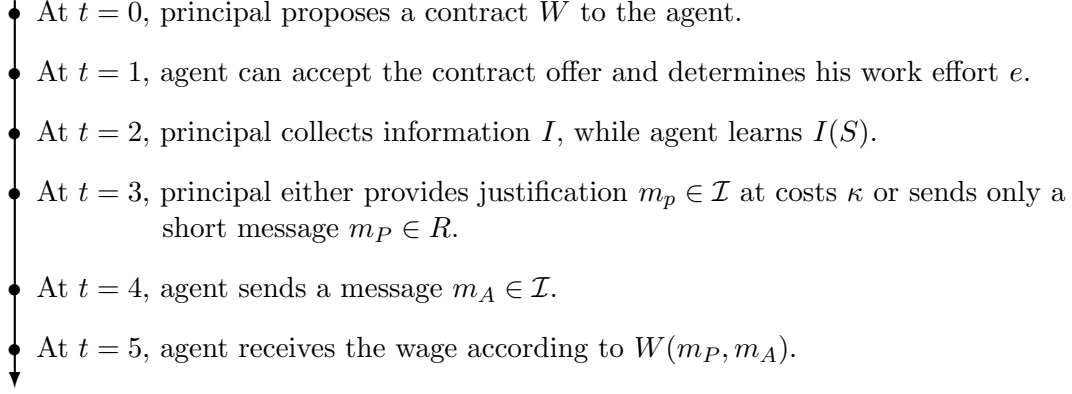


Figure 1: Timing of the Model

The agent is represented by a utility function  $U(W, e) = u(W) - d(e)$  if he chooses effort  $e$  and is paid a wage  $W$ . The function  $u$  is increasing and strictly concave with the limit  $\lim_{w \rightarrow 0} u(w) = -\infty$  and the derivative  $u' > \epsilon > 0$ . The function  $d$ , the disutility of performing effort, is also increasing, but strictly convex with the limit  $\lim_{e \rightarrow 1} d(e) = \infty$ . Both functions are twice differentiable. The agent receives a reservation utility  $\bar{u}$  if he rejects the principal's offer. The principal's benefit is

$$\begin{cases} B\mu - W - \kappa & \text{if she provides justification} \\ B\mu - W & \text{if she does not provide justification} \end{cases}$$

with a constant  $B$  if she pays a wage  $W$ .

Suppose the principal wants to implement positive work effort  $e > 0$  and the communication costs  $\kappa$  are small.<sup>8</sup> Then the agent is paid a wage  $w_H$  if the average of principal's message is 1. If the principal's message indicates a low performance,  $1/2$ , the wage depends also on the agent's message. If both messages coincide in at least one element with the value 1, the principal will pay the agent a wage  $w_L$ . If the messages differ completely, the agent receives  $w_L$  while the principal has to pay  $2w_H$ . Moreover,  $w_H > w_L$ .<sup>9</sup>

This contract yields a number of observations that are confirmed by the results in the next sections. First, there is truth-telling although the agent does not know the correct evaluation. Second, the principal justifies the evaluation if and only if the result of the evaluation is bad,  $\mu = 1/2$ . Third, the agent's wage does not depend on her message. Fourth, the agent understands a justification, i.e., he learns the result of the evaluation and can partially verify the message. Furthermore, on the equilibrium path, there are no payments to third parties. To understand these features, consider the principal's incentives if she has learned  $I = (1, 1)$ . If she just reports the result of the evaluation, her payoff is either  $B - w_H$  for a high message or  $B - w_L/2 - 2w_H/2$  for a low message as she expects the agent to send the message  $(1, 0)$  or  $(0, 1)$  with probability  $1/2$  each. Providing justification only reduces her payoffs by  $\kappa$ . Hence, it is optimal for the principal to report

<sup>8</sup>Formally, there is a  $\bar{\kappa}(e) > 0$  such that the following statements hold for all  $\kappa < \bar{\kappa}(e)$ .

<sup>9</sup>These statements follow from the more general results in the next sections. Hence, no proof is provided.

truthfully and not to justify the evaluation in this case. If she has learned  $I = (1, 0)$  or  $(0, 1)$ , reporting the evaluation's result yields her a payoff of  $B/2 - w_H$  for a high message or  $B/2 - w_L/2 - 2w_H/2$  for a low message. If she provides justification and sends a truthful message, her payoff is  $B/2 - w_L - \kappa$ . The other cases are analogous. Therefore it is optimal for the principal to report truthfully and to justify the evaluation in this case. Yet the stylized nature of the example makes it impossible to study more complex features of the contract, like wage compression at the top. For this purpose, the next section generalizes the model making the evaluation continuous and reducing the fraction of sources that the agent learns to 0.

### 3 Justify Bad Evaluations

#### 3.1 The Main Model

This time, the principal proposes a contract that specifies the agent's wage  $W$  depending on any information that is available at the time of the wage payments and enforceable by a third party. After the agent's effort choice  $e \in [0, 1]$ , the principal collects subjective information  $I(t) \in \{0, 1\}$  about the agent's work from different sources or performance areas  $t \in T = [0, 1]$ . Every source  $t \in T$  is independent and identically distributed and declares success with probability  $p$  and failure with probability  $1 - p$ . The probability  $p$  is drawn from the distribution  $F(p|e) = eF^H(p) + (1 - e)F^L(p)$  and therefore depends on the agent's effort  $e$ . The cumulative distribution functions  $F^H(p)$  and  $F^L(p)$  admit continuous densities  $f^H(p), f^L(p) > \epsilon > 0$ . In addition, the distribution  $F$  of  $p$  satisfies the monotone likelihood ratio property, i.e., the ratio  $f^H(p)/f^L(p)$  is strictly increasing in  $p$ . Notice that the evaluation  $\mu$  is a sufficient statistics for the agent's effort, as  $\mu = \int_T I(t)dt = p$ .<sup>10</sup> The monotone likelihood ratio property ensures that a higher evaluation  $\mu$  indicates higher work effort.

The agent observes only the reports of a finite subset  $S \subset T$  of positive sources with  $|S| = n$ . In Section 6 the agent also learns the reports of sources that report 0. This assumption does not matter for the results. Assume that the agent's sample  $S$  is large and consists of a random draw with full support over the positive sources, no atoms and a density bounded away from zero.<sup>11</sup> The set  $S$  is private information of the agent, while  $n$  is common knowledge. Hence, the principal does not know which information is observed by the agent. For later reference, denote by  $P(S|I, e)$  the conditional distribution and by  $P(I, S|e)$  the joint distribution of the agent's sample  $S$  and the principal's information  $I$ :  $t \mapsto I(t)$  conditional on the agent's effort  $e$ .

<sup>10</sup>I assume a law of large number here. Judd (1985) constructs a probability measure that allows avoiding measurability problems in formulating a law of large numbers for a continuum of random variables. Sun (2006) proves such a law of large numbers assuming essential pairwise independence.

<sup>11</sup>All results are such that  $\exists \bar{N} \in \mathbb{N}$  and the results are valid for all  $n > \bar{N}$ . If  $n$  is small, e.g.,  $n = 1$ , the results also hold, but require higher off-equilibrium payments.

Let  $\beta \in \{0, 1\}$  denote the principal's communication decision. For  $\beta = 0$ , no justification is provided and the principal's message space is restricted to  $R = [0, 1]$ . For  $\beta = 1$ , she justifies her evaluation of the agent's work at cost  $\kappa$  by sending a message  $m_P \in \mathcal{I} = \{0, 1\}^T$ .<sup>12</sup> The agent replies with a cheap-talk message  $m_A \in T^n$ . Finally, the contract  $W$  is performed according to the available enforceable information, the messages  $m_P$  and  $m_A$ . Therefore, the contract is a function  $W: \mathcal{I} \times T^n \rightarrow \mathbb{R}_+$ . The message spaces are sufficiently rich to encode  $\beta, e$  and  $\mu$ . Thus, it is unnecessary to include them explicitly in the contract. The principal has no commitment power other than the contract.

### 3.2 Analysis

It is crucial here that the principal has to pay communication costs  $\kappa$  to transmit the information  $I$ . The model ensures this by making it impossible to encode the information  $I$  in a message from the restricted message set  $R$ . The reason is that the cardinality of  $\mathcal{I}$  and  $R$  differs. Due to the restriction of the message space for  $\beta = 0$ , the classical revelation principle does not apply here. See Green and Laffont (1986) for an example. Hence, it is unclear whether truthful revelation is optimal in my setting. Define truthful revelation by the principal as

$$\theta(I, \beta) = \begin{cases} I & \text{if } \beta = 1 \\ \int_T I(t) dt & \text{if } \beta = 0. \end{cases}$$

Lemma 3 in the appendix proves that truthful revelation is optimal if an optimal contract exists. Thus, in the optimal contract principal and agent send a truthful message that reveals their private information, the principal's information  $I$  or its average  $\mu$  and the agent's sample  $S$  respectively.<sup>13</sup> As Proposition 2 in MacLeod (2003) and Proposition 1 in Fuchs (2007) demonstrate, some surplus has to be destroyed in this kind of model to implement positive effort of the agent. To account for this, I denote by  $W(m_P, m_A)$  the wage paid by the principal following messages  $m_P$  and  $m_A$ . On the other hand,  $c(m_P, m_A)$  is the wage earned by the agent. Proposition 5 shows how stochastic payments make the optimal contract ex-post budget-balanced. Grossman and Hart (1983) prove that the model can be solved in two steps. First, for every level of effort  $e$ , the optimal wage schedule  $W$  and its expected costs  $C(e)$  for the principal are computed. The second step determines the optimal effort level  $e$  by

$$\max_{e \in [0, 1]} B \int p dF(p|e) - C(e).$$

<sup>12</sup> $\{0, 1\}^T$  denotes the set of all functions  $T \rightarrow \{0, 1\}$ . For  $\beta = 0$ , I identify a message  $m_P \in R$  in the restricted message space  $R$  with the step function:  $T \rightarrow \{0, 1\}$  with 1 for  $t \leq m_p$  and 0 otherwise.

<sup>13</sup>Yet it is impossible to have the agent reveal his work effort  $e$  truthfully and make the wage dependent on his message about the effort in order to implement  $e > 0$ . Therefore it is without loss of generality for the messages to contain only the information the contracting parties collected at  $t = 2$ .

