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Predicting the Term Structure of Interest Rates: Incorporating Parameter Uncertainty, Model Uncertainty and Macroeconomic Information

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Predicting the Term Structure of Interest Rates*

Incorporating parameter uncertainty, model uncertainty
and macroeconomic information

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Abstract

We forecast the term structure of U.S. Treasury zero-coupon bond yields by analyzing a range of models that have been used in the literature. We assess the relevance of parameter uncertainty by examining the added value of using Bayesian inference compared to frequentist estimation techniques, and model uncertainty by combining forecasts from individual models. Following current literature we also investigate the benefits of incorporating macroeconomic information in yield curve models. Our results show that adding macroeconomic factors is very beneficial for improving the out-of-sample forecasting performance of individual models. Despite this, the predictive accuracy of models varies over time considerably, irrespective of using the Bayesian or frequentist approach. We show that mitigating model uncertainty by combining forecasts leads to substantial gains in forecasting performance, especially when applying Bayesian model averaging.

Key words: Term structure of interest rates, Nelson-Siegel model, Affine term structure model, forecast combination, Bayesian analysis

JEL Classification Code: C5, C11, C32, E43, E47, F47

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1 Introduction

Modelling and forecasting the term structure of interest rates is by no means an easy endeavor. As long yields are risk-adjusted averages of expected future short rates, yields of different maturities are intimately related and will therefore tend to move together, in the cross-section as well as over time. Long and short maturities are known to react quite differently, however, to shocks hitting the economy. Furthermore, monetary policy authorities such as the US Federal Reserve are actively targeting the short end of the term structure to help promote their national economic goals. Many forces are at work at moving interest rates and identifying these forces and understanding their impact is of crucial importance.

During the last decades significant progress has been made in modelling the term structure, which has come about mainly through the development of no-arbitrage factor models. The literature on these so-called affine models was kick-started by seminal papers of Vasicek (1977) and Cox *et al.* (1985), characterized by Duffie and Kan (1996) and classified by Dai and Singleton (2000)¹. Affine models identify a small number of latent factors that can be extracted from the panel of yields for different maturities and impose cross-equation restrictions that rule out arbitrage opportunities. Affine models, provided they are properly specified, have been shown to accurately fit the term structure, see for example Dai and Singleton (2000). The models are silent, however, about the links between the latent factors and macroeconomic forces.

The current term structure literature is actively progressing to resolve this missing link. A number of recent studies has yielded interesting approaches for studying the joint behavior of interest rates and macroeconomic variables. One approach that has been undertaken is to extend existing term structure models by adding in macroeconomic factors and to study the interactions between latent and observed factors. A key contribution to this strand of the literature is Ang and Piazzesi (2003) who were the first to extend a standard three-factor affine model with macroeconomic variables. Studies such as Bikhov and Chernov (2005), Kim and Wright (2005), Ang *et al.* (2006a), Dai and Philippon (2006) and DeWachter and Lyrio (2006) also include various macroeconomic variables and study their explanatory power for yield movements. Studies that take a more structural approach are, amongst others, Rudebusch and Wu (2003), Wu (2005) and Hordahl *et al.* (2006) who all combine a model for the macroeconomy with an arbitrage-free specification for the term structure. Moving away from the realm of no-arbitrage interest rate models to that of more ad-hoc models, in particular the Nelson and Siegel (1987) model, studies such as Diebold *et al.* (2006) and Mönch (2006b) have also shown that adding information

¹An excellent survey of issues involving the specification and estimation of affine models set in continuous time is Piazzesi (2003), whereas discrete models are discussed in Backus *et al.* (1998).

that reflects the state of the economy is beneficial².

Whereas modelling interest rates movements over time is already a strenuous task, accurately *forecasting* future rates is an equally difficult challenge. Yields of all maturities are close to being non-stationary, which makes it hard for any model to predict yields better than the simple random walk-based no-change forecast. Several studies have documented that beating the random walk forecast is indeed difficult, in particular for unrestricted yields-only based VAR and standard affine models, see Duffee (2002) and Ang and Piazzesi (2003). However, all does not seem lost as recently evidence for the predictability of yields has been reported. Whereas Duffee (2002) shows that more flexible affine specifications³ can beat the random walk, Krippner (2005) and Diebold and Li (2006) show that a dynamic Nelson-Siegel factor model forecasts particularly well. Results are even more promising with models that incorporate macroeconomic information. Ang and Piazzesi (2003) and Mönch (2006a) report improved forecasts for U.S. zero-coupon yields at various horizons using affine models augmented with principal component-extracted macrofactors. Hordahl *et al.* (2006) report similar results for German zero-coupon yields.

In spite of the powerful advances in term structure modelling and forecasting, the latter being the main topic studied here, we feel that a number of issues regarding estimation and forecasting have so far been left nearly unaddressed. This paper tries to fill in some of these gaps by investigating the relevance of parameter uncertainty and, in particular, model uncertainty. Especially for VAR and affine models, which are highly parameterized if we attempt to model the whole of the term structure, parameter uncertainty is likely to be substantial and needs to be accounted for. Regarding model uncertainty, when looking at the historical time series of (U.S.) interest rates we can easily identify subperiods across which yield dynamics are quite different. Likely reasons are for example the reigns of different Fed Chairmen, most notably that of Paul Volcker, or the strong decline in interest rate levels accompanied by a pronounced widening of spreads in the early 90's and after the burst of the Internet bubble. It will be unlikely that any individual model is capable of consistently producing accurate forecasts in each of these subperiods. As we demonstrate below, the forecasting performance of the models we consider in this study does indeed vary substantially across subperiods. In these situations, combining forecasts yields diversification gains and can therefore be an attractive alternative to relying on forecasts from a single model. Moreover, even if it would be possible to identify the

²Macrovariables mainly seem to help in capturing the dynamics of short rates. Modelling long-term bonds remains, however. Dai and Philippon (2006) show that fiscal policy can account for some of the unexplained long rate dynamics whereas DeWachter and Lyrio (2006) show that long-run inflation expectations are important for modelling long-term bond yields.

³Duffee (2002) denotes his preferred class of models “essentially affine” by allowing risk premia to depend on the entire state vector instead of being a multiple of volatility which is the assumption in standard affine models. Ang and Piazzesi (2003) remark that the essentially affine risk premia are not linear in the state vector and that using linear risk premia results in better forecasts.

best individual model, creating forecast combinations can be useful to scale down the magnitude of forecast errors.

In addition to these two focal points, we also further examine the use of macroeconomic diffusion indices in term structure models. Mönch (2006a,b) documents that using factors, which have been extracted from a large *panel* of macroseries instead of *individual* series works well, in both affine models and the Nelson-Siegel model. We complete the picture by also examining the use of diffusion indices in simpler AR and VAR models. Summarizing, the aim of this paper is threefold and consists of examining (i) parameter uncertainty, (ii) model uncertainty and (iii) the use of macro diffusion indices.

We analyze these objectives in the following manner. Using a relatively long time-series of U.S. zero-coupon bond yields, we examine the forecasting performance of a range of models that have been proposed in the literature to predict the term structure. We estimate each model and generate forecasts by applying frequentist maximum likelihood techniques as well as Bayesian techniques to gauge the effects of explicitly taking into account parameter uncertainty. Furthermore, we analyze each model both with and without macrofactors to assess the benefits of adding macroeconomic information. Finally, after showing the instability of the forecasting performance of the different models through subsample analysis, we consider several forecast combination approaches.

Our results can be summarized as follows. Using an out-of-sample period of 1994-2003 we show that the predictive ability of individual models varies considerably over time with a prime example being the Nelson and Siegel (1987) model which predicts interest rates accurately in the 90s but rather poorly in the early 2000s. We find that models which incorporate macroeconomic variables seem more accurate in subperiods during which the uncertainty about the future path of interest rates is substantial. This is especially the case for the early 2000s with the pronounced drop in interest rates and the widening of spreads. Models without macro information do particularly well in subperiods where the term structure has a more stable pattern such as in the early 90s with such models outperforming the random walk RMSPE by sometimes well over 30%.

That different models forecast well in different subperiods confirms ex-post that different model specifications play a complementary role in approximating the data generating process, which provides a strong claim for using forecast combination techniques as opposed to believing in a single model. Our forecast combination results confirm this conjecture. We show that combined forecasts, in particular when using Bayesian model averaging techniques, consistently outperform the random walk benchmark across individual models, maturities and subperiods.

The remainder of the paper is organized as follows. In Section 2 we discuss the set of U.S Treasury yields that we model and forecast, and we provide details about the panel of macro series that we employ to construct our macrofactors. We devote Section 3 to

present the different models we use to produce forecasts. In Section 4 we discuss results of the individual models across maturities and forecast horizons whereas in Section 5 we outline and discuss results of several forecasting combination approaches. Finally, in Section 6 we conclude.

2 Data

2.1 Yield Data

The term structure data we use in this study consists of end-of-month continuously compounded yields on U.S. zero-coupon bonds. These have been constructed from average bid-ask price quotes on U.S. Treasuries from the CRSP government bond files. CRSP filters the available quotes by taking out illiquid bonds and bonds with option features. The remaining quotes are used to construct forward rates using the Fama and Bliss (1987) bootstrap method as outlined in Bliss (1997). The forward rates are averaged to construct constant maturity spot rates⁴. Similar to Diebold and Li (2006) and Mönch (2006a), our dataset consists of unsmoothed Fama-Bliss yields. These unsmoothed yields exactly price the included US Treasury securities. Smoothed yields on the other hand, which can be obtained by fitting a Nelson-Siegel curve on the unsmoothed yields (see Bliss, 1997 for details), do not have this property, and, moreover, using these may give the Nelson-Siegel model an unfair advantage over the other models in terms of fitting and forecasting the term structure.

Throughout our analysis we use $N = 13$ maturities: 1, 3 and 6 months and 1, 2,..., 10 years. To estimate the Nelson-Siegel models we follow Diebold and Li (2006) and Diebold *et al.* (2006) by including additional maturities of 9, 15, 18, 21 and 30 months to increase the number of observations at the short end of the curve. Our sample period covers January 1970 till December 2003 for a total of 408 monthly observations. Similar to Duffee (2002) and Ang and Piazzesi (2003) we include data from well before the Volcker period, despite the reservations expressed in Rudebusch and Wu (2003) that the pricing of interest rate risk and the relationship between yields and macroeconomic variables are likely to have changed during such a long time span. We do so for two main reasons: (i) to have enough observations to sufficiently accurately identify the parameters of the models we consider some of which are highly parameterized, and (ii) to assess the forecasting performance of each model over (sub-)periods with strikingly different characteristics.

Figure 1 shows time-series plots for a subsample of the included yields whereas Table 1 reports summary statistics. The stylized facts common to yield curve data are clearly

⁴We kindly thank Robert Bliss for providing us with the unsmoothed Fama-Bliss forward rates and the programs to construct the spot rates.

present: the sample average curve is upward sloping and concave, volatility is decreasing with maturity, autocorrelations are very high and increasing with maturity and the null of normality for the full sample is rejected due to positive skewness and excess kurtosis. Correlations between yields of different maturities are high, especially for yields with maturities that are close to each other. The lowest correlation of 86% is that between the 1-month and 10-year maturities.

2.2 Macroeconomic Data

Our macro dataset originates from Stock and Watson (2005) and consists of 116 series⁵. The macrovariables are classified in the following 15 categories: (1) output and income, (2) employment and hours, (3) retail, (4) manufacturing and trade sales, (5) consumption, (6) housing starts and sales, (7) inventories, (8) orders, (9) stock prices, (10) exchange rates, (11) federal funds rate, (12) money and credit quantity aggregates, (13) price indexes, (14) average hourly earnings and (15) miscellaneous. Table 2 lists the series included in the macro dataset and their designated category.

We transform the monthly recorded macroseries, whenever appropriate, to ensure stationarity by using log levels, annual differences or annual log differences. Column 2 of Table 2 lists the applied transformation. We follow Ang and Piazzesi (2003), Mönch (2006a) and Diebold *et al.* (2006) in our use of annual growth rates. Monthly growth rates series turn out to be very noisy and are therefore expected to add little information when included in the various term structure models. Outliers in each individual series are replaced by the median value of the previous five observations, see Stock and Watson (2005) for details.

We need to be careful about the timing of the macroseries relative to the interest rate series to prevent the use of information that has not been released yet at the time when a forecast is being made. The interest rates we use are recorded at the end of the month. Although macro figures tend to be released at the beginning or in the middle of the month, they are usually released with a lag of one to sometimes several months. We accommodate for a potential look-ahead bias⁶ by lagging all macroseries by one month, except for S&P variables, exchange rates and the federal funds rate which are all monthly averages.

We extract a (small) number of factors from our dataset, similar to Mönch (2006a)

⁵We exclude all interest and spread related series (16 series in total), except for the federal funds rate, from the original 132 series in the panel dataset. The federal funds rate closely follows the federal fund target rate, which is the key monetary policy instrument for the US Federal Reserve, and should therefore be important for capturing the movements of especially the short end of the term structure.

⁶Note that Ang and Piazzesi (2003) and Mönch (2006a) use *contemporaneous* macro information to construct their term structure forecasts which may lead to the added value of including macro economic series being misperceived as too beneficial.

who, based on the work of Bernanke *et al.* (2005) builds a no-arbitrage Factor-Augmented VAR by four common factors from a large panel of macroeconomic variables. To this end we apply static principal component analysis, see Stock and Watson (2002a,b), to the full panel of macroseries, standardized to have zero mean, a variance of one and an Euclidean length of one. The use of common factors instead of individual macroseries allows us to incorporate information beyond that contained in commonly used variables such as CPI, PPI, employment, output gap or manufacturing capacity utilization, while at the same time ensuring that the number of model parameters remains manageable.

For the full sample period, the first common factor explains 35% of the variation in the macro panel. The second and third factors explain an additional 19% and 8%, whereas the first 10 factors together explain an impressive 85%. Figure 2 shows the R^2 when regressing each individual macroseries on each of three separate factors, which allows us to attach economic labels to the first three factors. The first factor closely resembles the series in the real output and employment categories (categories 1 and 2) and can therefore be labelled *business cycle* or *real activity* factor. The second factor loads mostly on inflation measures (category 13) which allows for the designation *inflation factor*. The third factor, although the correlations are much lower than for factors one and two, is mostly related to money stock and reserves (category 12) and could thus be labelled a *monetary aggregates* or *money stock* factor. Figure 3 corroborates these interpretations graphically through timeseries plots of the three macro factors with Industrial Production (total), Consumer Price Index (all items) and Money Stock (M1) respectively.

Since the first three factors explain over 60% of the variation in the macro panel, we have chosen to include these three factors as additional explanatory variables in the term structure models. In Section 3 we explain in detail how we model the macroeconomic factors.

3 Models

We assess the individual and combined forecasting performance of a range of models that are commonly used in the literature and in practice to forecast yields. Since previous studies have shown that more parsimonious models often outperform sophisticated models we consider models with different levels of complexity in our analysis. Our models range from unrestricted linear specifications for yield levels (AR and VAR models), models that impose a specific structure on factor loadings (the Nelson-Siegel class of models) to models that impose cross-sectional restrictions to rule out arbitrage opportunities (affine models). In this section we specify and discuss the different models. We defer all specific details regarding the frequentist and Bayesian techniques that we employ for drawing inference on model parameters and for generating (multi-step ahead) forecasts to the appendix.

3.1 Adding macrofactors

The approach we use to incorporate the first three macrofactors is the following. Denote M_t as the 3×1 vector containing the values of the macrofactors at time t , which have been extracted from the full panel of macroseries. We then add the factors to each of the term structure models contemporaneously⁷ as well as lagged by one month to capture any delayed effects of macroeconomic news on the term structure. The exogenous explanatory macro information that is added to the models, denoted by X_t , is therefore given by $X_t = (M_t' M_{t-1}')'$ which is a 6×1 vector.

