Pricing stock options under stochastic volatility and stochastic interest rates with efficient method of moments estimation

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Abstract

While the stochastic volatility (SV) generalization has been shown to improve the explanatory power compared to the Black-Scholes model, the empirical implications of the SV models on option pricing have not been adequately tested. The purpose of this paper is to first estimate a multivariate SV model using the efficient method of moments (EMM) technique and then investigate the respective effect of stochastic interest rate, systematic volatility and idiosyncratic volatility on option prices. We compute option prices using both underlying historical volatilities obtained through reprojection and volatilities implied from observed option prices and gauge each model's performance through direct comparison with observed market option prices. Our results suggest: (i) While theory predicts that the short-term interest rates are strongly related to the systematc volatility of the consumption process, our estimation results suggest that the short-term interest rate fails to be a good proxy of the systematic factor; (ii) While allowing for stochastic volatility of stock returns can in general reduce the pricing errors and allowing for asymmetry or "leverage effect" in the SV models does help to explain the skewness of the volatility "smile", allowing for stochastic interest rate has minimal impact on option prices in our case; (iii) Similar to Melino and Turnbull (1990), our empirical findings strongly suggest the existence of a non-zero risk premum for stochastic volatility of stock returns. Allowing for non-zero risk-premium of stochastic volatility and based on implied volatility, the SV models can largely reduce the option pricing errors, suggesting the importance of incorporating the information in the options market in pricing options.

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Preliminary, comments are welcome!

1 Introduction

Acknowledging the fact that volatility is changing over time in time-series of asset returns as well as in the empirical variances implied from option prices through the Black-Scholes model itself, there have been numerous recent studies on option pricing with time varying volatility. Many authors have proposed to model the asset return dynamics using the so-called *stochastic volatility* (SV) models. Examples of these models in continuous-time include Hull and White (1987), Johnson and Shannon (1987), Wiggins (1987), Scott (1987, 1991, 1997), Baily and Stulz (1989), Chesney and Scott (1989), Melino and Turnbull (1990), Stein and Stein (1991), Heston (1993), Bates (1996), and Bakshi, Cao and Chen (1997), and examples in discrete-time include Taylor (1986), Harvey, Ruiz and Shephard (1994), Amin and Ng (1993), Andersen (1994), and Kim, Shephard and Chib (1996). A review article on SV models is provided by Ghysels, Harvey and Renault (1996). Due to intractable analytical likelihood functions and hence the lack of readily available efficient estimation procedures, the general SV processes were viewed as an unattractive class of models in comparison to other time-varying volatility models, such as ARCH/GARCH models, see Shephard (1996) for a comparison of ARCH/GARCH models with SV models. Over the past few years, however, remarkable progress has been made in the field of statistics and econometrics regarding the estimation of nonlinear latent variable models in general and SV models in particular. Various estimation methods have been proposed, we mention Quasi Maximum Likelihood (QML, Harvey, Ruiz and Shephard (1994)), which has been improved to Monte Carlo Maximum Likelihood by Sandmann and Koopman (1997), GMM (Andersen and Sørensen (1996)), MCMC methods (to name a few: Jacquier, Polson and Rossi (1994) and Kim, Shephard and Chib (1996)) and the Efficient Method of Moments (EMM, Gallant and Tauchen (1996)).

While the stochastic volatility generalization has been shown to improve the explanatory power compared to the Black-Scholes model, the empirical implications of the SV models on option pricing have not yet been adequately tested. Can such generalization help resolve well-known systematic empirical biases associated with the Black-Scholes model, such as the volatility smiles (e.g. Rubinstein (1985)), asymmetry of such smiles (e.g. Stein (1989), Clewlow and Xu (1993), and Taylor and Xu (1993, 1994))? Is the gain, if any, from such generalization substantial compared to relatively simpler models? Or, in other words is the gain worth the additional complexity or implementational costs? The purpose of this paper is to study the empirical performance of stochastic volatility models in pricing stock options, and investigate the respective effect of stochastic interest rates, systematic volatility and idiosyncratic volatility on option prices in a multivariate SV model framework. We specify and implement a model extended in the line of Rubinstein (1976), Brennan (1979), and Amin and Ng (1993). The model incorporates both the effects of idiosyncratic volatility and systematic volatility of the underlying stock returns in option valuation and at the same time allows interest rates to be stochastic. In addition, we model the short-term interest rate dynamics and stock returns dynamics simultaneously and allow for the existence of the *leverage effect* through the correlation of shocks to stock returns and the conditional volatility. We also observe a substantial correlation between the interest rate changes and its conditional volatility and incorporate this effect in our model as well.

The first objective of this paper is to estimate the parameters of the multivariate SV model. Instead of implying the model parameters from market option prices through an option pricing formula in a risk-neutral specification, we directly estimate the model specified under the objective measure from the observations of underlying state variables. Doing so, the underlying model specification can be tested in the first hand for how well it represents the true DGP. In particular, we investigate the effects of the inclusion of systematic volatility components on the parameter estimates of both the stock return process and the latent volatility process. We employ the efficient method of moments (EMM) proposed by Gallant and Tauchen (1996) to estimate the multivariate SV model using historical observations of the state variables, namely the daily stock returns and daily short term interest rates. This method shares the advantage of being valid for a whole class of models, with moment based estimation techniques, and at the same time achieves the first order asymptotic efficiency of likelihood based methods. In addition, the method offers information about the model specification.

The second objective of this paper is to examine the effects of different elements considered in the model on stock option prices through direct comparison with observed market option prices, searching the balance between the costs of extensive modelling and gains of more complex models. Inclusion of both systematic components and idiosyncratic components in the model lend us the ability to judge whether extra predictability or uncertainty is more helpful for pricing options. In gauging the empirical performance of alternative option pricing models, we use both the relative difference and the implied Black-Scholes volatility as measures of systematic errors. Our setup contains a variety of option pricing models in the literature as special cases, for instance (i) the SV model of stock returns and stochastic interest rate which considers no systematic effects on option values; (ii) the SV model of stock returns with constant risk-free interest rate; (iii) the stochastic interest rate model with constant conditional volatility of stock returns; and (iv) the Black-Scholes model with both constant interest rate and constant conditional volatility of stock returns. We focus our comparison of the general model setup with the above four submodels.

Note that every option pricing model has to make at least two fundamental assumptions: the stochastic processes of underlying asset prices and efficiency of the markets. The latter assumption ensures the existence of market price of risk for each factor that leads to a "risk-neutral" specification. The joint hypothesis we aim to test in this paper is as following: the underlying model specification is correct and option markets are efficient. If the joint hypothesis holds, the option pricing formula derived from the underlying model under equilibrium should be able to correctly predict option prices, both in sample and out of the sample. Obviously such a joint hypothesis is testable by comparing the model predicted option prices with market observed option prices. The advantage of our framework is that since we emphasize on the underlying model specification in its objective measure and more importantly, its efficient estimation from the observations of the underlying variables, it lends us the ability to directly test whether the model specification is acceptable or not. Test of such a hypothesis, combined with the test of the above joint hypotheses, can lead us to make conclusions about whether the option markets are efficient or not, which is one of the most interesting issues to both practitioners and academics. The framework in this paper is different in spirit from the implied methodology often used in the finance literature in the following aspects. First, the implied methodology can at best offer a test of the joint hypotheses, it fails going any further to test the model specification or the efficiency of the market; Second, the reason for the first problem is due to the fact that implied in the option prices is only the risk-neutral specification of the underlying model, thus only a subset of the parameters can be estimated (or backed-out) from the option prices; Third, the implied methodology based on solely the information contained in the option prices is purely objective driven, it is rather a test of stability of certain relationship (the option pricing formula) between different input factors (the implied parameter values) and the output (the option prices).

In judging the empirical performance of alternative models in pricing options, we perform two alternative tests. First, we use in-sample historical volatility (through *reprojection*) to calculate a set of option prices with different maturities and terms to expiration. The model-generated option prices are compared to the observed market option prices in terms of relative percentage differences, as well as implied Black-Scholes volatility. Second, we perform an out-of-sample comparison using observed option prices to back out each day's implied volatility as well as a market price of risk for the stochastic volatility through fitting certain option pricing formula. Such values are used in the following day's volatility process to generate a set of option prices. Again, the model generated option prices are compared to the observed market option prices. In the first comparison, all models only use the information contained in the underlying state variables, while in the second comparison, the models use both information contained in the underlying state variables and in the observed (previous day's) market option prices. The major findings of this paper include: (i) While theory predicts that the short-term interest rates are strongly related to the systematic volatility of the consumption process, our empirical results suggest that the short-term interest rate fails to be a good proxy of the systematic factor; (ii) Overall, all models exhibit clear and significant discrepancies between model predicted option prices and market observed option prices for different terms to expiration and degrees of moneyness based on (reprojected) underlying volatility. In particular, the Black-Scholes model underprices short-term maturity call options and overprices long-term maturity call options, and underprices deep *in-the-money* options and overprices deep *out-of-the-money* options based on both underlying historical volatility and implied volatility; (iii) While allowing for stochastic volatility of stock returns can in general reduce the pricing errors, allowing for stochastic interest rates has minimal impact on option prices in our case; (iv) Similar to Melino and Turnbull (1990), our empirical findings strongly suggest the existence of a non-zero risk premium for stochastic volatility of stock returns. Allowing for non-zero risk-premium of stochastic volatility and based on implied volatility, the models with stochastic volatility of stock returns and in particular the asymmetric stochastic volatility models of stock returns can largely reduce the option pricing errors.

The plan of this paper is as follows. Section 2 outlines the general multivariate SV model; Section 3 describes the EMM estimation technique and the volatility reprojection method; Section 4 reports the estimation results of the general model and various submodels; Section 5 compares among different models the performance in pricing options and analyzes the effect of each individual factor; Section 6 concludes.

2 The Model

The uncertainty in the economy presented in Amin and Ng (1993) is driven by the realization of a set of random variables at each discrete date. Among them are a random shock to the consumption process, a random shock to the individual stock price process, a set of systematic state variables that determine the time-varying "mean", "variance", and "covariance" of the consumption process and stock returns, and finally a set of stock-specific state variables that determine the idiosyncratic part of the stock return "volatility". The investors' information set at time t is represented by the σ-algebra F_t which consists of all available information up to t. Thus the stochastic consumption process is driven by, in addition to a random noise, its mean rate of return and variance which are determined by the systematic state variables. The stochastic stock price process is driven by, in addition to a random noise, its mean rate of return and variance which are determined by both the systematic state variables and idiosyncratic state variables. In other words, the stock return variance can have a systematic component that is correlated and changes with the consumption variance. An important key relationship derived under the equilibrium condition is that the variance of consumption growth is negatively related to the interest rate, in other words interest rate can be used as a proxy of the systematic factor in the economy. Therefore a larger proportion of systematic volatility implies a stronger negative relationship between the individual stock return variance and interest rate. Given that the variance and the interest rate are two important inputs in the determination of option prices and that they have the opposite effects on call option values, the correlation between the variance and interest rate will therefore be important in determining the net effect of the two inputs. In the following, we will specify an empirically implementable SV models of the interest rate and stock returns for the purpose of pricing individual stock options.

2.1 The Basic Model

The basic model we specify in this paper is as follows. Let S_t denote the price of the stock at time t and r_t the interest rate at time t, where r_t is a systematic state variable and S_t is an individual state variable which is subject to both systematic and idiosyncratic shocks. We model the dynamics of daily stock returns and daily interest rate changes simultaneously as a multivariate stochastic volatility process. Suppose r_t is correlated to the trend or instantaneous mean of both state variables, then the de-trended or the unexplained stock return y_{st} is defined as

$$y_{st} := 100 \times \Delta \ln S_t - \mu_S - \phi_S r_{t-1} \tag{1}$$

and the de-trended or the unexplained interest rate change y_{rt} is defined as

$$y_{rt} := 100 \times \Delta \ln r_t - \mu_r - 100 \times \phi_r \ln r_{t-1} \tag{2}$$

Next, y_{st} and y_{rt} are modeled as stochastic volatility processes

$$y_{st} = \sigma_{st}\epsilon_{st} \tag{3}$$
$$y_{rt} = \sigma_{rt}\epsilon_{rt} \tag{4}$$

where

$$\ln \sigma^2_{st+1} = \alpha \ln r_t + \omega_s + \gamma_s \ln \sigma^2_{st} + \sigma_s \eta_{st} \qquad |\gamma_s| < 1 \tag{5}$$
$$\ln \sigma^2_{rt+1} = \omega_r + \gamma_r \ln \sigma^2_{rt} + \sigma_r \eta_{rt} \qquad |\gamma_r| < 1 \tag{6}$$

and

$$\begin{bmatrix} \epsilon_{st} \\ \epsilon_{rt} \\ \eta_{st} \\ \eta_{rt} \end{bmatrix} \sim IIN(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \lambda_1 & \lambda_2 & 0 \\ \lambda_1 & 1 & 0 & \lambda_3 \\ \lambda_2 & 0 & 1 & \lambda_4 \\ 0 & \lambda_3 & \lambda_4 & 1 \end{bmatrix}$$

so that $\mathsf{Cor}(\epsilon_{st}, \epsilon_{rt}) = \lambda_1, \mathsf{Cor}(\epsilon_{st}, \eta_{st}) = \lambda_2, \mathsf{Cor}(\epsilon_{rt}, \eta_{rt}) = \lambda_3$ and $\mathsf{Cor}(\eta_{st}, \eta_{rt}) = \lambda_4$. The model resembles the standard multivariate SV model in discrete time (see e.g. Andersen (1994), Harvey, Ruiz and Shephard (1994) and Taylor (1994)), but with a few extensions. For example, the inclusion of the systematic component in the stock return volatility process, the leverage effect through both λ_2 and λ_3. The interest rate model admits mean-reversion in the drift and allows for stochastic conditional volatility. We could also incorporate the "level effect" (see e.g. Andersen and Lund (1997)) into the conditional volatility. Since in this paper we focus on the pricing of stock options and we find that the specification of interest rate process is relatively less important in such applications, we do not incorporate this level effect.

The basic idea of the above SV model is guided by the empirical distribution implied in the option prices, i.e. to search alternative option pricing models which have the "right" distributional assumption. The SV model, for instance, offers a flexible distributional structure in which the correlation between volatility shocks and underlying stock returns serves to control the level of asymmetry and the volatility variation coefficient serves to control the level of kurtosis. But since volatility in the diffusion-type SV models only follows a continuous sample path, its ability to internalize enough short-term kurtosis and thus to price short-term options properly is limited. The above model setup is specified in discrete time and includes continuous-time models as special cases in the limit.

2.2 Statistical Properties

First of all, the above model is specified to catch the possible systematic effects through parameters ϕ_S in the trend and α in the conditional volatility; Second, we model the dynamics of logarithmic interest rates so that the nominal interest rates are restricted to be positive, as negative nominal interest rates are ruled out by a simple arbitrage argument; Third, the above model specification allows the movements in unconditional volatility to be correlated across the random noises ϵ_{st} and ϵ_{rt} via their correlation λ_1. We also include the systematic factor in the conditional volatility of stock returns. It is only the systematic state variable that affects the individual stock returns' volatility not the another way around. The parameters λ_2 and λ_3 are to measure the "leverage effects" for stock returns and interest rates. It is noted that when ϵ_{st} and η_{st} are allowed to be correlated with each other, the model can pick up the kind of asymmetric behavior which is often found for stock price changes. In particular, a negative correlation between η_{st} and ϵ_{st} ($\lambda_2 < 0$) induces a *leverage effect*. It is noted that the above model specification will be tested against alternative specifications and be investigated for its implications on option prices. The correlation λ_4 is to measure the correlation between the volatility shocks. Although we judge this correlation to be a potentially important parameter for option pricing we set this parameter a priori equal to zero. Since given the current state of knowledge of EMM, estimation of this parameter is not feasible using EMM. This will become clear from section 3. Danielsson (1996) claims that his Simulated Maximum Likelihood method can estimate this parameter. He finds estimates that are about .3 and significantly different from zero for stock indices and for major exchange rates.

Both the conditional volatility of stock returns and the change of logarithmic interest rates are assumed to be $\mathsf{AR}(1)$ processes except for the additional systematic effect in the stock return's conditional volatility. The main statistical properties of the above model can be summarized as: (i) Both y_{st} and y_{rt} are martingale differences, i.e. $\mathsf{E}[y_{st}|F_t] = 0, \mathsf{E}[y_{rt}|F_t] = 0$ and $\mathsf{Var}[y_{st}|F_t] = \sigma^2_{st}, \mathsf{Var}[y_{rt}|F_t] = \sigma^2_{rt}$; (ii) y_{rt} is stationary if and only if $\ln \sigma^2_{rt}$ is stationary; (iii) Since η_{rt} is assumed to be normally distributed, then $\ln \sigma^2_{rt}$ is also normally distributed, say $\ln \sigma^2_{rt} \sim N(\mu, \sigma^2)$, given certain initial conditions. Hence when ϵ_{rt} follows a standard normal distribution, using the fact $\mathsf{E}[\exp\{a \ln \sigma^2_{rt}\}] =$

$\exp\{a\mu + a^2\sigma^2/2\}$, it can be derived that the variance of y_{rt} is given by $\mathsf{Var}[y_{rt}] = \exp(\sigma^2/2)$ and the fourth moment or Kurtosis of y_{rt} is given by $3\exp(\sigma^2)$ which is greater than 3, so y_t exhibits more kurtosis and thus fatter tails than ϵ_{rt}; (iv) All the odd moments of y_{rt} are zero. Conditional on r_t or $\alpha = 0$, it can be shown that y_{st} has the above same statistical properties as y_{rt}.

2.3 Advantages of the Model

Advantages of the proposed model include: First, the model explicitly incorporate the effects of systematic volatility on option prices. Empirical evidence shows that the volatility of stock returns is not only stochastic, but that it is also highly correlated with the volatility of the market as a whole. That is, in addition to an idiosyncratic volatility for the returns of individual stock, there is also a systematic component that is related to the market volatility. (see e.g. Black (1975), Conrad, Kaul, and Gultekin (1991), Jarrow and Rosenfeld (1984), Jorion (1988), and Ng, Engle, and Rothschild (1992)). The empirical evidence also shows that the biases inherent in the Black-Scholes option prices are different for options on high and low risk stocks (see, e.g. Black and Scholes (1972), Gultekin, Rogalski, and Tinic (1982), and Whaley (1982)), inclusion of systematic volatility in the option prices valuation model has the potential contribution to reduce the empirical biases exhibited by prices computed from the Black-Scholes formula; Second, since the variance of consumption growth is negatively related to the interest rate in equilibrium, the dynamics of consumption process relevant to option valuation are embodied in the interest rate process. The model is thus naturally extended to allow for stochastic interest rates. Existing work of extending the Black-Scholes model has moved away from considering either stochastic volatility or stochastic interest rates (examples include Merton (1973), Rabinovitch (1989), Sandmann (1993)) but to considering both, examples include Baily and Stulz (1989), Amin and Ng (1993), Bakshi and Chen (1997a,b) and Scott (1997). Simulation results show that there can be a significant impact of stochastic interest rates on option prices. (see e.g. Rabinovitch (1989)). Furthermore, due to the relationship between interest rate and consumption process in the equilibrium, we only need to directly model the dynamics of interest rates; Third, the above proposed model allows the study of the simultaneous effects of both a stochastic interest rates and a stochastic stock return's volatility on the valuation of options. It is documented in the literature that when the interest rate is stochastic the Black-Scholes option pricing formula tends to underprice the European call options (Merton, 1973), while in the case that the stock return's volatility is stochastic, the Black-Scholes option pricing formula tends to overprice the at-the-money European call options (Hull and White, 1987). The combined effect of both factors depends on the relative variability of the two processes (Amin and Ng, 1993). Based on simulation, Amin and Ng (1993) show that stochastic interest rates cause option values to decrease if each of these effects acts by themselves. However, this combined effect should depend on the relative importance (variability) of each of these two processes. Finally, when the model is symmetric, i.e. there is no correlation between the shocks to stock returns and stock return conditional volatility, the closed form solutions of the option prices are available and are preference free in quite general conditions, i.e., the stochastic mean of the stock return process, the stochastic mean and variance of the consumption process, as well as the covariance between the changes of stock returns and consumption are predictable. These conditions are automatically satisfied in the continuous-time diffusion models, but requires slight constraint imposed on the model specification in the discrete time. In particular, when $\lambda_2 = 0$ in the general model setup, i.e. the assumption 2 in Amin and Ng (1993) is satisfied, the following option pricing formula can be derived. Let C_0 represent the value of a European call option at $t = 0$ with exercise price K and expiration date T, Amin and Ng (1993) derive that

$$C_0 = \mathsf{E}_0[S_0 \cdot N(d_1) - K\exp(-\sum_{t=0}^{T-1} r_t)N(d_2)] \tag{7}$$

where

$$\begin{aligned} d_1 &= \frac{\ln(S_0/(K\exp(\sum_{t=0}^{T} r_t)) + 0.5\sum_{t=1}^{T}\sigma_{st}}{(\sum_{t=1}^{T}\sigma_{st})^{1/2}} \\ d_2 &= d_1 - \sum_{t=1}^{T}\sigma_{st} \end{aligned}$$

where the expectation is taken w.r.t. the objective measure and can be calculated from simulations.