Now, returning to the first step, Program A below determines the optimal contract that implements effort  $e$  by choosing payments  $W(m_P, m_A)$  for the principal,  $c(m_P, m_A)$  for the agent, and which evaluations to justify,  $\beta(I)$ . The objective is to minimize the expected wage payment subject to several conditions. The participation constraint (PC) makes the agent willing to accept the proposed contract. The agent's incentive compatibility ( $IC_A$ ) guarantees that the agent chooses the desired level of effort. The principal's incentive compatibility ( $IC_P$ ) gives her an incentive to justify her evaluation if communication is desired. In addition, sending a truthful message has to be incentive compatible for the principal ( $IC_P$ ) and the agent ( $IC_{m_A}$ ). Finally, the principal's payment has to be higher than the wage received by the agent. To simplify the exposition, define the equilibrium payments  $W_{eq}(I, S) = W(\theta(I, \beta(I)), S)$  and  $c_{eq}(I, S) = c(\theta(I, \beta(I)), S)$ .

$$C(e) = \inf \int W_{eq}(I, S) + \kappa \beta(I) dP(I, S|e), \quad (A)$$

$$\text{subject to } \int u(c_{eq}(I, S)) dP(I, S|e) - d(e) \geq \bar{u}, \quad (PC)$$

$$e \in \arg \max \int u(c_{eq}(I, S)) dP(I, S|e) - d(e), \quad (IC_A)$$

$$\kappa \beta(I) + \int W_{eq}(I, S) dP(S|I, e) \leq \tilde{\kappa} + \int W(\hat{I}, S) dP(S|I, e) \quad \forall I \in \mathcal{I}, \forall (\tilde{\kappa}, \hat{I}) \in \{\{0\} \times R, \{\kappa\} \times \mathcal{I}\}, \quad (IC_P)$$

$$\int u(c(I, S)) dP(I|S, e) \geq \int u(c(I, \hat{S})) dP(I|S, e) \quad \forall S, \hat{S} \in T^n, \quad (IC_{m_A})$$

$$W(I, S) \geq c(I, S) \quad \forall I \in \mathcal{I}, \forall S \in T^n. \quad (1)$$

If the principal's information  $I$  were observable and contractible, the principal's and the agent's incentive for sending truthful messages ( $IC_P$ ) and ( $IC_{m_A}$ ) can be neglected. Denote this problem by  $A^*$ , the solution, the optimal complete wage, by  $w_e^*(\mu)$ , and the expected costs by  $C^*(e)$ .

**Lemma 1.** *If the principal's information  $I$  is contractible, the optimal contract to implement effort  $e$  offers a wage  $w_e^*(\mu)$ , that only conditions on the average  $\mu$  of the principal's information. Furthermore, the wage  $w_e^*(\mu)$  is almost surely continuous and increasing in  $\mu$  for positive effort,  $e > 0$ .*

The results of Holmström (1979) remain valid in this setting: The wage depends only on the sufficient statistics  $\mu$  instead of the entire information  $I$  and a better evaluation implies a higher wage.

If the principal's information is subjective and communication is a choice variable of the principal, the additional incentive constraints for the messages do matter. Yet these incentive constraints do not change the equilibrium wage in the absence of communication costs when  $\kappa = 0$ .

**Lemma 2.** *If the principal has no communication costs  $\kappa = 0$ , the optimal contract to implement effort  $e$  is*

$$c^+(m_P, m_A) = w_e^* \left( \int m_P(t) dt \right)$$

$$W^+(m_P, m_A) = \begin{cases} w_e^* \left( \int m_P(t) dt \right) & \text{if } m_P(t) = 1 \text{ for all } t \in m_A \\ w_e^*(1) + k & \text{otherwise} \end{cases}$$

with a large penalty  $k$ . The equilibrium wage is the same as in Lemma 1.

Whenever communication is costly and  $\kappa > 0$ , however, the equilibrium wage differs from  $w_e^*(\mu)$ . To gain some intuition, suppose for the moment that the principal justifies all evaluations. Then the optimal contract implements wage payments  $w_e^*(\mu)$  on the equilibrium path as in Lemma 2. Yet, the principal can modify this contract to save on communication costs. The reason is that the agent does not suspect a distorted evaluation by the principal and does not demand an explanation for the highest wages. Therefore it is suboptimal to justify all evaluations. For ease of exposition, denote the communication set by  $\mathcal{I}_C = \{I \in \mathcal{I} | \beta(I) = 1\}$ .

**Proposition 1.** *Justifying all evaluations is never optimal. Thus,  $\Pr(\mathcal{I}_C) < 1$  in the optimum. Moreover, the expected costs of the optimal contract are below  $C^*(e) + \kappa$ .*

The proof in the appendix shows that the principal's total costs decrease if the principal refrains from justifying evaluations that yield the highest wages. To further determine the communication set, it is necessary to know more about the optimal contract. The next proposition provides a solution to Program A and characterizes the optimal contract.

**Proposition 2.** *Suppose  $\Pr(\mathcal{I}_C) > 0$ . In the optimal contract the wage is constant for evaluations outside the communication set. Otherwise, the wage changes in the principal's evaluation.*

$$c^{**}(m_P, m_A) = \begin{cases} w^{**} \left( \int m_P(t) dt \right) & \text{if } m_P \in \mathcal{I}_C \\ w^{**} & \text{if } m_P \notin \mathcal{I}_C \end{cases}$$

$$W^{**}(m_P, m_A) = \begin{cases} w^{**} \left( \int m_P(t) dt \right) & \text{if } m_P \in \mathcal{I}_C \text{ and } m_P(t) = 1 \text{ for all } t \in m_A \\ w^{**} & \text{if } m_P \notin \mathcal{I}_C \\ w^{**} + \kappa & \text{otherwise.} \end{cases}$$

The principal justifies low wage payments, as  $w^{**} \left( \int m_P(t) dt \right) < w^{**}$  for all  $m_P \in \mathcal{I}_C$ . The values of  $w^{**}$  and  $w^{**}(\mu)$  are determined in Program (C) the proof.

There is pooling of wages, but evaluations are unbiased in the optimal contract. Yet, this is for ease of exposition only. It is easy to introduce also pooling of evaluations for

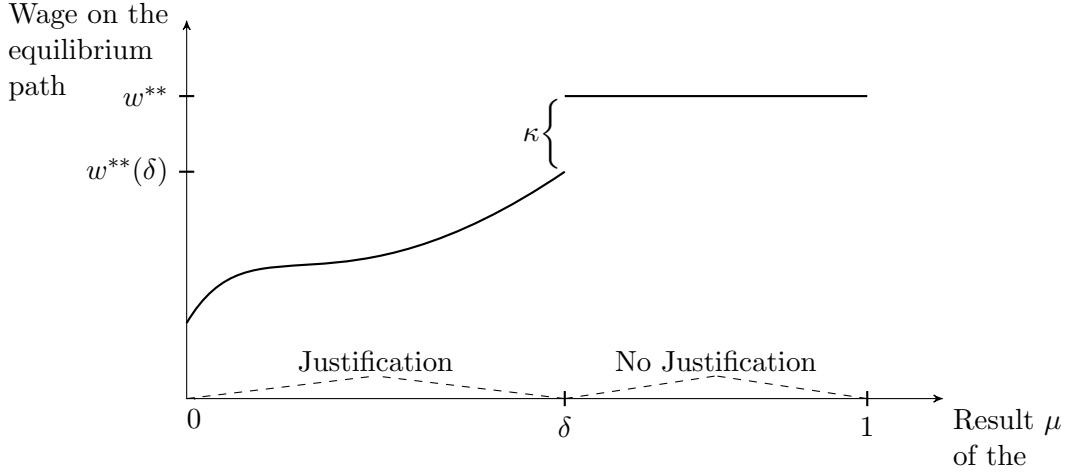


Figure 2: Proposition 3 and the Equilibrium Wage

In the optimal contract there is pooling of wages, but evaluations are unbiased. It is easy, however, to have pooling of evaluations, too. Then the principal sends  $m_P = 1$  if  $\mu \geq \delta$ . No change to the optimal contract is required.

$I \notin \mathcal{I}_C$ . This requires only the principal sending a pooling message for  $I \notin \mathcal{I}_C$ . In the contract the agent's wage is either constant or depends only on the average of the principal's message  $m_P$ . Additionally, the principal's payments are higher in the absence of justification. Otherwise the principal would deviate and would not justify the evaluation, because the agent cannot verify such a deviation. Furthermore, the agent's wage equals the principal's payments in the absence of a justification. This results from the interaction of the participation constraint with the agent's incentive compatibility. Varying the agent's wage in the absence of a justification eases the agent's incentive compatibility ( $IC_A$ ), but makes it more difficult to satisfy the agent's participation constraint (PC). The proof shows that the second effect dominates the first one due to the monotone likelihood ratio property. Therefore the agent's wage equals the principal's payments in the absence of a justification.

If the principal provides justification, the agent can detect deviations by the principal, as deviations trigger a mismatch in the messages. Therefore payments can vary in the principal's message  $m_P$ . A disagreement in the messages is punished by making the principal pay the highest wage. If the messages agree, the principal's payment equals the agent's wage. This structure of the contract allows characterizing the optimal communication strategy, that follows a threshold rule. The principal justifies only bad evaluations. In these cases, the agent suspects a distortion by the principal and insists on a justification of bad evaluations.

**Proposition 3.** *Suppose  $\Pr(\mathcal{I}_C) > 0$ . In the optimal contract there is a threshold  $\delta$ , such that the principal justifies evaluations  $\mu$  below  $\delta$ , while she does not justify evaluations  $\mu$  above  $\delta$ . Moreover,  $0 < \delta < 1$  and, on the equilibrium path, the wage is increasing in the evaluation for evaluations below  $\delta$ .*

The proposition exhibits the communication pattern described in the introduction and summarized in Figure 2. There is partial communication. The principal justifies



only bad evaluations and low wages, while she remains silent on good performance. This confirms empirical observations, like the leniency bias and the centrality bias that there is less distinction in subjective evaluations than in the underlying performance measure, in particular at the top. Yet this behavior is not the result of a bias, but part of the optimal contract, which pools several evaluations and rewards them with the same wage. Thus, the contract eliminates wage differences that the principal would have to justify.

The proof is by contradiction. Assume there is a threshold  $\mu^*$  such that the principal justifies some evaluations above  $\mu^*$ , but does not provide justification for some evaluations below  $\mu^*$ . This communication pattern implies a constant wage for the evaluations above  $\mu^*$  in the communication set. To show this, the proof establishes that the Lagrange multiplier of the modified principal's incentive compatibility ( $IC_P$ ) is positive on this set of evaluations.

The second step splits the set of the evaluations above  $\mu^*$  in the communication set into a lower and an upper half. Then the contract is adjusted so that the principal justifies evaluations in the lower half, but not in the upper half. At the same time the agent's wage is decreased in the lower half and increased in the upper half such that the agent's expected utility remains constant. Hence, the agent's participation constraint (PC) remains satisfied. Finally, I show that the agent's incentive compatibility ( $IC_A$ ) becomes slack by this contract modification due to the monotone likelihood ratio property. Yet, in this case the initial contract cannot be optimal.

Communication is indeed optimal if communication costs are not prohibitively high as specified in the next proposition. Then the optimal contract makes the principal justify some evaluations.

**Proposition 4.** *In the optimal contract there is communication with positive probability,  $\Pr(\mathcal{I}_C) > 0$ , if the principal wants to implement positive effort  $e > 0$  of the agent and the communication costs are not too high, i.e.,*

$$\kappa \leq u^{-1} \left( \bar{u} + d(e) + \frac{f(0|e)}{f^L(0) - f^H(0)} d'(e) \right) - \int w_e^*(p) dF(p|e). \quad (2)$$

Communication is beneficial, although it is costly and conveys no additional information about the agent's effort. Communication is also not about providing the principal with information for her decision making or giving the agent instructions in the sense of learning or which tasks to perform. Instead, communication makes the principal's promise of incentives to the agent credible and allows the principal to assure the agent that her evaluation is not distorted. Thus, it is in the principal's interest to be transparent about her evaluations, even if communication is costly and takes place after the agent's effort choice. Finally, consider two extensions to simplify the contract and make it ex-post budget balanced.