Our approach implies that when we forecast yields, we also need to model and forecast the macrofactors. We tackle this issue by following Ang and Piazzesi (2003) in only allowing for a *unidirectional* link from macrovariables to yields. Although this can be argued to be a restrictive assumption as it does not allow for a potentially rich *bidirectional* feedback⁸, it enables us to model the time-series behavior of the macrofactors separately, which considerably facilitates estimation. In particular, information criteria suggest to model and forecast M_t separately using a VAR(3) model:

$$M_t = c + \Phi_1 M_{t-1} + \Phi_2 M_{t-2} + \Phi_3 M_{t-3} + H \xi_t, \quad \varepsilon_t \sim \mathcal{N}(0, I) \quad (1)$$

where c is a 3×1 vector, Φ_i for $i = 1, \dots, 3$ is a 3×3 matrix and H a 3×3 lower triangular Cholesky matrix. We estimate the VAR using both frequentist and Bayesian techniques as we also use both types of inference for the term structure models.

3.2 Models

Random walk

The first model that we consider is a random walk for maturity τ_i , $i = 1, \dots, N$,

$$y_t^{(\tau_i)} = y_{t-1}^{(\tau_i)} + \sigma^{(\tau_i)} \varepsilon_t^{(\tau_i)}, \quad \varepsilon_t^{(\tau_i)} \sim \mathcal{N}(0, 1) \quad (2)$$

In this model any h -step ahead forecast $\hat{y}_{T+h}^{(\tau_i)}$ is equal to the most recently observed value $y_T^{(\tau_i)}$. It is natural to qualify this no-change model as the benchmark against which to judge the predictive power of all other models. Duffee (2002), Ang and Piazzesi (2003), Mönch

⁷Contemporaneous in the sense of same-month values for stock prices, exchange rates and the federal funds but one-month lagged values for the remaining macro series.

⁸In a forecasting exercise using German zero-coupon yields, Hordahl *et al.* (2006) show that term-structure information helps little in forecasting macro-economic variables (more specifically (i) inflation and (ii) the output gap) which is a justification for forecasting macro-variables outside the term structure models. The authors note, however, that this might be due to the fact that their proposed macroeconomic model has an imperfect ability to describe the joint dynamics of German macro economic variables. Diebold *et al.* (2006) and Ang *et al.* (2006a) allow for bi-directional effects between macro and latent yield factors but both studies find that the causality from macro to yields is much higher than from yields to macro.

(2006a) and Diebold and Li (2006) all show, using different models and different forecast periods, that beating the random walk is quite an arduous task. Table 1 confirms that yields are potentially non-stationary by means of the reported first order autocorrelation coefficients which are all very close to unity.

AR model

Although unreported results show that the null of a unit root for yield levels cannot be rejected statistically, the assumption of a random walk is difficult to interpret from an economic point of view. The random walk assumption implies that interest rates can roam around freely and do not revert back to a long-term mean, something which contradicts the Federal Reserve’s monetary policy targets. The second model that we therefore consider is a first-order univariate autoregressive model that allows for mean-reversion

$$y_t^{(\tau_i)} = c^{(\tau_i)} + \phi^{(\tau_i)} y_{t-1}^{(\tau_i)} + \psi^{(\tau_i)'} X_t + \sigma^{(\tau_i)} \varepsilon_t^{(\tau_i)}, \quad \varepsilon_t^{(\tau_i)} \sim \mathcal{N}(0, 1) \quad (3)$$

where $c^{(\tau_i)}$, $\phi^{(\tau_i)}$ and $\sigma^{(\tau_i)}$ are scalar parameters and $\psi^{(\tau_i)}$ is a 6×1 vector containing the coefficients on the macrofactors. We construct forecasts both with and without by setting $\psi^{(\tau_i)} = 0$. We denote the yield-only model by **AR** whereas the model with macrofactors is denoted by **AR-X**. For this and all other models we construct *iterated*⁹ forecasts.

VAR model

Vector autoregressive (VAR) models create the possibility to use the history of other maturities on top of any maturity’s own history as additional explanatory information. We use the following first-order VAR specification¹⁰,

$$Y_t = c + \Phi Y_{t-1} + \Psi X_t + H \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, I) \quad (4)$$

where Y_t contains the yield observations for all 13 maturities; $Y_t = [y_t^{(1m)}, \dots, y_t^{(10y)}]'$, c is a 13×1 vector, Φ a 13×13 matrix, Ψ is a 13×6 matrix and H is the lower triangular Cholesky decomposition of the (unrestricted) residual variance matrix $S = HH'$ containing $\frac{1}{2}N(N-1) = 78$ free parameters. As noted in the introduction, our approach is similar in spirit as

⁹Another approach would be to construct *direct* forecasts by regressing $y_t^{(\tau_i)}$ directly on its h -month lagged value $y_{t-h}^{(\tau_i)}$ as in Diebold and Li (2006). For the State Space form of Nelson-Siegel model and the affine model, such an approach is, however, infeasible. Therefore, and for matters of consistency, we choose to construct iterated forecasts for all the models. Whether using direct forecasts is better than iterated forecasts is a matter of ongoing debate, see the discussion in e.g. Marcellino *et al.* (2004).

¹⁰For both the AR and VAR models we examined the benefits of including more lags by using AR(p) and VAR(p) models with $p = 2, \dots, 12$. We found that using multiple lags resulted in nearly identical forecasts compared to the AR(1) and VAR(1) models and these results are therefore not reported nor were they included in the forecasting combination procedures in Section 4 and 5.

the VARs used in Evans and Marshall (1998, 2001) and also Ang and Piazzesi (2003) in the sense that we impose exogeneity of macroeconomic variables with respect to yields.

A well-known drawback of using unrestricted VAR model for yields is that forecasts can only be constructed for maturities that are used in the estimation of the model. As we want to construct forecasts for 13 maturities, this results in a considerable number of parameters that need to be estimated. As an attempt to mitigate estimation error, and subsequently, to reduce the forecasting error variance, we summarize the information contained in the explanatory vector Y_{t-1} by replacing it with a small number of common factors that drive yield curve dynamics. Similar to Litterman and Scheinkman (1991) and many other studies, we find that the first 3 principal components explain almost all the variation in yields (over 99%). We replace Y_{t-1} in (4) accordingly with the 13×3 factor matrix F_{t-1} ¹¹ giving,

$$Y_t = c + \Phi F_{t-1} + \Psi X_t + H\varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, I) \quad (5)$$

where Φ is now a 13×3 matrix. The VAR model without and with macroeconomic variables is denoted by **VAR** and **VAR-X** respectively.

Nelson-Siegel model

Diebold and Li (2006) show that a reinterpretation of the in essence one-period Nelson and Siegel (1987) functional form as a dynamic factor model produces interest rates forecasts which are highly accurate. The Nelson-Siegel, compared to the unrestricted VAR model (5), imposes structure on the factor loadings Φ by specifying these as exponential functions governed by a single parameter. Following Diebold *et al.* (2006) the State-Space representation of the three-factor model, with a first-order autoregressive representation for the dynamics of the state vector, is given by

$$\begin{aligned} y_t^{(\tau_i)} &= \beta_{1,t} + \beta_{2,t} \left(\frac{1 - \exp(-\tau_i/\lambda)}{\tau_i/\lambda} \right) + \beta_{3,t} \left(\frac{1 - \exp(-\tau_i/\lambda)}{\tau_i/\lambda} - \exp(-\tau_i/\lambda) \right) + \varepsilon_t^{(\tau_i)} \quad (6) \\ \beta_t &= a + \Gamma \beta_{t-1} + u_t \quad (7) \end{aligned}$$

where β_t is the state vector of latent factors at time t , $\beta_t = (\beta_{1,t}, \beta_{2,t}, \beta_{3,t})'$, λ the parameter that governs the exponential decay towards zero of the factor loadings on $\beta_{2,t}$ and $\beta_{3,t}$ (see Diebold and Li, 2006 for details), a a 3×1 vector of parameters and Γ a 3×3 matrix of parameters. The measurement equation errors in (6) and the state equation errors in (7) are assumed to be normally distributed and orthogonal to each other

$$\begin{bmatrix} \varepsilon_t \\ u_t \end{bmatrix} \sim \mathcal{N} \left(\begin{bmatrix} 0_{18 \times 1} \\ 0_{3 \times 1} \end{bmatrix}, \begin{bmatrix} H & 0 \\ 0 & Q \end{bmatrix} \right) \quad (8)$$

¹¹The time subscript ' $t-1$ ' indicates that we extract the common factors using the history of yields up until $t-1$, the vector of observations for time t is not used.

where H is a diagonal 18×18 matrix and Q a full 3×3 matrix.

We follow Diebold and Li (2006) by adding five maturities (9, 15, 18, 21 and 30 months) to the short end of the yield curve to estimate the Nelson-Siegel model in (6)-(8). We use two different estimation procedures: a *two-step* approach and a *one-step* approach.

The *two-step* approach is discussed in Diebold and Li (2006) and involves fixing λ and estimating the factors β_t in a first step using the cross-section of yields for each month t . Given the estimated time-series for the factors from the first step, the second step consists of modelling the factors in (7) by fitting either separate AR(1) models, thereby assuming that both Γ and Q are diagonal, or a single VAR(1) model. We denote these approaches by **NS2-AR** and **NS2-VAR** respectively.

The *one-step* approach follows from Diebold *et al.* (2006) and involves jointly estimating (6)-(8) as a State-Space model using the Kalman filter. In this approach we assume that Γ and Q are both full matrices and that λ now needs to be estimated alongside the other parameters. We denote the 1-step model by **NS1**.

With the frequentist approach we apply both the 2-step and 1-step estimation procedure whereas with Bayesian analysis we consider only the 1-step procedure.

Diebold *et al.* (2006) show how to extend the Nelson-Siegel model to incorporate macroeconomic variables by adding these as observable factors to the state vector and writing the model in companion form as:

$$\begin{aligned} y_t^{(\tau_i)} &= \beta_{1,t} + \beta_{2,t} \left(\frac{1 - \exp(-\tau_i/\lambda)}{\tau_i/\lambda} \right) + \beta_{3,t} \left(\frac{1 - \exp(-\tau_i/\lambda)}{\tau_i/\lambda} - \exp(-\tau_i/\lambda) \right) + \varepsilon_t^{(\tau_i)}(9) \\ f_t &= a + \Gamma f_{t-1} + \eta_t \end{aligned} \quad (10)$$

$$\begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix} \sim \mathcal{N} \left(\begin{bmatrix} 0_{18 \times 1} \\ 0_{12 \times 1} \end{bmatrix}, \begin{bmatrix} H & 0 \\ 0 & Q \end{bmatrix} \right) \quad (11)$$

where now $f_t = (\beta_{1,t}, \beta_{2,t}, \beta_{3,t}, M_t, M_{t-1}, M_{t-2})$ with the dimensions of a , Γ and Q increased as appropriate¹² and where η_t is given by $\eta_t = (u'_t, \xi'_t, 0, \dots, 0)'$. The companion form enables us to incorporate the VAR(3) specification for the macrofactors. We impose structure on Γ and Q to accommodate for the effects of macrofactors while maintaining the unidirectional causality from macrofactors to yields. In particular, the lower left (9×3) part of Γ consists of zeros whereas Q is block diagonal with a non-zero (3×3) block Q_1 for the yield factors and a non-zero (3×3) block Q_2 for the macrofactors. All other blocks on the diagonal contain only zeros. The Nelson-Siegel model with macrofactors can again be estimated using either a 2-step approach with AR or VAR dynamics for the yield factors, denoted by **NS2-AR-X** and **NS2-VAR-X**, or a 1-step approach, denoted by **NS1-X**.

¹²Note that the macrofactors are prevented from entering the yield equations directly by only allowing the factor loadings of β_t to be non-zero. Diebold *et al.* (2006) impose this restriction to maintain the assumption that three factors are sufficient for describing interest rate behavior. Relaxing this restriction would result in a substantial number of additional parameters.

Affine model

Models that impose no-arbitrage restrictions have been examined for their forecast accuracy in for example Duffee (2002), Ang and Piazzesi (2003) and Mönch (2006a). The attractive property of the class of no-arbitrage models is that sound theoretical cross-sectional restrictions are imposed on factor loadings to rule out arbitrage opportunities. In this study we consider a Gaussian-type discrete time affine no-arbitrage model using the set-up from Ang and Piazzesi (2003).

In particular, we assume that the vector of K underlying latent factors, or state variables, Z_t , which are assumed to drive movements in the yield curve, follow a Gaussian VAR(1) process

$$Z_t = \mu + \Psi Z_{t-1} + u_t \quad (12)$$

where $u_t \sim \mathcal{N}(0, \Sigma \Sigma')$ with Σ being a lower triangular Choleski matrix, μ a $K \times 1$ vector and Ψ a $K \times K$ matrix. The short interest rate is assumed to be an affine function of the factors

$$r_t = \delta_0 + \delta_1' Z_t \quad (13)$$

where δ_0 is a scalar and δ_1 a $K \times 1$ vector. Furthermore, we adopt a standard form for the pricing kernel, which is assumed to price all assets in the economy,

$$m_{t+1} = \exp\left(-r_t - \frac{1}{2} \lambda_t' \lambda_t - \lambda_t' u_{t+1}\right)$$

and we specify market prices of risk to be time-varying and affine in the state variables

$$\lambda_t = \lambda_0 + \lambda_1 Z_t \quad (14)$$

with λ_0 a $K \times 1$ vector and λ_1 a $K \times K$ matrix¹³. Under the assumption that bond prices are an exponentially affine function of the state variables,

$$P_t^{(\tau)} = \exp[A^{(\tau)} + B^{(\tau)'} Z_t] \quad (15)$$

we can recursively estimate the price of a τ -period bond using

$$P_t^{(\tau)} = \mathbb{E}_t[m_{t+1} P_{t+1}^{(\tau-1)}] \quad (16)$$

where the expectation is taken under the risk-neutral measure. Ang and Piazzesi (2003) show that doing so results in the following recursive formulas for the bond pricing coefficients:

$$A^{(\tau+1)} = A^{(\tau)} + B^{(\tau)'} [\mu - \Sigma \lambda_0] + \frac{1}{2} B^{(\tau)'} \Sigma \Sigma' B^{(\tau)} - \delta_0 \quad (17)$$

¹³With λ_1 equal to 0, risk premia are constant over time and if also λ_0 equals zero then risk premia are non-existent altogether.

$$B^{(\tau+1)'} = B^{(\tau)'}[\Psi - \Sigma\lambda_1] - \delta_1' \quad (18)$$

starting from $A^{(0)} = 0$ and $B^{(0)} = 0$. If bond *prices* are exponentially affine in the state variables then *yields* are affine in the state variables since $P_t^{(\tau)} = \exp[-y_t^{(\tau)}\tau]$. Consequently, it follows that $y_t^{(\tau)} = a^{(\tau)} + b^{(\tau)'}Z_t$ with $a^{(\tau)} = -A^{(\tau)}/\tau$ and $b^{(\tau)} = -B^{(\tau)}/\tau$. To estimate the model we deviate from the popular Chen and Scott (1993) approach and assume that every yield is contaminated with measurement error.

Summarizing, we specify the following affine model

$$y_t^{(\tau_i)} = a^{(\tau_i)} + b^{(\tau_i)'}Z_t + \varepsilon_t^{(\tau_i)} \quad (19)$$

$$Z_t = \mu + \Psi Z_{t-1} + u_t \quad (20)$$

$$\begin{bmatrix} \varepsilon_t \\ u_t \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0_{13 \times 1} \\ 0_{3 \times 1} \end{bmatrix}, \begin{bmatrix} H & 0 \\ 0 & Q \end{bmatrix}\right) \quad (21)$$

with $Q = \Sigma\Sigma'$ and $a^{(\tau_i)}$ and $b^{(\tau_i)}$ being recursive functions of the parameters that govern the dynamics of the state variables and of the risk premia parameters. We denote this model by **ATSM**.