As Amin and Ng (1993) point out, several option-pricing formulae in the available literature are special cases of the above option formulae. They include the Black-Scholes (1973) formula, the Hull-White (1987) stochastic variance stock option valuation formula, the Bailey-Stultz (1989) stochastic variance index option pricing formula, and the Merton (1973), Amin and Jarrow (1992), and Turnbull and Milne (1991) stochastic interest rate option valuation formulae. When the interest rate is constant, (7) is then the Hull-White (1987) formula. If the stock variance is also constant, then we obtain the Black-Scholes formula. In the above option pricing formula, the European call option prices depend on the average expected volatility over the length of the option contract. Since averaging should reduce standard errors, even relatively large standard errors of the volatility estimates do not necessarily carry over to option prices. In the limit, the average volatility over a long horizon converges to the unconditional variance when conditional on the parameters of the process and the interest rate process.

3 Estimation and Reprojection

The reason why an SV model cannot be estimated by standard maximum likelihood lies in the fact that the time varying volatility is modeled as a latent or unobserved variable which has to be integrated out of the likelihood. This is not a standard problem since the dimension of this integral equals the number of observations, which is typically large in financial time-series. Standard Kalman filter techniques cannot be applied either since the latent process is non-Gaussian and the resulting state-space form does not have a conjugate filter. Therefore stochastic volatility models have become quite popular in the econometrics literature to try new estimation techniques. Key references on estimation of stochastic volatility models include: Harvey, Ruiz and Shephard (1994), Harvey and Shephard (1996), Fridman and Harris (1997) and Sandmann and Koopman (1997) for Kalman filter techniques[1]. Jacquier, Polson and Rossi (1994), Schotman and Mahieu (1994), Kim, Shephard and Chib (1997) on Bayesian methods[2], Danielsson (1994) and Danielsson and Richard (1993) on Simulated Maximum Likelihood methods[3]. Finally, Gallant and Tauchen (1996) and Gallant, Hsieh and Tauchen (1994) are the main references for EMM methods[4]. These have been the most successful techniques. We also mention Wiggins (1987), Scott (1987), Chesney and Scott (1987), Melino and Turnbull (1990) and Andersen and Sørensen (1996) for GMM techniques and Monfardini (1996) for an application to SV models using indirect inference (Gourieroux, Monfort and Renault (1993)). For foundations of stochastic volatility models see Clark (1973), Tauchen and Pitts (1983), Taylor (1986) and Hull and White (1987). Review articles on stochastic volatility models have been provided by Ghysels, Harvey and Renault (1996) and Shephard (1996b).[5]. Recent techniques proposed by Jacquier,Polson and Rossi (1994), Kim, Shephard and Chib (1997) have made tremendous improvements in the estimation of SV models compared to the early GMM and standard Kalman filter techniques. Although these latter methods are essentially Bayesian, with standard arguments the posterior modes and the maximum likelihood estimators converge, see e.g. Barndorff-Nielsen and Cox (1994, pp136). A strictly classical estimation method is EMM of Gallant and Tauchen (1996). The main practical advantage of this technique is its flexibility, a property this technique inherits of other moment-based techniques. Once the moments are chosen one may estimate a whole class of SV models. Theoretically this method is ML efficient. In a stochastic volatility context recent Monte Carlo studies in Andersen, Chung and Sørensen (1997) and van der Sluis (1997b) confirm this for sample sizes larger than 2,000, which is rather small for financial time-series. For lower sample sizes there is a small loss in small sample efficiency compared to the likelihood based techniques such as Kim, Shephard and Chib (1997), Sandmann and Koopman (1997) and Fridman and Harris (1996). This is mainly caused by the imprecise estimate of the weighting matrix for samples of size smaller than 2,000. The same phenomenon occurs in ordinary GMM estimation.

Important criticism on EMM and on moment based estimation in general has been that the method does not provide a representation of the observables in terms of their past, which we do get from, for example, the prediction-error-decomposition in likelihood based techniques. Gallant & Tauchen (1997) overcome this problem by proposing *reprojection*. The main idea is to get a representation of the ob-

[1]On Kalman filter techniques in this context we also mention Ruiz (1994).

[2]On Bayesian methods in this context we also mention Shephard (1996)'s SvPack.

[3]See also Danielsson (1996a,b), Richard and Zhang(1995a,b) for more on SML methods in this context. Danielsson (1996a) actually contains the source code.

[4]Other references on EMM in this context are Andersen and Lund (1996), Andersen and Lund (1997), Gallant and Long (1997) and van der Sluis (1997a,b,c,d & 1998) and Andersen, Chung and Sørensen (1997)

[5]See also Taylor (1994), Andersen (1994) and Andersen (1992).

served process in terms of observables. In the same manner one can also get a representation for unobservables in terms of the past and present observables. This is important in our application where the unobservable volatility is needed in the option pricing formula. Using reprojection we are able to get a representation of the unobserved variables.

3.1 Estimation

In short the EMM method goes as follows[6]: the sequence of densities for the structural model will be denoted

$$\{p_1(x_1 \mid \theta), \{p_t(y_t \mid x_t, \theta)\}_{t:=1}^{\infty}\}$$

The sequence of densities for the auxiliary process will be denoted as

$$\{f_1(w_1 \mid \beta), \{f_t(y_t \mid w_t, \beta)\}_{t:=1}^{\infty}\}$$

where x_t and w_t are observable endogenous variables. In particular the x_t will be a vector of lagged y_t and the w_t will also be a vector of lagged y_t. The lag-length may differ, therefore a different notation is used. We impose assumptions 1 and 2 from Gallant and Long (1997) on the structural model, these are technical assumptions that imply standard properties of quasi maximum likelihood estimators and properties of estimators based on *Hermite expansions* which will be explained below. Let us define

$$m(\theta, \beta) := \int \int \frac{\partial}{\partial \beta} \ln f(y \mid w, \beta) p(y \mid x, \theta) dy p(x \mid \theta) dx$$

the expected score of the auxiliary model under the dynamic model. The expectation is written in integral form to anticipate on the fact the we will approximate this integral by standard Monte Carlo techniques. The simulation approach solely consists of calculating this function as

$$m_N(\theta, \beta) := \frac{1}{N} \sum_{\tau:=1}^{N} \frac{\partial}{\partial \beta} \ln f(y_\tau(\theta) \mid w_\tau(\theta), \beta)$$

Let n denote the sample size, the EMM estimator is defined as

$$\widehat{\theta}_n(\mathcal{I}_n) := \arg \min_{\theta \in \Theta} m_N^{'}(\theta, \widehat{\beta}_n)(\mathcal{I}_n)^{-1} m_N(\theta, \widehat{\beta}_n)$$

where $\mathcal{I}_n$ is a weighting matrix and $\widehat{\beta}_n$ denotes an estimator for the parameter of the auxiliary model. The optimal weighting matrix here is obviously

$$\mathcal{I}_0 = \lim_{n \to \infty} V_0[\frac{1}{\sqrt{n}} \sum_{t:=1}^{n} \{\frac{\partial}{\partial \beta} \ln f_t(y_t \mid w_t, \beta^*)\}]$$

where β^* is a (pseudo) true value.

[6]We discuss case 2 from Gallant and Tauchen (1996).

With the theory of misspecified models (White (1994)) one can prove consistency for the parameters of the auxiliary model under several assumptions posed in Gallant and Tauchen (1996) and in Gallant and Long (1997),

$$\lim_{n\to\infty}(\widehat{\beta}_n - \beta^*) = 0 \text{ a.s.}$$

and asymptotic normality

$$\sqrt{n}(\widehat{\beta}_n - \beta^*) \xrightarrow{d} N(0, (\mathcal{J}_0)^{-1}(\mathcal{I}_0)(\mathcal{J}_0)^{-1})$$

Here

$$\begin{aligned}\mathcal{I}_n &= V_0[\frac{1}{\sqrt{n}}\sum_{t:=1}^{n}(\frac{\partial}{\partial\beta}\ln f_t(\tilde{y}_t \mid \tilde{w}_t, \widehat{\beta}_n)] \\ \mathcal{J}_n &= -\frac{\partial}{\partial\beta}m_N^{'}(\theta_0, \widehat{\beta}_n)\end{aligned}$$

where θ_0 denotes the (pseudo) true value. Under standard regularity assumptions we have that

$$\begin{aligned}\lim_{n\to\infty}\mathcal{I}_n &= \mathcal{I}_0 \\ \lim_{n\to\infty}\mathcal{J}_n &= \mathcal{J}_0\end{aligned}$$

One can also prove for the scores

$$\sqrt{n}m_N(\theta_0, \widehat{\beta}_n) \xrightarrow{d} N(0, \mathcal{I}_0)$$

Hence consistency and asymptotic normality of the estimator of the structural parameters $\widehat{\theta}_n$ follows:

$$\sqrt{n}(\widehat{\theta}_n(\mathcal{I}_0) - \theta_0) \xrightarrow{d} N(0, [\mathcal{M}_0^{'}(\mathcal{I}_0)^{-1}\mathcal{M}_0]^{-1})$$

where $\mathcal{M}_0 := \frac{\partial}{\partial\theta'}m(\theta_0, \beta^*)$.

In order to obtain *maximum likelihood efficiency*[7] it is required that the auxiliary model in some sense embeds the structural model. The semi-nonparametric (SNP) density of Gallant and Nychka (1987)[8] may be a good choice, see Gallant and Tauchen (1996) and Gallant and Long (1997). The auxiliary model is built as follows. The process $y_t(\theta_0)$ is the process under investigation, $\mu_t(\beta^*) := \mathsf{E}_{t-1}[y_t(\theta_0)]$, is the conditional mean of the auxiliary model, $\sigma_t^2(\beta^*) := \mathsf{Cov}_{t-1}[y_t(\theta_0) - \mu_t(\beta^*)]$ the conditional variance matrix and $z_t(\beta^*) := R_t^{-1}(\theta)[y_t(\theta_0) - \mu_t(\beta^*)]$ the standardized process. Here R_t will typically be a lower or upper triangular matrix. The SNP density now takes the following form

$$f(y_t;\theta) = \frac{1}{|\det(R_t)|}\frac{[P_K(z_t, x_t)]^2\phi(z_t)}{\int[P_K(u, x_t)]^2\phi(u)du}$$

[7] Maximum likelihood efficiency is used throughout meaning first order asymptotic efficiency.

[8] Building on earlier work of Phillips (1983). See also Fenton & Gallant (1996a, b) for recent results on SNP densities.

where ϕ denotes the standard multinormal density, $x := (y_{t-1}, ..., y_{t-L})$ and the polynomials

$$P_K(z, x_t) := \sum_{i:=0}^{K_z} a_i(x_t) z^i := \sum_{i:=0}^{K_z} [\sum_{j:=0}^{K_x} a_{ij} x_t^j] z^i$$

We have to be careful what is exactly meant by z^i in case z is a vector. Here we mean that i is a *multi-index*, so for the k-vector $z = (z_1, \ldots, z_k)'$ we have $z^i := z_1^{i_1} \cdot z_2^{i_2} \cdots z_k^{i_k}$ under the condition $\sum_{j=1}^{k} i_j = i$ and $i_j \geq 0$ for $j \in \{1, ..., k\}$. A specific form for the polynomials is taken, namely orthogonal Hermite polynomials (see Gallant, Hsieh and Tauchen (1991) and Andersen and Lund (1997)). Relevant formulae for the derivatives can be found in Abramowitz and Stegun (1972) and Fenton and Gallant (1996a). The model $\sigma_t^2(\beta)$ and $\mu_t(\beta)$ is chosen as a *leading term* in the Hermite expansion to relieve the expansion of some of its task, dramatically improving its small sample properties.

In this paper we will take $p := \dim(\theta)$, $q := \dim(\beta)$. The number of moment conditions q may be determined using several criteria. For EMM, it is necessary that q increases with n. Note in this respect the conceptual difference with GMM. It will automatically happen that q increases with n using any of the model specification criteria such as the Akaike Information Criterion (AIC, Akaike (1973)), the Schwarz Criterion (BIC, Schwarz (1978)) or the Hannan-Quinn Criterion (HQC, Hannan and Quinn (1979) and Quinn (1980)). The theory of model selection in the context of SNP models is not very well developed yet. Results in Eastwood (1991) may lead to believe AIC is optimal in this case. However, as for multivariate ARMA models, the AIC may overfit the model to noise in the data so we may be better off by following the BIC or HQC. The same findings were reported in Andersen and Lund (1997). In their paper Gallant and Tauchen (1996) rely on the BIC in their applications. We will return on this issue in section 4.1.

Under the null that the structural model is true one may deduce.

$$n \cdot m_N'(\widehat{\theta}_n, \widehat{\beta}_n)(\widehat{\mathcal{I}}_n)^{-1} m_N(\widehat{\theta}_n, \widehat{\beta}_n) \xrightarrow{d} \chi^2_{q-p}$$

This gives rise to the Hansen J-test for overidentifying restrictions that is well known in the GMM literature. The direction of the misspecification may be indicated by the quasi-t ratios

$$\widehat{T}_n := \widehat{S}_n^{-1} \sqrt{n} m_N(\widehat{\theta}_n, \widehat{\beta}_n)$$
$$\widehat{S}_n := [\text{diag}(\widehat{\mathcal{I}}_n - \widehat{\mathcal{M}}_n(\widehat{\mathcal{M}}_n' \widehat{\mathcal{I}}_n^{-1} \widehat{\mathcal{M}}_n)^{-1} \widehat{\mathcal{M}}_n')]^{1/2}$$

Here $\widehat{T}_n$ is distributed as t_{q-p}.

Estimation was done using EmmPack (van der Sluis, 1998), and procedures from van der Sluis (1997b) and van der Sluis (1997c). In the latter paper some encouraging Monte Carlo results for EMM are given. The leading term in the SNP expansion is a(n) (multivariate) EGARCH model. Of course one can simultaneously estimate all the parameters so including $\mu_S, \mu_r, \phi, \rho_1, ..., \rho_l$ and the volatility parameters of $y_{1,t}$ and $y_{2,t}$. This is optimal but too cumbersome and probably not necessary. Estimation will be carried out in the following sub-optimal way:

(i) Estimate μ_S and ϕ, retrieve $y_{1,t}$, Estimate $\mu_r, \rho_1, ..., \rho_l$, retrieve $y_{2,t}$. Both using standard regression techniques.

(ii) Simultaneously estimate parameters of stochastic volatility model, including λ via EMM

As we have mentioned, the EMM estimation of stochastic volatility models is rather cumbersome and time-consuming. Moreover many of the above stochastic volatility models are never estimated in practice. Therefore we use a multivariate variant of the EGARCH model of Nelson (1990) as a guide to which of the above SV models is worth looking at. The EGARCH model is a convenient choice since (i) it is an a very good approximation to the stochastic volatility model, see Nelson and Foster (1994), (ii) we use the EGARCH model as a leading term in the auxiliary model of the EMM estimation methodology and (iii) direct maximum likelihood techniques are admitted by this class of models. We can thus view the following M-EGARCH model as a pendant to the structural SV models that are proposed in section 2.1.

$$\begin{aligned}
\begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} &= \begin{bmatrix} \sigma_{1,t} & 0 \\ 0 & \sigma_{2,t} \end{bmatrix} \begin{bmatrix} z_{1,t} \\ z_{2,t} \end{bmatrix} \\
\begin{bmatrix} \ln \sigma^2_{1,t} \\ \ln \sigma^2_{2,t} \end{bmatrix} &= \begin{bmatrix} \alpha_{01} \\ \alpha_{02} \end{bmatrix} + \sum_{i=1}^{r} L^i \begin{bmatrix} \gamma_{11,i} & \gamma_{12,i} \\ \gamma_{21,i} & \gamma_{22,i} \end{bmatrix} \begin{bmatrix} \ln \sigma^2_{1,t} \\ \ln \sigma^2_{2,t} \end{bmatrix} + (1 + \sum_{j=1}^{q} L^j \begin{bmatrix} \alpha_{11,j} & \alpha_{12,j} \\ \alpha_{21,j} & \alpha_{22,j} \end{bmatrix}) \\
&\quad \times [\begin{bmatrix} \kappa_{1,11} & \kappa_{1,12} \\ \kappa_{1,21} & \kappa_{1,22} \end{bmatrix} \begin{bmatrix} z_{1,t-1} \\ z_{1,t-1} \end{bmatrix} + \begin{bmatrix} \kappa_{2,11} & \kappa_{2,12} \\ \kappa_{2,21} & \kappa_{2,22} \end{bmatrix} \begin{bmatrix} (b(z_{1,t-1}) - \sqrt{2/\pi}) \\ (b(z_{1,t-1}) - \sqrt{2/\pi}) \end{bmatrix}] \\
\mathsf{E}[\epsilon_t \epsilon_t^{'}] &= \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}
\end{aligned}$$

where some parameters may be restricted. In the application the $\alpha_{ij,k}$, $\kappa_{ij,1}$ and $\kappa_{ij,2}$ for $i \neq j$ will be set zero. So we will have

$$\begin{aligned}
\begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} &= \begin{bmatrix} \sigma_{1,t} & 0 \\ 0 & \sigma_{2,t} \end{bmatrix} \begin{bmatrix} z_{1,t} \\ z_{2,t} \end{bmatrix} \\
\begin{bmatrix} \ln \sigma^2_{1,t} \\ \ln \sigma^2_{2,t} \end{bmatrix} &= \begin{bmatrix} \alpha_{01} \\ \alpha_{02} \end{bmatrix} + \sum_{i=1}^{r} L^i \begin{bmatrix} \gamma_{11,i} & 0 \\ 0 & \gamma_{22,i} \end{bmatrix} \begin{bmatrix} \ln \sigma^2_{1,t} \\ \ln \sigma^2_{2,t} \end{bmatrix} + (1 + \sum_{j=1}^{q} L^j \begin{bmatrix} \alpha_{11,j} & 0 \\ 0 & \alpha_{22,j} \end{bmatrix}) \\
&\quad \times [\begin{bmatrix} \kappa_{1,11} & 0 \\ 0 & \kappa_{1,22} \end{bmatrix} \begin{bmatrix} z_{1,t-1} \\ z_{1,t-1} \end{bmatrix} + \begin{bmatrix} \kappa_{2,11} & 0 \\ 0 & \kappa_{2,22} \end{bmatrix} \begin{bmatrix} (b(z_{1,t-1}) - \sqrt{2/\pi}) \\ (b(z_{1,t-1}) - \sqrt{2/\pi}) \end{bmatrix}] \\
\mathsf{E}[\epsilon_t \epsilon_t^{'}] &= \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}
\end{aligned}$$

The ρ parameter corresponds to λ_1. The κ's, possibly in combination with some of the parameters of the polynomial, correspond to λ_2 and λ_3. This latter correspondence is further investigated in a

Monte Carlo study in van der Sluis (1997b) with very encouraging results. In the same way an auxiliary model is specified for the SV model with interest level r_t included in the volatility process

$$\begin{aligned}
\begin{bmatrix} y_{s,t} \\ y_{r,t} \end{bmatrix} &= \begin{bmatrix} \sigma_{1,t} & 0 \\ 0 & \sigma_{2,t} \end{bmatrix} \begin{bmatrix} z_{1,t} \\ z_{2,t} \end{bmatrix} \\
\begin{bmatrix} \ln\sigma^2_{s,t} \\ \ln\sigma^2_{r,t} \end{bmatrix} &= \begin{bmatrix} \pi & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} r_t \\ r_t \end{bmatrix} + \begin{bmatrix} \alpha_{01} \\ \alpha_{02} \end{bmatrix} + \sum_{i=1}^{r} L^i \begin{bmatrix} \gamma_{11,i} & \gamma_{12,i} \\ \gamma_{21,i} & \gamma_{22,i} \end{bmatrix} \begin{bmatrix} \ln\sigma^2_{1,t} \\ \ln\sigma^2_{2,t} \end{bmatrix} + \\
& +(1+\sum_{j=1}^{q} L^j \begin{bmatrix} \alpha_{11,1} & \alpha_{12,1} \\ \alpha_{21,1} & \alpha_{22,1} \end{bmatrix})[\begin{bmatrix} \kappa_{1,11} & \kappa_{1,12} \\ \kappa_{1,21} & \kappa_{1,22} \end{bmatrix} \begin{bmatrix} z_{1,t-1} \\ z_{1,t-1} \end{bmatrix} + \\
& + \begin{bmatrix} \kappa_{2,11} & \kappa_{2,12} \\ \kappa_{2,21} & \kappa_{2,22} \end{bmatrix} \begin{bmatrix} (b(z_{1,t-1}) - \sqrt{2/\pi}) \\ (b(z_{1,t-1}) - \sqrt{2/\pi}) \end{bmatrix}] \\
\mathsf{E}[\epsilon_t \epsilon_t^{'}] &= \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}
\end{aligned}$$

In this model the π corresponds to the α in the SV model.