## 4 Stochastic Contracts

For stochastic payments, the expected value for the principal is higher than the agent's certainty equivalent. Therefore it is possible to replace payments to a third party by stochastic payments. This does not require a risk-neutral principal. As long as the degree of risk aversion differs between the principal and the agent, the optimal contract can achieve ex-post budget balance.

**Proposition 5.** *The optimal contract can be ex-post budget-balanced if stochastic contracts are feasible. Then the optimal contract is*

$$\bar{W}^{**}(m_P, m_A) = \begin{cases} w^{**}(\int m_P(t)dt) & \text{if } m_P \in \mathcal{I}_C \text{ and } m_P(t) = 1 \text{ for all } t \in m_A \\ w^{**} & \text{if } m_P \notin \mathcal{I}_C \\ w^{**} + \Lambda_z(\int m_P(t)dt) & \text{otherwise.} \end{cases}$$

The lotteries  $\Lambda_z$  have a mean of  $\mathbb{E}(\Lambda_z) = \kappa$  and a certainty equivalent for the agent of  $u^{-1}(\mathbb{E}[u(w^{**} + \Lambda_z(\mu))]) = w^{**}(\mu)$ . The values of  $w^{**}$  and  $w^{**}(\mu)$  are determined in Proposition 2.

Stochastic contracts ensure ex-post budget balance. Examples are stock options whose valuation is influenced by external shocks to the financial sector or uncertain arbitration procedures. The contracting parties might be uncertain how a disagreement is interpreted and which wage payment is appropriate.

## 5 Indirect Mechanism

The contract described in Proposition 2 can be simplified by changing the agent's message space. Hence, reduce his message space to the binary decision whether to accept or reject the evaluation by the principal. If he accepts the evaluation  $\mu$ , the principal pays him the corresponding wage  $w^{**}(\mu)$ . If he rejects, the principal has to pay  $w^{**} + \Lambda(z(\mu))$ . The principal values this payment at  $\mathbb{E}(w^{**} + \Lambda(z(\mu))) = w^{**} + \kappa$ , while the agent's certainty equivalent is  $w^{**}(\mu)$ .

Formally, the contract is now a function  $W: \mathcal{I} \times \{Y, N\} \rightarrow \mathbb{R}_+$  and depends on the principal's evaluation,  $m_P \in \mathcal{I}$ , and the agent's decision,  $m_A \in \{Y, N\}$ . Thus, the agent has the possibility to object to the principal's evaluation. This conflict resolution might be quite realistic, as Bretz et al. (1992, p.332) state that “most organizations report having an informal dispute resolution system (e.g., open door policies) that employees may use to contest the appraisal outcome. About one-quarter report having formalized processes.”

The indirect mechanism leaves the incentives of the contracting parties unchanged. If the principal receives information in the communication set, any deviation makes her

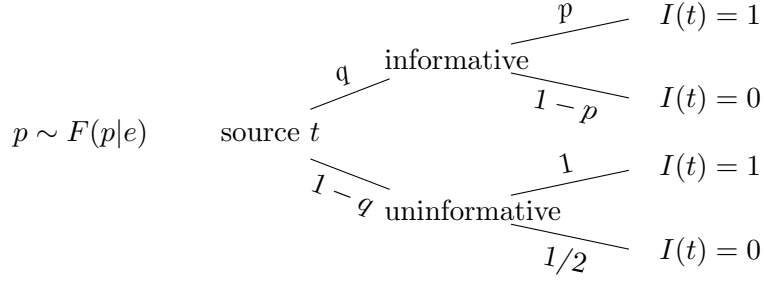


Figure 3: Alternative Distribution of the Reports of a Source

worse off, as the deviation increases her payments to at least  $w^{**}$ . For evaluations outside the communication set, it is also unprofitable to deviate, as any evaluation yielding a lower wage will be rejected. For the agent, on the other hand, the following strategy is a best reply: accept an evaluation if and only if the principal proposed  $\mu \geq \delta$  or she provides justification that matches the agent's information. Consequently, the modified setting also implements effort  $e$  of the agent at optimal costs. Hence, the relevant part of the model is the principal's decision to justify her evaluation to the agent at  $t = 3$ .

## 6 Robustness of the Results

In the model, the agent learns the reports of a finite subset  $S$  of positive signals. It is possible, however, to extend the model to include negative signals into  $S$ . For this purpose assume, that there are informative and uninformative sources. A source is informative with probability  $1 > q > 0$ . In this case, the source reports a success with probability  $p$  as before. With probability  $1 - q$  the source is uninformative and reports statistical noise, i.e., success or failure with probability  $1/2$  each. The quality of a source is unobservable. Figure 3 depicts the distribution for each source. The set  $S$  is drawn randomly from the uninformative sources. Then the agent learns

$$\begin{cases} I(t) & \text{if } t \in S \\ 0 & \text{if } t \notin S. \end{cases}$$

In this case my results remain valid even if  $S$  is unobservable by the agent. There are some adaptations to the optimal contract in this case. In particular, the consensus wage is paid, if the messages agree, i.e.,  $m_P(t)m_A(t) = m_A(t)$  for all  $t \in T$ .

Alternatively, assume that the agent is biased and systematically overestimates his performance. Hence, he understands some sources to report success, although they indeed report a failure. However, as long as the bias is systematical, it is possible to adapt the definition of an agreement in the messages. Then the results of this paper remain valid. Suppose, for example, that the agent perceives a finite number of additional sources  $S'$  to report success independently of their true value. If  $S'$  is known at the time the contract is proposed, the contract just neglects these elements of the agent's message. If  $S'$  is

unknown at the time the contract is proposed, agreement is defined as  $m_P(t) = 1$  for all  $t \in m_A$  with  $|S'|$  exceptions.

So far there is only one messaging stage, i.e., both the agent and the principal are allowed to send just one message. This assumption ensures that pure (communication) strategies are optimal. If there were an alternating sequence of messages, random communication might become optimal. In this case, the principal first sends the result of the evaluation as a costless message. Then the agent with some small probability requests justification. After that the principal has the possibility to reply with a justification which has to match a subsequent message by the agent. This sequence allows to save on communication costs but requires high punishments out of equilibrium.

In the main model, I assume that the principal can commit to a contract that specifies the wage payments depending on the principal's and the agent's messages. These payments have to be designed in such a way that they give the principal incentives to follow the communication pattern characterized in Proposition 3. The optimal contract indeed provides these incentives. It is important, however, that renegotiations are impossible. Otherwise, the principal would have an incentive to insure the risk-averse agent ex-post.

Finally, consider the messages. In the model, the agent can learn the principal's communication decision  $\beta$  from the message, as the restricted message set  $R$  has mass zero in  $\mathcal{I}$ . This is unnecessary, however. Assume that a message in  $m_P \in R = [0, 1]$  is identified with a uniformly distributed lottery over all messages  $\{I \in \mathcal{I} \mid \int I(t) dt = m_P\}$  with an average of  $m_P$ . Nevertheless the contract proposed in the paper still remains optimal in this setting. In addition, alternative message spaces are possible. For the mechanism to work, however, the message space  $\mathcal{I}$  has to have a strictly bigger cardinality than  $R$ . Otherwise, it would be possible to save communication costs  $\kappa$  by encoding the principal's information  $I$  in a message from the restricted messages set  $R$ . Additionally, having an (at least countable) infinite message space  $R$  helps to avoid integer problems.

## 7 Conclusion

This article discusses communicating a subjective performance measure in a principal-agent model. The principal can justify her evaluation of the agent's work. Providing justification is costly, does not convey additional information about the agent's effort, and does not serve a learning or instructing purpose. Nevertheless, in the optimal contract the principal justifies some evaluations. This allows the agent to detect distorted evaluations. Therefore providing a justification makes the incentives for the agent credible. In the optimal contract, the principal justifies only bad evaluations. This communication pattern results in pooling and wage compression at the top, as illustrated in Figure 2 on page 13. These results fit well with empirical observations, often referred to as leniency bias and

centrality bias.<sup>14</sup> The paper argues that this pattern of evaluations is a feature of the optimal contract with unbiased agents and no proof of biased behavior per se.

The principal's justification convinces the agent that the principal evaluates her appropriately ex-post. In addition, they motivate him ex-ante to implement the specified work effort. Compare this to a naive contract that does not give the principal an incentive to provide justification. In this naive contract, the principal does not justify the evaluation and always reports the evaluation associated with the lowest wage. Anticipating this behavior the agent is unmotivated to implement any positive work effort. This partially explains the concern of the management literature to ensure credible feedback provision. In addition, the problem of credible evaluations provides a partial answer to Fuchs (2007, p. 1446), who emphasizes the importance of exploring "possible reasons for the existence of communication" between agents and principals. Communication at the interim stages might be explained by training and instruction reasons, but credibility problems are responsible for communication in the last stages of the principal-agent relation.

The results of this paper are important for the design of incentives systems. First, the systems have to ensure the credible provision of appropriate feedback by institutionalizing the feedback process or using multi-source feedback. Second, the pooling at the top could cause the costs of an incentives scheme to be substantial if there is a bonus attached to receiving a positive evaluation and many employees receive a positive evaluation due to the compression at the top.<sup>15</sup> Third, my result provides a rationale, why forced distribution systems, requiring supervisors to match a given distribution with their evaluations, are an uncommon response to lenient evaluations.<sup>16</sup> These systems ensure dispersion in the results of the evaluation, but are suboptimal, as they require too much communication.

This paper assumes that the principal incurs costs for communicating with the agent. I would get similar results if the principal's costs instead concerned the acquisition of information. In this case, the principal only learns the result of the evaluation, i.e., the average  $\mu = \int I(t)dt$  of the information  $I$ . Then she decides whether to spend  $\kappa$  to acquire the entire information  $I$ . Independently of her choice of information acquisition, the principal can send a cheap-talk message in the unrestricted message set  $\mathcal{I}$ . Both settings have some merits; in reality, there could be a mixture of these two polar cases.

<sup>14</sup> "The distribution of ratings is typically both concentrated and biased." (Gibbs, 1991, p. 5)

<sup>15</sup> Bernardin and Orban (1990, p. 199) provide the example of the Small Business Administration and NASA introducing a bonus scheme based on subjective evaluations. After more than 50% of eligible employees should receive a bonus, Congress responded with the requirement that no more than 25% of employees shall receive a bonus.

<sup>16</sup> Bretz et al. (1992) and Gibbs (1991) show that the use of forced distributions is very limited. According to Murphy (1993, p. 47), forced distribution systems "mitigate managerial tendencies to assign uniform ratings but may generate important counterproductive side effects."

## A Appendix

### A.1 The Optimal Complete Contract

Lemma 1 characterizes the optimal complete contract and states the solution to Program A\* if the principal's information  $I$  is public and verifiable. This yields a benchmark solution  $w_e^*(\mu)$ , the optimal complete wage. Additionally, the lemma shows that every effort  $e \in [0, 1]$  is implementable at finite costs  $C^*(e)$ .