Extending the model to include observable macroeconomic factors can be done in a similar way as for the Nelson-Siegel model

$$y_t^{(\tau_i)} = a^{(\tau_i)} + b^{(\tau_i)'}f_t + \varepsilon_t^{(\tau_i)} \quad (22)$$

$$f_t = \mu + \Psi f_{t-1} + \eta_t \quad (23)$$

$$\begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0_{13 \times 1} \\ 0_{12 \times 1} \end{bmatrix}, \begin{bmatrix} H & 0 \\ 0 & Q \end{bmatrix}\right) \quad (24)$$

where $f_t = (Z_t, M_t, M_{t-1}, M_{t-2})$. The dimensions of $a^{(\tau_i)}$, $b^{(\tau_i)}$, μ , Ψ and Q are again increased as appropriate and (23) is written in companion form. As in the Nelson-Siegel model, Q is block diagonal with only two non-zero blocks, Q_1 and Q_2 . The affine model with macroeconomic factors is denoted by **ATSM-X**.

Adding macroeconomic variables to affine models can cause problems as it further increase the number of parameters in these already highly parameterized models¹⁴. To speed up and facilitate the estimation procedure, we therefore use the two-step approach of Ang *et al.* (2006b) by making the latent yield factors observable. Contrary to Ang *et al.* (2006b) who directly use the observed short rate and the term-spread as measures of the level and slope of the yield curve, we use principal component analysis to extract the first three common factors from the full set of yields and use these as our observable state variables.

¹⁴Contrary to the reduced form affine model of Ang and Piazzesi (2003), Hordahl *et al.* (2006) use a structural affine model with macroeconomic variables in which the number of parameters can be kept down. They show that their model leads to better longer horizon forecasts compared to the Ang-Piazzesi model, which indicates that instead of only imposing no-arbitrage restrictions, which is the case in affine models, imposing also *structural* equations seems to mitigate overparameterization problems due to adding macro economic variables.

4 Forecasting

In this section we examine the forecasting performance of each individual model over different (sub)samples. This section more or less serves as a prelude to Section 5 where we investigate forecast combinations.

4.1 Forecast procedure

We divide our dataset in an initial estimation period which covers the sample 1970:1 - 1988:12 (228 observations) and a forecasting period which is comprised of the remaining sample 1989:1 - 2003:12 (180 observations). The forecasting sample is further divided in three 60-month subperiods; 1989:1 - 1993:12, 1994:1 - 1998:12 and 1999:1 - 2003:12. The initial subperiod is primarily used as a training sample to start up the forecast combinations. Consequently, we report forecast results for the sample 1994:1 - 2003:12 (120 observations) and the last two subsamples (60 observations each). The vertical lines in Figure 1 enclose the subperiods.

We recursively estimate all models using an expanding window of all data from 1970:1. We construct point forecasts for four different horizons equal to $h = 1, 3, 6$ and 12 months ahead. As mentioned in the previous section, for horizons beyond $h = 1$ month we compute iterated forecasts when using frequentist techniques whereas for Bayesian inference we compute the mean of each model's predictive density.

4.2 Forecast evaluation

To evaluate out-of-sample forecasts we use a number of different popular error metrics. We compute the Root Mean Squared Prediction Error (RMSPE) and the Mean Prediction Error (MPE) per maturity and forecast horizon. Similar to Hordahl *et al.* (2006) we also summarize the forecasting performance for each model over *all* maturities by computing the Trace Root Mean Squared Prediction Error (TRMSPE), see Christoffersen and Diebold (1998) for details. The final metric we consider¹⁵ is the Hit Rate (HR) which is computed as the percentage of the correctly predicted signs of changes in interest rates. As a benchmark for the HR we use 0.50.

The reason behind using the Hit Rate is that it provides us with a metric to judge forecasts from a more economic point view, which may in particular be of interest to for example bond investors who take directional bets in the bond market. The RMSPE, which is widely used to assess forecast accuracy, takes into account the magnitude of

¹⁵Other forecast performance statistics, in particular the Mean Absolute Prediction Error (MAPE) and the R^2 when regressing observed h -month ahead yields on the corresponding forecasts are not reported but are available upon request.

forecast errors but does not distinguish between positive and negative errors. The Hit Rate does the opposite and it is interesting to see if, and to what extent, the two metrics lead to conflicting conclusions¹⁶.

To test the statistical accuracy of (combined) forecasts of all models relative to our benchmark model, the random walk, we apply, like Hordahl *et al.* (2006) and Mönch (2006a), the White (2000) “reality check” test with the stationary bootstrap approach of Politis and Romano (1994). We carry out the test using 1000 block-bootstraps of the forecast error series with an average block-length of 12 months¹⁷.

4.3 Forecasting results: individual models

Tables 3-6 report out-of-sample results for the sample 1994:1-2003:12 for each of the four selected forecast horizons. Panels A and B of each table contain the results when using the frequentist approach for the models with and without macrofactors. Panels D and E show the results using Bayesian inference. Subsample results are reported in Tables 7-10 for the sample 1994:1-1999:12 and Tables 11-14 for the sample 1999:1-2003:12.

The first row in each table shows the value of the different forecast evaluation metrics for the random walk (reported in basis point errors) whereas all other rows show values relative to the random walk. Relative values for any forecast that are below one are highlighted in bold to indicate that these forecasts are on average more accurate than those of the random walk. Lower relative values for the (T)RMSPE indicate progressively more accurate forecasts relative to the random walk. Stars indicate statistically significant outperformance according to White’s test. A Hit Rate above 0.50 indicates a model’s ability to forecast directional changes in yields.

4.3.1 Full sample results

Sample 1994:1 - 2003:12

The results for the 1-month horizon are not very encouraging. For nearly all maturities the random walk shows better statistics than any of the models based on yields only, even when parameter uncertainty is incorporated. The results are in line, however, with other studies showing that it is very difficult to outperform the RW for short term forecasts. Especially for short horizons the near unit root behavior of yields seems to dominate, such that model-based yield forecasts add little.

¹⁶It would be interesting to evaluate the different forecasting models from a truly economic point of view by gauging the performance of bond portfolios but such an analysis is beyond the scope of this paper and is therefore left for further research.

¹⁷An alternative approach would be to use the Diebold and Mariano (1995) test, either directly as used in Diebold and Li (2006), or with the small sample correction proposed by Harvey *et al.* (1997). However, West (2006) shows that using Diebold-Mariano type tests can be problematic in the case of comparing nested models, which holds true for at least a subset of our models, so we do not apply these tests here.

Incorporating macroeconomic information as an additional source of information improves forecasts for the AR and VAR models. The (T)RMSPE statistics are now very close and often marginally better than those of the RW. The largest improvements are shown for the shortest maturities, in particular the 3-month maturity where the relative RMPSE is now 0.95. From the MPE we see that macroeconomic information helps to reduce the forecast bias. However, the improvements do not appear substantial enough for the AR-X model to produce significantly better forecasts, as judged by the White test. The evidence for more complex model specifications is mixed but, in general, adding macroeconomic information worsens accuracy. Especially when Bayesian inference is used, the forecasts that are produced with the Nelson-Siegel and affine models now suffer from a severe bias. For example, for the 6-month maturity the relative RMSPE increases from 1.10 to 1.71 for the Nelson-Siegel model when adding macrofactors.

The results for the 3-month forecast horizon are very similar to those for the 1-month horizon, although the RMSPE is now higher in absolute terms. The latter is expected since the yield curve is subject to more new information when the forecast horizon lengthens. It still proves to be very difficult for any of the models to provide forecasts that are more accurate than the random walk forecast. The AR-X model is again the only model that shows promising results, which can again be attributed to the macrofactors, and it gives a TRMSPE statistic lower than that of the random walk. The improvement is, however, again not statistically significant. What is striking though is that whereas with the frequentist approach without macrofactors the RMSPE goes up and the MPE even more so for $h = 3$ compared to $h = 1$, with the Bayesian approach the RMSPE actually goes down for some models, in particular the Nelson-Siegel model.

For a 6-month horizon more models start to outperform the random walk for more maturities, as indicated by the increase in relative RMSPEs below 1, although the results are still by no means impressive, and, in general, the best model only improves the random walk by a few basis points. Taking into account macro-economic information as well as parameter uncertainty results in reasonably accurate forecasts although there is still no significant outperformance according to the White reality check. Incorporating parameter uncertainty is very beneficial for the Nelson-Siegel model. Estimating the State-Space form of the model with Bayesian analysis substantially reduces the relative RMSPE compared to their frequentist counterparts. The same conclusion holds for the MPE if macrofactors are not taken into account. With macrofactors, the forecast bias is actually larger for the Bayesian NS-X specification. Models that keep struggling are the VAR and affine models. In both cases this is most likely due to the large number of yields (compared to for example Duffee (2002) and Ang and Piazzesi (2003)) that we use in

estimation, resulting in a large number of parameters¹⁸. Note that the VAR model with Bayesian inference does worse than when estimated using maximum likelihood. This can be explained by realizing that Bayesian analysis entails drawing inference on the variance parameters of each of the 13 maturities in addition to all the other parameters, which is not necessary with maximum likelihood as we are only generating point forecasts.

The longest horizon that we consider is $h = 12$. Two models produce forecasts that consistently outperform the random walk across all separate maturities: the frequentist VAR-X model and the Bayesian NS1-X model. For both models, the TRMSPEs are lower compared to the random walk. RMSPEs are on average five basis points lower, although for the NS1-X the differences are not significant. For all other models, the benefits of adding macrofactors are evident with all relative MSPE going down considerably. Compared to the frequentist results, the Bayesian VAR model still struggles.

It is interesting to compare our results with those of Mönch (2006a) since he uses an almost identical forecasting sample (1994:1 - 2003:9) but a much shorter estimation period for the VAR, NS2-AR and NS2-VAR model (1983:1 - 1993:12). Our results for the RW are all but identical, as they should be, which is a convenient check on our results. The RMSPEs we find for the VAR(1) on yields and a 1-month horizon are somewhat higher for shorter maturities (below 5 years) whereas for longer maturities they are very similar. For a 12-month horizon the differences are larger with Mönch who reports RMPSEs which are roughly 20% lower than ours. The differences will partly be due to using a slightly different set of maturities and our use of yield-factors when estimating the VAR instead of using lagged yields directly. The main reason for the different sets of results will, however, be due to our much longer estimation sample. It seems that including the 1970s and beginning of 1980s leads to poorer yield forecasts compared to those obtained when starting the sample after the Volcker period. For the NS2-AR and NS2-VAR the 1-month ahead results are again very similar. However, whereas Mönch finds that the NS2-AR outperforms the NS2-VAR for a 6- and 12-month horizon we find that NS2-VAR is usually more accurate. Our affine model without macrovariables provides similar results as for the $A_0(3)$ model that Mönch considers for $h = 1$ but less accurate results for $h = 6$ and $h = 12$ ¹⁹. It is interesting to note that none of the models we consider here have an out-of-sample performance which is as good as that of the FAVAR model advocated by Mönch. It would therefore be worthwhile to add this model to the model consideration

¹⁸An obvious solution to this problem would be to estimate the affine models using a smaller set of yields. The reason we do not follow this strategy here is because we want to use a similar number of yields as in Mönch (2006a).

¹⁹However, we forecast the 1-month maturity (not reported in the tables but available upon request) much more accurately which is most likely due to the fact that we estimate the short rate parameters δ_0 and δ_1 using *only* data on the 1-month yield instead of estimating simultaneously with the other model parameters.

set but we leave this for further research.

As we mentioned earlier, it is interesting to see whether the Hit Rate provides different results than the commonly applied RMSPE and MPE. Judging from the results for the Hit Rate in Tables 15 and 16 we can draw a number of conclusions. Adding macrofactors and taking into account parameter uncertainty adds little to improve the Hit Rate across maturities and across forecast horizons. Only for the 1-month horizon macrofactors seem to lead to more correctly predicted yield change signs. For example, the Hit Rate for the 6-month yield using a 1-month horizon, when moving from the AR to the AR-X model, improves from 0.54 to 0.57 with the frequentist approach and from 0.54 to 0.58 with the Bayesian approach.

For a 1-month horizon sign predictability is in general higher for longer maturities although the sign of yield changes for the 10-year maturity seems harder to predict than for the 5-year and 7-year maturities.

For the remaining horizons, sign predictability is better for the short end of the curve and macrofactors do little to improve the Hit Rate. Results for a 3-month horizon seem to be the most promising with Hit Rates that are well above 0.50 and sometimes as high as 0.70, although the Hit Rates for 6-months out are still very reasonable as well. From the point of view of an investor who is attempting to make money using slope investments these results are very encouraging. For a one-year horizon the Hit Rate results are somewhat problematic. The only model that seems to provide directional guidance is the VAR-X model when parameter uncertainty is incorporated. However, also for the 10-year maturity the Hit Rate falls below 0.5.

As an overall summary for the 1994:1-2003:12 period we can remark that our results for the individual models are not very encouraging as interest rate predictability appears to be rather low. This may be attributed to a number of possible causes with one main reason, being the out-of-sample period we select. Except for Mönch (2006a) who reports very promising out-of-sample results for his FAVAR model for nearly the same period as we use, Duffee (2002), Ang and Piazzesi (2003), Diebold and Li (2006) and Hordahl *et al.* (2006) all use an out-of-sample period that ranges from roughly the mid 1990s till 2000. As we also include the period from 2000 onwards, a possible explanation for our poor forecasting results seems to be locked up in that period. Figure 1 surely indicates that the interest rate behaviour during that period with its pronounced widening of spreads is rather different from the stable second half of the 1990s. The subsample results reported in Mönch (2006a) for the 2000:1-2003:9 indicate the VAR, NS2-AR and NS2-VAR models perform poorly compared to the RW which is evidence that model forecastability is indeed low during that period. Through analyzing the subsamples 1994:1-1998:12 and 1999:1-2003:12 we hope get insights.

4.3.2 Subsample results

Sample 1994:1 - 1998:12

The five years that this subsample consists of are the years that have been most heavily investigated in other forecasting studies, with positive results found for different models. For example, Duffee (2002) reports forecast results for essentially affine models that hold up favorably against the random walk for the sample 1995:1-1998:12. Similarly, Ang and Piazzesi (2003) show that a no-arbitrage Gaussian VAR model predicts well 1-month ahead for the sample 1996:1-2000:12 while Diebold and Li (2006) report outperforming forecasts for the Nelson-Siegel model for the sample 1994:1-2000:12²⁰. These studies suggest that there should be a high degree of predictability for this subperiod. Tables 7-10 confirm this claim. Compared to the corresponding tables for the first subperiod the level of the RMSPEs are (often) lower. Even for a 1-month horizon it is already possible to outperform the random walk. The AR-X model in particular performs well across all maturities with results for the frequentist approach being slightly better than for the Bayesian approach. The latter is most likely due to the fact that the prior information based solely on the initial sample does not fit well with this period of smooth interest rates. The TRMSPEs are lower than that of the random walk but the White test does not indicate that the forecasts are significantly better. The NS2-AR and VAR-X models also do well although the 2-year and 10-year maturities still seem difficult to forecast. The affine models render poor forecasts so in this subsample, except for the 5- and 7-year maturities. This differs from Ang and Piazzesi (2003) who show that an affine model augmented with inflation and real activity factors forecasts better than the random walk for maturities up to and including five years. This difference in results could be due to the substantially larger number of yields that we use in estimation. Furthermore, Ang and Piazzesi (2003) do not forecast beyond a 1-month horizon.

Moving to the 3-month horizon, other models also start to predict well, but especially for 6- and 12-months ahead predictability is evident. The VAR-X model but and the NS2-AR model in particular now produce forecasts that are significantly better than the no-change forecast with relative RMSPE being lower by sometimes as much as 30-40%. Adding macrofactors seems to lower forecast accuracy, in particular by introducing a negative forecast bias. Except for the VAR-X model, incorporating parameter uncertainty does not seem to help either. The performance of the affine models also improves. Interestingly, for shorter maturities simple affine models do better than their counterparts with macro information, but the evidence is just the opposite for longer forecast horizons.

²⁰Hordahl *et al.* (2006) construct one through 12 months ahead forecasts for the period 1995:1-1998:12 but these authors apply their structural model to German zero-coupon data and their results might therefore not be directly comparable to the results for the U.S data that we report here.