It should be clear that the EGARCH model does not have a counterpart of the correlation parameter λ_4 from the SV model. Asymptotically the cross-terms in the Hermite polynomial should account for this. In practice, with no counterpart of the parameter in the leading term, we have strong reasons to believe that the small sample properties of an EMM estimator for λ_4 will not be very satisfying. Therefore, as was argued in section 2.2, we reluctantly set this parameter a priori equal to zero.

Here at this moment the innovations in the SV models to both the stock returns and spot interest rates are assumed to be normally distributed random variables, which is related to the fact that our sampling observations are daily data. The EGARCH model is expanded with a semiparametric density which allows for nonnormality. This is a consequence of the EMM methodology. In section 4.1 we will argue how to pick a suitable order for the Hermite polynomial for a Gaussian SV model. The efficient moments for the SV model will come initially from the auxiliary model: bi-variate SNP density with bi-variate EGARCH leading term. For an extensive evaluation of this bi-variate EGARCH model and even of higher dimensional EGARCH models see van der Sluis (1997c). This model will also serve as a guide in the specification of the structural SV model. Once the SV models is estimated the moments of the M-EGARCH(p,q)-H(K_x, K_z) model will serve as diagnostics by considering the $\widehat{T}_n$ test-statistics.

3.2 Reprojection

After the model is estimated we use recent results from Gallant and Tauchen (1997) to obtain estimates of the unobserved volatility process $\{\sigma_t\}_{t=1}^n$, as we need this series in our option pricing formula. Gallant and Tauchen (1997) propose *reprojection* as a general purpose technique for characterizing the dynamic response of a partially observed nonlinear system to its observable history. Reprojection is the third step in the EMM methodology. First data is summarized by projecting on Hermite polynomials. Next system parameters are estimated where the criterion is based on these Hermite polynomials. Reprojection can now be seen as projecting a long simulated series from the estimated system on the Hermite polynomials. In short reprojection goes as follows. We calculate another set of auxiliary parameters as

$$\widetilde{\beta} = \arg \max_{\beta \in B} \mathsf{E}_{\widehat{\theta}_n} f(y_0|y_{-L}, ..., y_{-1}, \beta)$$

note $\mathsf{E}_{\widehat{\theta}_n} f(y_0|y_{-L}, ..., y_{-1}, \beta)$ is calculated using one set of simulations $y(\widehat{\theta}_n)$. Doing so, we reproject a long simulation from the estimated model on the auxiliary model. Results in Gallant & Long (1997) show that

$$\lim_{K \to \infty} f(y_0|y_{-L}, ..., y_{-1}, \widetilde{\beta}_K) = p(y_0|y_{-L}, ..., y_{-1}, \widehat{\theta})$$

where K is the overall order of the Hermite polynomials and should grow with the sample size n, either adaptively as a random variable or deterministic. Therefore the following conditional moments under the structural model can be calculated using the auxiliary model as follows

$$\begin{aligned} \mathsf{E}(y_0|y_{-L}, ..., y_{-1}) &= \int y_0 f(y_0|x_{-1}, \widetilde{\beta}) dy_0 \\ \mathsf{Var}(y_0|y_{-L}, ..., y_{-1}) &= \int (y_0 - \mathsf{E}(y_0|y_{-L}, ..., y_{-1}))^2 f(y_0|x_{-1}, \widetilde{\beta}) dy_0 \end{aligned}$$

For an estimate of the unobserved volatility we could use $\sqrt{\mathsf{Var}(y_0|y_{-L}, ..., y_{-1})}$. A more common notion of filtration is to use the information on the observable y up to time t, instead of $t-1$, since we want a representation for unobservables in terms of the past and present observables. Indeed for option pricing it is more natural to include the present observables y_t. For today's option value we want to include todays' stock price and todays' interest rate in the information set. We follow Gallant and Tauchen (1997) in their modification and repeat the above derivation replacing y with y^*, where $y^* = \sigma_0$. Doing so we need to specify a different auxiliary model from the one we used in the estimation stage. More precisely, we need to specify an auxiliary model for $\ln \sigma_t^2$ using information up till time t,instead of $t-1$, as in the auxiliary EGARCH model. Since for the sample size we encounter in this application projection on pure Hermite polynomials may not be a good idea due to small sample distortions and issues of non-convergence, we use the following intuition to build a useful leading term. We can write the SARMAV$(1,0)$ model as

$$\ln y_t^2 = \ln \sigma_t^2 + \ln \zeta_t^2$$

where $\zeta_t \sim N(0,1)$ and the $\ln \sigma_t^2$ follow a AR(1) process. Observe that the above process is a non-Gaussian ARMA$(1,1)$ process. We therefore consider

$$\ln \sigma_t^2 = \alpha_0 + \alpha_1 \ln y_t^2 + \alpha_2 \ln y_{t-1}^2 + ... + \alpha_r \ln y_{t-r-1}^2 + error$$

where the lag-length r will be determined by AIC. It may be an idea to project on

$$\ln \sigma_t^2 = \alpha_0 + \alpha_1 \ln y_t^2 + \alpha_2 \ln \sigma_{t-1}^2 + error$$

In figure 3 we see that projection on this model is not a good idea, in the sense that it gives a very flat projection of the volatility, which *on average* has good properties but does not capture the dynamics of the underlying stochastic volatility model. In this picture a series of $5,000$ was simulated from an SV model with realistic parameter values. Since we know the volatilities we can compare them with the reprojected volatilities. Further research should be made on this issue because see that the projection on the autoregressive series still can be improved. It should be noted that in the application two series of antithetic variables of size $50,000$ were used instead of $5,000$ as in figure 3. This improves the figure slightly.

For the asymmetric model, we would like to include the z_t as well like in the EGARCH model. Therefore we propose to consider

$$\ln \sigma_t^2 = \alpha_0 + \sum_{i=0}^{r} \alpha_{i+1} \ln y_{t-i}^2 + \sum_{j=1}^{s} \beta_i \frac{y_{t-j}}{\sigma_{t-j}} + error$$

In the original set-up of Gallant and Tauchen (1997) one should as well project on the Hermite polynomials. The auxiliary model here is then the SNP density, modified to include information up to time t. So from an asymptotic point of view projection on the autoregressive series is sub-optimal. As in the estimation stage we could specify an SNP density for the error term. We chose not to do so. In part because we think that as in the estimation stage a good leading term will pick up all the salient features in the data (see section 4.1), in part because the resulting formulae get rather complicated, but would not affect much of the resulting estimates of the volatility. From the above, $\ln \widehat{\sigma}_0^2 = \mathsf{E}(\ln \sigma_0^2 | y_0, ..., y_{-L})$ follows straightforward. In both the symmetric and asymmetric case, we have thus a projection on an autoregressive series.

4 Empirical Results

4.1 Description of the data

Summary statistics of both interest rates and stock returns are reported in Table 1. The interest rates used in this paper as proxy of riskless rates are daily U.S. 3-month Treasury bill rates and the underlying stock considered in this paper is 3 Com Corporation which is listed in NASDAQ. Both the stock and its options are actively traded. The stock claims no dividend and thus theoretically all options on the stock can be valued as European type options. The data covers the period from March 14, 1986 to July 14, 1997 providing 2,836 observations. From Table 1, we can see that both the first difference of logarithmic interest rates and that of the logarithmic stock prices (i.e. the daily stock returns) are skewed to the left and have positive excess kurtosis ($>> 3$) suggesting fat tails of the unconditional distributions. Similarly, the filtered interest rates Y_{r_t} as well as the filtered stock returns $Y1_{s_t}$ (with systematic effect) and $Y2_{s_t}$ (without systematic effect) are also skewed to the left and have positive excess kurtosis. However, the logarithmic squared filtered series, as proxy of the logarithmic conditional volatility, all have negative excess kurtosis and appear to justify the normal driving noise as we specified in the general model. As far as dynamic properties, the filtered interest rates and stock returns as well as logarithmic squared filtered series are all temporally correlated. For the logarithmic squared filtered series, the first order autocorrelations are in general low, but higher order autocorrelations are of similar magnitudes as the first order autocorrelations. This would suggest that all series are roughly ARMA$(1, 1)$ or equivalently AR(1) with measurement error, which is consistent with the first order autoregressive SV model specification. Estimates of trend parameters in the general model are reported in 2. For stock returns, it appears that interest rate has significant explanatory power, suggesting the presence of systematic effect or certain predictability of stock returns. For logarithmic interest rates, there is an insignificant linear mean-reversion, which is consistent with many findings in the literature. [9]

We further look at the data through specification of the score generator or auxiliary model, since the score-generator is thought to give a good description of the data. Therefore we deal with the specification of the score-generator in this section. We use the score-generator as a guide for the structural model, since there is a clear relationship between the parameters of the auxiliary model and the structural model. If some parameters in the score-generator or not significantly different form zero, we set the corresponding parameters in the SV model a priori equal to zero. Various model selection criteria and t-statistics of individual parameters of a wide variety of different auxiliary models that were proposed in section 3 indicate (i) multivariate models are all clearly rejected on basis of the model selection criteria and by looking at the t–values of the parameter ρ. We therefore set its corresponding SV parameter λ_1 a priori equal to zero; (ii) The parameter π was marginally significant at a 5% level. On basis of the BIC however inclusion of this parameter is not justified. We therefore set its corresponding parameter α a priori equal to zero; (iii) The cross terms $\gamma_{12,1}$ and $\gamma_{21,1}$ were significantly different from zero albeit small, again on basis of the BIC inclusion of these parameters was not justified; (iv) As it comes to picking a suitable order of the Hermite polynomial of the SNP expansion we observe that for all models K_x should be equal to zero, more importantly according to the most conservative criterion, which is the BIC, $K_z > 10$. For picking the size of K_z we argue as follows. In van der Sluis (1997a,d) sample sizes of about 1,000 and 1,500 were under study. For these sample sizes K_z of 4 or 5 was found to be BIC optimal. In this paper sample size of 3,000 are under study, it is found here that BIC is in favour of Hermite polynomials of order K_z larger than 10. With recent results of Andersen, Chung and Sørensen (1997) and van der Sluis (1997b) in mind, which tell that for sample sizes of 3,000, convergence problems can occur in a substantial number of cases for such

[9]For instance, Stanton (1997) and Jiang (1997) both suggest that the U.S. short rate process has a non-linear mean-reverting property.

high order polynomials and that under the null of a *Gaussian* SV model, setting $K_z = 0$ will yield virtually efficient EMM estimates, which are not necessarily dominated by setting $K_z > 0$[10] Still we can learn something from the fitted SNP densities with $K_z > 0$. We plotted the conditional density implied by the ML estimates of the parameters and compared the shape of these densities. See figures 4 till 9. We plotted the conditional densities implied by the ML estimates for $K_z = 4, 6, 8$ and 10 for both datasets. Clearly, there is evidence in the data that a *Gaussian* EGARCH model is not good enough as was also indicated by the model selection criteria. One can also perform Likelihood Ratio tests for a Gaussian EGARCH model against an Seminonparametric EGARCH model. This is clearly in favour of a high order seminonparametric EGARCH model. In turn this indicates that a Gaussian SV model is not adequate and one should consider a fat-tailed SV model or a *jump diffusion* process. This can also be seen by comparing the sample properties of the data with the sample properties of the SV model in the optimum. It also appears that for $K_z > 6$ the SNP density starts to put probability mass at outliers. It remains an issue whether we should include such high orders in the auxiliary model. For descriptive purposes such high orders can be desirable. However, since under the null of Gaussian SV we cannot get such outliers, there is no need to consider them. Therefore we decided for these sample sizes to set the Hermite polynomial equal to zero. It is interesting to see whether the parameter values of the auxiliary EGARCH model are sensitive to the order of the polynomial may also be an issue therefore in Table 3 and Table 4 we provide ML estimates for the leading term in the EGARCH(1,1) model for both datasets for different K_z. We observe that the estimated parameters values are sensitive to the choice of the Hermite polynomials. We performed EMM estimation for the series using H(6,0) to see whether the results would differ from the ones with H(0,0). We shall later see that the parameter estimates also differ somewhat. However under non-Gaussian SV the option pricing formula will differ and is also sensitive to the choice of the distribution, so it is not only sensitive to the parameters. Further research should therefore include this fact by using a structural model with jump diffusions or fat-tails. For estimating fat-tailed SV models we would clearly need $K_z > 0$. However such a non-Gaussian SV model will make the resulting option pricing formula even more complicated. We therefore postpone the use of such structural models to future research.

4.2 Structural model and Estimation Results

The general model: the model specified in section 2.1 assumes stochastic volatility for both the stock returns and interest rate dynamics as well as systematic effect on stock returns. This model nests the Amin and Ng (1993) model as a special case when $\lambda_2 = 0$. We will investigate the implications of leverage effect in particular on the skewness of implied Black-Scholes volatility.

Following are four alternative model specifications:

[10] Although in van der Sluis (1997b) for samples of size $4,000$ with $K_z = 4$ rather good results were obtained regarding both convergence and efficiency.

- Submodel 1: No systematic effect, i.e. $\phi_s = 0$ and $\alpha = 0$, i.e. a bi-variate stochastic volatility model;
- Submodel 2: No stochastic interest rates, i.e. interest rate is constant. That is, $r_t = r$, the H-W model, the Baily and Stulz (1989) model.
- Submodel 3: Constant stock return volatility but stochastic interest rate. That is, $\sigma_{st} = \sigma$, the Merton (1973), Turnbull and Milne (1991) and Amin and Jarrow (1992) models.
- Submodel 4: Constant stock return volatility and constant interest rate. That is, $\sigma_{st} = \sigma, r_t = r$, the Black-Scholes model.

Estimation was mainly done using EmmPack (van der Sluis (1998)). The interested reader is referred to this paper for details. The results reported here are all for H(0,0). These results have been used in the paper. The models have also been estimated using higher order Hermite polynomials. No big differences have been encountered between using $\mathsf{H}(6,0)$ and $\mathsf{H}(0,0)$.

- General model: The estimates for the mean terms are given in Table 2. We obtained the following estimates for the symmetric SV model using the EGARCH(1,1)-H(0,0) score generator with the asymmetry parameter κ_1 a priori set equal to zero ,

$$\begin{aligned} y_t &= \sigma_t \epsilon_t \\ \ln \sigma_{t+1}^2 &= \underset{(.065)}{.005} + \underset{(30.1)}{.955} \ln \sigma_t^2 + \underset{(16.2)}{.218} \eta_t \end{aligned}$$

  for the interest rates and

$$\begin{aligned} y_t &= \sigma_t \epsilon_t \\ \ln \sigma_{t+1}^2 &= \underset{(30.8)}{.161} + \underset{(66.1)}{.940} \ln \sigma_t^2 + \underset{(17.5)}{.161} \eta_t \end{aligned}$$

  for the stock prices. In order to obtain the filtered series, we used an autoregressive model with 34 lags for the interest rate and an autoregressive model with 29 lags for the stock prices. For the asymmetric model we used the EGARCH(1,1)-H(0,0) with an unrestricted asymmetry parameter κ_1 as a score generator to obtain the following estimates

$$\begin{aligned} y_t &= \sigma_t \epsilon_t \\ \ln \sigma_{t+1}^2 &= \underset{(.107)}{.004} + \underset{(47.4)}{.959} \ln \sigma_t^2 + \underset{(31.8)}{.222} \eta_t \\ \mathsf{Cor}(\epsilon_t, \eta_{t+1}) &= \underset{(-156)}{-.270} \end{aligned}$$

  for the interest rates and

$$\begin{aligned} y_t &= \sigma_t \epsilon_t \\ \ln \sigma_{t+1}^2 &= \underset{(121)}{.175} + \underset{(233)}{.935} \ln \sigma_t^2 + \underset{(35.2)}{.161} \eta_t \\ \mathsf{Cor}(\epsilon_t, \eta_{t+1}) &= \underset{(-164)}{-.424} \end{aligned}$$

for the stock prices. We observe common estimates for the parameters for these type of data. The persistence parameter is close to unity. The asymmetry is moderate for both series and significantly different from zero. The leverage effect is somewhat higher for the stock returns than it is for the interest rate returns. It is noted that in both the symmetric and asymmetric models, the estimator of α is insignificantly different from zero, rejecting that the short-term interest rate is correlated with conditional volatility of the stock returns. The explanation of this finding can be that either the stochastic volatility of the stock returns truly does not have a systematic component or the short-term interest rate serves as a poor proxy of the systematic factor. We believe the latter conjecture to be true as we re-ran the model with other stock returns and invariably we found α insignificantly different from zero. For the reprojection we incorporated the asymmetry and the AIC advocates to use 31 lagged $\ln y_t^2$ and 20 lagged z_t. for the interest rates and 28 lagged $\ln y_t^2$ and 28 lagged z_t for the stock prices. The filtered series for the stock-prices using the symmetric and asymmetric models are displayed in figure 10. Filtered series for the interest rates are displayed in figure 11.

- Submodel 1: The mean terms are given in 2. We obtained the following estimates for the symmetric SV model using the EGARCH(1,1)-H(0,0) score generator again with $\kappa_1 = 0$,

$$
\begin{aligned}
y_t &= \sigma_t \epsilon_t \\
\ln \sigma_t^2 &= \underset{(.094)}{.004} + \underset{(51.8)}{.959} \ln \sigma_{t-1}^2 + \underset{(31.3)}{.217}\, \eta_t
\end{aligned}
$$

for the interest rates and

$$
\begin{aligned}
y_t &= \sigma_t \epsilon_t \\
\ln \sigma_t^2 &= \underset{(83.5)}{.149} + \underset{(192)}{.944} \ln \sigma_{t-1}^2 + \underset{(31.3)}{.148}\, \eta_t
\end{aligned}
$$

for the stock prices. In order to obtain the filtered series, we used an autoregressive model with 34 lags for the interest rate and an autoregressive model with 29 lags for the stock prices. For the asymmetric model we used the EGARCH(1,1)-H(0,0) as a score generator to obtain the following estimates

$$
\begin{aligned}
y_t &= \sigma_t \epsilon_t \\
\ln \sigma_t^2 &= \underset{(.110)}{.004} + \underset{(47.8)}{.959} \ln \sigma_{t-1}^2 + \underset{(31.9)}{.223}\, \eta_t \\
\mathrm{Cor}(\epsilon_t, \eta_{t+1}) &= \underset{(-158)}{-.275}
\end{aligned}
$$

for the interest rates and

$$
\begin{aligned}
y_t &= \sigma_t \epsilon_t \\
\ln \sigma_t^2 &= \underset{(86.6)}{.154} + \underset{(186)}{.944} \ln \sigma_{t-1}^2 + \underset{(25.4)}{.147}\, \eta_t \\
\mathrm{Cor}(\epsilon_t, \eta_{t+1}) &= \underset{(-247)}{-.557}
\end{aligned}
$$

for the stock prices. The estimates do not differ much from the ones obtained for the general model. For the reprojection we incorporated the asymmetry and the AIC advocates to use 31 lagged $\ln y_t^2$ and 20 lagged z_t. for the interest rates and 28 lagged $\ln y_t^2$ and 28 lagged z_t for the stock prices. For reasons of space the filtered series for the submodel have not been displayed. The series resemble the series for the general model very much as displayed in figure 10 and in figure 11.