**Proof of Lemma 1:** Holmström (1979) shows that the optimal wage only conditions on  $\mu = \int I(t)dt$ , because the average of the principal's information  $I$  is a sufficient statistics for the agent's effort,  $\Pr(I, S|e, \int I(t)dt) = \Pr(I, S|\int I(t)dt)$ . In order to implement no effort,  $e = 0$ , set  $w_0^*(\mu) = u^{-1}(\bar{u} + d(0))$  for all  $\mu$ .

If, on the other hand, the desired effort is positive,  $e > 0$ , the agent's incentive compatibility matters. The first-order approach is valid here, because  $F(p|e)$  is a linear combination of distribution functions. This implies that the convex distribution function condition is satisfied. According to Grossman and Hart (1983) and Rogerson (1985), the convex distribution function condition in combination with the convexity of  $d(\cdot)$  and the monotone likelihood ratio property guarantees the validity of the first-order approach. Therefore, the agent's incentive compatibility reads

$$\int u(w(\mu))(f^H(\mu) - f^L(\mu))d\mu = d'(e) \quad (\text{IC}_A)$$

In addition, the constraint set is nonempty. Take for example any  $\bar{w} > 0$  and the contract

$$w(\mu) = \begin{cases} \bar{w} & \text{if } f^H(\mu) - f^L(\mu) \geq 0 \\ h(\bar{w}) & \text{otherwise} \end{cases}$$

with  $h(\bar{w})$  positive, but small enough, such that the agent's incentive compatibility ( $\text{IC}_A$ ) is satisfied. This implicitly defines an increasing function  $h(\cdot)$ . Consequently, there is a  $\bar{w}$  fulfilling the participation constraint (PC) with equality. Therefore the constraint set of Program A\* is nonempty. Moreover, the costs of the contract are  $h(\bar{w})F(\zeta|e) + \bar{w}(1 - F(\zeta|e)) < \bar{w} < \infty$  with  $\zeta = \inf\{\mu \in [0, 1] | f^H(\mu) - f^L(\mu) \geq 0\}$ .

Holmström (1979) proves that the Lagrange multipliers of the participation constraint  $\lambda_1$  and of the incentive compatibility  $\lambda_2$  are positive. Pointwise optimization<sup>17</sup> determines the optimal contract as

$$\begin{aligned} f(\mu|e) - \lambda_1 u'(w(\mu))f(\mu|e) - \lambda_2 u'(w(\mu))(f^H(\mu) - f^L(\mu)) &= 0 \quad \text{a.s.}, \\ \frac{1}{u'(w(\mu))} = \lambda_1 + \lambda_2 \frac{f^H(\mu) - f^L(\mu)}{f(\mu|e)} &= \lambda_1 + \lambda_2 \frac{\frac{f^H(\mu)}{f^L(\mu)} - 1}{e \frac{f^H(\mu)}{f^L(\mu)} + 1 - e} \quad \text{a.s.} \end{aligned} \quad (3)$$

<sup>17</sup>This technique allows for piecewise continuous functions, as Kamien and Schwartz (1991) show in Part II, Section 12. Therefore bonus wages are possible and there is no restriction to continuous contracts.

Since the fraction  $\frac{l-1}{el+1-e}$  is increasing in  $l$ , the right-hand side of above equation is increasing in  $\mu$  due to the monotone likelihood ratio property. Therefore the concavity of  $u(\cdot)$  implies that  $w_e^*(\mu)$  is increasing in  $\mu \in [0, 1]$  almost surely. Moreover,  $w_e^*(\mu)$  is continuous almost surely and any discontinuity is removable, because the densities  $f^H$  and  $f^L$  are continuous.  $\square$

## A.2 The Optimal Contract

I begin by considering the optimal contract in the absence of communication costs,  $\kappa = 0$ .

**Proof of Lemma 2:** First, I show that the contract proposed in Lemma 2 satisfies the additional incentive constraints  $(IC_P)$  and  $(IC_{m_A})$ . As the agent's wage is independent of  $m_A$ ,  $(IC_{m_A})$  is trivially satisfied. Now consider a deviation by the principal. For this purpose, consider a subset  $D \subset T$  with  $\int_D 1dt > 0$ . Then there is a  $\bar{N} \in \mathbb{N}$ , such that a deviation on  $D$  is unprofitable for the principal for all  $n > \bar{N}$ , because the probability of a mismatch in the messages is sufficiently close to 1. Additionally, deviations on larger subsets of  $T$  are also unprofitable given the assumption of full support and no atoms for the distribution of the agent's sample  $S$ . What about deviating on a subset  $D$  with infinitesimal mass  $\int_D 1dt$ ? Consider a parameterization of such deviations  $D(v)$ , which maps  $[0, 1]$  to the Borel algebra on  $T$ , with  $v = \int_{D(v)} 1dt$ . In the limit for  $v \rightarrow 0$  the potential gain of a deviation *and* the probability of detection converge to 0. Yet, for  $v \rightarrow 0$  such a deviation is unprofitable if

$$\frac{\partial w_e^*(\mu)}{\partial \mu} < k \frac{\partial \Pr(S \cap D(v) \neq \emptyset | I, e)}{\partial v}(0)$$

for all  $\mu \in [0, 1]$ . The left-hand side of this inequality captures the potential gain of a deviation and is determined by Eq. (3). The right-hand side of this inequality equals the penalty  $k$  multiplied with the density of at least one source from the agent's sample being an element of  $D(0)$ . This density is increasing in the size  $n$  of the agent's sample. Therefore there is a  $\bar{N} \in \mathbb{N}$ , such that a deviation on  $D$  is unprofitable for the principal for all  $n > \bar{N}$ . Hence, the additional incentive constraints  $(IC_P)$  and  $(IC_{m_A})$  are satisfied.

Second, the contract proposed in Lemma 2 is the optimal contract, because it satisfies the additional incentive constraints  $(IC_P)$  and  $(IC_{m_A})$  and implements the equilibrium wage  $w_e^*(\mu)$ . As  $w_e^*(\mu)$  is a solution to Problem A\* according to Lemma 1, we have found a solution to Problem A for  $\kappa = 0$ .  $\square$

According to Proposition 1, it is suboptimal to provide justification almost surely.

**Proof of Proposition 1:** Suppose the principal communicates almost surely, i.e.,  $\Pr(\mathcal{I}_C) = 1$ . Then the expected communication costs are  $\kappa \mathbb{E}(\beta(I)) = \kappa$  and it just remains to minimize the wage costs. Yet it is possible to implement payments  $w_e^*(\mu)$  defined in

Program A\* by the following contract.

$$\bar{W}(m_P, m_A) = \begin{cases} w_e^*(\mu) & \text{if } m_P(t) = 1 \text{ for all } t \in m_A \\ w_e^*(1) + 2\kappa & \text{if } m_P(t) \neq 1 \text{ for some } t \in m_A \end{cases}$$

and  $\bar{c}(m_P, m_A) = w_e^*(\mu)$  with  $\mu = \int m_P(t)dt$ . For the same reasons as in the proof of Lemma 2, the additional incentive constraints (IC<sub>P</sub>) and (IC<sub>m<sub>A</sub></sub>) are satisfied. Furthermore, the equilibrium wage is  $w_e^*(\mu)$ , as the parties' messages agree in equilibrium.

It is possible, however, to implement a certain work effort  $e$  of the agent even cheaper by partial communication. For this purpose, modify the contract  $\bar{W}$  to

$$\bar{W}'(m_P, m_A) = \begin{cases} w_e^*(\int m_P(t)dt) & \text{if } m_P(t) = 1 \text{ for all } t \in m_A \\ w_e^*(1) + 2\kappa & \text{if } \int m_P(t)dt < \delta \text{ and } m_P(t) \neq 1 \text{ for some } t \in m_A \\ w_e^*(\delta) + \kappa & \text{if } \int m_P(t)dt \geq \delta \text{ and } m_P(t) \neq 1 \text{ for some } t \in m_A \end{cases}$$

with a  $\delta < 1$ , such that

$$f^H(\delta) - f^L(\delta) \geq 0 \quad \text{and} \quad w_e^*(\delta) + \kappa \geq w_e^*(1). \quad (4)$$

Lemma 1 proves that  $w_e^*(\mu)$  is almost surely continuous and any discontinuity is removable. Consequently, there exists a continuous function that almost surely equals  $w_e^*(\mu)$ . Replacing  $w_e^*(\mu)$  by that function in the definition of  $\bar{W}'$  also yields a solution to Program A. This procedure guarantees that the conditions (4) on  $\delta$  are feasible.

In the contract  $\bar{W}'$ , for  $n > \bar{N}$ , the principal justifies all evaluations except the highest ones and the communication set is

$$\mathcal{I}_C = \left\{ I \in \mathcal{I} \mid \int I(t)dt \leq \delta \right\}.$$

If the principal's information indicates a very good performance,  $\mu > \delta$ , communication would increase her total costs consisting of wage and communication costs as according to Lemma 1  $w_e^*(\cdot)$  is increasing and, hence,  $w_e^*(\delta) + \kappa < w_e^*(\mu) + \kappa$ . In addition, the conditions in (4) guarantee that constraints (PC) and (IC<sub>A</sub>) are still satisfied by choosing the agent's wage appropriately. Therefore contract  $\bar{W}'$  implements effort  $e$  of the agent and is cheaper than the contract  $\bar{W}$ . This shows that the principal will not justify all evaluations.  $\square$

Lemma 3 shows that I can concentrate on truthful messages without loss of generality.

**Lemma 3.** *For every contract  $W$  there is a contract  $W'$ , such that*

- *$W'$  gives the agent incentives to implement the same effort  $e$  as in  $W$ ,*
- *$W'$  has (weakly) lower costs for the principal than  $W$  and*
- *$W'$  gives the agent and the principal incentives to send truthful messages.*

*In addition, contract  $W'$  has the following structure*



$$c'(m_P, m_A) = \begin{cases} c(m_P) & \text{if } m_P \in \mathcal{I}'_C \\ \bar{w} & \text{if } m_P \notin \mathcal{I}'_C \end{cases} \quad (5)$$

$$W'(m_P, m_A) = \begin{cases} w(m_P) & \text{if } m_P \in \mathcal{I}'_C \text{ and } m_P(t) = 1 \text{ for all } t \in m_A \\ \bar{w} + \kappa & \text{if } m_P \in \mathcal{I}'_C \text{ and } m_P(t) \neq 1 \text{ for some } t \in m_A \\ \bar{w} & \text{if } m_P \notin \mathcal{I}'_C \end{cases} \quad (6)$$

with the communication set  $\mathcal{I}'_C = \{I \in \mathcal{I} | \beta'(I) = 1\}$ .

**Proof:** The proof consists of four parts. The first part characterizes the equilibrium utilities in contract  $W$ . In the second part, the contract  $W'$  is determined in such a way that the parties have an incentive to send truthful messages. The third part analyzes the agent's incentive compatibility ( $\text{IC}_A$ ). The fourth part ensures that the new contract  $W'$  satisfies the agent's incentive compatibility and participation constraint.