However, the affine models are never the best performing models for any maturity, which is a result also found by Diebold and Li (2006).

Comparing our results to those of Diebold and Li (2006) makes sense, since that study has the largest overlap in the set of models considered²¹ Results for $h = 1$ for the RW, AR, VAR and NS2-AR models are nearly identical in terms of RMSPE although we find slightly different MPEs (in our case the MPE is positive in general whereas Diebold and Li report mainly negative values). For $h = 6$ we find lower RMSPEs for the short and middle end of the curve (maturities below 5 years) whereas for the AR and VAR models results are very similar, despite the different way we estimate the VAR model. The differences in the reported MPEs are more pronounced now as well. Still we find MPEs that are positive, as opposed to negative values in Diebold and Li. A detailed analysis of the prediction errors reveals that for the sample period 1999:1-2000:12, during the yield hike, all the models are consistently producing forecasts that are too low resulting in substantially negative forecasting errors, which explains why Diebold and Li find negative MPEs. For the 12-month horizon we also find that the NS2-AR model substantially outperforms the RW, AR and VAR model. Whereas for the NS2-VAR model Diebold and Li find that this specification performs the worst, we find that although it performs indeed worse than the NS2-AR, its forecast accuracy is comparable to, and is sometimes even better than that of the AR and VAR models. Overall, our findings corroborate the superior performance of the NS2-AR model for this subsample.

Given the favorable RSMPE results it is curious to see that Hit Rates for this subsample are disappointingly low for the longer horizons. For the 6-month but especially for the 12-month horizon the Hit Rate is seldom higher than 0.5, see Table 18. A possible explanation for this phenomenon could be that interest rates are fairly stable during this subperiod, making it hard for the model to determine a clear long-term up or down movement.

Sample 1999:1 - 2003:12

During this subperiod, interest rates initially go up until the end of 2000 after which they decline sharply by roughly 5% from 6% to 1% for the short rate accompanied by a substantial widening of spreads between long and short rates. Forecasts results are shown in Table 11 - 14. Although adding macrofactors again improves forecasts, the only model that seems to be able to compete with the RW is the Bayesian NS1-X model and only consistently for the longest horizons. The frequentist AR-X model does well for shorter maturities. The VAR model shows a strikingly poor performance with very large positive

²¹Although the forecast period of Diebold and Li contains 24 more months, a comparison still seems interesting to conduct.

MPEs indicating that the VAR model cannot cope with the downward sloping trend of interest rates. The Bayesian ATSM-X model does better than the Bayesian VAR and predicts the short end of the curve reasonably well. This shows that imposing no-arbitrage restrictions helps but not enough to beat simple univariate models.

Since the downward trend in yields is so persistent throughout this subsample it should not be surprising that Hit Rates are very high, especially for the 6-month and 12-month horizons. Long rates are more volatile than short rates which results in higher Hit Rates for the short and medium end of the curve as compared to the long end. Bayesian analysis and macrofactors again do hardly anything to improve sign predictability.

4.3.3 Rolling TRMSPE

The subsample results clearly show that different models perform well during different subsamples. A obvious example is the NS2-AR model which comfortably outperforms all other models for the first subsample but produces disappointing forecasts the second subsample. Similar conclusions can be drawn for other models as well. To further illustrate how the forecasting performance of different models varies over time we compute TRMSPEs using a 60-month rolling window. Figures 4-7 show these results for all forecast horizons considered and for a subset of models²². Each graph shows the rolling TRMSPE of the RW, AR, VAR, NS1 and ATSM models, either with (left panels) or without macrofactors (right panels)

The patterns for the two five year subsamples reappear. TRMSPEs are fairly stable until 1997 after which a decreasing trend sets in lasting until mid 2000. The high degree of interest rate predictability during the 1994-1998 subperiod is the cause of the decreasingly low TRMSPEs for 1998-2001. From 2001 onwards a sharp increase is visible in TRMSPEs indicating large forecasting errors due the sharp decline in interest rate levels and widening of spreads during this period.

Zooming in on the performance of individual models, we notice that the random walk is one of the best models at the beginning and at the end of the forecasting period. During the 1998-2001 period the random walk tends to outperformed by the AR-X, VAR-X and NS1-X models. An opposite pattern is visible for the ATSM model which performs well only in the middle of the out-of-sample period. The main point to take from these graphs is that the performance of individual models varies substantially over time and establishing a clear-cut ordering of the models which holds across the entire 1994-2003 seems infeasible. Therefore, believing in a single model may be dangerous. In the next section, we discuss several forecast combination techniques.

²²Note that the graphs only depict model specifications that were estimated using both frequentist and Bayesian inference. As a result, the NS2-AR and NS2-VAR are not included but these results are available on request.

5 Forecast combination

Our subsample and rolling TRMPSE analysis reveals that there does not seem to be a single model that consistently outperforms the random walk across all subperiods. The forecasting ability of individual models differs considerably over time. It seems that each model may play a complementary role in approximating the data generating process, at least during subperiods. Model uncertainty is troublesome if one has hopes of believing in a single model for forecasting or investment purposes. A worthwhile endeavor for cushioning the effects of model uncertainty is to *combine* forecasts that have been generated with different models. In this section we examine several forecast combination schemes. Two combination methods are standard approaches and can be applied to combine frequentist as well as Bayesian forecasts. The third combination method we investigate is a truly Bayesian approach which can only be applied to Bayesian forecasts. We first discuss the different methods before moving on to examining the forecast combination results in comparison to the results of the individual models.

5.1 Forecast combination: schemes

Scheme 1: Equally weighted forecasts

The first forecast combination method assigns an equal weight to the forecast from all individual models. Assuming we are combining forecasts from M different models, each weight is the same and equal to $w_{T+h,m}^{(\tau_i)} = 1/M$ for $m = 1, \dots, M$. The equally weighted combined forecast for a h -month horizon for any maturity τ_i is therefore given by $\hat{y}_{T+h}^{(\tau_i)} = \sum_{m=1}^M w_{T+h,m}^{(\tau_i)} \hat{y}_{T+h,m}^{(\tau_i)}$ which we denote as the Forecast Combination - Equally Weighted forecast (**FC-EW**). As explained in Timmermann (2006) this method is likely to work well if forecasts from different models are highly correlated, which certainly holds for the models we consider in this study.

Scheme 2: Inverted MSPE-weighted forecasts

The second forecast combination scheme we examine uses weights that take into account historical relative performance. Model weights are based on each model's (inverted) MSPE relative to those of all other models, computed over a window of the previous v months²³. The weight for model m is computed as $w_{T+h,m}^{(\tau_i)} = \frac{1/\text{MSPE}_{T+h,m}^{(\tau_i)}}{\sum_{m=1}^M (1/\text{MSPE}_{T+h,m}^{(\tau_i)})}$ where $\text{MSPE}_{T+h,m}^{(\tau_i)} = \frac{1}{v} \sum_{r=1}^v (\hat{y}_{T+h-r|T-r,m}^{(\tau_i)} - y_{T+h-r}^{(\tau_i)})^2$. A model with a lower MSPE will get a relatively larger weight than a worse performing model, see Timmermann (2006) for discussion and Stock

²³Note that whereas in the tables we report results for the Root MSPE we use the MSPE to construct weights since MSPE is a direct measure of forecast accuracy.

and Watson (2004) for an application to forecasting GDP growth²⁴. We use a window of $v = 60$ months when computing MSPEs and denote the resulting forecasts by Forecast Combination - MSPE (**FC-MSPE**).

Scheme 3: Bayesian predictive likelihood

The third and final combination scheme we consider is a purely Bayesian model averaging approach, which we denote by **BMA**²⁵, and which is based on the idea that any model is as good as its predictions, as proposed by Geweke and Whiteman (2006). The idea behind this method is to consider the out-of-sample, or predictive in Bayesian language, performance of individual models. The probability of the *realized* value at time $T + h$ is evaluated under the Bayesian predictive density for $T + h$ conditional on the information at time T and the specific model m . If the predictive density is accurate the realized value will fall near the center of the density and will be assigned a large value relative to the case where the realization ends up being far out in the tail of the density.

Averaging over the predictive likelihood is an alternative approach to the most common BMA method based on the marginal likelihood which was (re-)introduced in an empirical application by Madigan and Raftery (1994). We choose the predictive likelihood BMA for three reasons. First, the predictive likelihood is an out-of-sample performance measure. Second, the marginal likelihood of highly nonlinear models, such as the Nelson-Siegel and affine models, cannot be derived analytically and may be very difficult to compute by Monte Carlo simulation. Third, Eklund and Karlsson (2005) show, in a simulation setting and in an empirical application to forecast the Swedish inflation rate, that model weights based on the predictive likelihood have better small sample properties and result in better out-of-sample performance than weights based on the traditional marginal likelihood measure.

Whereas we refer to the appendix for specific details, we do want to briefly discuss a major difference between our forecast combination approach and that of Eklund and Karlsson (2005). Unlike in their study, we do not apply the system of updating and probability forecasting *prequential*, as defined by Dawid (1984). We compute the predictive density for month $T + h$ using information up until month T and we evaluate the *realized* value for time $T + h$ using the same density. The resulting probability is then used to

²⁴The weights applied in this and the previous forecast combination scheme will always be bounded between 0 and 1. Other approaches for which this does not necessarily need to be the case, in particular OLS-based and Kalman Filter-based weights, see again Timmermann (2006), proved to be problematic here due to multicollinearity problems between the different forecasts. This resulted in often extreme (offsetting) weights and was therefore not pursued further.

²⁵In the remainder of the text, we often refer to case 3, BMA, as forecast combination. With a slight abuse of denotation we share BMA in the class of forecast combination methods which, strictly speaking, is incorrect since BMA *averages* models instead of *combining* models.

compute the weight for model m in constructing the forecast for $T + 2h$ made at time $T + h$. Eklund and Karlsson (2005) on the other hand evaluate the fit of the predictive density over more observations, by means of the predictive likelihood, and then update the probability density for the forecasts. The latter approach results in weights which are based more on the fit of the model, even when using out-of-sample data, than on the probability of out-of-sample realized values. In an unreported simulation exercise we find that our approach reacts faster to out-of-sample uncertainty since it is not constrained to give more probability to the model which fits ex-post predicted values best. Our approach incorporates the uncertainty that future out-of-sample values may also differ from historical out-of-sample realizations.

5.2 Forecast combination results

Before setting the weights in the combination forecast, the question which models to include in the combination should be answered. Here we combine forecasts from two different sets of models. First we include only those specifications that incorporate macrofactors ($M = 7$ for the models estimated with frequentist methods and $M = 5$ for the Bayesian counterpart) and second, we simply combine all specifications ($M = 13$ and $M = 9$ respectively)²⁶. This allows us to assess the added value of including macro information also in the combined forecasts. We always include the random walk in the forecast combinations.

5.2.1 Full sample 1994:1 - 2003:12

The results of forecast combinations for the 1994-2003 period are reported in Panels C and F of Tables 3-6. The first point to notice is that for all horizons combining forecasts seems to be a valuable alternative compared to selecting a single model, especially when Bayesian forecasts are combined. The reported TRMSPE values show that the forecast combination methods perform as well as the best individual model and nearly always outperform the random walk. Panel F of Tables 3-6 show that combining forecasts works increasingly well for longer forecast horizons, in particular when using BMA. Indeed, for a 6- and 12-month horizons, BMA outperforms the random walk by several percentage points in terms of relative RMSPEs and results are often very similar to the best performing individual model. Judging from the MPE, which are of a similar order of magnitude as

²⁶Many other subsets can of course be selected. Aiolfi and Timmermann (2005) suggest filtering out the worst performing model(s) in an initial step. Preliminary analysis suggests that doing so does not lead to much improvement in forecasting performance in our case. However, a more thorough selection procedure than simply including all available models as applied here, will most likely lead to better results for the forecast combination methods. Although this is a very interesting issue to examine in more detail, here we merely strive to show the benefits of combining forecasts as an alternative to putting all one's eggs in a single model basket.

for the macrofactor-augmented models, the improvement in RMSPE is due to a decrease in forecast error volatility thereby leading to more stable forecast series.

When comparing the different forecast combination methods amongst each other, it is apparent that equally weighted and MSPE-based weighted forecasts nearly always give results that are very similar. In some cases, using MSPE-based weights works slightly better, but overall the benefits of weights that are based on relative historical performance seem limited. Bayesian model averaging often provides the most accurate forecasts. Bayesian model averaging incorporates two sources of uncertainty: parameter uncertainty as well as model uncertainty. Contrary to the equally weighted and MSPE weighted approaches, Bayesian model averaging tends to assign near-zero weights to the worst performing models and thereby effectively eliminating the worst performing models, as these models often have higher levels of parameter uncertainty. For the first two approaches models can effectively only be eliminated manually.

It is ambiguous whether forecasts should be combined using the smaller or the larger set of models. Combining models that only incorporate macrofactors seems the best strategy for the 1-month and 3-month horizons, whereas combining all the models provides more accurate forecasts for longer horizons but only with the frequentist approach, not with Bayesian inference. This finding is not surprising ex-post, because for shorter horizon forecasts it is mostly the AR-X model that has predictive power, and for longer horizons the Bayesian NS1-X model does particularly well.

In term of Hit Ratios, reported in Panels C and F of Table 15 and 16, conclusions are partially similar. Forecast combinations render high hit rates, often close to the best individual models. However, it is less clear which combination method should be preferred, and in particular Bayesian model averaging is no longer the dominant combination approach. It is still evident though that only combining models that incorporate macro variables is better for shorter horizons whereas combining all models provides more accurate forecasts for the 6- and 12-month horizons.

Subsample 1994:1 - 1998:12

For this period, which is characterized by a high level of predictability, forecast combinations are attractive although single models, mainly the NS2-AR, give better results. Applying equal weights provides the most accurate forecasts which can be explained by the high correlations (not reported) between forecast series. Forecast combinations from models estimated with frequentist methods do better than when taking into account parameter uncertainty, mainly due to the NS2-AR model which is only included in the former case. As we noticed in Section 4.3.2, macroeconomic information decreases the performance of single models which translated into better relative results for the combination methods that include all model specifications.

Subsample 1999:1 - 2003:12

The results for this subsample are almost the complete opposite from those reported for 1994-1998. Combined forecasts are at least very similar, but in most cases more accurate than those produced by individual models. Bayesian model averaging in particular performs well. Ex-post, these results can be explained by the forecast accuracy of the Bayesian NS1-X and AR-X models, and, most likely also because of the ability of this approach to assign near-zero weights to the worst performing models.

Hit Rates for this subsample remain high also for the forecast combination methods, especially when combining the forecasts from all model specifications.

6 Conclusion

This paper addresses the task of forecasting the term structure of interest rates. Several recent studies have shown that significant steps forward are being made in this area. We contribute to the existing literature by assessing the importance of incorporating macroeconomic information, parameter uncertainty, and, in particular, model uncertainty. Our results show that these issues are worth addressing since each improves interest rate forecasts.

We examine the forecast accuracy of a range of models with varying degrees levels of complexity. We assess model forecasts over a ten-year out-of-sample period, using the entire period as well as several subperiod to show that the predictive ability of individual models varies over time considerably. Models that incorporate macroeconomic variables seem more accurate in subperiods during which the uncertainty about the future path of interest rates is substantial. As an example we mention the period 2000-2003 when spreads were high. Models without macro information do particularly well in subperiods where the term structure has a more stable pattern such as in the early 1990s.

The fact that different models forecast well in different subperiods confirms ex-post that different model specifications play a complementary role in approximating the data generating process. Furthermore, our subsample results provide a strong claim for using forecast combination techniques as opposed to believing in a single model. Our model combination results, in particular when using Bayesian model averaging techniques, show that recognizing model uncertainty and mitigating the likely effects, leads to substantial gains in interest rate forecastability. In particular, we show that combined forecasts are superior to those generated using individual models and the random walk benchmark.