- Other Submodels: Estimation of the other submodels is fairly straightforward. Submodel 2 takes the SV part of the stock returns. Submodel 3 takes the SV part from the interest rate returns.

In Table 5 the results for the Hansen J-test have been displayed for the models using EMM. As we see all the models have been accepted at a 5% level[11]. Although a P-value is a monotonic function of the actual evidence against H_0, it is very dangerous to choose the best model of these specifications on basis of the P-values (see Berger and Delampady (1987)). An LR test to test the asymmetric SV model versus the symmetric SV model cannot be deduced from the difference in criterion values, since the criterion values are based on different moment conditions. However from the $t-$values corresponding to the asymmetry parameter we can deduce that the null hypothesis of symmetry will certainly be rejected in favour of the alternative asymmetric model. For the submodel 1 we obtain similar results. Using J-tests from the EGARCH(1,1)-H(6,0) model leads to rejection of the null hypothesis of the J-tests. Monte Carlo results in van der Sluis (1997b) indicate that for this auxiliary model the null hypothesis of an *Gaussian(!)* SV is rejected too often.

5 The Pricing of Stock Options

The presence of SV has important implications on option pricing. The effects of SV on stock option prices have been examined by Hull and White (1987), Johnson and Shanno (1987), Scott (1987), and Wiggins (1987), among others. These authors demonstrate that model predicted European option prices tend to be less than Black-Scholes options for at the money options and greater than Black-Scholes option prices for deep in the money options. For deep out of the money options, the results are sensitive to the parameters of the stochastic process describing changes in volatility and the correlation between changes in volatility and stock prices. For instance, Chesney and Scott (1989) compared the performance of the modified Black-Scholes model and a random variance option pricing model in pricing European currency options through examination of model fit and the biases with respect to the strike price, time to maturity, and volatility. They find that there is some evidence of mispricing but the gains are small by trading with the random variance model. Melino and Turnbull (1990) found

[11] The fact that the individual $t-$values are all about same, is a consequence of the fact that there is only one degree of freedom in the test. Asymptotically they should therefore be equal with probability one.

that the SV model did reduce the average and root mean squared pricing errors on predicted Canadian dollar option prices over February 1983 to January 1985 relative to the continuously readjusted and ad hoc Black-Scholes model. Most of the improvement appears attributable to superior predictions of the term structure of implicit volatilities relative to the Black-Scholes assumption of a flat term structure. They found evidence to support the notion that a non-zero risk premium on the variance process exists in the Canadian $/U.S. $ exchange rate option markets. They restricted the price of variance risk to be a constant. Lamoureux and Lastrapes (1993) suggested that such a risk premium is time-varying in the stock market.

In this paper we will investigate the implications of model specification on option prices through direct comparison with observed market option prices, while the Black-Scholes model is treated as a benchmark model. Empirical evidence suggests systematic mispricing of the Black-Scholes call option pricing models. These biases have been documented with respect to the call option's exercise prices, its time to expiration, and the underlying common stock's volatility. Since there is a one-to-one relationship between volatility and option price through the Black-Scholes formula, the volatility is often used to quote the value of an option. An equivalent measure for the mispricing of Black-Scholes model is thus the implied or implicit volatility, i.e. the volatility which generates the corresponding option price. The Black-Scholes model imposes a flat term structure of volatility, i.e. the volatility is constant across both maturity and strike prices of options. If option prices in the market were confirmable with the Black-Scholes formula, all the Black-Scholes implied volatilities corresponding to various options written on the same asset would coincide with the volatility parameter σ of the underlying asset. In reality this is not the case, and the Black-Scholes implied volatility heavily depends on the calendar time, the time to maturity, and the moneyness of the options.[12] The price distortions, well-known to practitioners, are usually documented in the empirical literature under the terminology of the *smile* effect, referring to the U-shaped pattern of implied volatilities across different strike prices.

5.1 Description of the Option Data

The sample of market option quotes covers the period of June 19, 1997 through August 18, 1997, with first half of the sample overlaps with the sample of stock returns, which will be used for the study of the models' in-sample performance. Since we do not rely solely on option prices to obtain the parameter estimates through fitting the option pricing formula, such a sample size is adequate for our comparison purpose. The intradaily bid-ask quotes for the stock options are extracted from the CBOE database. To ease computational burden, for each business day in the sample only one reported bid-ask quote during the last half hour of the trading session (i.e. between 3:30 – 4:00 PM Eastern standard time) of each option contract is used in the empirical test. The main considerations for the choice of the particular

[12] This may produce various biases in option pricing or hedging when Black-Scholes implied volatilities are used to evaluate new options with different strike prices and maturities.

bid-ask quote include: i) The movements of stock price is relatively stable around the point of time so that the option quotes are well adjusted; ii) Option quotes which do not satisfy arbitrage restrictions are excluded. The stock prices are calculated as average of bid-ask quotes which are simultaneously observed as the option's bid-ask quote. Therefore they are not transaction data and the data set used in this study avoids the issue of non-synchronous prices.

The sampling properties of the option data set are reported in Table 6. The data only include options with at least 5 days to expiration to reduce biases induced by liquidity-related issues. We divide the option data into several categories according to either moneyness or time to expiration. In this paper, we use a slight different definition of moneyness for options from the conventional one [13] following Ghysels, Harvey and Renault (1996), we define

$$x_t = \ln(S_t/Ke^{-\int_t^T r_\tau d\tau}) \tag{8}$$

Technically if $x_t = 0$, the current stock price S_t coincides with the present value of the strike price K, the option is called at-the-money; if $x_t > 0$ (respectively $x_t < 0$), the option is called in-the-money (respectively out-of-the-money). In our partition, a call option is said to be *at-the-money* (ATM) if $-0.03 < x \leq 0.05$; *out-of-the-money* (OTM) if $x \leq -0.03$; and *in-the-money* (ITM) if $x > 0.05$. A finer partition resulted in six moneyness categories as in 6. According to the time to expiration, an option contract can be classified as: i) short-term ($T - t \leq 30$ days); ii) medium-term ($30 < T - t < 180$ days); and iii) long-term ($T - t \geq 180$ days). The partition according to moneyness and maturity results in 18 categories as in 6. For each category, the average bid-ask midpoint price and its standard error, the average effective bid-ask spread (i.e. the ask price minus the bid-ask midpoint) and its standard deviation, as well as the number of observations in the category are reported. Note that among 2120 total observations, about 26.56% are OTM options, 12.69% are ATM options, 60.75% are ITM options; 26.23% are short-term options, 49.01% are medium-term options, and 24.76% are long-term options. The average price ranges from \$0.223 for short-term deep out-of-the-money options to \$25.93 for long-term deep in-the-money options, and the average effective bid-ask spread ranges from \$0.066 for short-term deep out-of-the-money options to \$0.375 for log-term deep in-the-money options.

Figure 12 plots the implied Black-Scholes volatility against moneyness for options with different terms of maturity. The implied Black-Scholes volatilities are backed out from each option quote using the corresponding stock price, time to expiration, and the current yield of U.S. treasury instruments with maturity closest to the maturity of the option. Namely, we use 3-month T-bill rates for options with maturity less than 4 months, and 6-month T-bill rates for options with maturity longer

[13]In practice, it is more common to call an option as at-the-money/in-the-money/out-of-the-money when $S_t = K/S_t > K/S_t < K$ respectively. For American type options with possibility of early exercise, it is more convenient to compare S_t with K, while for European type options and from an economic point of view, it is more appealing to compare S_t with the present value of the strike price K.

than 4 months. The yields are hand-collected from the *Wall Street Journal* over the sample period and the discount rates are converted to annualized compound rates. It is noted that the Black-Scholes implied volatility exhibits an obvious U-shaped patterns (*smiles*) as the call option goes from deep OTM to ATM and then to deep ITM, with the deepest ITM call option implied volatilities taking the highest values. Furthermore, the volatility smiles are more pronounced and more sensitive to the term to expiration for short-term options than for the medium-term and long-term options. On the contrast, the volatility smiles are more obviously skewed to the left for the long-term and medium-term options than for the short-term options. The former observation indicates that short-term options are the mostly severely mispriced ones by the Black-Scholes model and present perhaps the greatest challenge to any alternative option pricing model, while the later observation indicates a possible skewness due to such as the leverage effect is expected by the option traders on the dynamics of stock returns. These findings of clear moneyness- and maturity-related biases associated with the Black-Scholes model are consistent with the findings for many other securities in the literature. For instance, the following stylized facts are extensively documented (see e.g. Rubinstein (1985), Clewlow and Xu (1993), Taylor and Xu (1993)): i) The U-shaped pattern of implied volatility as a function of moneyness (or log moneyness) has its minimum centered at near-the-money options; ii) The volatility smile is often but not always symmetric as a function of log moneyness; iii) The amplitude of the smile increases quickly when time to maturity decreases. Indeed, for short maturities the smile effect is very pronounced while it almost completely disappears for longer maturities. However, as Ghysels, Harvey and Renault (1996) point out, we have to be cautious about the temptation of interpreting asymmetric smile as evidence of negatively-skewed stock return distribution with excess kurtosis.

5.2 Testing Option Pricing Models

As Bates (1997) points out, fundamental to testing option pricing models against time series data is the issue of identifying the relationship between the *true* process followed by the underlying state variables in the objective measure and the “risk-neutral” processes implied through option prices in a artificial measure. Representative agent equilibrium models such as Rubinstein (1976), Brennan (1979), Cox, Ingersoll and Ross (1985), Ahn and Thompson (1988), Bates (1988, 1991), and Amin and Ng (1993) among others indicate that European options that pay off only at maturity are priced as if investors priced options at their expected discounted payoffs under an equivalent “risk-neutral” representation that incorporates the appropriate compensation for systematic asset, volatility, interest rate, or jump risk. Thus, the corresponding “risk-neutral” specification of the general model specified in section 2 involves compensation for various factor risk. Namely, the “mean” of stock return in the ”risk-neutral” specification will be equal to the risk-free rate, the “mean” of the interest rate process as well as the “means” of the stochastic conditional volatilities for both interest rate and stock return will be adjusted for the interest rate risk and systematic volatility risk. Standard approaches for pricing systematic volatility risk, interest rate risk, and jump risk have typically involved either assuming the risk is nonsystematic and therefore has zero price, or by imposing a tractable functional form on the risk premium (e.g. the factor risk premiums are proportional to the respective factors) with extra (free) parameters to be estimated from observed options prices or bond prices (for interest rate risk).

Under the "risk-neutral" distribution of the general framework, a European call option on a non-dividend paying stock that pays off $\max(S_T - X, 0)$ at maturity T for exercise price X is priced as

$$C_0(S_0, r_0, \sigma_{r0}, \sigma_{S0}; T, X) = \mathsf{E}_0^*[e^{-\int_0^T r_t dt} \max(S_T - X, 0) | S_0, r_0, \sigma_{r0}, \sigma_{S0}]$$

where E_0^* is the expectation using the "risk-neutral" specification for the state variables conditional on all information at $t = 0$. In particular, when $\lambda_2 = 0$ in the general model setup, i.e. the assumption 2 in Amin and Ng (1993) is satisfied, the option pricing formula can be derived as in (7). The call option price is the expected Black-Scholes price with the expectation taken with respect to the stochastic variance over the life of the option, i.e. the European call option prices depend on the average expected volatility over the length of the option contract. Furthermore, if the stock variance is also constant, then we obtain the Black-Scholes formula. Since the underlying stock we consider in this paper claims no dividend, all options on the stock can be valued as European type options. [14] Option prices given in the formula can be computed based on direct simulations. Since our model is essentially in a discrete-time framework, the only approximation involved in the simulation is the Monte Carlo error. Such error can be reduced to any desired level buy increasing the number of path simulations. The estimation error involved in our study is also minimal as we rely on large number of observations over long sampling period to estimate model parameters. Our analysis for the implications of model specification on option prices is outlined as follows:

First, we distinguish two types of volatility: volatility estimated from the actual stock returns, we call it the underlying or historical volatility; and volatility implied from observed option prices through certain option pricing formula, we call it the implied or implicit volatility. The underlying volatilities are directly estimated for submodels 3 and 4 as constants, and are obtained through projection methods for the general model and submodels 1 and 2;

Second, in our comparison, all option prices at certain time, say t, are calculated based on information available at or before time t. For instance, when historical volatilities are used for each model a set of European call option prices are generated based on Monte Carlo or the close form solution with various maturities and degrees of moneyness using the current volatilities, while when the implied volatilities are used, we use the previous day's $(t-1)$ observed option prices to back out the volatility level at that day $(t-1)$ which will then be used as the starting values for the volatility processes of

[14] For options with early exercise potential, i.e. the American options, one way to approximate its price is to compute the Barone-Adesi and Whaley (1987) early-exercise premium, treating it as if the stock volatility and the yield-curve were time-invariant. Adding this early-exercise adjustment component to the European option price should result in a reasonable approximations of the corresponding American option price (e.g. Bates (1996a)).

current day (t) in simulating option prices. The option prices calculated from the underlying volatility use only information contained in the underlying processes, while those calculated from the implied volatility use the information contained in both the underlying process and the observed option prices. The use of implied volatility as the information extracted from the option market mimics the practice of option traders who often quote option prices in terms of implied (Black-Scholes) volatilities;

Third, we measure the biases of model option prices with the observed market option prices in terms of relative percentage difference and implied Black-Scholes volatilities.

5.3 Comparison based on In-Sample Historical Volatilities

The in-sample comparison of alternative option pricing models is different from those who use option prices to imply all parameter values of the "risk-neutral" model, e.g. Bakshi, Cao and Chen (1997). In their analysis, all the parameters and underlying volatility are estimated through fitting the option pricing model into observed option prices. Then these implied parameters and underlying volatility are used to predict the same set of option prices. It is rather a test of the flexibility of alternative models in fitting option prices with various maturities and moneyness. Obviously models with more factors (or more parameters) are given extra advantage. In our comparison, we use the reprojected in-sample historical volatility, i.e., $\{\hat{\sigma}_{rt}, \hat{\sigma}_{St}\}$, which are known information to price options. Since we aim to compare among different models, in order to give each model an equal chance we will assume that the risk premium in both interest rate and stock return processes as well as the conditional volatility processes are all zero. It is noted that such an assumption will be tested in our empirical analysis.

Option pricing biases are compared to the observed market prices based on the mean relative percentage option pricing error (MRE) and the mean absolute relative option pricing error (MARE), given by

$$MRE = \frac{1}{n}\sum_{i=1}^{n}\frac{C_i^M - C_i}{C_i}$$

$$MARE = \frac{1}{n}\sum_{i=1}^{n}\frac{|C_i^M - C_i|}{C_i}$$

where n is the number of options used in the comparison, C_i and C_i^M represents respectively the observed market option price and the theoretical model option price. The MRE statistic measures the average relative biases of the model option prices, while the MARE statistic measures the dispersion of relative biases of the model prices. The difference between MARE and MRE suggests the direction of the bias of the model prices, namely when MARE and MRE are of the same absolute values, it suggests that the model systematically misprices the options to the same direction as the sign of MRE, while when MARE is much larger than MRE in absolute magnitude, it suggests that the model is inaccurate in pricing options but the mispricing is less systematic. Since the percentage errors are very sensitive to the magnitude of option prices which are determined by both moneyness and length of maturity, we also calculate MRE and MARE for each of the 18 moneyness-maturity categories in 6.

Table 7 reports the relative pricing errors (%) based on underlying volatility for alternative models. In each cell, from top to bottom are the MRE (mean relative error) and MARE (mean absolute relative error) statistics for: 1. the asymmetric general SV model with $\lambda_2 \neq 0, \lambda_3 \neq 0$; 2. the symmetric general SV model with $\lambda_2 = \lambda_3 = 0$; 3. the asymmetric submodel I with $\lambda_2 \neq 0, \lambda_3 \neq 0$; 4. the symmetric submodel I with $\lambda_2 = \lambda_3 = 0$; 5. the asymmetric submodel II with $\lambda_3 \neq 0$; 6. the symmetric submodel II $\lambda_3 = 0$; 7. the submodel III; and 8. the submodel IV. The conclusions we draw from the above comparison are summarized as following.

First, the symmetric SV models in general outperform the Black-Scholes model, which we believe is due to the fact that the Black-Scholes model uses the average volatility over the sampling period, while the SV models use the instantaneous volatility reprojected from the estimated process. It is obvious from Figures 10 and 11 that both the stock return volatility and interest rate volatility are time-varying and sometimes clustered, i.e. with bunching of high and low episodes. Therefore, the SV model provides a better prediction of the volatility. While the reduction of pricing errors is more significant for deep OTM options than for deep ITM options, for short maturity options within the symmetric model framework, the pricing errors are actually higher for stochastic volatility models;

Second, surprisingly the asymmetric SV models do not outperform the Black-Scholes model and actually underperform the symmetric SV models, i.e. it actually has higher relative option pricing errors than their symmetric counterparts. Bearing in mind the possible Monte Carlo errors associated with in particular the tail behaviour of the distribution, a similar mispricing pattern of the asymmetric SV models as that of the Black-Scholes model for the OTM options but a different mispricing pattern than the Black-Scholes model for the ITM options suggests the asymmetry of asset return distributions under correlated shocks to asset return and conditional volatility. A further look at the implied Black-Scholes volatility of the asymmetric model prices, however, reveal that the implied volatility curve of the asymmetric models against maturity, reported in Figure 13, has a curvature closer to the implied volatility from observed market options prices in its shape, suggesting such pricing biases may be easier to correct;

Third, consistent with simulation results in e.g. Hull and White (1987) and others, the symmetric SV models tend to predict lower prices than the Black-Scholes model for ATM options and higher prices than the Black-Scholes model for deep ITM options. However, such findings are not observed for the asymmetric SV models;

Fourth, the effect of stochastic interest rates on option prices is minimal in both cases of stochastic stock return volatility and constant stock return volatility, i.e. the simulation results between submodels I and II and those between submodels III and IV;

Fifth, the systematic effect on the “mean” of stock returns, namely the additional predictability of stock returns, has a clear effect on option prices as evidenced in the simulation results between the general model and submodel I. This is due to the fact that the reprojected underlying volatilities are different in magnitude under alternative specifications of the ”mean” functions. In the asymmetric model framework, the predictability tends to reduce the overall option pricing errors. As discussed in Lo and Wang (1995), predictability of asset returns can have significant impact on option prices, even though the exact effect is far from being clear;

Finally, in general all the models share the similar pattern of mispricing as the Black-Scholes model, i.e. underpricing of short-maturity options and overpricing of long-maturity options, overpricing of deep OTM options and underpricing of deep ITM options. Since the simulation results in next section suggest the existence of a non-zero risk premium for the stochastic volatility, the overall overpricing of all SV models may be due to our assumption of zero risk premium for conditional volatility.