**Step 1** Denote the expected payment conditional on information  $I$  given equilibrium strategies in contract  $W$  by  $w(I)$  for the principal and the corresponding certainty equivalent by  $c(I)$  for the agent. If in contract  $W$  the principal provides no justification in equilibrium,  $I \notin \mathcal{I}_C$ , the agent cannot verify the evaluation. The reason is that the cardinality of  $\mathcal{I}$  and  $R$  differ. According to Cantor's theorem, the set of all subsets of a set  $A$  has a strictly greater cardinality than the set  $A$  itself. Here, the cardinality of  $\{0, 1\}^T$  equals the cardinality of the power set of  $T$  and is bigger than the cardinality of  $T$  and the one of  $R$ . Hence, it is impossible to encode the information  $I$  into a message in  $R$ . No matter which information the principal tells in her message  $m_P \in R$ , the probability that it matches the agent's information  $S$  is 0. Consequently, the agent cannot verify the principal's message. Therefore the principal's payments have to be constant or  $w(I) = w(I')$  for all  $I, I' \notin \mathcal{I}_C$ . Moreover, they have to be higher than the principal's payments in the communication set including communication costs.

$$w(I) \geq \kappa + \sup_{I' \in \mathcal{I}_C} w(I') \quad \forall I \notin \mathcal{I}_C$$

Otherwise, the principal would not provide justification and act as if  $I \notin \mathcal{I}_C$ , because the agent could not observe this deviation.

**Step 2** Consider the contract  $W'$  given by equations (5) and (6) with  $\mathcal{I}'_C = \mathcal{I}_C \setminus R$  and  $\bar{w} = w(I)$  for an  $I \notin \mathcal{I}_C$ .<sup>18</sup> The agent's wage does not depend on his message. Therefore he is indifferent between sending any message and the incentive compatibility for his message is satisfied. If the principal should communicate,  $I \in \mathcal{I}_C$ , any disagreement in the messages shows a deviation by the principal and the payment  $W'(I, S)$  with  $I \in \mathcal{I}_C$  and  $I(t) \neq 1$  for some  $t \in S$  matters only for the right-hand side of the principal's incentive compatibility ( $\text{IC}_P$ ). Therefore I can increase this payment to satisfy ( $\text{IC}_P$ )

<sup>18</sup>This requires identifying a message  $m_P \in R$  with the step function:  $T \rightarrow \{0, 1\}$  with 1 for  $t \leq m_p$  and 0 otherwise. Additionally, if  $\mathcal{I}_C = \mathcal{I}$ , set  $\bar{w} = \kappa + \sup_{\hat{I} \in \mathcal{I}_C} w(\hat{I})$ .

without affecting any other constraint or the objective function. Accordingly, there will be a penalty for  $I \in \mathcal{I}_C$  and  $I(t) \neq 1$  for some  $t \in S$ . By setting the principal's payment in this case to

$$W(I, S) = \bar{w} + \kappa$$

there is a  $\bar{N} \in \mathbb{N}$  such that for  $n > \bar{N}$  the principal will never deviate to another message in the communication set  $\mathcal{I}_C$ . The reason is that a deviation at least weakly increases payments as

$$\bar{w} + \kappa > \int W'(\hat{I}, S) dP(S|I, e) \geq \bar{w} > w(I) = \int W'(I, S) dP(S|I, e)$$

for all  $I, \hat{I} \in \mathcal{I}_C$  with  $\int_{\{t \in T | \hat{I}(t) \neq I(t)\}} 1 dt > 0$ . The reasoning is the same as in the proof of Proposition 1. For large deviations the probability of a mismatch in the messages is sufficiently close to 1. For small deviations increasing the number of sources in the agent's sample increases the density of one of these sources being in the deviation set. On the margin, the density matters and not the probability. Hence, a deviation is unprofitable.

**Step 3** Denote by  $M(\mu) = \{I \in \mathcal{I} | \int I(t) dt = \mu\}$  the set of all information with an average  $\mu$ . For the agent's incentives only the expected wage in  $M(\mu)$  matters, because the agent's information,  $S$ , does not depend on her effort choice and the average of the principal's information is a sufficient statistics for the agent's effort,  $\Pr(I, S | e, \int I(t) dt) = \Pr(I, S | \int I(t) dt)$ . In addition, Lemma 1 shows that the first-order approach is valid here. Therefore, the agent's incentive compatibility (IC<sub>A</sub>) equals

$$\int \left( \int u(c(I)) dP(I, S | e, I \in M(\mu)) \right) (f^H(\mu) - f^L(\mu)) d\mu = d'(e) \quad (7)$$

Consequently, the agent's incentives remain unchanged if

$$\int u(c(I)) dP(I, S | e, I \in M(\mu)) = \int u(c'(\theta(I, \beta(I)), S)) dP(I, S | e, I \in M(\mu)) \quad (8)$$

for all  $\mu \in [0, 1]$ . By the definition of  $c'$  in (5), the right-hand side equals

$$\int \beta'(I) u(c(I)) dP(I, S | e, I \in M(\mu)) + u(\bar{w}) \int 1 - \beta'(I) dP(I, S | e, I \in M(\mu))$$

Hence, the equality in (8) is not guaranteed for all  $\mu$ , as  $c(I) \leq c'(I, m_A) = \bar{w}$  for  $I \notin \mathcal{I}'_C$ .

**Step 4** If (8) is not satisfied, it is necessary to reduce the expected wage of the agent. For this purpose, let the principal justify every evaluation  $I \notin \mathcal{I}_C \cup R$  with  $c(I) \leq \bar{w} - \kappa$ . Denote the set of these  $I$  by  $\mathcal{I}'$ . Now set  $\mathcal{I}'_C = \mathcal{I}'_C \cup \mathcal{I}'$ ,  $c'(I, m_A) = c(I)$  and

$$W'(I, m_A) = \begin{cases} c(I) & \text{if } I(t) = 1 \quad \forall t \in m_A \\ \bar{w} + \kappa & \text{if } \exists t \in m_A: I(t) \neq 1 \end{cases} \quad \forall I \in \mathcal{I}', \forall m_A \in T^n.$$

Thus, for any remaining evaluations that are not justified,  $I \notin \mathcal{I}'_C \cup R$ , the agent's wage is  $\bar{w} - \kappa < c(I) \leq \bar{w}$  in contract  $W$ . Finally, increase the communication set and make the principal justify a fraction  $\alpha$  of the evaluations  $M(\mu) \setminus \mathcal{I}'_C$ , such that

$$\begin{aligned} \int u(c(I))dP(I, S | e, I \in M(\mu)) &= \int \beta'(I)u(c(I))dP(I, S | e, I \in M(\mu)) + \\ &+ (\alpha u(\bar{w} - \kappa) + (1 - \alpha)u(\bar{w})) \int 1 - \beta'(I)dP(I, S | e, I \in M(\mu)) \end{aligned}$$

As the agent's wage is  $\bar{w} - \kappa < c(I) \leq \bar{w}$  for all  $I \notin \mathcal{I}'_C \cup R$ , it is possible to find such an  $\alpha \in [0, 1]$ . Set  $\alpha = 1$  if  $\alpha$  is not uniquely determined, as  $M(\mu) \setminus \mathcal{I}'_C$  has mass 0. Denote the addition evaluations that are justified by  $\mathcal{I}''$ . To make communication optimal, adjust contract  $W'$  by  $\mathcal{I}'_C = \mathcal{I}'_C \cup (\mathcal{I}'' \setminus R)$ ,  $c'(I, m_A) = \bar{w} - \kappa$  and

$$W'(I, m_A) = \begin{cases} \bar{w} - \kappa & \text{if } I(t) = 1 \quad \forall t \in m_A \\ \bar{w} + \kappa & \text{if } \exists t \in m_A: I(t) \neq 1 \end{cases} \quad \forall I \in (\mathcal{I}'' \setminus R), \forall m_A \in T^n.$$

Repeat these steps for every  $\mu$ . Then the agent's incentives in the new contract  $W'$  are the same as in contract  $W$ . In addition, the agent's participation constraint is also satisfied in contract  $W'$ , so that contract  $W'$  implements effort  $e$  at (weakly) lower costs than contract  $W$ .  $\square$

Lemma 4 proves that contracts in which the agent's wage payment depends only on the average of the principal's information are no loss of generality.

**Lemma 4.** *For every contract  $W$  there is a contract  $W'$  implementing the same effort  $e$  at the same costs as  $W$ . Moreover, in contract  $W'$  the communication choice  $\beta$  and the agent's wages just depend on the average of the principal's information, i.e.,  $\beta(m_P) = \beta(m'_P)$  and  $c'(m_P, m_A) = c'(m'_P, m_A)$  for all  $m_A \in T^n, m_P, m'_P \in \mathcal{I}_C$  with  $\int m_P(t)dt = \int m'_P(t)dt$ .*

**Proof:** Lemma 3 shows that the agent's wage does not depend on his message  $m_A$ . In addition, according to equation (7), the agent's incentive compatibility ( $IC_A$ ) depends only on the expected utility of the agent given the average of the principal's information. The same is valid for the agent's participation constraint in Program A. Therefore it is possible without violating these constraints to set  $c'(m_P, m_A) = \tilde{c}(\mu)$  for all  $m_P \in \mathcal{I}_C$  and  $m_A \in T^n$  with

$$u(\tilde{c}(\mu)) = \int u(c(\hat{I}, S))dP(\hat{I}, S | e, \hat{I} \in M(\mu) \cap \mathcal{I}_C), \quad \mu = \int m_P(t)dt,$$

and  $M(\mu) = \{I \in \mathcal{I} | \int I(t)dt = \mu\}$ . This reduces at least weakly the expected wage, because the agent is risk-averse. Hence,

$$\tilde{c}(\mu) \leq \int c(\hat{I}, S)dP(\hat{I}, S | e, \hat{I} \in M(\mu) \cap \mathcal{I}_C). \quad (9)$$

Yet, the agent's new wage  $c'(m_P, m_A) = \tilde{c}(\mu)$  might be higher than the principal's payment  $W(m_P, m_A)$  for some  $m_A$  and  $m_P \in \mathcal{I}_C$ . To make the contract feasible and satisfy constraint (1) in Program A, set

$$W'(m_P, m_A) = \int W(\hat{I}, S) dP(\hat{I}, S | e, \hat{I} \in M(\mu) \cap \mathcal{I}_C) \quad \forall m_P \in \mathcal{I}_C, \forall m_A \in T^n$$

with  $\mu = \int m_P(t) dt$ . This ensures that  $W'(m_P, m_A) \geq \tilde{c}(\mu)$  for all  $m_P, m_A$ , because  $W(m_P, m_A) \geq c(m_P, m_A)$  in contract  $W$  and the new wage,  $\tilde{c}(\mu)$ , is lower than the previous expected wage according to equation (9). In addition, the principal's expected payments in the new contract  $W'$  are the same as in contract  $W$ , as

$$\int W'_{eq}(I, S) dP(I, S | e, I \in M(\mu)) = \int W_{eq}(I, S) dP(I, S | e, I \in M(\mu)) \quad \forall \mu.$$

Consequently, there exists a contract that gives the principal and the agent the same utility as the old contract  $W$ , but makes the agent's utility depend only on the average of the principal's information, i.e.,  $c(m_P, m_A) = \tilde{c}(\mu)$  for all  $m_A$  and all  $m_P \in \mathcal{I}_C$ . It remains to ensure constraint (IC<sub>P</sub>) in contract  $W'$ . For  $I \in \mathcal{I}_C$  in the communication set,  $W'(I, S)$  with  $I(t) \neq 1$  for some  $t \in S$  matters only on the right-hand side of constraint (IC<sub>P</sub>). Therefore increasing  $W'(I, S)$  to  $\kappa + \sup_{\bar{I}, \bar{S}} W(\bar{I}, \bar{S})$  does not affect the objective function or other constraints, but gives the principal incentives to communicate truthfully. Additionally, the wage in the communication set is lower than outside this set including the communication costs,  $W'(I, S) + \kappa < W'_{eq}(\hat{I}, \hat{S})$  for all  $I \in \mathcal{I}_C, \hat{I} \notin \mathcal{I}_C$  and all  $S, \hat{S} \in T^n$ , because contract  $W$  meets this condition according to Lemma 3. This guarantees that there is a  $\bar{N} \in \mathbb{N}$  such that for all  $n > \bar{N}$  any deviation in the communication choice,  $\beta(I)$ , and/or the message  $m_P$  makes the principal worse off. Therefore also the new contract  $W'$  satisfies the principal's incentive compatibility.