We feel that our results open up exciting avenues for further research. In this study we have only considered very generic models, in particular in our use of a three-factor Gaussian affine model. It would therefore be interesting to expand the model consideration set with more sophisticated models such as the FAVAR models of Mönch (2006a) or

the structural model by Hordahl *et al.* (2006) both of which have been found to forecast well. More sophisticated ways of combining forecasts are worth addressing as well, see e.g. Guidolin and Timmermann (2007) who use a combination scheme with time-varying weights where weights have regime switching dynamics. In terms of incorporating parameter uncertainty, much more work can be done on the use of sensible priors. As an example we mention the use of adaptive priors that could take into account likely changes in yield dynamics due to clear political or economic reasons. Other, technical issues that could be addressed are more specifically related to estimation and forecasting procedures. For example, changes in yield dynamics could also be accounted for by using rolling estimation windows instead of expanding windows as we have used here.

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Appendix

A Individual models

In this appendix we provide details on how we perform inference on the parameters of the models in Section 3. We discuss each model separately and we distinguish between frequentist and Bayesian inference.

A.1 AR model

Frequentist Inference

We estimate the parameters c , ϕ and ψ using standard OLS. Given the parameter estimates we construct iterated forecasts as

$$\hat{y}_{T+h}^{(\tau_i)} = \hat{c}^{(\tau_i)} + \hat{\phi}^{(\tau_i)} \hat{y}_{T+h-1}^{(\tau_i)} + \hat{\psi}^{(\tau_i)'} \hat{X}_{T+h} \quad (\text{A-1})$$

with $\hat{y}_T^{(\tau_i)} = y_T^{(\tau_i)}$. We construct forecast both with and without the macroeconomic factors. The forecasts \hat{X}_{T+h} are iterated forecasts constructed from the VAR(3) model for the macrofactors.

Bayesian Inference

For the Bayesian inference, we use a Normal-Gamma conjugate prior for the parameters

$$(c^{(\tau_i)}, \phi^{(\tau_i)}, \psi^{(\tau_i)}, \sigma^{2,(\tau_i)}), \quad (c^{(\tau_i)}, \phi^{(\tau_i)}, \psi^{(\tau_i)})', \sigma^{2,(\tau_i)} \sim NG(\underline{b}, \underline{v}, \underline{s}^2, \underline{\nu}) \quad (\text{A-2})$$

The marginal posterior densities for parameters $(c^{(\tau_i)}, \phi^{(\tau_i)}, \psi^{(\tau_i)}, \sigma^{(\tau_i)})$ and the predictive density of $y_{T+h}^{(\tau_i)}$ conditional on $y_T^{(\tau_i)}$ and X_{T+h} to draw inference on parameters and to forecast $y_{T+h}^{(\tau_i)}$ can be derived using standard Bayesian results, see for example Koop (2006).

A.2 VAR model

Frequentist Inference

We estimate the equation parameters in (5) using equation-by-equation OLS. Forecasts are constructed as

$$\hat{Y}_{T+h} = \hat{c} + \hat{\Phi} \hat{F}_{T+h-1} + \hat{\Psi} \hat{X}_{T+h} \quad (\text{A-3})$$

We construct yield factor forecasts, \hat{F}_{T+h-1} , by applying the factor loadings obtained using data up until month T to the iterated yields forecasts.

Bayesian Inference

We apply direct simulation to infer the VAR model in equation (5) differing from other literature which often uses MCMC algorithms. Direct simulation gains in time and in precision, in particular when the forecast horizon is very long, since it applies independent draws. Our derivation is based on Zellner (1971), who provides all the necessary computations with diffuse priors, and we extend the analysis with informative priors²⁷.

Prior Specification We apply informative prior densities for the parameter matrices $\Pi = [c \ \Phi \ \Psi]$ and S in (5). For computational tractability we select the following conjugate priors:

$$\Pi | S \sim MN(\underline{B}, S \otimes \underline{V}) \quad (\text{A-4})$$

and

$$S \sim IW(\underline{S}, \underline{\mu}) \quad (\text{A-5})$$

where MN indicates the mactrivariate normal distribution with mean \underline{B} and variance matrix $S \otimes \underline{V}$ and where IW indicates the Inverted Wishart distribution.

²⁷We present the main results. Details of the derivations are available upon request.

Posterior Simulation The likelihood function of Y_T for the model in (5) is given by

$$p(Y_T|F_{T-1}, X_T, \Pi, S) = (2\pi)^{-TN/2} |S|^{-T/2} \exp\left[-\frac{1}{2} \text{tr}(S^{-1}(Y_T - Z_T \Pi)'(Y_T - Z_T \Pi))\right] \quad (\text{A-6})$$

where T is the number of in-sample observations, $Z_T = (e_N, F_{T-1}, X_T)$, and e_N is a $(N \times 1)$ vector of ones. If we combine (A-6) together with the prior densities in (A-4)–(A-5) the joint posterior density for (Π, S) is

$$\begin{aligned} p(\Pi, S|Y_T, F_{T-1}, X_T) &= p(Y_T|F_{T-1}, X_T, \Pi, S)p(\Pi|S)p(S) \\ &\propto |S|^{-(T+N+\nu+1)/2} \exp\left(-\frac{1}{2} \text{tr}(S^{-1}(\underline{S} + (Y_T - Z_T \Pi)'(Y_T - Z_T \Pi) + (\Pi - \underline{B})' \underline{V}^{-1}(\Pi - \underline{B})))\right) \end{aligned} \quad (\text{A-7})$$

where $\nu = K + \underline{\nu}$ with K the number of columns of Π . If we define $W_T = (Y_T, \underline{V}^{-1/2} \underline{B})'$ and $V_T = (Z_T, \underline{V}^{-1/2})'$, applying the decomposition rule and the Inverted Wishart integration step, the posterior density for Π conditional on (Y_T, F_{T-1}, X_T) is a generalized t -distribution with location parameter $\hat{\Pi} = (V_T' V_T)^{-1} V_T' W_T$, scale parameters $\underline{S} + (W_T - V_T \hat{\Pi})'(W_T - V_T \hat{\Pi})$ and $(Z_T' Z_T + \underline{V}^{-1})$, and $T + \underline{\nu}$ degrees of freedom. That is,

$$\Pi|Y_T, F_{T-1}, X_T \sim |\underline{S} + (W_T - V_T \hat{\Pi})'(W_T - V_T \hat{\Pi}) + (\Pi - \hat{\Pi})'(Z_T' Z_T + \underline{V}^{-1})(\Pi - \hat{\Pi})|^{-(T+\nu)/2} \quad (\text{A-8})$$

The posterior density of S conditional on (Y_T, F_{T-1}, X_T) is:

$$S|Y_T, F_{T-1}, X_T \sim IW(\underline{S} + (W_T - V_T \hat{\Pi})'(W_T - V_T \hat{\Pi}), T + \nu) \quad (\text{A-9})$$

Forecasting The predictive density conditional on (Y_T, X_T) and (F_{T+h-1}, X_{T+h}) is defined as:

$$\begin{aligned} p(Y_{T+h}|Y_T, X_T, F_{T+h-1}, X_{T+h}) &= \int \int p(Y_{T+h}, \Pi, S|Y_T, X_T, F_{T+h-1}, X_{T+h}) d\Pi dS \\ &= \int \int p(Y_{T+h}|F_{T+h-1}, X_{T+h}, \Pi, S) p(\Pi, S|Y_T, X_T) d\Pi dS \end{aligned} \quad (\text{A-10})$$

If we apply the inverted Wishart step to (A-10), and integrate with respect to Π , we have:

$$\begin{aligned} p(Y_{T+h}|Y_T, X_T, F_{T+h-1}, X_{T+h}) &\propto |\underline{S} + (W_T - V_T \hat{\Pi})'(W_T - V_T \hat{\Pi}) + \\ &\quad (Y_{T+h} - Z_{T+h} \hat{\Pi})'(I - Z_{T+h} L^{-1} Z_{T+h}') (Y_{T+h} - Z_{T+h} \hat{\Pi})|^{-(T+\nu+h)/2} \end{aligned} \quad (\text{A-11})$$

where $Z_{T+h} = (I_h, F_{T+h-1}, X_T)$ with I_h is a $(h \times h)$ identity matrix, and where $L = (Z_{T+h}' Z_{T+h} + \underline{V}^{-1})$.

The predictive density of Y_{T+h} conditional on $(Y_T, X_T, F_{T+h-1}, X_{T+h})$ is thus a generalized t -distribution with location parameter $Z_{T+h} \hat{\Pi}$, scale parameters $\underline{S} + (W_T - V_T \hat{\Pi})'(W_T - V_T \hat{\Pi})$ and $(I_N - Z_{T+h} L^{-1} Z_{T+h}')$, and $T + \nu$ degrees of freedom.

Following Zellner (1971) we can express equation (A-11) as:

$$p(Y_{T+h}|Y_T, X_T, F_{T+h-1}, X_{T+h}) = p(Y_{T+1}|F_T, X_{T+1}) \dots p(Y_{T+h}|F_T, X_{T+1}, \dots, F_{T+h-1}, X_{T+h}) \quad (\text{A-12})$$

We apply direct simulation. F_{T+h-1} and X_{T+h} are generated from their densities conditional on their previous values, and then substituted in equation (A-12) to derive the density for Y_{T+h} conditional on Y_T and X_T :

$$p(Y_{T+h}|Y_T, X_T) = \iint p(Y_{T+h}|Y_T, X_T, F_{T+h-1}, X_{T+h}) p(F_{T+h}|F_{T+h-1}) p(X_{T+h}|X_{T+h-1}) dF_{T+h-1} dX_{T+h} \quad (\text{A-13})$$

Note that we integrate with respect to the forecast distribution of the macroeconomic variables X_{T+h} given X_T .

A.3 Nelson-Siegel model

Frequentist Inference

With the frequentist approach we estimate the Nelson-Siegel model using the two-step approach of Diebold and Li (2006) and the one-step approach of Diebold *et al.* (2006).

In the two-step approach we fix λ to 16.42, which, as shown in Diebold and Li (2006), maximizes the curvature factor loading at a 30-month maturity. Per t the vector of β 's is then estimated by applying OLS on the cross-section of 18 maturities. From this first step we obtain time-series for the three factors, $\{\beta_t\}_{t=1}^T$. The second step consists of modelling the factors in (7) by fitting either separate AR(1) models or a single VAR(1) model.

In the one-step approach we estimate the unknown parameters and latent factors by means of the Kalman Filter using the prediction error decomposition for the State-Space model in (6)-(7). For each sample in the recursive estimation procedure, we first run the two-step approach with a VAR(1) specification for the state vector to obtain starting values. The unconditional mean and covariance matrix of $\{\beta_t\}_{t=1}^T$ are used to start the Kalman Filter. We discard the first 12 observations when evaluating the likelihood. All variance parameters of the diagonal matrix H and the full matrix Q are initialized to 1. The covariance terms in Q are initialized to 0. In the optimization, we maximize the likelihood using the standard deviations as parameters to ensure positivity for all variances. Finally, λ is initialized to 16.42.

Iterated forecasts for the factors are obtained from (7) as

$$\hat{f}_{T+h} = \hat{a} + \hat{\Gamma} \hat{f}_{T+h-1} \quad (\text{A-14})$$

where $\hat{f}_{T+h} = (\hat{\beta}_{1,T+h}, \hat{\beta}_{2,T+h}, \hat{\beta}_{3,T+h}, \hat{M}_{T+h}, \hat{M}_{T+h-1}, \hat{M}_{T+h-2})$. These are then be inserted in (7) to give interest rate forecasts

$$\hat{y}_{T+h}^{(\tau_i)} = \hat{\beta}_{1,T+h} + \hat{\beta}_{2,T+h} \left(\frac{1 - \exp(-\tau_i/\hat{\lambda})}{\tau_i/\hat{\lambda}} \right) + \hat{\beta}_{3,T+h} \left(\frac{1 - \exp(-\tau_i/\hat{\lambda})}{\tau_i/\hat{\lambda}} - \exp(-\tau_i/\hat{\lambda}) \right) \quad (\text{A-15})$$

Bayesian Inference

The joint posterior density for parameters of the Nelson-Siegel and affine models does not have a known closed-form expression as in the AR and VAR models. Therefore, marginal densities for model parameters as well as marginal predictive densities cannot be computed analytically and we need to use Monte Carlo methods. We discuss our simulation approach below.

Prior Specification The model parameters are summarized by $\theta = (\lambda, \sigma, a, \Gamma, Q)$, where $\sigma = (\sigma^{(\tau_1)}, \dots, \sigma^{(\tau_N)})$ is the $(N \times 1)$ vector of diagonal elements of H in (8). For the ease of posterior simulation we take independent conjugate priors for the model parameters. For the variance parameters $\sigma^{(\tau_i)}$ we take the Inverted Gamma-2 prior

$$\sigma^{(\tau_i)} \sim \text{IG-2}(\underline{\nu}^{(\tau_i)}, \underline{\delta}^{(\tau_i)}) \quad (\text{A-16})$$

For the matrix of covariance-variance Q_1 and Q_2 we assume the Inverted Wishart distributions,

$$Q_1 \sim \text{IW}(\underline{\mu}_1, \underline{\Delta}_1) \quad (\text{A-17})$$

$$Q_2 \sim \text{IW}(\underline{\mu}_2, \underline{\Delta}_2) \quad (\text{A-18})$$

For the linear regression parameters we take matrixvariate Normal distribution,

$$[a, \Gamma] \sim MN(\underline{\Gamma}, Q \otimes \underline{V}_\Gamma) \quad (\text{A-19})$$

Finally for λ we assume an uniform distribution,

$$\lambda \sim U(\underline{a}_\lambda, \underline{b}_\lambda) \quad (\text{A-20})$$

The parameters \underline{a}_λ and \underline{b}_λ cqn be chosen to reflect the prior belief about the shape of the loading factors.

Posterior Simulation Posterior results are obtained using the Gibbs sampler of Geman and Geman (1984) with the technique of data augmentation of Tanner and Wong (1987). The latent variables $B_T = \{\beta_{1,t}, \beta_{2,t}, \beta_{3,t}\}_{t=1}^T$ are simulated alongside the model parameters θ .

The complete data likelihood function is given by

$$p(Y_T, F_T | \theta) = \prod_{t=1}^T \prod_{i=1}^N p(y_t^{(\tau_i)} | f_t, \lambda, \sigma^{(\tau_i)}) p(f_t | f_{t-1}, a, \Gamma, Q) \quad (\text{A-21})$$

where $Y_T = \{y_t^{(\tau_1)}, \dots, y_t^{(\tau_N)}\}_{t=1}^T$ and where $F_T = \{\beta_{1,t}, \beta_{2,t}, \beta_{3,t}, M_t, M_{t-1}, M_{t-2}\}_{t=1}^T$. The terms $p(y_t^{(\tau_i)} | f_t, \lambda, \sigma^{(\tau_i)})$, and $p(f_t | f_{t-1}, a, \Gamma, Q)$ are Normal density functions which follow directly from (6)–(7). If we combine (A-21) together with the prior density $p(\theta)$ implied by (A-16)–(A-20) we obtain the posterior density

$$p(\theta, B_T | Y_T, M_T, M_{T-1}, M_{T-2}) \propto p(Y_T, F_T | \theta) p(\theta) \quad (\text{A-22})$$

The full conditional posterior density for the latent regression parameters B_T is computed using the simulation smoother as in Carter and Kohn (1994, Section 3). The Kalman smoother is applied to derive the conditional mean and variance of the latent factors; for the initial value β_0 a multivariate normal prior with mean 0 is chosen.

To sample the θ parameters, excluding λ , we can use standard results. Hence, the variance parameters $\sigma^{(\tau_i)}$ are sampled from inverted Gamma-2 distributions, the matrix Q_1 is sampled from an Inverted Wishart distribution, and the parameters a_1, Γ_1 are sampled from matrix-variate Normal distributions, where (a_1, Γ_1) are the appropriate elements of a and Γ respectively. In our framework the macro variables have a VAR(3) structure independent from the latent factors. Therefore, we simulate a_2, Γ_2 , and Q_2 from their marginal densities, respectively generalized t -distributions and an Inverted Wishart distribution to improve the speed of convergence.