5.4 Out-of-Sample Comparison based on Implied Volatility

For out-of-sample analysis, since the underlying volatility is no longer observable, we will use market option prices observed in the previous day (t-1) to imply the conditional volatilities at $t-1$ which are used as the initial values of the volatility processes at day t. In addition, we also assume a simple functional form for the risk premium of stock return stochastic volatility, namely $\lambda_t \sigma_s$, i.e. the risk premium is proportional to the conditional volatility of the stochastic volatility process. For each given option pricing model, we imply a parameter set, $\theta_t = (\sigma_{st}, \sigma_{rt}, \lambda_t)$ or its subset depending on the model specification, by minimizing the sum of squared error (SSE), i.e.

$$\tilde{\theta}_{t-1} = \text{Argmin}_{\theta_{t-1}} \sum_i (C^M_{t-1}(S_{t-1}, r_{t-1}, \theta_{t-1}; T_i, X_i) - C_{t-1}(T_i, X_i))^2 \qquad (9)$$

where $C_{t-1}(T_i, X_i)$ is the option price observed at $t-1$ with maturity date T_i and strike price X_i. The implied volatilities and risk premium at $t-1$ are then used to price the options at t

$$C^M_t(S_t, r_t, \sigma_{rt}, \sigma_{St}; T, X) = \mathsf{E}^*_t[e^{-\int_0^T r_t dt} \max(S_T - X, 0) | S_t, r_t, \tilde{\theta}_{t-1}] \qquad (10)$$

Our simulation results using the same data set as in the in-sample comparison show that, based on implied volatility instead of underlying volatility, but maintaining the assumption of zero risk premium for stock return stochastic volatility, can slightly reduce the relative pricing errors (not reported), and the pricing errors for long-term options are still high. While allowing for non-zero risk premium for stochastic volatility but relying on the historical underlying volatility can largely reduce the overall relative pricing errors (not reported), but the relative pricing errors for short-maturity options remain high. When both conditional volatility and risk premium are implied from previously observed market option prices following the above procedure, the relative pricing errors (not reported) are very similar to our findings in this section. The above evidence suggests that, similar to the findings in Melino and Turnbull (1990), there exists a non-zero risk premium for stochastic volatility of stock returns. However, the risk premium parameter λ_t appears to be rather stable over time. Due to the relatively small number of time periods over short time span involved in our study, such finding is not surprising and a rigorous test of the constant risk premium parameter is not performed.

Table 8 reports the relative pricing errors (%) based on implied volatility and/or non-zero risk premium of stochastic volatility for alternative models. In each cell, from top to bottom are the MRE (mean relative error) and MARE (mean absolute relative error) statistics for various models as listed in Table 7. The basic conclusions we draw from the comparison are summarized as following. First, all models have substantially reduced the pricing errors due to the use of implied volatility. The Black-Scholes model exhibits similar pattern of mispricing, namely underpricing of short-maturity options and over-pricing of long-maturity options, and overpricing of deep OTM options and underpricing deep ITM options; Second, all SV models outperform non-SV models due to the use of implied volatility and more importantly the introduction of non-zero risk premium for conditional volatility. The asymmetric model captures the curvature of the implied volatility curve better than the symmetric models, as reflected in Figure 14. Its direct implication is the lower pricing errors for deep ITM options, which are of important economic meaning in terms of absolute price differences. However, the asymmetric models still exhibit systematic pricing errors, namely underpricing of short-term deep OTM options, overpricing of long-term deep OTM options, and underpricing of deep ITM options. Furthermore, the MARE statistics, a measure of the dispersion of the relative pricing errors, are not reduced as much as the MRE statistics; Third, the interest rate still only has minimal impact on option prices; Fourth, the impact of the systematic effect in the "mean" of stock returns is much less obvious. This is due to the fact that under risk-neutral specification, the models are equivalent regardless of the specification of "mean" functions.

6 Conclusion

In this paper, we specify a SV asset pricing model in a multivariate framework to simultaneously describe the dynamics of stock returns, stock return conditional volatility, interest rates, and interest rate conditional volatility. In addition, the model assumes a systematic component in the stock return volatility and "leverage effect" for both stock return and interest rate processes. The proposed model is first estimated using the EMM technique based on observations of underlying state variables. The estimated model is then utilized to investigate the respective effect of systematic volatility, idiosyncratic volatility, and stochastic interest rates on option prices. The empirical results are summarized as follows.

While theory predicts that the short-term interest rates are strongly related to the systematic volatility of the consumption process, our empirical results suggest that the short-term interest rate fails to serve as a good proxy of the systematic factor. However, the short-term interest rate is significantly correlated with the "mean" of the stock returns, suggesting stock return is predictable to certain extent. Such predictability, as shown in the empirical results based on the underlying volatility, has a clear impact on option prices as the reprojected underlying volatilities are different in magnitude in alternative model specifications. It is noted that when volatilities are implied from previously observed option prices, such an effect disappears as the "risk-neutral" specifications under different conditional "means" are essentially equivalent.

Overall, all models exhibit clear and significant discrepancies between model predicted option prices and market observed option prices for different terms to expiration and degrees of moneyness based on reprojected underlying volatility. In particular, the Black-Scholes model underprices short-term maturity call options and overprices long-term maturity call options, and underprices deep ITM options and overprices deep OTM options based on both underlying historical volatility and implied volatility. While allowing for stochastic volatility of stock returns can in general reduce the pricing errors, allowing for stochastic interest rates has minimal impact on option prices in our case.

Similar to Melino and Turnbull (1990), our empirical findings strongly suggest the existence of a non-zero risk premium for stochastic volatility of stock returns. When a non-zero risk-premium of stochastic volatility is introduced into the model and the implied volatility is used to price options, the models with stochastic volatility of stock returns and in particular the asymmetric stochastic volatility models of stock returns can largely reduce the option pricing errors.

Finally, the failure of short-term interest rate as a valid proxy of systematic volatility component suggests that in the future study, an alternative state variable, say market index, should be used to study the impact of systematic volatility on option prices. Our empirical results also suggest that normality of the stochastic volatility model may not be adequate for this data set and other data sets as well. We leave it in our future research to explore a richer structural model, for example the jump-diffusion and/or the SV model with Student-t disturbances, to describe the dynamics of asset returns.

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Table 1: Summary Statistics of Interest Rates and Stock Returns

(a) Static Properties of Original Series

(100×)	N	Mean	Std. Dev.	Skewness	Kurtosis	Max	Min	Corr($\cdot, r_{t-1}$)
$\Delta ln(r_t)$	2835	-9.759 10^{-3}	1.042	-0.549	11.848	6.936	-9.137	
$\Delta ln(S_t)$	2835	1.051 10^{-1}	3.719	-1.109	14.655	21.48	-36.67	-0.043

(b) Static Properties of Filtered Interest Rates and Stock Returns

	N	Mean	Std. Dev.	Skewness	Kurtosis	Max	Min	$Corr(., r_{t-1})$
Y_{r_t}	2834	-6.10 10^{-10}	1.042	-0.556	11.87	6.948	-9.154	
$ln(Y_{r_t}^2)$	2834	-2.199	2.995	-1.241	1.806	4.429	-20.16	
$Y1_{S_t}$	2834	3.86 10^{-4}	3.716	-1.107	14.56	21.34	-36.56	
$ln(Y1_{S_t}^2)$	2834	0.766	2.525	-1.201	2.032	7.198	-14.19	-0.063
$Y2_{S_t}$	2834	4.38 10^{-4}	3.719	-1.110	14.65	21.37	-36.77	
$ln(Y2_{S_t}^2)$	2834	0.794	2.409	-0.850	0.279	7.209	-6.673	-0.046

(c) Dynamic Properties of Filtered Interest Rates and Stock Returns (autocorrelations ($\times 10^{-1}$))

	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(4)$	$\rho(5)$	$\rho(10)$	$\rho(15)$	$\rho(20)$
Y_{r_t}	1.270	-0.118	-0.441	0.292	-0.271	0.250	0.837	0.197
$ln(Y_{r_t}^2)$	1.120	0.981	0.662	0.795	1.180	0.216	0.199	0.419
$Y1_{S_t}$	0.653	-0.469	-0.554	0.050	0.011	-0.316	0.414	-0.233
$ln(Y1_{S_t}^2)$	0.644	0.479	-0.064	0.460	0.455	0.219	0.305	0.782
$Y2_{S_t}$	0.670	-0.451	-0.537	0.066	0.028	-0.346	0.430	-0.218
$ln(Y2_{S_t}^2)$	0.594	0.495	-0.090	0.363	0.285	0.193	0.247	0.653

Note: $Y1$ represents the filtered series with systematic effects on stock returns, while $Y2$ without systematic effects.

Table 2: Estimates of trend

Model	Stock Return Parameter		Interest Rate Parameter	
	μ_S	ϕ_S	μ_r	ϕ_r
With Systematic Effect	0.667	-10.29	-0.215	-6.98 10^{-4}
	(2.60)	(-2.28)	(-1.119)	(-1.075)
Without Systematic Effect	0.105		-0.215	-6.98 10^{-4}
	(1.505)		(-1.119)	(-1.075)

Note: The numbers in brackets are t-ratios of the above estimates. The blank cell indicates the parameter is pre-set as zero in the corresponding model.

K_z	α_0	α_1	γ	κ_1	κ_2
0	$.118$ $(1.91e{-3})$	$-.305$ $(5.90e{-2})$	$.957$ $(7.50e{-4})$	$-.080$ $(1.41e{-2})$	$.123$ $(1.44e{-2})$
1	$.118$ $(6.18e{-4})$	$-.305$ $(1.79e{-2})$	$.957$ $(2.52e{-4})$	$-.080$ $(7.41e{-3})$	$.123$ $(3.92e{-3})$
2	$.118$ $(6.18e{-4})$	$-.305$ $(1.79e{-2})$	$.957$ $(2.52e{-4})$	$-.080$ $(7.41e{-3})$	$.123$ $(3.92e{-3})$
3	$..097$ $(5.45e{-4})$	$-,433$ $(1.40e{-2})$	$.968$ $(8.61e{-3})$	$-.098$ $(8.61e{-3})$	$.135$ $(4.00e{-3})$
4	$.190$ $(1.32e{-3})$	$-.256$ $(2.02e{-2})$	$.949$ $(4.50e{-4})$	$-.075$ $(1.45e{-2})$	$.183$ $(5.26e{-3})$
5	$.182$ $(1.27e{-3})$	$-.254$ $(1.95e{-2})$	$.952$ $(4.35e{-4})$	$-.079$ $(1.43e{-2})$	$.183$ $(5.06e{-3})$
6	$.183$ $(1.37e{-3})$	$-.382$ $(1.82e{-2})$	$.955$ $(4.38e{-4})$	$-.071$ $(2.04e{-2})$	$.201$ $(6.11e{-3})$
7	$.174$ $(1.36e{-3})$	$-.365$ $(1.90e{-2})$	$.957$ $(4.43e{-4})$	$-.072$ $(1.94e{-2})$	$.193$ $(5.97e{-3})$
8	$.177$ $(1.37e{-3})$	$-.364$ $(1.85e{-2})$	$.957$ $(4.41e{-4})$	$-.073$ $(1.98e{-2})$	$.196$ $(5.89e{-3})$
9	$.180$ $(1.47e{-3})$	$-.384$ $(1.94e{-2})$	$.956$ $(4.69e{-4})$	$-.072$ $(2.10e{-2})$	$.202$ $(6.56e{-3})$
10	$.183$ $(1.69e{-3})$	$-.297$ $(2.62e{-2})$	$.954$ $(5.42e{-4})$	$-.062$ $(1.99e{-2})$	$.172$ $(6.60e{-3})$

Table 3: Sensitivity of the EGARCH(1,1) leading term parameters to the order of the polynomial for the Stock returns. Standard errors are below between brackets.

K_z	α_0	α_1	γ	κ_1	κ_2
0	$.020$ $(3.35e{-}3)$	$.240$ $(8.09e{-}2)$	$.949$ $(7.30e{-}3)$	$-.029$ $(9.62e{-}3)$	$.191$ $(1.21e{-}2)$
1	$.020$ $(1.33e{-}3)$	$.241$ $(3.33e{-}2)$	$.949$ $(3.16e{-}3)$	$-.029$ $(4.13e{-}3)$	$.191$ $(4.49e{-}3)$
2	$.020$ $(1.33e{-}3)$	$.241$ $(3.33e{-}2)$	$.949$ $(3.16e{-}3)$	$-.029$ $(4.13e{-}3)$	$.191$ $(4.48e{-}3)$
3	$.030$ $(1.24e{-}3)$	$.141$ $(2.82e{-}2)$	$.957$ $(3.50e{-}3)$	$-.028$ $(4.67e{-}3)$	$.195$ $(4.34e{-}3)$
4	$.030$ $(1.42e{-}3)$	$.050^*$ $(3.18e{-}2)$	$.959$ $(4.94e{-}3)$	$-.041$ $(6.53.e{-}3)$	$.185$ $(4.91e{-}3)$
5	$.031$ $(1.42e{-}3)$	$.051^*$ $(3.16e{-}2)$	$.960$ $(5.01e{-}3)$	$-.039$ $(6.56e{-}3)$	$.185$ $(4.91e{-}3)$
6	$.056$ $(1.26e{-}3)$	$-.100$ $(1.88e{-}2)$	$.976$ $(2.54e{-}3)$	$-.024$ $(8.82e{-}3)$	$.232$ $(4.50e{-}3)$
7	$.057$ $(1.32e{-}3)$	$-.122$ $(1.87e{-}2)$	$.977$ $(2.49e{-}3)$	$-.024$ $(9.07e{-}3)$	$.246$ $(4.84e{-}3)$
8	$.060$ $(1.32e{-}3)$	$-.115$ $(1.83e{-}2)$	$.977$ $(2.42e{-}3)$	$-.024$ $(9.17e{-}3)$	$.248$ $(4.74e{-}3)$
9	$.045$ $(2.10e{-}3)$	$.161$ $(3.48e{-}2)$	$.934$ $(7.14e{-}3)$	$-.028$ $(6.04e{-}3)$	$.208$ $(5.46e{-}3)$
10	$.054$ $(1.75e{-}3)$	$.045^*$ $(3.22e{-}2)$	$.964$ $(5.62e{-}3)$	$-.014^*$ $(7.79e{-}3)$	$.204$ $(5.80e{-}3)$

Table 4: Sensitivity of the EGARCH(1,1) leading term parameters to the order of the polynomial for the Interest Rates. Standard errors are below between brackets. The values marked with an asterix are insignificantly different from zero at a 5% level

Table 5: Test statistics for the SV models

(a) Interest rate returns

	Asymmetric General Model	Symmetric General Model	Asymmetric Submodel 1	Symmetric Submodel 1
J-test	.471	2.96	.471	2.96
df	1	1	1	1
P-value	.493	.085	.493	.085
α_0	-.709	-1.72	-.686	-1.72
α_1	.715	-1.72	.685	-1.72
γ	.696	1.72	.685	1.72
κ_1	-.588	-	-.684	-
κ_2	.704	1.72	.686	1.72

(b) Stocks returns

	Asymmetric General Model	Symmetric General Model	Asymmetric Submodel 1	Symmetric Submodel 1
J-test	.034	.416	.202	.841
df	1	1	1	1
P-value	.854	.519	.653	.359
α_0	.185	-.645	.451	-.917
α_1	.180	-.645	.450	-.917
γ	.185	-.645	.451	-.917
κ_1	.193	-	-.450	-
κ_2	-.187	-.645	-.450	-.917

Table 6: Sample Properties of Stock Call Option

es

	Moneyness $x = ln(S/KB(t,T))$ $[-0.68,\ 1.11]$	Days-to-Expiration T-t [5, 215]			
		≤30	$30 - -180$	≥ 180	Subtotal
OTM	$x \leq -0.20$	0.223 (0.112) 0.066 (0.035) {33}	1.760 (0.819) 0.137 (0.033) {85}	1.892 (0.911) 0.274 (0.038) {146}	264
	$-0.20 < x \leq -0.03$	0.817 (0.577) 0.090 (0.040) {76}	3.140 (1.364) 0.149 (0.047) {164}	5.231 (0.989) 0.173 (0.065) {59}	299
ATM	$-0.03 < x \leq 0.00$	1.783 (1.023) 0.114 (0.054) {47}	4.911 (1.440) 0.183 (0.061) {48}	7.042 (0.584) 0.250 (0.000) {46}	151
	$0.00 < x \leq 0.05$	2.997 (0.876) 0.143 (0.050) {32}	5.843 (1.318) 0.195 (0.058) {65}	7.976 (0.559) 0.190 (0.060) {21}	118
ITM	$0.05 < x \leq 0.30$	9.114 (3.030) 0.264 (0.091) {182}	10.61 (2.736) 0.294 (0.089) {283}	11.90 (2.178) 0.306 (0.109) {101}	566
	$x > 0.30$	21.23 (5.403) 0.361 (0.053) {176}	23.99 (6.004) 0.369 (0.049) {394}	25.93 (6.135) 0.375 (0.066) {152}	722
Subtotal		556	1039	525	2120(total)

Note: In each cell from top to bottm are: the average bid-ask midpoint call option prices with standard error in parentheses; the average effective bid-ask spread (ask price minus the bid-ask midpoint) with standard error in parentheses; and the number of option price observations (in curly brackets) for each moneyness-maturity category. The option price sample covers the period of June 19, 1997 through August 18, 1997 with total 2120 observations. In calculating the moneyness, we use U.S. 3-month T-bill rates for options with maturity less than 4 months and 6-month T-bill rates for options with maturity longer than 4 months.

Table 7: Relative Pricing Errors (%) of Alternative Models based on Underlying Volatility

	Moneyness $x = ln(S/KB(t,T))$ $[-0.68,\ 1.11]$	Days-to-Expiration T-t [5, 215]			
		≤30	$30 - -180$	≥ 180	
OTM	$x \leq -0.20$	-23.83 23.90	23.32 30.19	57.84 57.84	42.53 49.91
		-26.47 26.47	17.45 17.45	47.09 47.09	33.46 41.63
		-18.89 20.04	38.32 43.22	61.68 61.68	51.90 55.11
		-27.12 27.12	16.68 24.01	46.38 46.38	32.76 42.11
		-18.89 20.05	38.32 43.22	61.67 61.67	51.90 55.12
		-27.12 27.12	16.68 24.01	46.38 46.38	32.76 42.11
		-40.71 42.66	24.82 36.36	59.97 59.97	41.96 54.81
		-40.72 42.67	24.77 36.31	59.64 49.64	41.47 54.56
	$-0.20 < x \leq -0.03$	-5.00 14.27	15.41 16.33	22.91 22.91	12.48 17.71
		-6.26 12.56	12.27 13.30	17.76 17.76	9.22 14.40
		-1.20 14.16	23.54 24.38	34.02 34.02	20.39 24.61
		-6.41 12.16	11.62 13.05	17.52 17.52	8.82 14.21
		-1.20 14.16	23.54 24.38	34.02 34.02	20.39 24.61
		-6.41 12.16	11.61 13.05	17.52 17.52	8.82 14.21
		6.21 25.61	18.78 20.71	24.28 24.28	17.23 22.96
		5.35 26.04	19.20 20.42	24.09 24.09	17.15 22.89
ATM	$-0.03 < x \leq 0.00$	0.22 4.18	8.21 9.03	16.45 16.45	7.73 9.22
		0.31 3.73	6.31 7.26	12.13 12.13	5.83 7.29
		0.90 4.62	14.02 14.02	24.28 24.28	12.57 13.56
		-0.95 4.43	6.90 6.90	11.96 11.96	5.82 7.26
		0.90 4.62	14.02 14.02	24.28 24.28	12.57 13.56
		-0.95 4.43	6.91 6.91	11.96 11.96	5.82 7.26
		9.54 15.71	10.86 10.86	16.33 16.33	11.60 13.25
		11.50 15.49	9.85 10.94	16.18 16.18	11.55 13.20
	$0.00 < x \leq 0.05$	1.15 4.45	7.38 7.73	13.70 13.70	7.63 8.61
		0.51 4.11	5.38 5.79	9.80 9.80	5.43 6.52
		2.47 4.90	12.64 12.86	20.58 20.58	11.90 12.61
		0.32 3.97	5.23 5.63	9.66 9.66	5.27 6.36
		2.47 4.90	12.64 12.86	20.58 20.58	11.90 12.61
		0.32 3.97	5.23 5.63	9.66 9.66	5.27 6.36
		6.99 11.47	8.25 8.72	12.94 12.94	9.27 10.61
		6.99 11.47	8.21 8.69	12.80 12.80	9.22 10.56

ITM	$0.05 < x \leq 0.30$	-0.25 1.10	3.32 3.50	9.03 9.03	3.77 4.22
		-0.59 1.16	1.89 2.16	5.87 5.87	2.21 2.81
		0.08 1.08	5.59 5.70	13.67 13.67	6.10 6.43
		-0.63 1.16	1.82 2.09	5.78 5.78	2.14 2.76
		0.08 1.08	5.59 5.70	13.67 13.67	6.10 6.43
		-0.63 1.16	1.82 2.09	5.78 5.78	2.14 2.76
		-0.14 2.25	2.46 3.04	7.37 7.37	2.98 3.90
		-0.13 2.25	2.44 3.02	7.26 7.26	2.94 3.87
	$x > 0.30$	-0.14 0.22	0.43 0.60	2.44 2.44	0.82 1.00
		-0.30 0.33	-0.62 0.71	0.44 0.97	-0.21 0.61
		-0.10 0.21	1.11 1.17	4.02 4.02	1.55 1.67
		-0.31 0.33	-0.63 0.72	0.42 0.96	-0.23 0.66
		-0.10 0.21	1.11 1.17	4.02 4.02	1.55 1.67
		-0.31 0.33	-0.63 0.72	0.42 0.96	-0.23 0.66
		-0.54 0.61	-1.10 1.14	-0.04 1.13	-0.61 0.98
		-0.54 0.62	-1.12 1.20	-0.10 1.13	-0.64 0.99
Overall		-2.60 4.53	5.73 6.37	25.38 25.38	10.29 12.43
		-3.18 4.54	3.88 5.00	19.74 19.89	7.47 10.01
		-1.53 4.25	9.31 9.75	39.51 39.51	16.84 18.54
		-3.33 4.58	3.74 4.86	19.43 19.59	7.27 9.86
		-1.53 4.25	9.31 9.75	39.51 39.51	16.84 18.54
		-3.33 4.58	3.74 4.86	19.44 19.60	7.27 9.86
		-2.08 8.78	5.74 7.54	25.17 25.51	10.36 14.06
		-2.08 8.07	5.71 7.51	24.99 25.34	10.29 13.99

Note: In each cell, from top to bottom are the MRE (mean relative error) and MARE (mean absolute relative error) statistics for:

1. the asymmetric general SV model with $\lambda_2 \neq 0, \lambda_3 \neq 0$;
2. the symmetric general SV model with $\lambda_2 = \lambda_3 = 0$;
3. the asymmetric submodel I with $\lambda_2 \neq 0, \lambda_3 \neq 0$;
4. the symmetric submodel I with $\lambda_2 = \lambda_3 = 0$;
5. the asymmetric submodel II with $\lambda_3 \neq 0$;
6. the symmetric submodel II with $\lambda_3 = 0$;
7. the submodel III; and
8. The sunmodel IV.