It remains to prove that the communication choice does not change within the set  $M(\mu)$  for any  $\mu$ . **1.** Suppose there is a  $\mu$  such that  $\hat{c}(\mu) \leq \bar{w} - \kappa$  with  $\bar{w}$  the wage outside the communication set according to Lemma 3 and  $\hat{c}$  defined by

$$u(\hat{c}(\mu)) = \int u(c(\hat{I}, S)) dP(\hat{I}, S | e, \hat{I} \in M(\mu)).$$

Then it is possible to justify all  $I \in M(\mu)$  by setting  $\mathcal{I}'_C = \mathcal{I}'_C \cup M(\mu)$  and by adapting the wage as in the first part of the proof.

**2.** If, on the other hand,  $\hat{c}(\mu) = \bar{w}$ , the principal does not need to provide justification for all  $I \in M(\mu)$  by setting  $\mathcal{I}'_C = \mathcal{I}'_C \setminus M(\mu)$  and  $W(I, m_A) = c(I, m_A) = \bar{w}$  for all  $m_A \in T^n$ .

**3.** Finally, in the last case  $\bar{w} - \kappa < \hat{c}(\mu) < \bar{w}$ . Denote by  $A$  the set of all  $\mu$  with this property,

$$A = \{\mu \in [0, 1] | \bar{w} - \kappa < \hat{c}(\mu) < \bar{w}\}.$$

If the set  $A$  has no mass,  $\int_A f(\mu|e)d\mu = 0$ , set  $\mathcal{I}'_C = \mathcal{I}'_C \cup M(\mu)$  and  $\hat{c}(\mu) = \bar{w} - \kappa$  for all  $\mu \in A$  and adapt the wage as in the first part of the proof. If  $\int_A f(\mu|e)d\mu > 0$ , there exists a unique  $\delta$ , such that justifying only evaluations below  $\delta$  does not change the agent's expected utility, i.e.,

$$\int_A u(\hat{c}(\mu))dF(\mu|e) = u(\bar{w} - \kappa) \int_{A^1} dF(\mu|e) + u(\bar{w}) \int_{A^2} dF(\mu|e)$$

with the sets  $A^1 = \{\mu \in A | \mu \leq \delta\}$  and  $A^2 = \{\mu \in A | \mu > \delta\}$ . For this purpose, modify the contract to

$$\begin{aligned} \beta'(m_P) &= \begin{cases} 1 & \text{if } \bar{\mu} \in A^1 \\ 0 & \text{if } \bar{\mu} \in A^2 \end{cases} & c'(m_P, m_A) &= \begin{cases} \bar{w} - \kappa & \text{if } \bar{\mu} \in A^1 \\ \bar{w} & \text{if } \bar{\mu} \in A^2 \end{cases} \\ W'(m_P, m_A) &= \begin{cases} \bar{w} & \text{if } \bar{\mu} \in A^2 \\ \bar{w} - \kappa & \text{if } \bar{\mu} \in A^1 \text{ and } m_P(t) = 1 \text{ for all } t \in m_A \\ \bar{w} + \kappa & \text{if } \bar{\mu} \in A^1 \text{ and } m_P(t) \neq 1 \text{ for some } t \in m_A \end{cases} \end{aligned}$$

for all  $m_P \in M(\bar{\mu})$ , all  $m_A \in T^n$  and all  $\bar{\mu} \in A$ . As the agent is risk-averse and  $W(m_P, m_A) \geq c(m_P, m_A) \in (\{\bar{w}\} \cup (0, \bar{w} - \kappa])$ , this reduces the principal's payments:

$$\int_A \left( \int W(\hat{I}, S) dP(\hat{I}, S | e, \hat{I} \in M(\mu)) \right) dF(\mu|e) \geq (\bar{w} - \kappa) \int_{A^1} dF(\mu|e) + \bar{w} \int_{A^2} dF(\mu|e)$$

In addition, the left-hand side of the agent's incentive compatibility (7) increases:

$$\begin{aligned} & \int_{A^1} (u(\bar{w} - \kappa) - u(\hat{c}(\mu))) \Delta^f(\mu) d\mu + \int_{A^2} (u(\bar{w}) - u(\hat{c}(\mu))) \Delta^f(\mu) d\mu = \\ &= \int_{A^1} \underbrace{(u(\bar{w} - \kappa) - u(\hat{c}(\mu)))}_{<0} \frac{\Delta^f(\mu) f(\mu|e)}{f(\mu|e)} d\mu + \int_{A^2} \underbrace{(u(\bar{w}) - u(\hat{c}(\mu)))}_{>0} \frac{\Delta^f(\mu) f(\mu|e)}{f(\mu|e)} d\mu > \\ &> \frac{f^H(\delta) - f^L(\delta)}{f(\delta|e)} \left( \int_{A^1} u(\bar{w} - \kappa) dF(\mu|e) + \int_{A^2} u(\bar{w}) dF(\mu|e) - \int_A u(\hat{c}(\mu)) dF(\mu|e) \right) = 0 \end{aligned}$$

with  $\Delta^f(\mu) = f^H(\mu) - f^L(\mu)$ . The monotone likelihood ratio property ensures the strict inequality and the constant value of the participation constraint yields the final equality. This shows that the modified contract satisfies the agent's incentive compatibility ( $IC_A$ ).

Consequently, contract  $W'$  satisfies all the constraints of Program A and the communication decision just depends on the average of the principal's information.  $\square$

Proposition 2 characterizes the optimal contract.

**Proof of Proposition 2:** Lemma 3 determines the basic structure of the optimal contract and this proof uses the notation introduced there. Lemma 4 shows that the communication decision and the agent's wage just depend on the average of the principal's

message. It remains to calculate the agent's wage. First, I prove that there are no payments to third parties on the equilibrium path. For this purpose, simplify Program A according to Lemma 3 and 4 to

$$\begin{aligned}
& \inf \int w(I) + \kappa\beta(I)dP(I, S|e), & (B) \\
& \text{subject to } \int u(c(\mu))dF(\mu|e) - d(e) \geq \bar{u}, & (PC) \\
& \int u(c(\mu))(f^H(\mu) - f^L(\mu))d\mu = d'(e), & (IC_A) \\
& \bar{w} \geq w(I) + \beta(I)\kappa \geq (1 - \beta(I))\bar{w} & \forall I \in \mathcal{I}, & (IC_P) \\
& w(I) \geq c\left(\int I(t)dt\right) & \forall I \in \mathcal{I}. & (10)
\end{aligned}$$

(IC<sub>P</sub>) ensures that the wage is constant outside the communication set. Furthermore, the condition requires that the principal's costs in the communication set, accounting for the communication costs, is lower than outside this set. According to Lemma 3 this is equivalent to condition (IC<sub>P</sub>) in Program A. Condition (10) is the equivalent to (1) in the initial program and guarantees that payments to third parties are nonnegative. Assume to the contrary that in the optimal contract there is an  $I \in \mathcal{I}$ , such that  $w(I) > c(\int I(t)dt)$ . Then  $\beta(I) = 1$ , because Lemma 3 and 4 show that  $c(\int I(t)dt) = \bar{w} = w(I)$  for  $I \notin \mathcal{I}_C$ . In addition, it is possible to (weakly) decrease the objective function without violating any constraint by setting  $w(I) = c(\int I(t)dt)$ . Therefore the principal's consensus payment just depends on the average of her message and  $w(I) = c(\int I(t)dt)$  for all  $I \in \mathcal{I}$ . This results in payments to a third party of

$$W(m_P, m_A) - c\left(\int m_P(t)dt\right) = \begin{cases} \bar{w} - w(m_P) + \kappa & \text{if } m_P \in \mathcal{I}_C \text{ and } \exists t \in S: I(t) \neq 1 \\ 0 & \text{otherwise.} \end{cases}$$

As the principal's consensus payments and her communication decision just depend on the average of the principal's information, I write them as functions of the average  $\mu$  instead of the information  $I$  in the following. Finally, it remains to determine the values of  $w^{**}(\mu)$  and  $w^{**}$ , which solve the following program:

$$\begin{aligned}
& C^c(e) = \inf \int w(\mu) + \kappa\beta(\mu)dF(\mu|e), & (C) \\
& \text{subject to } \int u(w(\mu))dF(\mu|e) - d(e) \geq \bar{u}, & (PC) \\
& \int u(w(\mu))(f^H(\mu) - f^L(\mu))d\mu = d'(e), & (IC_A) \\
& (1 - \beta(\mu))(w(\mu) - \bar{w}) = 0 & \forall \mu, & (11) \\
& w(\mu) + \kappa\beta(\mu) \leq \bar{w} & \forall \mu. & (12)
\end{aligned}$$

As before, the objective is to minimize the expected costs, here communication costs

and the principal's payments. The agent's participation constraint (PC) and incentive compatibility ( $IC_A$ ) remain unchanged. Constraints (11) and (12) replace condition ( $IC_P$ ). Program C determines the optimal contract and completes the proof.  $\square$

### A.3 The Optimal Communication Pattern

Proposition 3 verifies that a threshold strategy is optimal, with justification below a threshold  $\delta$  and pooling above  $\delta$ .