Finally, the posterior density for λ conditional on Y_T, F_T, H is:

$$p(\lambda | Y_T, F_T, H) \propto \prod_{t=1}^T \prod_{i=1}^N p(y_t^{(\tau_i)} | f_t, \lambda, \sigma^{\tau_i}) p(\lambda) \quad (\text{A-23})$$

Equation (A-23) is not proportional to a known density. Therefore, λ may be drawn by applying MCMC methods. We use the Griddy Gibbs algorithm. The Griddy Gibbs sampler was developed by Ritter and Tanner (1992) and is based on the idea to form a simple approximation of the inverse cumulative distribution function of the target density on a grid of points²⁸. More formally and referring to equation (A-23), we perform the following steps:

- We evaluate $p(\lambda | Y_T, F_T, H)$ at points $V_i = v_1, \dots, v_n$ to obtain w_1, \dots, w_n ;
- We use w_1, \dots, w_n to obtain an approximation to the inverse cdf of $p(\lambda | Y_T, F_T, H)$;
- We sample a uniform (0,1) deviate and we transform the observation via the approximate inverse cdf.

Forecasting The h -step ahead predictive density of Y_{T+h} conditional on Y_T and F_T is given by

$$p(Y_{T+h} | Y_T, F_T) = \iint p(y_{T+h}^{(\tau_i)} | f_{T+h}, \lambda, \sigma^{(\tau_i)}) p(f_{T+h} | f_{T+h-1}, a, \Gamma, Q) p(\theta, B_T | Y_T, M_T, M_{T-1}, M_{T-2}) df_{T+h} d\theta \quad (\text{A-24})$$

where $p(y_{T+h}^{(\tau_i)} | f_{T+h}, \lambda, \sigma^{(\tau_i)})$ and $p(f_{T+h} | f_{T+h-1}, a, \Gamma, Q)$ follow directly from (6)–(7) and where $p(\theta, B_T | Y_T, M_T, M_{T-1}, M_{T-2})$ is the posterior density.

Simulating Y_{T+h} from the h -step ahead distribution (A-24) is straightforward. In each step of the Gibbs sampler, we use the simulated values of (a, Γ, Q) to draw the out-of-sample values of f_{T+h} . Then, f_{T+h} in combination with the current Gibbs draws of H and λ provide a simulated value for $y_{T+h}^{(\tau_i)}$.

²⁸Mönch (2006b) applies a random walk Metropolis Hastings algorithm to draw λ . We choose the Griddy-Gibbs since the space of λ is well defined and only the cdf must be estimated in these points.

A.4 Affine model

Frequentist Inference

To estimate the affine model we assume that yields of every maturity are contaminated with measurement error. We estimate the parameters in the resulting State-Space model by applying the two-step approach used in Ang *et al.* (2006b). We make the latent factors Z_t observable by extracting the first three principal components from the panel of yield for different maturities. The first step of the estimation procedure consists of estimating the equation and variance parameters of the state equations (23). In the second step we estimate the remaining parameters $(\delta_0, \delta_1, \lambda_0, \lambda_1)$. We first estimate (δ_0, δ_1) by applying OLS to the short rate equation (13) where we use the 1-month yield as the observable short rate. We then estimate the risk premia parameters (λ_0, λ_1) by minimizing the sum of squared yields errors in the measurement equations (22), giving the parameter estimates from the first step, $(\hat{\mu}, \hat{\Psi}, \hat{\Sigma})$ and the short rate parameters $(\hat{\delta}_0, \hat{\delta}_1)$. In the second step we initialize all risk premia parameters to 0. Common approaches for obtaining starting values for the risk premia parameters by first estimating either λ_0 or λ_1 in a separate step yielded unsatisfactory results.

Yield forecasts are generated by iterating forward the state-equations (23)

$$\hat{f}_{T+h} = \hat{\mu} + \hat{\Psi} \hat{f}_{T+h-1} \quad (\text{A-25})$$

where $\hat{f}_{T+h} = (\hat{Z}_{1,T+h}, \hat{M}_{T+h-1}, \hat{M}_{T+h-2})$. With the estimated parameters substituted in $a^{(\tau_i)}$ and $b^{(\tau_i)}$ we then construct interest rate forecasts as

$$\hat{y}_{T+h}^{(\tau_i)} = \hat{a}^{(\tau_i)} + \hat{b}^{(\tau_i)} \hat{f}_{T+h} \quad (\text{A-26})$$

Bayesian Inference

Bayesian inference on model (22)-(23) is very complex due to the high nonlinearity of the structure of the parameters and, above all, the large set of yields we forecast. In particular, it may be difficult to define the space of the parameters δ_0 and δ_1 . The likelihood is very sensitive to those parameters and small perturbations may give very different and unrealistic results. Therefore, a Bayesian approach as in Ang *et al.* (2006a) may not be the optimal solution. We opt for a normal approximation of the full posterior density around frequentist parameter estimates. The aforementioned choice implies that it is not necessary to derive posterior densities based on the full likelihood function for the parameters in (22)-(23) in order to compute the predictive density of Y_{T+h} conditional on Y_T and $F_t = \{f_t\}_{t=1}^T$.

Forecasting The h -step ahead predictive density of $y_{T+h}^{(\tau_i)}$ made at time T conditional on Y_T and F_T is given by

$$p(y_{T+h}^{(\tau_i)} | Y_T, F_T) = \iint p(y_{T+h}^{(\tau_i)} | f_{T+h}, a^{(\tau_i)}, b^{(\tau_i)}, \sigma^{(\tau_i)}) p(f_{T+h} | f_{T+h-1}, \mu, \Psi, Q) p(\theta | Y_T, F_T) df_{T+h} d\theta \quad (\text{A-27})$$

where $p(y_{T+h}^{(\tau_i)} | x_{T+h}, a^{(\tau_i)}, b^{(\tau_i)}, \sigma^{(\tau_i)})$, and $p(f_{T+h} | f_{T+h-1}, \mu, \Psi, Q)$ are the conditional predictive densities given model (22)-(23) and where $p(\theta | Y_T, X_T)$ is the posterior density for parameters $\theta = (\mu, \Psi, Q, a, b, \lambda_0, \lambda_1)$. As we discussed in the previous paragraph we approximate $p(\theta | Y_T, X_T)$ in (A-27), with a normal distribution around frequentist estimates $q(\hat{\theta} | Y_T, X_T)$. Direct simulation is then applied to computed the following predictive density of Y_{T+h} conditional on (Y_T, F_T) given by

$$p(y_{T+h}^{(\tau_i)} | Y_T, X_T) = \iint p(y_{T+h}^{(\tau_i)} | f_{T+h}, \hat{a}^{(\tau_i)}, \hat{b}^{(\tau_i)}, \hat{\sigma}^{(\tau_i)}) p(f_{T+h} | f_{T+h-1}, \hat{\mu}, \hat{\Psi}, \hat{Q}) df_{T+h} d\theta \quad (\text{A-28})$$

B Bayesian Model Averaging

The predictive density of $y_{T+h}^{(\tau_i)}$ conditional on Y_T and information at time T , which is denoted in this section by D_T , given M individual models is:

$$p(y_{T+h}^{(\tau_i)} | Y_T, D_T) = \sum_{i=1}^M P(m_j^{(\tau_i)} | Y_T, D_T) p(y_{T+h}^{(\tau_i)} | Y_T, D_T, m_j^{(\tau_i)}) \quad (\text{B-1})$$

where $j = 1, \dots, M$ where $P(m_j^{(\tau_i)}|Y_T, D_T)$ is the posterior probability of model m_j for maturity τ_i conditional on data at time T , where $p(y_{T+h}^{(\tau_i)}|Y_T, D_T, m_j^{(\tau_i)})$ is the predictive density of $y_{T+h}^{(\tau_i)}$ conditional on Y_T and D_T given model m_j . $p(y_{T+h}^{(\tau_i)}|Y_T, D_T, m_j^{(\tau_i)})$ is computed separately for each model j as outlined in the way in Appendix A. The posterior probability of model m_j for maturity τ_i is computed as:

$$P(m_j^{(\tau_i)}|Y_T, D_T) = \frac{p(y_{T,o}^{(\tau_i)}|Y_T, D_T, m_j^{(\tau_i)})P(m_j^{(\tau_i)})}{\sum_{s=1}^k p(y_{T,o}^{(\tau_i)}|Y_T, D_T, m_s^{(\tau_i)})P(m_s^{(\tau_i)})} \quad (\text{B-2})$$

where $P(m_j^{(\tau_i)})$ is the prior probability of model m_j for maturity τ_i . $p(y_{T,o}^{(\tau_i)}|Y_T, D_T, m_j^{(\tau_i)})$ is the predictive likelihood given model m_j which is the number derived by substituting the realized value $y_{T,o}^{(\tau_i)}$ in the predictive density $p(y_T^{(\tau_i)}|Y_T, D_T, m_j^{(\tau_i)})$ given model m_j .

As the notation indicates, individual models are averaged independently for any maturity.

C Prior specification

In the literature uninformative priors or diffuse informative priors are often chosen to derive posterior densities that depend only on data information (the likelihood). We do not follow this approach as we apply informative priors in our estimation and forecasting procedures. There are several motivations to do so. Firstly, for models which have non-linear structures in the parameters, such as the NS1 or ATSM models, it is very difficult to define what is non-informative. Secondly, the simulation algorithm might get stuck in some (nonsensical) regions of the parameter space and it may require a substantial number of simulations to converge, thereby enormously increasing estimation time. Thirdly, we think that market operators have priors in their minds and apply these when forecasting interest rates. Finally, we want to study and underline differences between frequentist and Bayesian inference in forecasting yields, and the use of priors is one of the main differences if not the major difference.

We briefly discuss the specification of the prior densities for the parameters of the models presented in Appendix A. We start with the AR model and the Normal-Gamma conjugate prior in (A-2) for parameters $(c^{(\tau_i)}, \phi^{(\tau_i)}, \psi^{(\tau_i)}, \sigma^{(\tau_i)})$. We choose $\underline{v} = 0.01$ to have a prior density for the vector $(c^{(\tau_i)}, \phi^{(\tau_i)}, \psi^{(\tau_i)})$ concentrated around the mean value. The mean vector value \underline{b} is chosen by calibration with the initial in-sample data (1970:1-1993:12) and to prevent unit root type behavior. The prior for $\sigma^{(\tau_i)}$ is less informative. $\underline{\nu}$ is fixed equal to 20 and \underline{s}^2 is calibrated with in-sample data.

The calibration of the prior for the VAR model is more complex due to the high dimensionality of Π and S . Therefore, we relax our prior assumption and we choose a wider region for \underline{V} and $\underline{\nu}$ in (A-4)–(A-5). \underline{B} is again calibrated with initial in-sample data and values imply plausible factor loadings of the PCA factors.

The order of prior information in the NS1 model is comparable to the VAR model. What it is new is the parameter λ . We choose the following prior density:

$$\lambda \sim U(3.34, 33.45) \quad (\text{C-1})$$

By restricting λ in the interval $[3.34, 33.45]$ the curvature factor loading on β_{3t} is at its maximum for a maturity between 6 months and 5 years.

For the ATSM model we do not apply prior densities. We use a normal approximation of the conditional predictive density around maximum likelihood parameter estimates. Indeed, due to the large number of yields that we consider and the resulting large number of parameters that need to be estimated, the speed of convergence of MCMC algorithms such as the Gibbs sampler is very slow. Moreover, some parameters do not converge at all, and unrealistic values are simulated. However, we believe the approximation is rational and satisfactory; therefore parameter uncertainty is still implemented in the ATSM model.

Finally, in BMA uninformative priors are applied, and each model has a prior probability of $1/M$.

Table 1: Summary Statistics

maturity	mean	stdev	skew	kurt	min	max	JB	ρ_1	ρ_{12}	ρ_{24}
1-month	6.049	2.797	0.913	4.336	0.794	16.162	85.671	0.968	0.690	0.402
3-month	6.334	2.896	0.871	4.237	0.876	16.020	76.380	0.974	0.708	0.415
6-month	6.543	2.927	0.788	4.016	0.958	16.481	58.796	0.976	0.723	0.444
1-year	6.755	2.860	0.661	3.763	1.040	15.822	38.907	0.975	0.733	0.474
2-year	7.032	2.724	0.644	3.672	1.299	15.650	35.240	0.978	0.748	0.526
3-year	7.233	2.594	0.685	3.663	1.618	15.765	38.796	0.979	0.763	0.560
4-year	7.392	2.510	0.728	3.607	1.999	15.821	41.640	0.980	0.771	0.582
5-year	7.483	2.449	0.759	3.478	2.351	15.005	42.454	0.982	0.786	0.607
6-year	7.611	2.406	0.791	3.437	2.663	14.979	45.236	0.983	0.797	0.626
7-year	7.659	2.344	0.841	3.488	3.003	14.975	51.562	0.983	0.787	0.623
8-year	7.728	2.320	0.841	3.365	3.221	14.936	49.798	0.984	0.809	0.651
9-year	7.767	2.317	0.877	3.427	3.389	15.018	54.765	0.985	0.813	0.656
10-year	7.745	2.266	0.888	3.496	3.483	14.925	57.117	0.985	0.796	0.647

Notes: The table shows summary statistics for end-of-month unsmoothed continuously compounded US zero-coupon yields. The results shown are for annualized yields. The sample period is January 1970 - December 2003 (408 observations). Reported are the mean, standard deviation, skewness, kurtosis, minimum, maximum, the Jarque-Bera test statistic for normality and the 1st, 12th and 24th sample autocorrelation.