Table 8: Relative Pricing Errors (%) of Alternative Models based on Implied Volatility

	Moneyness $x = ln(S/KB(t,T))$ $[-0.68,\ 1.11]$	Days-to-Expiration T-t [5, 215]			
		≤30	$30 - -180$	≥ 180	
OTM	$x \leq -0.20$	-17.16 17.16	-2.90 4.20	3.13 5.07	-4.77 10.72
		-27.33 27.33	-3.18 3.85	2.50 4.76	-7.61 11.25
		-17.20 17.20	-2.91 4.21	2.93 5.06	-4.87 10.73
		-27.34 27.34	-3.19 3.86	2.31 4.75	-7.92 11.26
		-17.15 17.15	-2.90 4.19	2.92 5.07	-4.86 10.71
		-27.32 27.32	-3.18 3.84	2.30 4.76	-7.91 11.24
		-40.01 40.80	3.92 8.18	22.41 22.55	5.16 15.47
		-39.95 40.75	3.89 8.16	22.38 22.48	5.12 15.43
	$-0.20 < x \leq -0.03$	0.75 3.39	-0.58 1.57	-0.60 2.38	-0.40 2.31
		0.81 3.41	-0.62 1.85	-0.66 2.76	-0.44 2.69
		0.74 3.38	-0.60 1.59	-0.66 2.38	-0.45 2.32
		0.82 3.43	-0.64 1.85	-0.69 2.79	-0.45 2.72
		0.75 3.39	-0.58 1.57	-0.65 2.36	-0.42 2.31
		0.81 3.41	-0.62 1.85	-0.68 2.77	-0.44 2.70
		-8.88 21.77	0.05 5.79	11.44 11.63	0.78 10.23
		-8.79 21.72	0.06 5.77	11.39 11.58	0.75 10.20
ATM	$-0.03 < x \leq 0.00$	0.60 2.44	-0.37 0.89	-0.24 0.86	0.12 1.46
		1.76 2.45	-0.42 0.88	-0.52 0.87	0.62 1.52
		0.63 2.46	-0.37 0.92	-0.24 0.87	0.13 1.48
		1.75 2.43	-0.43 0.89	-0.54 0.90	0.61 1.53
		0.59 2.44	-0.38 0.89	-0.24 0.85	0.12 1.46
		1.77 2.45	-0.42 0.88	-0.51 0.87	0.63 1.52
		-1.74 8.07	3.87 7.34	0.83 0.83	1.79 7.42
		-1.75 8.71	3.87 7.33	0.79 0.79	1.83 7.42
	$0.00 < x \leq 0.05$	1.13 1.45	-0.17 0.30	-.23 0.45	0.20 0.77
		1.84 3.96	-0.33 0.71	-.45 0.87	0.33 1.45
		1.11 1.44	-0.16 0.32	-.22 0.45	0.21 0.78
		1.80 3.95	-0.32 0.74	-.44 0.86	0.33 1.44
		1.13 1.45	-0.15 0.30	-.21 0.43	0.21 0.76
		1.84 3.96	-0.33 0.71	-.45 0.87	0.33 1.45
		-1.12 7.49	-1.99 3.96	9.59 8.11	0.57 5.18
		-1.17 7.50	-1.97 3.95	7.50 8.08	0.59 5.17

		-0.07 0.16	-.24 0.86	-0.19 1.74	-0.18 1.26
		-0.15 0.22	-.24 0.64	-0.22 1.01	-0.21 0.72
		-0.08 0.17	-.26 0.90	-0.22 1.79	-0.21 1.31
ITM	$0.05 < x \leq 0.30$	-0.14 0.22	-0.25 0.66	-0.22 0.90	-0.23 0.70
		-0.06 0.16	-0.27 0.88	-0.20 1.76	-0.19 1.28
		-0.15 0.22	-0.25 0.64	-0.23 0.89	-0.23 0.69
		-1.86 2.04	-1.33 2.40	1.48 3.23	-1.17 2.39
		-1.83 2.02	-1.33 2.39	1.45 3.23	-1.16 2.37
		-0.08 0.18	-0.04 1.06	-0.05 1.93	-0.06 1.16
		-0.13 0.15	-0.06 0.94	-0.07 1.18	-0.08 0.90
		-0.06 0.18	-0.04 1.10	-0.07 1.95	-0.05 1.19
	$x > 0.30$	-0.10 0.17	-0.06 0.95	-0.10 1.21	-0.08 0.91
		-0.06 0.16	-0.04 1.06	-0.06 1.93	-0.05 1.16
		-0.11 0.15	-0.06 0.94	-0.09 1.18	-0.08 0.89
		-1.19 1.19	-0.99 1.01	-0.95 1.48	-1.00 1.09
		-1.18 1.18	-0.99 1.01	-0.97 1.48	-1.00 1.10
		-0.07 1.36	-0.24 2.57	-0.35 3.80	-0.22 2.54
		-0.12 1.41	-0.27 2.75	-0.37 3.53	-0.24 2.45
		-0.08 1.37	-0.25 2.58	-0.37 3.80	-0.24 2.55
Overall		-0.13 1.41	-0.28 2.75	-0.41 3.54	-0.27 2.46
		-0.07 1.35	-0.25 2.57	-0.36 3.78	-0.23 2.52
		-0.13 1.39	-0.27 2.74	-0.39 3.51	-0.25 2.43
		-5.16 9.34	-0.32 3.22	7.28 8.62	-0.10 5.13
		-5.15 9.31	-0.33 3.21	7.26 8.59	-0.10 5.12

Note: In each cell, from top to bottom are the MRE (mean relative error) and MARE (mean absolute relative error) statistics for:

1. the asymmetric general SV model with $\lambda_2 \neq 0, \lambda_3 \neq 0$;
2. the symmetric general SV model with $\lambda_2 = \lambda_3 = 0$;
3. the asymmetric submodel I with $\lambda_2 \neq 0, \lambda_3 \neq 0$;
4. the symmetric submodel I with $\lambda_2 = \lambda_3 = 0$;
5. the asymmetric submodel II with $\lambda_3 \neq 0$;
6. the symmetric submodel II with $\lambda_3 = 0$;
7. the submodel III; and
8. The sunmodel IV.

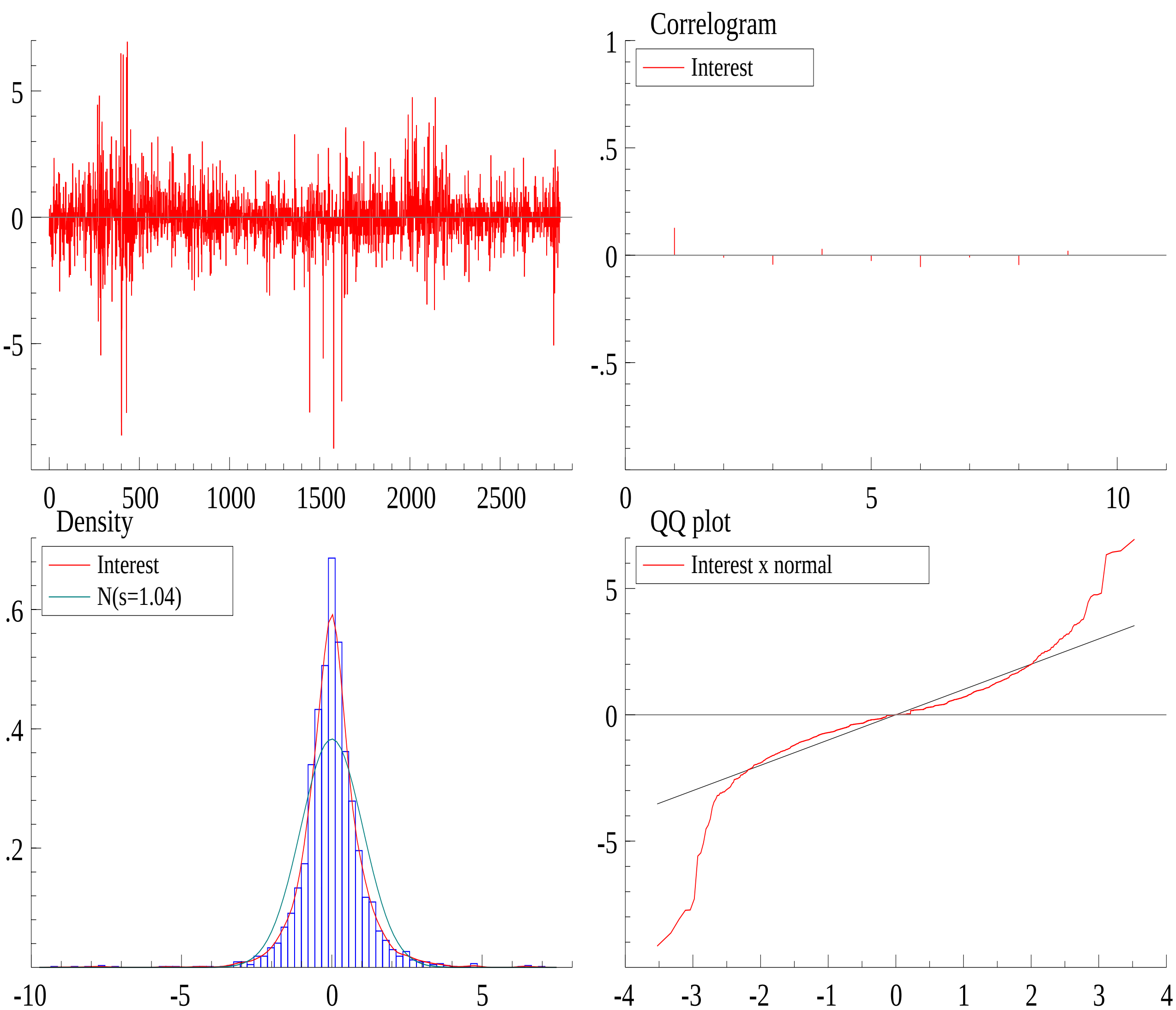

Figure 1: Salient feature of interest rate returns

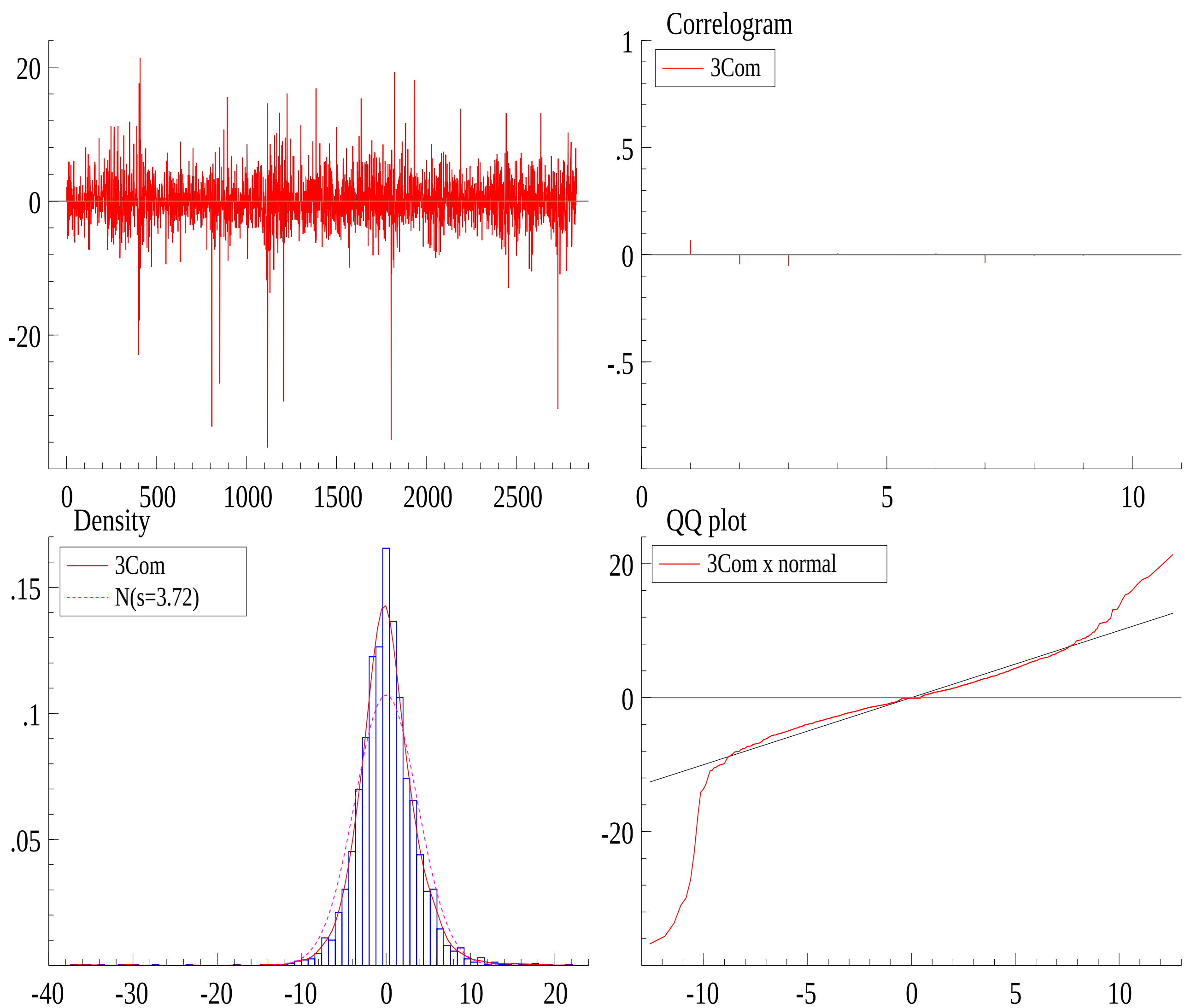

Figure 2: Salient feature of stock returns

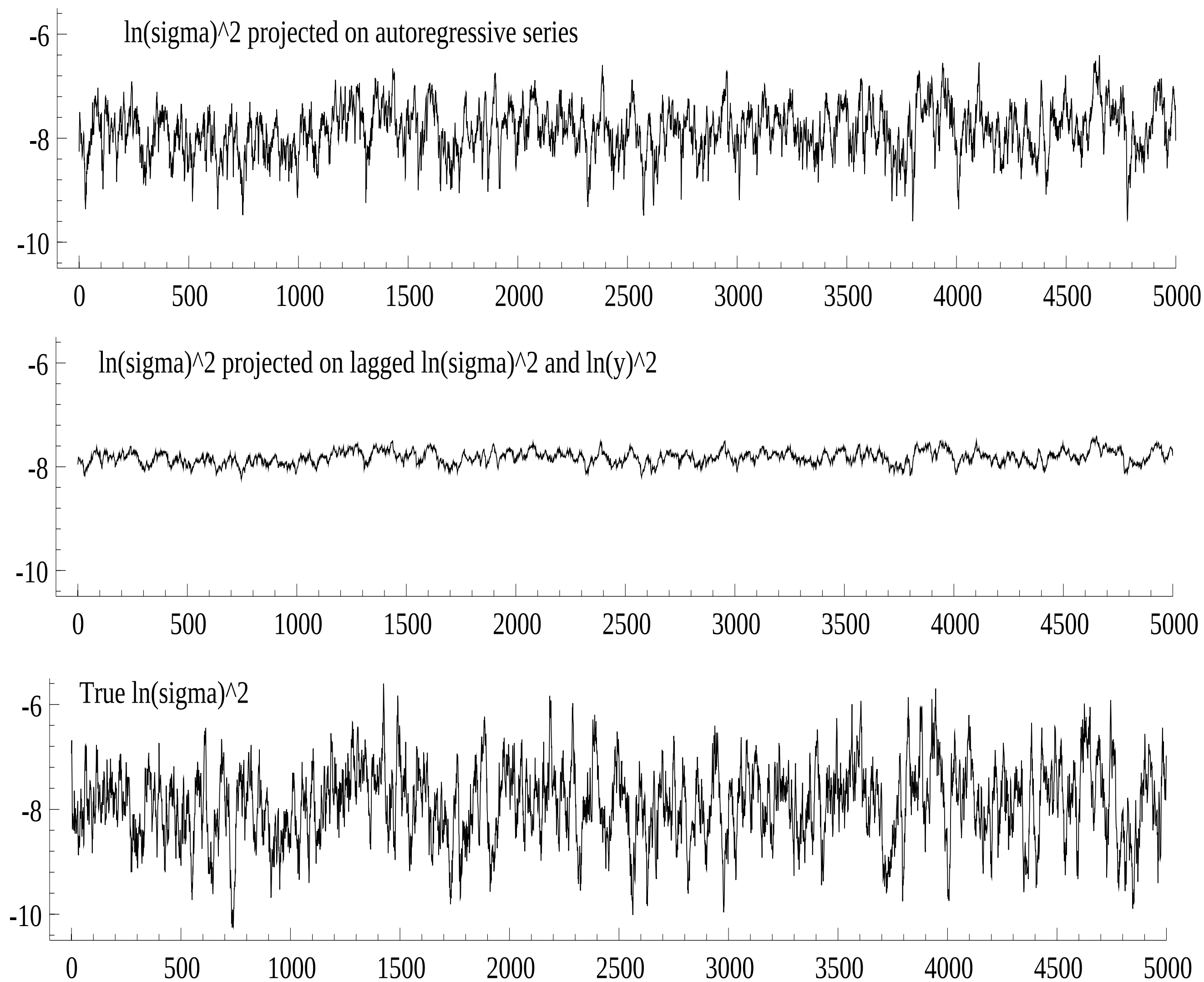

Figure 3: Upper panel: reprojected $\ln \sigma_t^2$ from reprojection on $\ln y_t^2$ and lagged $\ln y_t^2$. Middle panel: reprojected $\ln \sigma_t^2$ from reprojection on $\ln \sigma_{t-1}^2$ and $\ln y_t^2$. Lower panel: true $\ln \sigma_t^2$.