**Proof of Proposition 3:** Proposition 1 proves that complete feedback is suboptimal and  $\Pr(\mathcal{I}_C) < 1$ . In order to show that only bad evaluations will be justified, assume to the contrary that in the optimal contract there is a  $\mu^*$  with the following properties. With positive probability the principal justifies evaluations  $\mu$  above  $\mu^*$  and with positive probability the principal does not justify evaluations  $\mu$  below  $\mu^*$ . Denote the corresponding sets by  $A^K = \{\mu \in [0, 1] | \mu \geq \mu^* \text{ and } \beta(\mu) = 1\}$  for justified evaluations above  $\mu^*$  and evaluations without justification below  $\mu^*$  by  $A^N = \{\mu \in [0, 1] | \mu \leq \mu^* \text{ and } \beta(\mu) = 0\}$ . By assumption,  $\Pr(A^K), \Pr(A^N) > 0$ . Then rewrite program C in the following way. Change constraint (11) to

$$f(\mu|e)(1 - \beta(\mu))(w(\mu) - w)^2 \leq 0 \quad (13)$$

for all  $\mu$ . (13) is equivalent to (11), but simplifies the next steps of the proof. This condition guarantees that the wage is constant outside the communication set, in particular, in  $A^N$ . In the communication set and thus in the subset  $A^K$ , condition (13) is trivially fulfilled, as  $1 - \beta(\mu) = 0$ . Furthermore, multiply constraint (12) by  $f(\mu|e)$  to obtain

$$f(\mu|e)(w(\mu) + \kappa\beta(\mu) - w) \leq 0 \quad (14)$$

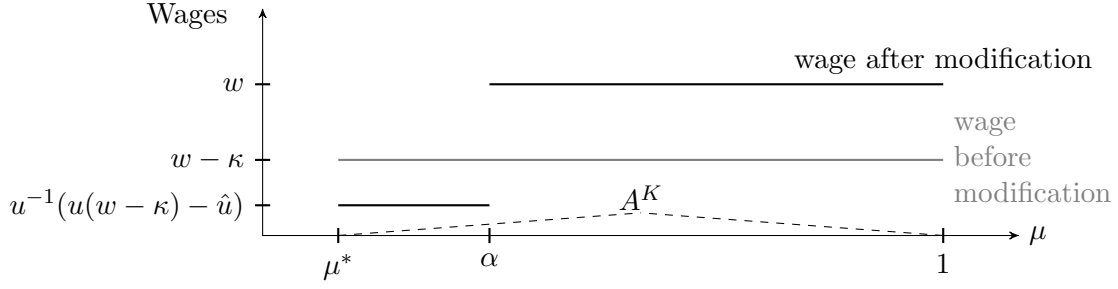
for all  $\mu$ . (14) is equivalent to (12) and guarantees that it is optimal to justify an evaluation if justification is required by the contract, i.e.,  $\beta(\mu) = 1$ . Together, both constraints make justification optimal, if and only if  $\beta(\mu) = 1$ . In addition, they ensure that the wage has to be lower in  $A^K$  than in  $A^N$ , as

$$w(\mu) < w = w(\mu') \quad \forall \mu \in A^K, \mu' \in A^N. \quad (15)$$

The inequality follows from condition (14), while the equality is given by condition (13). Define  $\lambda_1, \lambda_2, \nu_1(\mu)$  and  $\nu_2(\mu)$  to be the Lagrange multipliers of the constraints (PC), ( $IC_A$ ), (13) and (14) respectively. Pointwise optimization<sup>19</sup> with respect to  $w(\mu)$  yields

$$\begin{aligned} 1 - u'(w(\mu)) \left( \lambda_1 + \lambda_2 \frac{f^H(\mu) - f^L(\mu)}{f(\mu|e)} \right) + \nu_1(\mu)(1 - \beta(\mu))2(w(\mu) - w) + \nu_2(\mu) &= 0 \\ \Leftrightarrow 1 + \nu_2(\mu) = u'(w(\mu)) \left( \lambda_1 + \lambda_2 \frac{f^H(\mu) - f^L(\mu)}{f(\mu|e)} \right), &\text{ a.s.} \end{aligned} \quad (16)$$

<sup>19</sup>Cf. footnote 17 for the generality of this method.



In the figure the set  $A^K$  is an interval to ease exposition. In general,  $A^K$  need not be convex.

Figure 4: The Modification of the Wage Contract

because constraint (13) guarantees  $(1 - \beta(\mu))(w(\mu) - w) = 0$  for all  $\mu$ . The monotone likelihood ratio property ensures that

$$\frac{f^H(\mu) - f^L(\mu)}{f(\mu|e)} = \frac{x(\mu) - 1}{ex(\mu) + 1 - e} \quad \text{with } x(\mu) = \frac{f^H(\mu)}{f^L(\mu)}$$

is increasing in  $\mu$ . Together with (15) this proves that the right-hand side of equation (16) is higher for  $\mu \in A^K$  than for  $\mu' \in A^N$ , because  $\mu' < \mu$ . To match this increase,  $\nu_2(\mu)$  almost surely has to be positive in  $A^K$ . Hence,  $w(\mu) = w - \kappa$  almost surely for  $\mu \in A^K$ .

The next step modifies the contract to implement effort  $e$  cheaper than before. For this purpose, determine the median  $\alpha$  of  $A^K$ , such that

$$\int_{A_1^K} f(\mu|e) d\mu = \int_{A_2^K} f(\mu|e) d\mu \quad (17)$$

for  $A_1^K = A^K \cap [\mu^*, \alpha)$  and  $A_2^K = A^K \cap [\alpha, 1]$ . In the new contract, the principal does not justify elements of  $A^K$  above  $\alpha$ ,  $\beta(\mu) = 0$ , while below  $\alpha$ , she provides justification. Then, I add (resp. subtract) the monetary equivalent of  $\hat{u} = u(w) - u(w - \kappa)$  to the wage, so that

$$w'(\mu) = \begin{cases} u^{-1}(u(w - \kappa) + \hat{u}) = w & \text{for } \mu \in A_2^K \\ u^{-1}(u(w - \kappa) - \hat{u}) = u^{-1}(2u(w - \kappa) - u(w)) & \text{for } \mu \in A_1^K. \end{cases}$$

This means that the wage increases by  $\kappa$  for values in  $A^K$  above  $\alpha$  and decreases by an amount adjusted for the changes in marginal utility below  $\alpha$ . Figure 4 illustrates this modification of the wage. The modified wage still satisfies the condition (PC). Additionally, the left-hand side of condition (IC<sub>A</sub>) is now bigger than the marginal cost of effort,  $d'(e)$ , because the left-hand side of condition (IC<sub>A</sub>) increases by

$$\hat{u} \left( - \int_{A_1^K} f^H(\mu) - f^L(\mu) d\mu + \int_{A_2^K} f^H(\mu) - f^L(\mu) d\mu \right) =$$



$$\begin{aligned}
&= \hat{u} \left( - \int_{A_1^K} \frac{f^H(\mu) - f^L(\mu)}{f(\mu|e)} f(\mu|e) d\mu + \int_{A_2^K} \frac{f^H(\mu) - f^L(\mu)}{f(\mu|e)} f(\mu|e) d\mu \right) > \\
&> \hat{u} \frac{f^H(\alpha) - f^L(\alpha)}{f(\alpha|e)} \left( - \int_{A_1^K} f(\mu|e) d\mu + \int_{A_2^K} f(\mu|e) d\mu \right) = 0.
\end{aligned}$$

The last equality follows from the definition of  $\alpha$  in (17). The main inequality follows from the increasing likelihood ratio. By reducing the variance of the wage payments until condition (IC<sub>A</sub>) holds with equality, condition (PC) becomes slack. This allows decreasing the wage and shows a contradiction to the existence of  $\mu^*$ . Therefore the optimal contract does not require the principal to justify good evaluations, as the agent does not suspect a distortion of these evaluation results. On the other hand, the principal has to justify bad evaluations. The set  $A^K$  can be replaced by any set of justified evaluations and a wage of  $w - \kappa$ . Thus, in the optimal contract any set with these properties has no mass and, hence, has no influence on the optimal contract. Therefore, the wage and communication pattern can be adjusted to a threshold rule with respect to communication. Combining these results with Program C, allows me to prove analogously to Lemma 1 that  $w(\mu)$  is strictly increasing in  $\mu \in [0, \delta]$ .  $\square$

**Proof of Proposition 4:** The proof proceeds along the following lines. If  $e > 0$  and the principal justifies almost surely no evaluation, e.g., due to high communication costs  $\kappa$ , the optimal contract does not exist.<sup>20</sup> Yet, for every  $\epsilon$  there is a contract whose costs are at most  $\epsilon$  higher than the infimum costs to implement effort  $e > 0$  by the agent. Finally, the proof shows that condition (2) ensures that the optimal contract provides justification with positive probability.

In the absence of justification, the principal's payments have to be constant according to Lemma 3, as it is impossible to verify her message  $m_P$ . Denote her payments by  $\bar{w}$ . The next steps calculate the infimum costs to implement positive effort  $e > 0$  of the agent. For this purpose, the set of feasible contracts is reduced step by step, as I show that contracts with specific characteristics are suboptimal. First consider a contract with  $c(\mu) < \bar{w}$  almost surely. In this case, it is possible to reduce the principal's payments  $\bar{w}$  without violating any constraint. Therefore I only have to take contracts into account with a positive probability for  $\{\mu \in [0, 1] | c(\mu) = \bar{w}\}$ .

Second, consider an optimal contract  $W$  with a  $\mu^*$ , such that there are payments to a third party for  $\mu > \mu^*$  with positive probability, but with positive probability there are no such payments for  $\mu \leq \mu^*$ . Denote the corresponding sets by  $A^S = \{\mu \in [0, 1] | \mu > \mu^* \text{ and } c(\mu) < \bar{w}\}$  with third-party payments above  $\mu^*$  and the set of evaluations without such payments below  $\mu^*$  by  $A^D = \{\mu \in [0, 1] | \mu \leq \mu^* \text{ and } c(\mu) = \bar{w}\}$ . By assumption  $\int_{A^S} f(\mu|e) d\mu, \int_{A^D} f(\mu|e) d\mu > 0$ . Now reduce the bigger set, until both sets have the

<sup>20</sup>There is an optimal contract if the distributions  $F^H$  and  $F^L$  have atoms at 0.

same mass,  $\int_{A^S} f(\mu|e)d\mu = \int_{A^D} f(\mu|e)d\mu$ . In the next step, I modify the wage to

$$c'(\mu) = \begin{cases} u^{-1} \left( \frac{1}{\int_{A^D} f(\mu|e)d\mu} \int_{A^S} u(c(\mu)) f(\mu|e)d\mu \right) & \text{for } \mu \in A^D \\ \bar{w} & \text{for } \mu \in A^S \\ c(\mu) & \text{otherwise.} \end{cases} \quad (18)$$

On  $A^D$  the agent's wage is reduced to the average wage on  $A^S$  in contract  $W$ , while on  $A^D$  the wage increases to  $\bar{w}$ . Otherwise, the contract remains unchanged. In order to check whether this contract is feasible, I analyze the remaining constraints (PC) and (IC<sub>A</sub>). By the definition of the wage modification  $W'$  in (18), the agent's participation constraint (PC) is still satisfied. On the other hand, the left-hand side of the agent's incentive compatibility (IC<sub>A</sub>) increases by

$$\begin{aligned} & \int_{A^D} \underbrace{(u(c'(\mu)) - u(\bar{w}))}_{<0} (f^H(\mu) - f^L(\mu))d\mu + \int_{A^S} \underbrace{(u(\bar{w}) - u(c(\mu)))}_{>0} (f^H(\mu) - f^L(\mu))d\mu > \\ & > \frac{f^H(\mu^*) - f^L(\mu^*)}{f(\mu^*|e)} \left( \int_{A^D} u(c'(\mu)) f(\mu|e)d\mu - \int_{A^S} u(c(\mu)) f(\mu|e)d\mu \right) = 0. \end{aligned}$$

The monotone likelihood ratio property ensures the strict inequality and the constant value of the participation constraint yields the equality. Therefore the modified wage  $W'$  in (18) makes the left-hand side of condition (IC<sub>A</sub>) bigger than the marginal cost of effort,  $d'(e)$ . In addition, the contract satisfies condition (PC). By reducing the variance of the wage payments, until condition (IC<sub>A</sub>) holds with equality, condition (PC) becomes slack. This allows decreasing the wage. Consequently, restrict attention to contracts with a  $\delta < 1$ , such that  $c(\mu) = \bar{w}$  for all  $\mu > \delta$ .