Table 2: Macro-economic series

group code			description	group code			description
1	7		Personal Income (AR, Bil. Chain 2000 \$) (TCB)	6	4		Houses Authorized By Build. Permits:South(Thou.U.)S.A.
1	7		Personal Income Less Transfer Payments (AR, Bil. Chain 2000 \$) (TCB)	6	4		Houses Authorized By Build. Permits:West(Thou.U.)S.A.
1	7		Industrial Production Index - Total Index	7	1		Napm Inventories Index (Percent)
1	7		Industrial Production Index - Products, Total	7	7		Manufacturing And Trade Inventories (Bil. Chain 2000 \$) (TCB)
1	7		Industrial Production Index - Final Products	7	8		Ratio, Mfg. And Trade Inventories To Sales (Based On Chain 2000 \$) (TCB)
1	7		Industrial Production Index - Consumer Goods	8	1		Purchasing Managers' Index (Sa)
1	7		Industrial Production Index - Durable Consumer Goods	8	1		Napm New Orders Index (Percent)
1	7		Industrial Production Index - Nondurable Consumer Goods	8	1		Napm Vendor Deliveries Index (Percent)
1	7		Industrial Production Index - Business Equipment	8	7		Mfrs' New Orders, Consumer Goods And Materials (Bil. Chain 1982 \$) (TCB)
1	7		Industrial Production Index - Materials	8	7		Mfrs' New Orders, Durable Goods Industries (Bil. Chain 2000 \$) (TCB)
1	7		Industrial Production Index - Durable Goods Materials	8	7		Mfrs' New Orders, Nondefense Capital Goods (Mil. Chain 1982 \$) (TCB)
1	7		Industrial Production Index - Nondurable Goods Materials	8	7		Mfrs' Unfilled Orders, Durable Goods Indus. (Bil. Chain 2000 \$) (TCB)
1	7		Industrial Production Index - Manufacturing (Sic)	9	7		S&P's Common Stock Price Index: Composite (1941-43=10)
1	7		Industrial Production Index - Residential Utilities	9	7		S&P's Common Stock Price Index: Industrials (1941-43=10)
1	7		Industrial Production Index - Fuels	9	8		S&P's Composite Common Stock: Dividend Yield (% Per Annum)
1	1		Napm Production Index (Percent)	9	7		S&P's Composite Common Stock: Price-Earnings Ratio (%Nsa)
1	8		Capacity Utilization (Mfg) (TCB)	10	7		United States:Effective Exchange Rate(Merm)(Index No.)
2	1		Index Of Help-Wanted Advertising In Newspapers (1967=100;Sa)	10	7		Foreign Exchange Rate: Switzerland (Swiss Franc Per U.S.\$)
2	1		Employment: Ratio; Help-Wanted Ads:No. Unemployed Clf	10	7		Foreign Exchange Rate: Japan (Yen Per U.S.\$)
2	7		Civilian Labor Force: Employed, Total (Thous.,Sa)	10	7		Foreign Exchange Rate: United Kingdom (Cents Per Pound)
2	7		Civilian Labor Force: Employed, Nonagric.Industries (Thous.,Sa)	10	7		Foreign Exchange Rate: Canada (Canadian \$ Per U.S.\$)
2	1		Unemployment Rate: All Workers, 16 Years & Over (%Sa)	11	1		Interest Rate: Federal Funds (Effective) (% Per Annum,Nsa)
2	8		Unemploy.By Duration: Average(Mean)Duration In Weeks (Sa)	12	7		Money Stock: M1(Curr,Trav.Cks,Dep,Other Ck'able Dep)(Bil\$,Sa)
2	7		Unemploy.By Duration: Persons Unempl.Less Than 5 Wks (Thous.,Sa)	12	7		Money Stock:M2(M1+O'nite Rps,Euro\$,G/P&B/D Mmmfs&Sav&Sm Time Dep(Bil\$,Sa)
2	7		Unemploy.By Duration: Persons Unempl.5 To 14 Wks (Thous.,Sa)	12	7		Money Stock: M3(M2+Lg Time Dep,Term Rp's&Inst Only Mmmfs)(Bil\$,Sa)
2	7		Unemploy.By Duration: Persons Unempl.15 Wks + (Thous.,Sa)	12	7		Money Supply - M2 In 1996 Dollars (Bci)
2	7		Unemploy.By Duration: Persons Unempl.15 To 26 Wks (Thous.,Sa)	12	7		Monetary Base, Adj For Reserve Requirement Changes(Mil\$,Sa)
2	7		Unemploy.By Duration: Persons Unempl.27 Wks + (Thous.,Sa)	12	7		Depository Inst Reserves:Total, Adj For Reserve Req Chgs(Mil\$,Sa)
2	7		Average Weekly Initial Claims, Unemploy. Insurance (Thous.) (TCB)	12	7		Depository Inst Reserves:Nonborrowed,Adj Res Req Chgs(Mil\$,Sa)
2	7		Employees On Nonfarm Payrolls: Total Private	12	7		Commercial & Industrial Loans Outstanding In 1996 Dollars (Bci)
2	7		Employees On Nonfarm Payrolls - Goods-Producing	12	1		Wkly Rp Lg Com'l Banks:Net Change Com'l & Indus Loans(Bil\$,Saar)
2	7		Employees On Nonfarm Payrolls - Mining	12	7		Consumer Credit Outstanding - Nonrevolving(G19)
2	7		Employees On Nonfarm Payrolls - Construction	12	8		Ratio, Consumer Installment Credit To Personal Income (Pct.) (TCB)
2	7		Employees On Nonfarm Payrolls - Manufacturing	13	7		Producer Price Index: Finished Goods (82=100,Sa)
2	7		Employees On Nonfarm Payrolls - Durable Goods	13	7		Producer Price Index: Finished Consumer Goods (82=100,Sa)
2	7		Employees On Nonfarm Payrolls - Nondurable Goods	13	7		Producer Price Index:I ntermed Mat.Supplies & Components(82=100,Sa)
2	7		Employees On Nonfarm Payrolls - Service-Providing	13	7		Producer Price Index: Crude Materials (82=100,Sa)
2	7		Employees On Nonfarm Payrolls - Trade, Transportation, And Utilities	13	7		Spot market price index: bls & crb: all commodities(1967=100)
2	7		Employees On Nonfarm Payrolls - Wholesale Trade	13	7		Index Of Sensitive Materials Prices (1990=100)(Bci-99a)
2	7		Employees On Nonfarm Payrolls - Retail Trade	13	1		Napm Commodity Prices Index (Percent)
2	7		Employees On Nonfarm Payrolls - Financial Activities	13	7		Cpi-U: All Items (82-84=100,Sa)
2	7		Employees On Nonfarm Payrolls - Government	13	7		Cpi-U: Apparel & Upkeep (82-84=100,Sa)
2	7		Employee Hours In Nonag. Establishments (AR, Bil. Hours) (TCB)	13	7		Cpi-U: Transportation (82-84=100,Sa)
2	1		Avg Weekly Hrs of Prod or Nonsup Workers On Private Nonfarm Payrolls - Goods-Producing	13	7		Cpi-U: Medical Care (82-84=100,Sa)
2	8		Avg Weekly Hrs of Prod or Nonsup Workers On Private Nonfarm Payrolls - Mfg Overtime Hours	13	7		Cpi-U: Commodities (82-84=100,Sa)
2	1		Average Weekly Hours, Mfg. (Hours) (TCB)	13	7		Cpi-U: Durables (82-84=100,Sa)
2	1		Napm Employment Index (Percent)	13	7		Cpi-U: Services (82-84=100,Sa)
3	7		Sales Of Retail Stores (Mil. Chain 2000 \$) (TCB)	13	7		Cpi-U: All Items Less Food (82-84=100,Sa)
4	7		Manufacturing And Trade Sales (Mil. Chain 1996 \$) (TCB)	13	7		Cpi-U: All Items Less Shelter (82-84=100,Sa)
5	7		Real Consumption (AC) a0m224/gmde (a0m224 is from TCB)	13	7		Cpi-U: All Items Less Midical Care (82-84=100,Sa)
6	4		Housing Starts:Nonfarm(1947-58);Total Farm&Nonfarm(1959-)(Thous.,Saar)	13	7		Pce, Impl Pr Defl:Pce (1987=100)
6	4		Housing Starts:Northeast (Thous.U.)S.A.	13	7		Pce, Impl Pr Defl:Pce; Durables (1987=100)
6	4		Housing Starts:Midwest(Thous.U.)S.A.	13	7		Pce, Impl Pr Defl:Pce; Nondurables (1996=100)
6	4		Housing Starts:South (Thous.U.)S.A.	13	7		Pce, Impl Pr Defl:Pce; Services (1987=100)
6	4		Housing Starts:West (Thous.U.)S.A.	14	7		Avg Hourly Earnings of Prod or Nonsup Workers On Private Nonfarm Payrolls - Goods-Producing
6	4		Housing Authorized: Total New Priv Housing Units (Thous.,Saar)	14	7		Avg Hourly Earnings of Prod or Nonsup Workers On Private Nonfarm Payrolls - Construction
6	4		Houses Authorized By Build. Permits:Northeast(Thou.U.)S.A	14	7		Avg Hourly Earnings of Prod or Nonsup Workers On Private Nonfarm Payrolls - Manufacturing
6	4		Houses Authorized By Build. Permits:Midwest(Thou.U.)S.A.	15	8		U. Of Mich. Index Of Consumer Expectations(Bcd-83)

Notes: The table lists the individual macro series used in the macro PCA. These are categorized in groups using: (1) real output and income, (2) employment and hours, (3) real retail, (4) manufacturing and trade sales, (5) consumption, (6) housing starts and sales, (7) real inventories, (8) orders, (9) stock prices, (10) exchange rates, (11) federal funds rate, (12) money and credit quantity aggregates, (13) prices indexes, (14) average hourly earnings and (15) miscellaneous. The transformations applied to original series are coded as: 1:=no transformation (levels are used), 4:=logarithm of the level, 7:=annual first differences of the log levels and 8:=annual first differences of the levels. The sample period is January 1970 - December 2003 (408 observations).

Table 3: 1994:1 - 2003:12, $h = 1$

Maturity	TRMSPE	RMSPE							MPE						
	all	3m	6m	1y	2y	5y	7y	10y	3m	6m	1y	2y	5y	7y	10y
RW	101.59	21.18	21.82	25.71	29.12	30.48	29.30	27.95	1.78	1.88	2.01	1.97	1.62	1.41	1.35
FREQUENTIST INFERENCE															
Panel A: Model without macrofactors															
AR	1.02	1.07	1.06	1.05	1.03	1.01	1.01	1.01	8.17	8.05	8.56	7.35	5.52	5.27	4.78
VAR	1.06	1.03	1.23	1.14	1.13	1.04	1.05	1.11	8.23	13.28	11.41	7.04	6.95	6.70	9.52
NS2-AR	1.10	1.13	1.27	1.24	1.19	1.11	1.06	1.07	5.29	6.91	7.72	12.81	13.90	6.90	8.10
NS2-VAR	1.04	0.96*	1.10	1.10	1.11	1.06	1.03	1.06	5.46	5.93	5.15	8.83	10.23	3.91	5.82
NS1	1.06	1.09	1.08	1.05	1.10	1.07	1.04	1.06	11.83	9.39	5.43	7.66	9.44	2.47	3.37
ATSM	1.07	0.93	1.15	1.23	1.18	1.04	1.08	1.07	4.80	12.24	16.46	11.29	-2.87	10.17	9.72
Panel B: Models with macrofactors															
AR-X	0.99	0.95	0.96	0.98	0.98	0.99	1.00	0.99	2.10	2.26	2.82	1.98	0.71	0.66	0.07
VAR-X	1.02	0.99	1.03	1.01	1.12	1.02	1.02	1.03	-1.61	-2.31	-0.79	-3.27	-1.04	-0.80	2.46
NS2-AR-X	1.09	1.22	1.31	1.28	1.17	1.05	1.06	1.06	-8.90	-7.36	-6.42	-0.46	3.66	-2.03	0.36
NS2-VAR-X	1.05	1.05	1.17	1.20	1.13	1.03	1.05	1.05	-7.91	-7.44	-7.95	-3.12	1.89	-2.92	0.34
NS1-X	1.05	1.01	1.04	1.08	1.10	1.04	1.05	1.06	0.25	-1.83	-4.99	-1.19	3.54	-2.50	-0.87
ATSM-X	1.13	1.13	1.18	1.29	1.42	1.04	0.99	1.06	-12.35	-8.83	-6.32	-10.75	1.57	4.40	1.95
Panel C: Forecast combinations															
FC-EW-X	1.01	0.97	1.03	1.06	1.09	1.00	1.01	1.01	1.36	-3.38	-3.09	-2.12	1.71	-0.25	0.81
FC-MSPE-X	1.00	0.96	1.01	1.04	1.07	1.00	1.01	1.01	-3.80	-2.61	-2.20	-1.48	1.69	-0.35	0.86
FC-EW-ALL	1.00	0.94	1.02	1.05	1.08	1.01	1.01	1.02	1.32	2.47	2.55	3.09	4.24	2.59	3.61
FC-MSPE-ALL	1.00	0.94	1.01	1.04	1.07	1.01	1.01	1.02	1.36	2.29	2.51	3.09	3.90	2.40	3.61
BAYESIAN INFERENCE															
Panel D: Models without macrofactors															
AR	1.02	1.05	1.05	1.04	1.02	1.01	1.01	1.01	7.18	7.00	7.43	6.10	4.05	3.72	3.17
VAR	1.04	1.10	1.30	1.15	1.11	1.01	1.02	1.03	8.33	12.95	10.89	5.45	3.03	1.94	3.52
NS1	1.08	1.05	1.10	1.07	1.15	1.10	1.04	1.08	9.54	7.61	5.87	11.36	11.75	3.10	2.60
ATSM	1.08	0.93	1.15	1.23	1.19	1.05	1.09	1.07	4.75	12.22	16.30	11.20	-2.88	10.17	9.68
Panel E: Models with macrofactors															
AR-X	0.99	0.95	0.96	0.98	0.98	0.99	1.00	0.99	1.44	1.45	1.68	0.88	0.21	0.14	-0.17
VAR-X	1.02	1.00	1.04	1.01	1.12	1.02	1.03	1.04	-2.01	-2.64	-0.98	-3.22	-1.21	-1.38	1.40
NS1-X	1.28	1.66	1.71	1.58	1.31	1.11	1.22	1.21	-29.48	-32.17	-33.28	-24.12	-14.52	-19.88	-17.41
ATSM-X	1.12	1.15	1.16	1.26	1.39	1.03	1.01	1.11	-11.64	-9.5	-7.53	-11.41	1.26	4.96	2.43
Panel F: Forecast Combinations															
FC-EW-X	1.00	0.98	1.00	1.02	1.08	0.99	0.99	0.99	-7.98	-8.20	-7.62	-7.18	-2.53	-2.95	-2.48
FC-MSPE-X	0.99	0.95	0.97	1.00	1.05	0.99	1.00	0.99	-4.14	-4.26	-4.02	-5.23	-2.22	-2.36	-1.75
BMA-X	1.01	0.98	1.00	1.04	1.10	0.99	0.99	1.00	-7.47	-7.00	-5.94	-6.42	-1.17	0.24	-0.36
FC-EW-ALL	0.98	0.91	0.95	0.99	1.05	0.99	1.00	1.00	-1.12	-0.13	0.27	-0.20	0.37	0.46	0.73
FC-MSPE-ALL	0.99	0.93	0.97	1.00	1.04	0.99	1.00	1.00	1.01	1.44	1.83	0.67	0.37	0.73	1.22
BMA-ALL	0.99	0.91	0.95	1.00	1.06	1.00	1.01	1.02	-0.06	1.41	2.13	1.78	1.54	3.19	2.99

Notes: The table reports the [Trace] Root Mean Squared Prediction Error ([T]RMPSE) and the Mean Prediction Error (MPE) for *individual* yield models, with and without macrofactors, estimated using the frequentist approach (Panels A and B) and using Bayesian inference (Panels D and E). Panels C and F show results for different forecast *combination* methods for the frequentist and Bayesian estimated models. All results are for a 1-month horizon for the out-of-sample period 1994:1 - 2003:12. The following model abbreviations are used: RW stands for the Random Walk, (V)AR for the first-order (Vector) Autoregressive Model, NS2-(V)AR for two-step Nelson-Siegel model with a (V)AR specification for the factors, NS1 for the one-step Nelson-Siegel model, ATSM for the affine model, FC-EW and FC-MSPE for the forecast combination based on equal weights and MSPE-based weights respectively, BMA for the Bayesian Model Averaging forecast. The affix 'X' indicates that macrofactors have been added as additional variables. The first line lists the value of the forecast accuracy metrics, reported in basis points, for the Random Walk model (RW) while all other lines reports statistics relative to those of the RW. Bold numbers indicate outperformance relative to the RW whereas *, ** and *** indicate significant outperformance at the 90%, 95% and 99% level respectively according to the White (2000) reality check using 1000 block-bootstraps and an average block-length of 12.