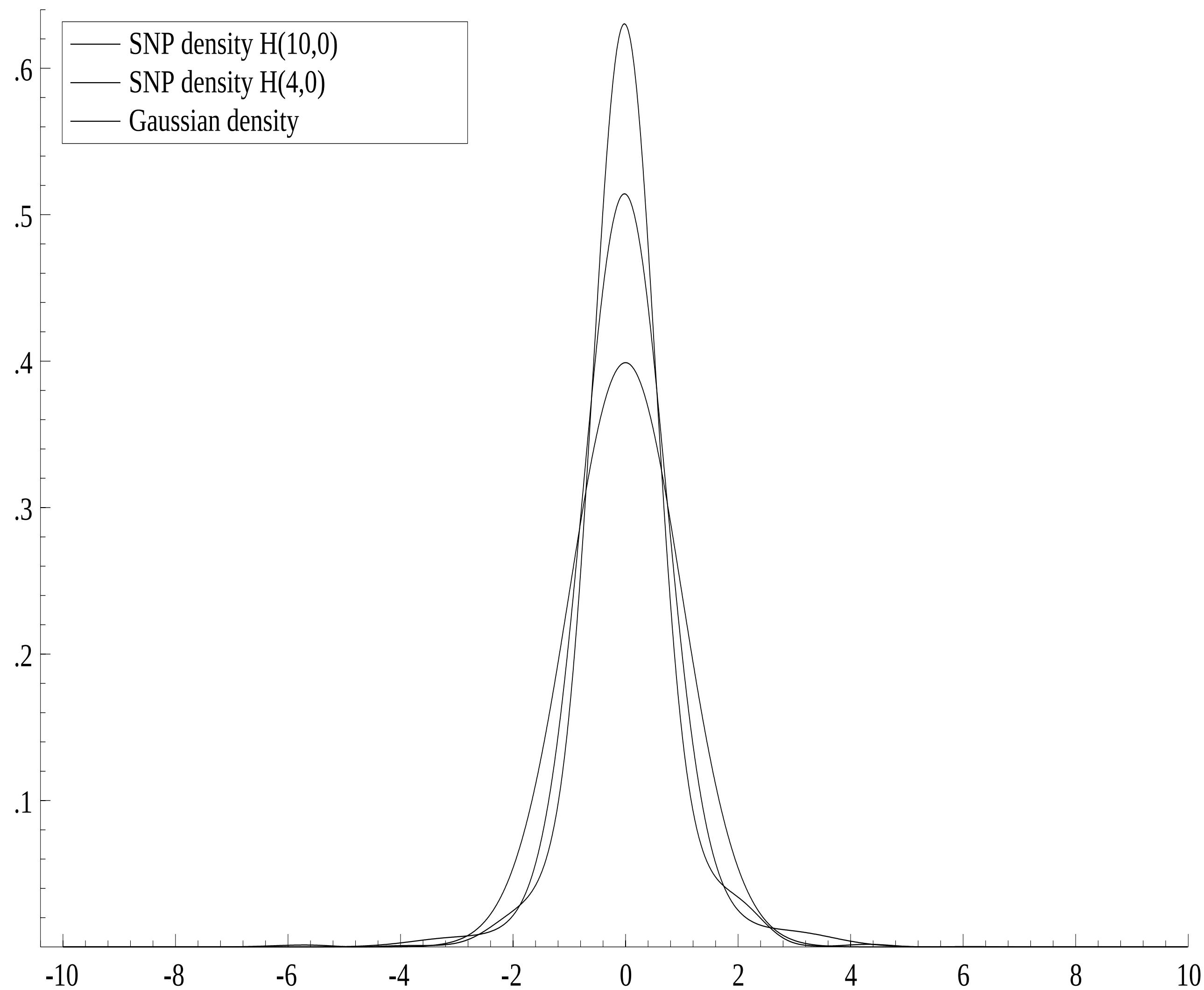

Figure 4: Estimated conditional density for EGARCH(1,1)-H(4,0) model for interest rate returns

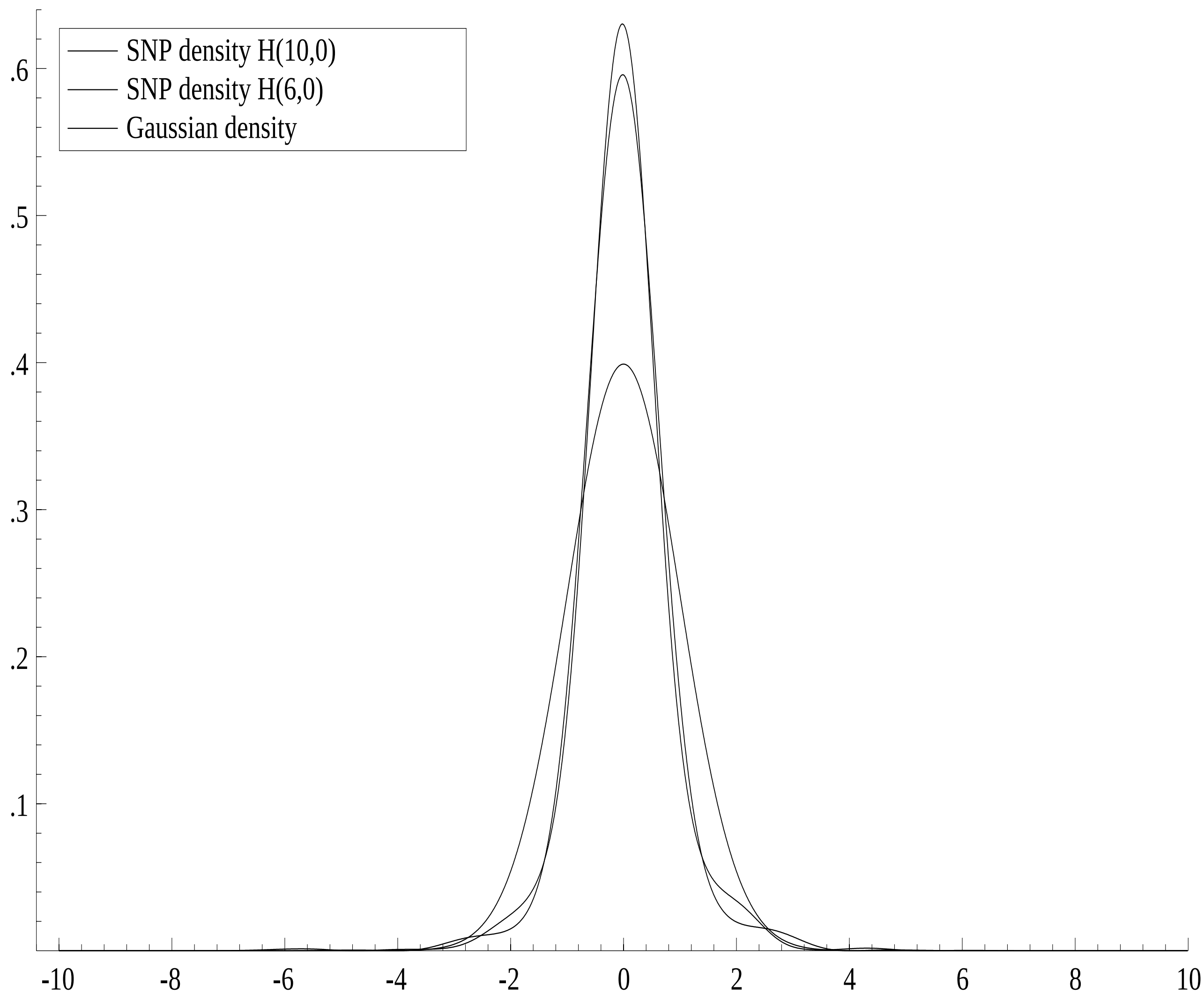

Figure 5: Estimated conditional density for EGARCH(1,1)-H(6,0) model for interest rate returns

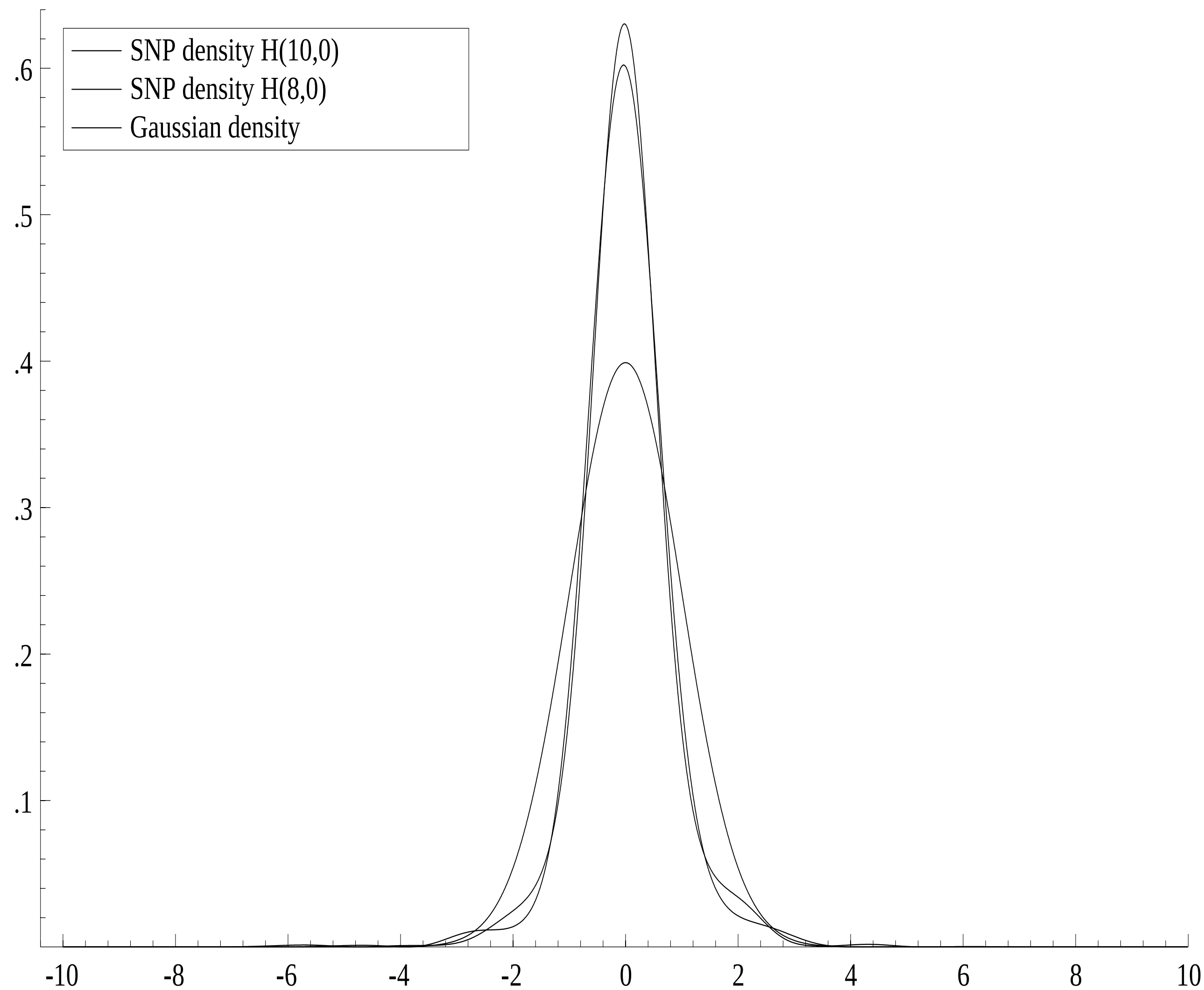

Figure 6: Estimated conditional density for EGARCH(1,1)-H(8,0) model for interest rate returns

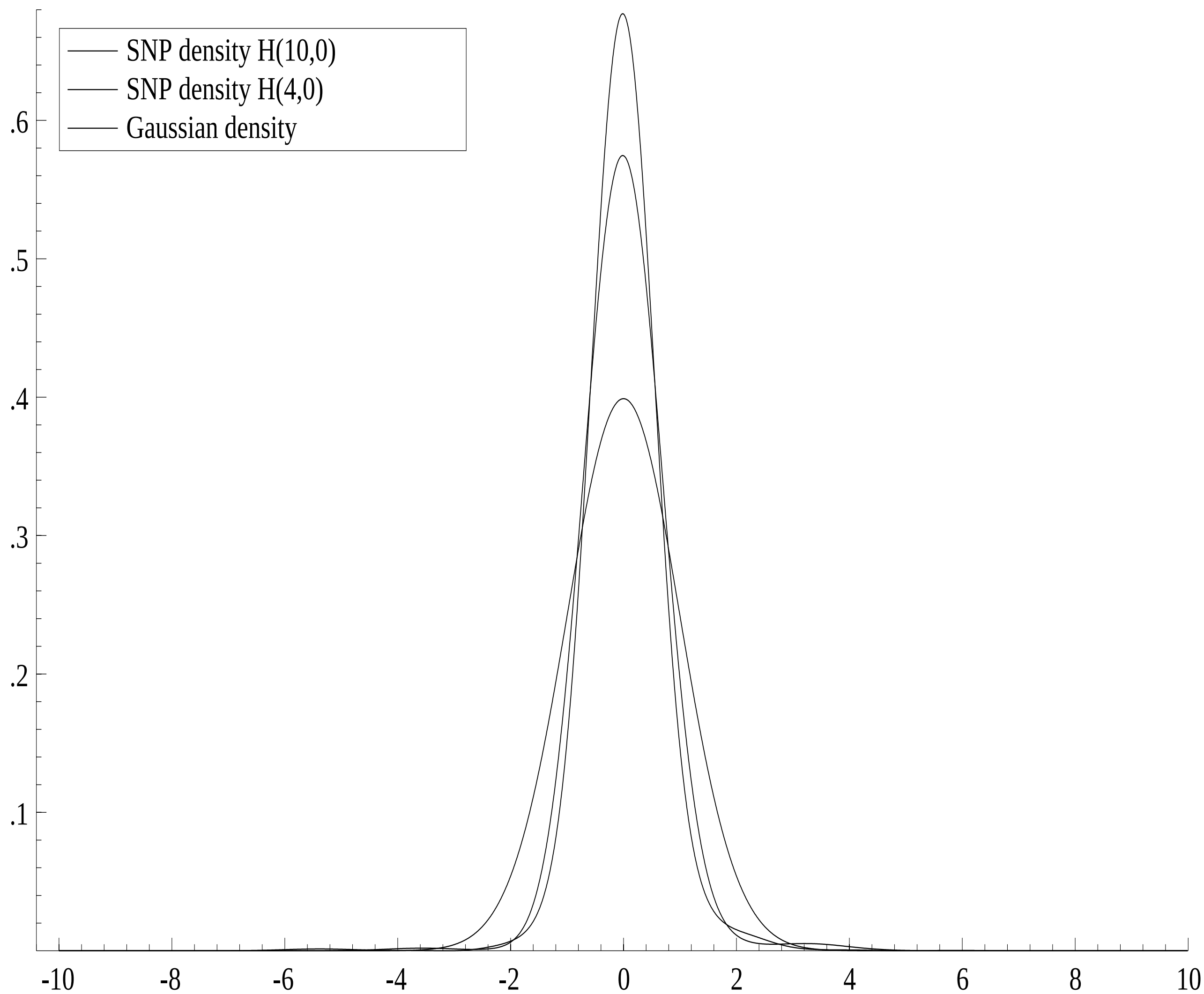

Figure 7: Estimated conditional density for EGARCH(1,1)-H(4,0) model for stock returns

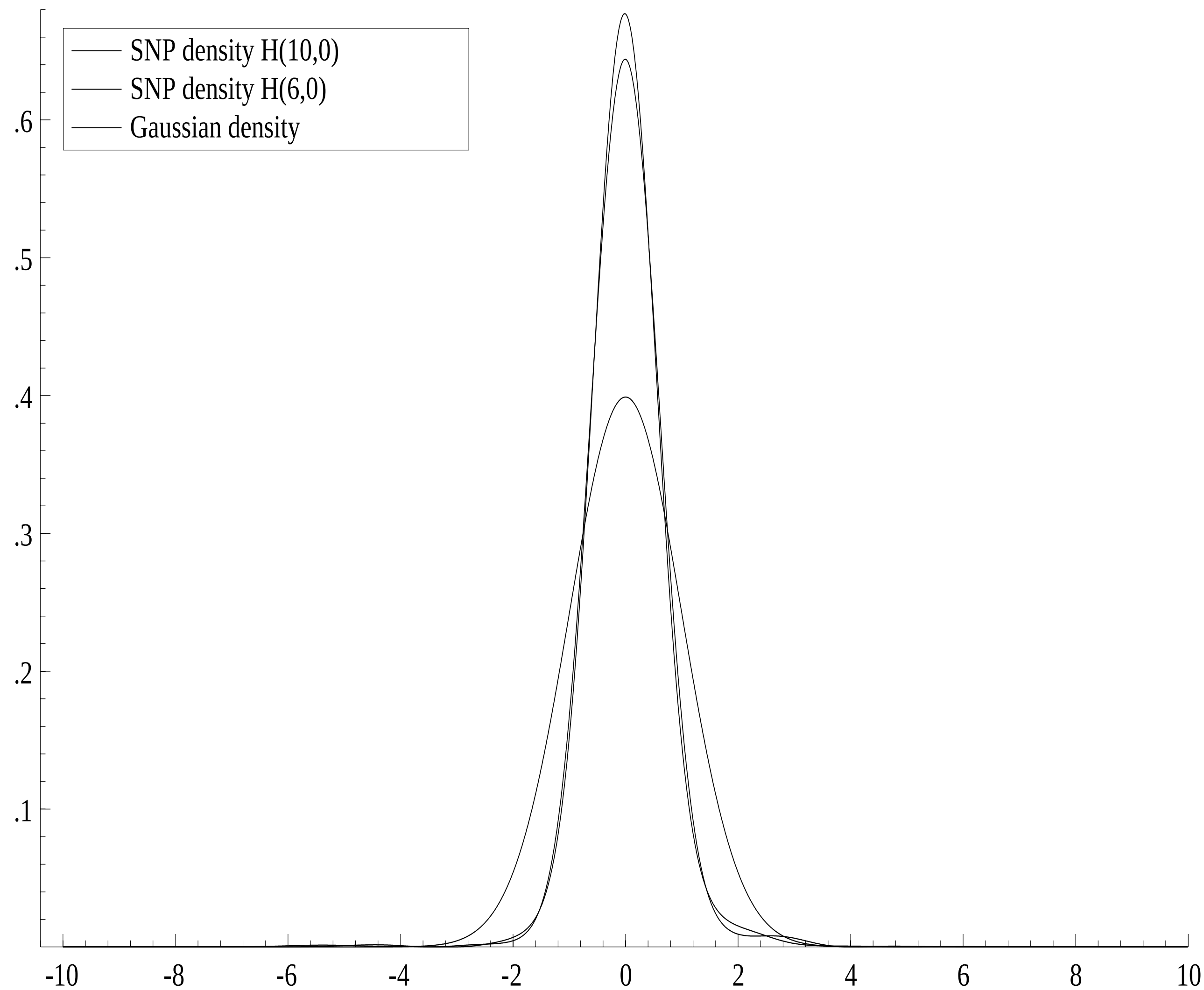

Figure 8: Estimated conditional density for EGARCH(1,1)-H(6,0) model for stock returns