Third, consider such a contract  $W$  with  $\hat{\delta}$ . To satisfy the agent's incentive compatibility, there has to be a payment to a third party, i.e.,  $c(\mu) < \bar{w}$ , with positive probability. Denote the corresponding set by  $A^S = \{\mu \in [0, 1] | c(\mu) < \bar{w}\}$  and its median by  $\alpha$ . By the previous remarks, these values are below  $\hat{\delta}$ , i.e.,  $\mu \leq \hat{\delta}$  for all  $\mu \in A^S$ . Now, modify the wage to

$$c''(\mu) = \begin{cases} u^{-1}(\hat{u}(\mu)) & \text{for } \mu \leq \alpha \text{ and } \mu \in A^S \\ \bar{w} & \text{otherwise} \end{cases}$$

$$\hat{u}(\mu) = u(c(\mu)) - u(\bar{w}) + \frac{2}{\int_{A^S} f(\bar{\mu}|e)d\bar{\mu}} \int_{\bar{\mu} \in A^S \text{ and } \bar{\mu} > \alpha} u(c(\bar{\mu})) f(\bar{\mu}|e)d\bar{\mu}.$$

For evaluations above the median  $\alpha$ , the agent's wage increases to  $\bar{w}$ , while below the median the wage is reduced to balance the utility gain above the median. Otherwise, the contract remains unchanged. The proof is now analogous to the last case. Therefore it is

possible to improve the contract, as long as  $\delta > 0$ . Yet the contract with a constant wage for the agent and  $\delta = 0$  does not satisfy the agent's incentive compatibility. Therefore an optimal contract does not exist and I have to consider a sequence of contracts.

For this purpose, construct a sequence of feasible contracts that satisfy the properties derived in this proof. For those contracts, I derive upper and lower bounds for their costs. Consider the following contracts for a small  $\delta > 0$ :  $W_\delta(m_P, m_A) = \bar{w}(\delta)$  and

$$\bar{c}_\delta(m_P, m_A) = c_\delta \left( \int m_P(t) dt \right) = \begin{cases} \bar{w}(\delta) & \text{if } \int m_P(t) dt > \delta \\ \bar{w}(\delta) - \Lambda_\delta(\int m_P(t) dt) & \text{if } \int m_P(t) dt \leq \delta, \end{cases}$$

with  $\bar{w}(\delta)$  and third-party payments  $\Lambda_\delta(\mu)$  such that the agent's incentive compatibility and his participation constraint are satisfied. This requires

$$\int_0^\delta u(c_\delta(\mu)) f(\mu|e) d\mu + (1 - F(\delta|e)) u(\bar{w}(\delta)) = \bar{u} + d(e) \quad (\text{PC})$$

$$\int_0^\delta u(c_\delta(\mu)) (f^H(\mu) - f^L(\mu)) d\mu - u(\bar{w}(\delta)) \int_0^\delta f^H(\mu) - f^L(\mu) d\mu = d'(e) \quad (\text{IC}_A)$$

as  $\int_\delta^1 f^H(\mu) - f^L(\mu) d\mu = \int_0^\delta f^L(\mu) - f^H(\mu) d\mu$ . The constraint  $(\text{IC}_A)$  implies  $u(c_\delta(\mu)) < 0$  for all  $\mu \leq \delta$  with  $\delta$  sufficiently small, because  $\int_0^\delta f^H(\mu) - f^L(\mu) d\mu \rightarrow 0$  for  $\delta \rightarrow 0$  and  $f^H(\mu) - f^L(\mu) < 0$  for all  $\mu \leq \delta$ . Rearranging the agent's incentive compatibility and approximating it from above results in

$$d'(e) \leq -u(\bar{w}(\delta)) \int_0^\delta f^H(\mu) - f^L(\mu) d\mu + \frac{f^H(0) - f^L(0)}{f(0|e)} \int_0^\delta u(c_\delta(\mu)) f(\mu|e) d\mu.$$

In the next step, insert the participation constraint for  $\int_0^\delta u(c_\delta(\mu)) f(\mu|e) d\mu$  to get

$$u(\bar{w}(\delta)) \geq \frac{d'(e) + \frac{f^L(0) - f^H(0)}{f(0|e)} (\bar{u} + d(e))}{\int_0^\delta f^L(\mu) - f^H(\mu) d\mu + \frac{f^L(0) - f^H(0)}{f(0|e)} (1 - F(\delta|e))}$$

as a lower bound or

$$u(\bar{w}(\delta)) \leq \frac{d'(e) + \frac{f^L(\delta) - f^H(\delta)}{f(\delta|e)} (\bar{u} + d(e))}{\int_0^\delta f^L(\mu) - f^H(\mu) d\mu + \frac{f^L(\delta) - f^H(\delta)}{f(\delta|e)} (1 - F(\delta|e))}$$

as an upper bound. For  $\delta \rightarrow 0$ , both bounds converge to

$$\bar{u} + d(e) + \frac{f(0|e)}{f^L(0) - f^H(0)} d'(e).$$

This also coincides with the optimal contract in the case of atoms at 0, which guarantee

existence of an optimal contract. To ensure that the set of these contracts  $W_\delta$  is nonempty, consider the following contract:

$$\bar{w}(\delta) = u^{-1} \left( \bar{u} + d(e) + \frac{F(\delta|e)}{\int_0^\delta f^L(\mu) - f^H(\mu) d\mu} d'(e) \right) \quad \text{and}$$

$$u(\bar{w}(\delta) - \Lambda_\delta(\bar{\mu})) = \bar{u} + d(e) - \frac{1 - F(\delta|e)}{\int_0^\delta f^L(\mu) - f^H(\mu) d\mu} d'(e), \quad \forall \bar{\mu} \leq \delta.$$

This contract satisfies all the properties derived in this proof as well as constraints (PC) and (IC<sub>A</sub>) with equality. The costs of this contract are  $\bar{w}(\delta)$  which by l'Hôpital's rule converges to  $C^0(e) = u^{-1} \left( \bar{u} + d(e) + \frac{f(0|e)}{f^L(0) - f^H(0)} d'(e) \right)$  for  $\delta \rightarrow 0$ . The convergence is also monotone in the costs of the contract, because  $\bar{w}(\delta)$  is strictly increasing in  $\delta$ . As the densities are continuous, for every  $\epsilon > 0$  there exists a  $\delta' > 0$ , such that the costs of the contract  $W_\delta$  for all  $0 < \delta \leq \delta'$  are lower than  $C^0(e) + \epsilon$ . Yet it is impossible to approximate first-best, because  $C^0(e) > u^{-1}(\bar{u} + d(e))$ .

For the final step of the proof, compare the principal's costs with and without justification. If she provides justification with probability 0,  $\Pr(\mathcal{I}_C) = 0$  and the principal's costs are  $C^0(e)$ . On the other hand, the expected wage costs are

$$C^*(e) = \int w_e^*(\mu) dF(\mu|e)$$

with justification almost surely,  $\Pr(\mathcal{I}_C) = 1$ , as shown in Proposition 1 in combination with Proposition 2. Then the optimal contract implements the second-best benchmark wage defined by Program A\*. It is now possible to show that assumption (2) is feasible, which is equivalent to the difference between  $C^0(e)$  and  $C^*(e)$  being positive. Neglecting the communication costs, the costs for implementing the justification contract  $\bar{W}$  are lower than for any contract without justifications. The reason is the informativeness principle of Holmström (1979). This proves that condition (2) is indeed feasible and provides a lower bound for the savings, i.e.,  $C^c(e) - C^0(e)$ , which are possible by providing justification. Hence, as long as the communication costs are lower than this bound, the principal will justify evaluations with positive probability.  $\square$

#### A.4 Budget-Balanced Contracts

It is possible to make the optimal contract ex-post budget-balanced.

**Proof of Proposition 5:** In order to capture stochastic payments, change the interpretation of the notation. Now  $w(m_P, m_A)$  denotes the expected wage after principal and agent sent messages  $m_P$  and  $m_A$ . On the other hand,  $c(m_P, m_A)$  is the agent's certainty equivalent of the wage payment. Finally, lotteries with the corresponding mean and certainty equivalent will be specified. This formalization captures any stochastic payment without loss of generality. The principal could pay the agent a lottery or could discard

certain messages with some probability by ‘turning a blind eye’.<sup>21</sup>

Program A still describes the problem. Yet constraint (1) now captures the agent’s risk aversion. Proposition 2 states the solution to Program A. The solution  $W^{**}$  gives the principal and the agent the same utilities as the contract  $\bar{W}^{**}$  in Proposition 5 if the lotteries are chosen accordingly. For the principal this is obvious, as the lotteries have mean  $\kappa$ . The agent’s expected utility also remains unchanged, because the certainty equivalent equals his former wage.

Finally, specify a lottery with the desired properties. Let  $\Lambda_z$  denote a lottery that pays  $\kappa + z$  and  $\kappa - z$  with probability 1/2, respectively.<sup>22</sup>  $z(\mu)$  is determined, such that

$$\mathbb{E}u(w^{**} + \Lambda_{z(\mu)}) = \frac{1}{2}u(w^{**} + \kappa + z(\mu)) + \frac{1}{2}u(w^{**} + \kappa - z(\mu)) = u(w^{**}(\mu)).$$

It is possible to find such a  $z(\mu)$ . Additionally,  $z(\mu)$  is unique for every  $\mu$  due to the strict concavity of  $u$ . The lottery does not change the principal’s expected payments, but reduces the agent’s certainty equivalent. The mean preserving spread introduced by the lottery is the reason for this loss of utility. Once the lottery is realized, the party who gains in the lottery has an incentive to avoid renegotiations. Therefore the lottery ought to be realized as soon as the messages are available in order to make the contract renegotiation-proof.<sup>23</sup>  $\square$

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<sup>21</sup>This is optimal for example in Herweg et al. (2010).

<sup>22</sup>If the agent has limited liability, the lottery  $\Lambda_z$  can be redefined to limit the negative realization, but to assign it a higher probability.

<sup>23</sup>Rasmusen (1987) and Maskin (2002) provide a good discussion of this problem.

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