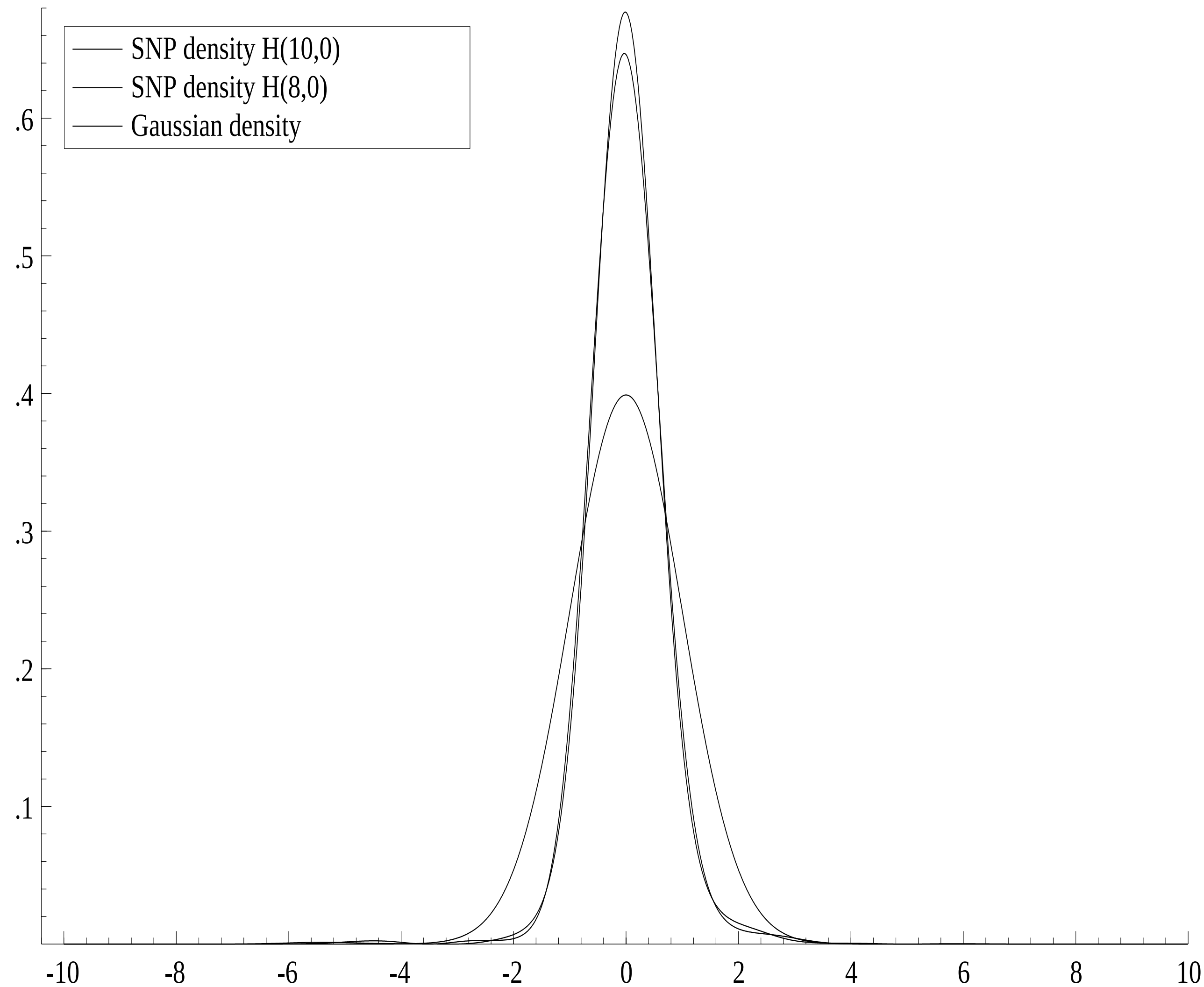

Figure 9: Estimated conditional density for EGARCH(1,1)-H(8,0) model for interest rate returns

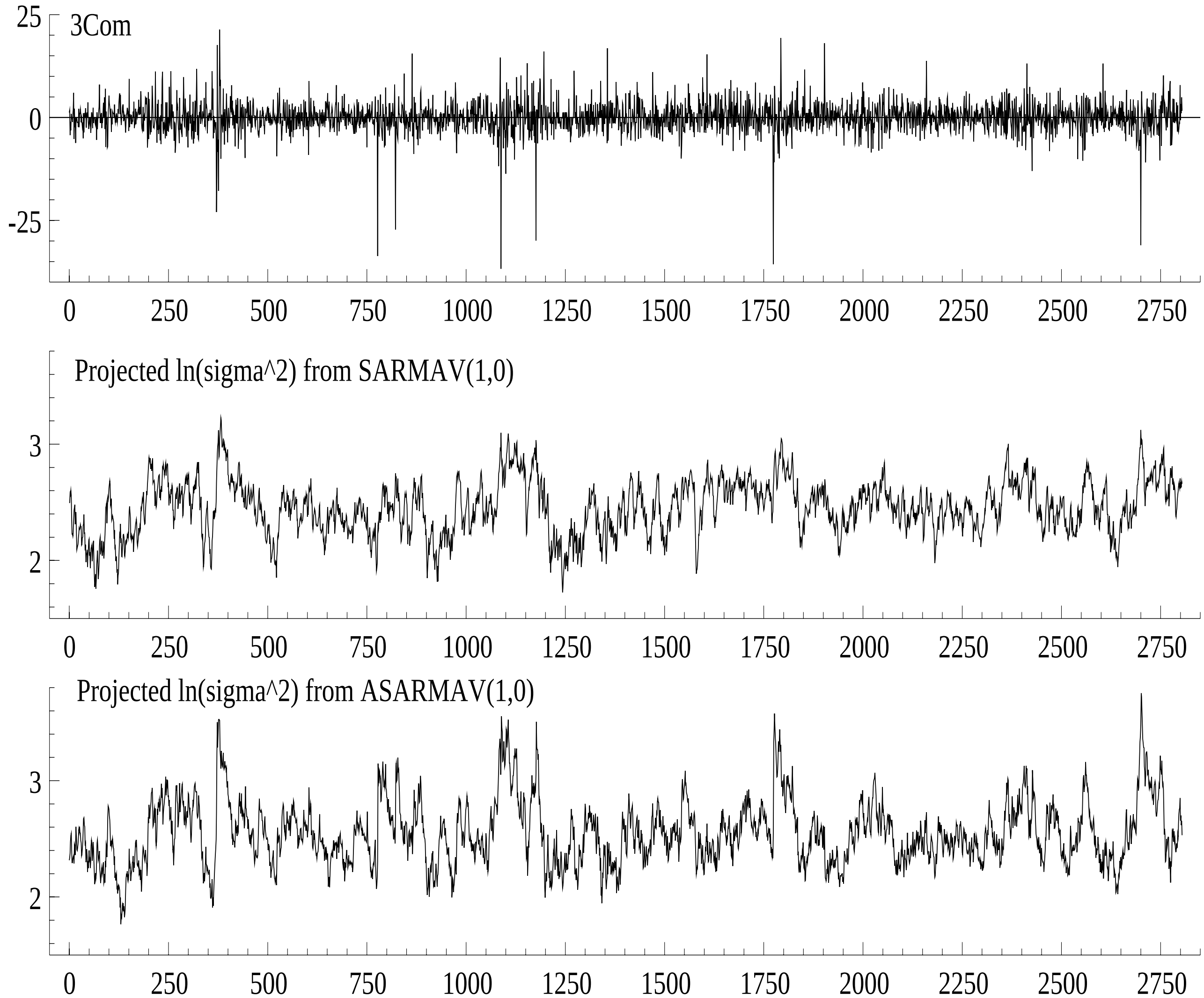

Figure 10: Filtered stock returns volatility for the SARMAV(1,0) and ASARMAV(1,0) models using reprojection

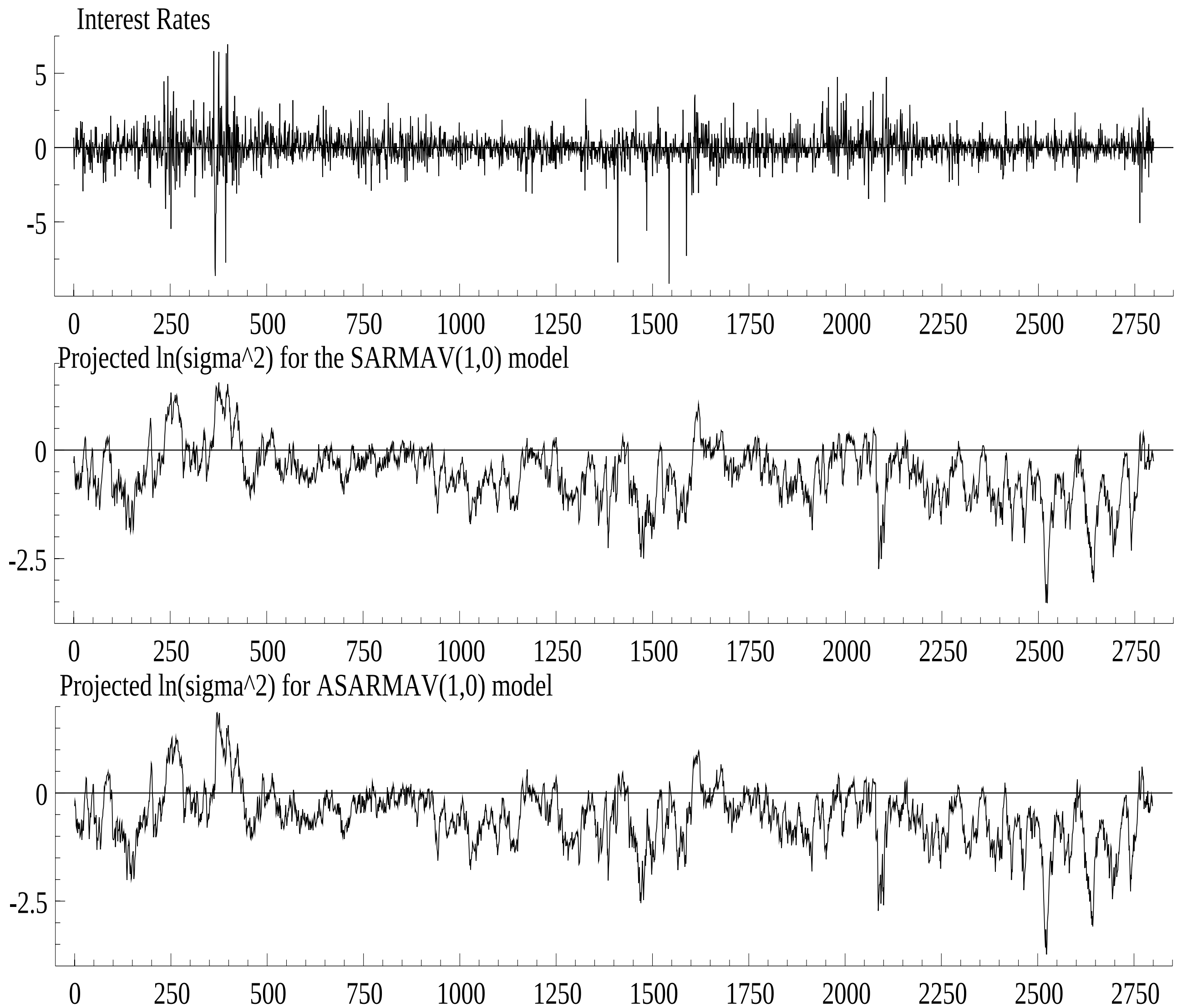

Figure 11: Filtered interest returns volatility for the SARMAV(1,0) and ASARMAV(1,0) models using reprojection

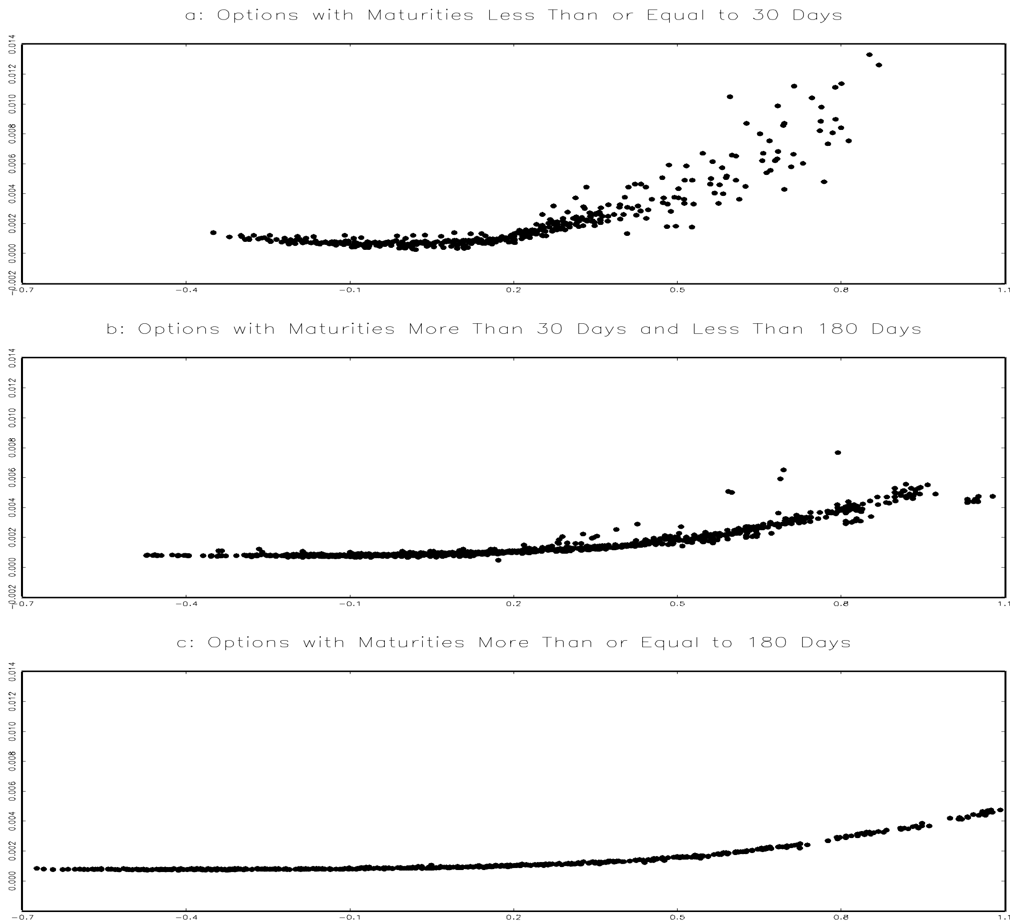

Figure 12: Implied Black-Scholes Volatility from Observed Option Prices

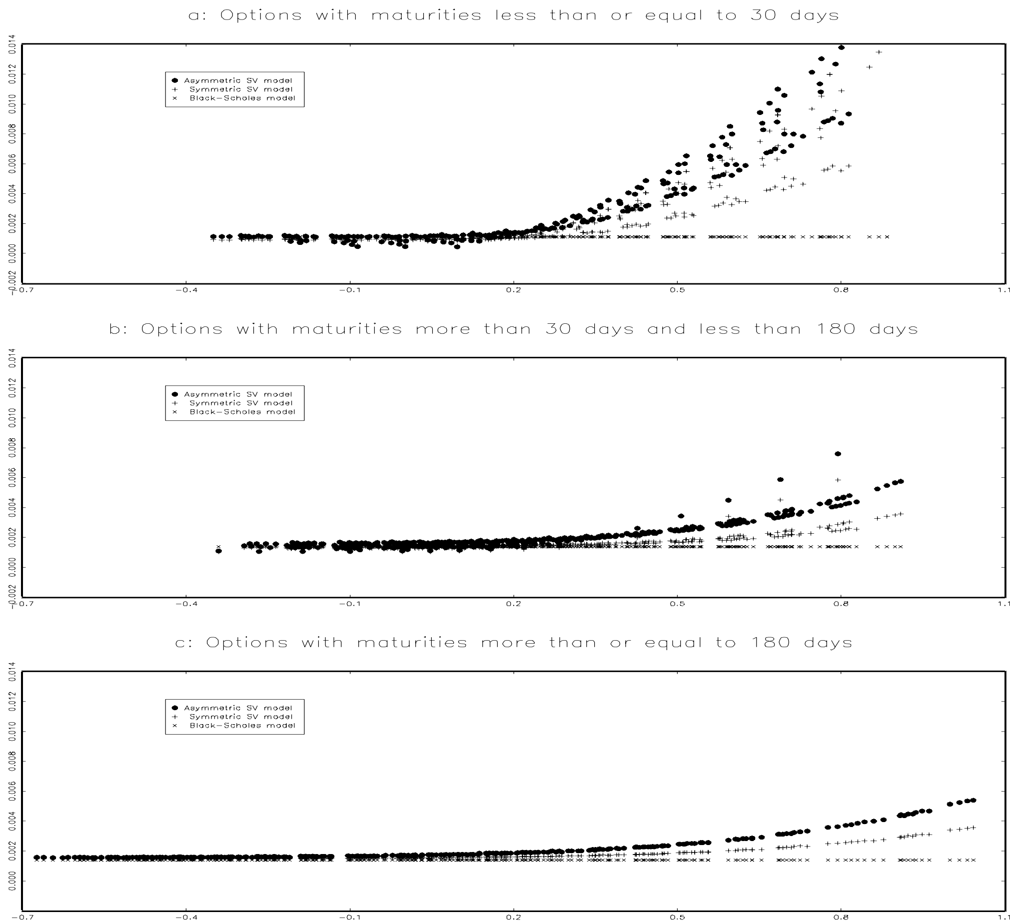

Figure 13: Implied Black-Scholes Volatility from Model Predicted Option Prices based on Underlying Volatility

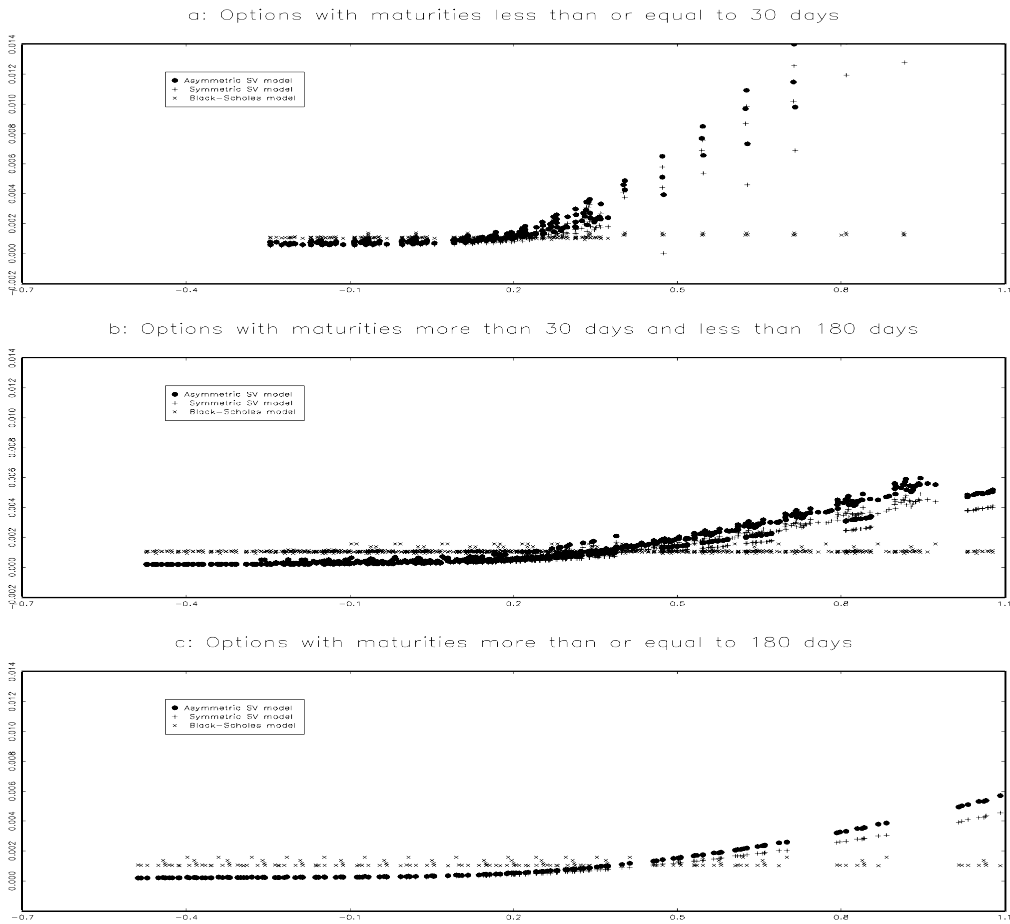

Figure 14: Implied Black-Scholes Volatility from Model Predicted Option Prices based on Implied Volatility