Table 4: 1994:1 - 2003:12, $h = 3$

Maturity	TRMSPE	RMSPE							MPE						
	all	3m	6m	1y	2y	5y	7y	10y	3m	6m	1y	2y	5y	7y	10y
RW	195.81	48.24	50.71	55.36	59.86	57.25	53.47	49.72	5.45	5.66	5.68	5.52	4.36	3.89	3.63
FREQUENTIST INFERENCE															
Panel A: Model without macrofactors															
AR	1.05	1.10	1.09	1.08	1.04	1.02	1.03	1.03	24.26	23.86	24.99	21.44	15.94	15.37	13.84
VAR	1.10	1.08	1.21	1.20	1.16	1.09	1.08	1.13	29.77	35.64	33.70	28.22	25.00	23.36	24.92
NS2-AR	1.13	1.16	1.24	1.26	1.23	1.13	1.07	1.06	18.75	22.00	24.90	31.29	29.75	20.85	20.23
NS2-VAR	1.05	0.99	1.08	1.11	1.11	1.06	1.03	1.05	20.68	21.03	19.86	22.52	21.30	13.94	14.94
NS1	1.06	1.09	1.11	1.10	1.10	1.06	1.02	1.03	30.07	26.59	20.94	20.81	19.30	11.39	11.52
ATSM	1.06	0.96	1.11	1.18	1.14	1.02	1.07	1.06	20.35	28.19	32.42	26.36	9.35	20.92	18.77
Panel B: Models with macrofactors															
AR-X	0.98	0.95	0.96	0.98	0.98	0.99	0.99	0.99	5.05	5.33	6.66	4.49	0.87	0.98	-0.92
VAR-X	0.99	0.98	1.00	1.00	1.03	0.99	0.99	1.00	3.92	3.35	4.98	2.41	4.08	3.85	6.70
NS2-AR-X	1.13	1.24	1.27	1.28	1.20	1.08	1.07	1.04	-22.02	-18.80	-15.27	-6.22	0.62	-4.73	-2.20
NS2-VAR-X	1.07	1.04	1.13	1.19	1.16	1.05	1.05	1.03	-19.00	-18.28	-18.11	-11.69	-2.76	-6.04	-1.44
NS1-X	1.03	0.96	1.04	1.10	1.10	1.04	1.03	1.03	-3.68	-5.98	-9.16	-4.70	2.00	-3.41	-1.29
ATSM-X	1.04	0.94	1.04	1.14	1.20	1.03	1.00	1.01	-15.91	-12.73	-9.90	-13.32	1.31	5.06	3.20
Panel C: Forecast combinations															
FC-EW-X	1.00	0.96	1.01	1.05	1.06	1.00	1.00	0.99	6.41	-5.92	-5.02	-3.36	1.50	-0.06	1.10
FC-MSPE-X	0.99	0.94	1.00	1.03	1.05	1.00	1.00	0.99	-6.60	-4.11	-2.99	-2.35	1.62	0.12	1.30
FC-EW-ALL	1.00	0.93	1.01	1.05	1.06	1.01	1.00	1.00	7.51	8.91	9.36	9.78	10.09	8.11	8.61
FC-MSPE-ALL	0.99	0.93	1.00	1.04	1.05	1.01	1.00	1.00	6.41	7.61	8.47	8.87	9.29	7.62	8.35
BAYESIAN INFERENCE															
Panel D: Models without macrofactors															
AR	1.04	1.08	1.07	1.05	1.03	1.03	1.03	1.01	22.16	20.88	21.37	18.14	12.11	9.91	8.44
VAR	1.10	1.22	1.31	1.23	1.14	1.04	1.03	1.03	28.92	34.29	31.26	23.12	15.36	12.32	12.33
NS1	1.01	0.99	1.03	1.02	1.04	1.01	1.00	1.02	13.01	11.24	9.55	14.78	14.45	5.52	4.74
ATSM	1.06	0.96	1.11	1.19	1.15	1.03	1.08	1.06	20.38	28.22	32.39	26.34	9.31	20.88	18.74
Panel E: Models with macrofactors															
AR-X	0.98	0.95	0.95	0.98	0.98	0.99	1.00	1.00	0.56	0.34	1.43	-0.68	-1.78	-1.91	-2.54
VAR-X	1.02	0.96	1.03	1.07	1.11	1.03	1.01	1.01	-3.73	-4.41	-2.29	-2.87	1.58	2.15	5.52
NS1-X	1.02	1.01	1.06	1.09	1.04	0.99	1.04	1.05	-26.61	-29.13	-30.22	-21.20	-12.03	-17.67	-15.39
ATSM-X	1.04	0.95	1.02	1.14	1.19	1.03	1.00	1.02	-15.23	-13.20	-10.87	-14.06	0.98	5.23	3.46
Panel F: Forecast Combinations															
FC-EW-X	0.97	0.91	0.96	1.00	1.02	0.98	0.98	0.98	-7.91	-8.15	-7.25	-6.66	-1.38	-1.66	-1.07
FC-MSPE-X	0.97	0.92	0.96	1.00	1.02	0.98	0.98	0.98	-7.34	-7.42	-6.59	-6.31	-1.54	-1.76	-0.96
BMA-X	0.97	0.92	0.96	1.00	1.01	0.98	0.97	0.97	-6.94	-6.90	-5.96	-5.48	-0.80	-0.05	0.21
FC-EW-ALL	0.98	0.92	0.97	1.00	1.02	0.99	0.99	0.98	4.99	5.99	6.48	5.46	4.93	4.48	4.32
FC-MSPE-ALL	0.97	0.92	0.97	1.00	1.01	0.99	0.99	0.98	3.28	3.86	5.24	5.03	4.77	4.02	4.14
BMA-ALL	0.96	0.89	0.94	0.98	1.00	0.97	0.98	0.98	3.53	3.94	4.45	4.54	4.29	4.72	4.65

Notes: The table reports forecast results for a 3-month horizon for the out-of-sample period 1994:1 - 2003:12. See Table 3 for further details.

Table 5: 1994:1 - 2003:12, $h = 6$

Maturity	TRMSPE	RMSPE							MPE						
	all	3m	6m	1y	2y	5y	7y	10y	3m	6m	1y	2y	5y	7y	10y
RW	300.94	82.31	85.20	89.24	92.74	86.36	79.23	72.50	10.65	11.00	11.23	11.03	8.51	7.46	6.94
FREQUENTIST INFERENCE															
Panel A: Model without macrofactors															
AR	1.07	1.12	1.10	1.10	1.06	1.03	1.04	1.04	47.22	46.56	48.88	42.28	31.41	30.14	27.17
VAR	1.20	1.22	1.31	1.31	1.24	1.14	1.15	1.21	69.27	76.19	73.78	66.58	58.78	55.14	54.93
NS2-AR	1.12	1.12	1.18	1.22	1.20	1.11	1.06	1.06	36.29	41.06	45.85	53.30	48.84	37.99	35.49
NS2-VAR	1.05	1.03	1.09	1.11	1.10	1.04	1.02	1.06	43.61	43.72	41.92	43.02	37.89	29.00	28.65
NS1	1.06	1.12	1.13	1.11	1.08	1.02	1.00	1.03	56.81	52.17	44.51	41.30	35.00	25.65	24.61
ATSM	1.06	1.02	1.12	1.17	1.12	1.01	1.07	1.07	44.01	52.03	55.91	48.34	27.22	36.70	32.16
Panel B: Models with macrofactors															
AR-X	1.00	0.96	0.97	0.99	1.00	1.01	1.00	1.01	7.82	8.15	11.00	7.23	0.26	0.43	-3.35
VAR-X	0.98	0.98	1.00	1.01	1.00	0.97	0.97	0.99	12.59	12.25	14.12	11.27	11.93	10.82	12.97
NS2-AR-X	1.13	1.22	1.24	1.26	1.18	1.06	1.05	1.04	-40.75	-35.62	-29.10	-16.33	-5.76	-10.47	-7.53
NS2-VAR-X	1.07	1.07	1.14	1.19	1.15	1.04	1.03	1.03	-34.82	-33.31	-31.66	-22.68	-9.06	-10.74	-4.78
NS1-X	1.02	0.96	1.03	1.09	1.08	1.01	1.00	1.01	-7.93	-10.18	-13.01	-7.59	0.98	-3.91	-1.40
ATSM-X	1.02	0.95	1.04	1.11	1.12	0.99	0.98	1.01	-20.74	-17.54	-13.99	-16.01	1.35	5.90	4.58
Panel C: Forecast combinations															
FC-EW-X	0.99	0.96	1.01	1.04	1.04	0.99	0.98	0.99	12.94	-9.32	-7.35	-4.73	1.17	-0.07	1.06
FC-MSPE-X	0.99	0.95	1.00	1.03	1.03	0.99	0.98	0.99	-10.45	-6.25	-3.94	-2.70	1.64	0.41	1.51
FC-EW-ALL	0.99	0.95	1.00	1.04	1.03	0.98	0.98	0.99	17.23	18.96	19.96	20.13	19.03	16.47	16.19
FC-MSPE-ALL	0.99	0.95	1.00	1.03	1.03	0.98	0.98	0.99	12.94	14.64	16.57	17.24	17.02	15.00	15.14
BAYESIAN INFERENCE															
Panel D: Models without macrofactors															
AR	1.06	1.10	1.09	1.08	1.05	1.06	1.05	1.08	43.11	42.73	42.73	37.02	23.16	21.58	18.40
VAR	1.24	1.43	1.47	1.38	1.23	1.10	1.13	1.22	83.78	84.30	73.68	59.39	46.56	43.69	44.29
NS1	1.00	0.99	1.02	1.02	1.01	0.99	0.99	1.02	18.60	17.02	15.39	20.25	18.53	9.24	8.19
ATSM	1.07	1.02	1.13	1.18	1.13	1.02	1.07	1.08	43.85	51.95	55.91	48.39	27.26	36.65	32.19
Panel E: Models with macrofactors															
AR-X	0.98	0.96	0.96	0.99	0.99	1.00	1.00	0.99	-2.59	-2.47	-0.41	-3.48	-5.79	-5.24	-6.87
VAR-X	1.03	0.99	1.06	1.11	1.11	1.01	0.99	1.00	5.97	5.69	8.74	9.97	16.49	17.20	20.52
NS1-X	0.97	0.93	0.97	1.00	0.98	0.96	0.99	1.01	-22.14	-24.31	-25.13	-16.39	-8.20	-13.89	-12.20
ATSM-X	1.02	0.96	1.04	1.11	1.12	0.99	0.98	1.01	-19.57	-17.27	-14.55	-16.56	1.12	6.34	5.08
Panel F: Forecast Combinations															
FC-EW-X	0.97	0.93	0.97	1.00	1.00	0.97	0.96	0.97	-5.54	-5.47	-4.03	-3.09	2.42	2.37	2.69
FC-MSPE-X	0.97	0.93	0.97	1.00	1.00	0.97	0.97	0.97	-6.45	-5.92	-4.46	-3.48	1.88	2.07	2.76
BMA-X	0.96	0.95	0.97	0.99	0.99	0.96	0.96	0.96	-3.07	-3.06	-2.49	-1.86	1.98	2.44	2.56
FC-EW-ALL	0.98	0.95	0.99	1.02	1.01	0.97	0.98	1.00	17.96	18.74	18.62	16.63	14.18	13.67	12.95
FC-MSPE-ALL	0.98	0.94	0.98	1.01	1.00	0.98	0.98	1.00	11.49	11.91	12.69	13.03	12.87	12.44	12.08
BMA-ALL	0.96	0.95	0.97	0.98	0.98	0.96	0.96	0.97	13.39	13.62	13.87	14.40	11.25	10.56	10.39

Notes: The table reports forecast results for a 6-month horizon for the out-of-sample period 1994:1 - 2003:12. See Table 3 for further details.

Table 6: 1994:1 - 2003:12, $h = 12$

Maturity	TRMSPE	RMSPE							MPE						
	all	3m	6m	1y	2y	5y	7y	10y	3m	6m	1y	2y	5y	7y	10y
RW	452.51	140.61	145.03	146.89	141.77	121.21	108.58	98.96	20.19	21.12	22.31	23.63	21.28	19.89	19.46
FREQUENTIST INFERENCE															
Panel A: Model without macrofactors															
AR	1.10	1.11	1.09	1.10	1.09	1.07	1.09	1.10	89.35	88.94	93.87	83.71	65.93	63.88	58.80
VAR	1.43	1.41	1.44	1.42	1.40	1.41	1.46	1.55	155.02	163.93	161.27	153.28	140.45	134.11	131.37
NS2-AR	1.10	1.04	1.06	1.10	1.14	1.13	1.12	1.13	68.58	74.76	81.40	90.09	83.69	71.35	67.40
NS2-VAR	1.08	1.07	1.08	1.08	1.09	1.07	1.07	1.12	86.90	87.04	84.89	84.54	74.75	63.89	61.78
NS1	1.09	1.15	1.13	1.10	1.09	1.05	1.04	1.08	107.50	101.82	92.12	85.39	72.84	61.54	58.92
ATSM	1.10	1.07	1.11	1.14	1.12	1.04	1.12	1.13	89.05	97.48	101.13	91.91	65.84	72.52	64.33
Panel B: Models with macrofactors															
AR-X	1.02	0.95	0.98	1.00	1.03	1.06	1.05	1.06	11.07	11.61	17.73	12.34	1.92	3.15	-3.65
VAR-X	0.98	0.97	0.98	0.99	0.99	0.96	0.97	0.99	28.96	29.34	32.20	30.36	31.29	29.39	30.70
NS2-AR-X	1.14	1.19	1.18	1.19	1.16	1.09	1.09	1.08	-67.91	-60.28	-49.68	-31.37	-13.85	-16.83	-12.59
NS2-VAR-X	1.11	1.13	1.14	1.17	1.15	1.07	1.06	1.05	-59.80	-56.35	-51.31	-37.22	-15.69	-14.98	-7.08
NS1-X	1.01	0.96	1.00	1.05	1.06	1.01	1.00	1.01	-11.20	-12.66	-13.99	-6.11	6.09	2.14	5.35
ATSM-X	1.02	0.99	1.04	1.07	1.08	0.99	1.00	1.02	-26.24	-21.77	-16.85	-16.25	6.08	12.33	11.92
Panel C: Forecast combinations															
FC-EW-X	1.00	0.97	0.99	1.02	1.03	1.00	1.00	1.00	26.49	-12.71	-8.51	-3.52	5.30	5.01	6.30
FC-MSPE-X	1.00	0.97	0.99	1.01	1.02	1.00	1.00	1.00	-14.99	-8.97	-4.50	-0.89	5.79	5.70	6.88
FC-EW-ALL	0.99	0.95	0.97	1.00	1.01	0.99	1.00	1.01	37.80	40.38	42.70	43.41	41.59	38.64	37.44
FC-MSPE-ALL	0.99	0.95	0.98	1.00	1.01	0.99	0.99	1.01	26.49	29.52	33.62	35.52	35.30	33.10	32.60
BAYESIAN INFERENCE															
Panel D: Models without macrofactors															
AR	1.07	1.10	1.08	1.08	1.06	1.06	1.06	1.10	81.12	79.70	82.42	70.27	48.86	48.59	42.00
VAR	1.64	1.77	1.76	1.68	1.58	1.52	1.57	1.65	215.49	220.22	208.07	186.39	155.81	145.11	138.65
NS1	0.99	0.99	1.00	1.00	1.00	0.97	0.98	1.01	28.14	27.49	27.08	32.77	31.35	21.92	20.64
ATSM	1.10	1.08	1.12	1.14	1.12	1.05	1.13	1.13	89.24	97.67	101.21	91.96	65.83	72.47	64.27
Panel E: Models with macrofactors															
AR-X	1.01	0.96	0.98	1.01	1.03	1.04	1.03	1.02	-9.81	-10.10	-5.55	-9.29	-10.46	-7.79	-9.99
VAR-X	1.14	1.09	1.10	1.13	1.14	1.13	1.15	1.20	74.36	75.14	79.24	81.04	85.27	83.28	84.07
NS1-X	0.96	0.94	0.96	0.97	0.97	0.95	0.96	0.98	-13.08	-14.32	-13.86	-4.09	4.45	-1.39	0.38
ATSM-X	1.02	0.99	1.03	1.07	1.07	0.99	1.00	1.04	-25.79	-22.52	-17.99	-17.20	5.53	12.55	12.22
Panel F: Forecast Combinations															
FC-EW-X	0.96	0.93	0.96	0.98	0.99	0.96	0.97	0.98	9.17	9.86	12.83	14.82	21.21	21.31	21.23
FC-MSPE-X	0.97	0.95	0.97	0.99	1.00	0.97	0.97	0.98	3.90	5.45	8.86	11.15	18.31	19.66	20.80
BMA-X	0.96	0.96	0.97	0.98	0.97	0.96	0.96	0.96	7.93	8.17	9.29	11.69	15.55	15.70	15.99
FC-EW-ALL	1.00	0.98	1.00	1.01	1.01	0.99	1.01	1.03	51.10	52.71	53.66	50.61	45.32	43.85	41.30
FC-MSPE-ALL	0.98	0.96	0.98	1.00	1.00	0.99	0.99	1.01	30.92	32.05	34.84	35.61	35.81	35.55	35.14
BMA-ALL	0.97	0.97	0.98	0.98	0.97	0.96	0.96	0.97	27.95	28.82	29.86	32.83	30.63	28.43	27.64

Notes: The table reports forecast results for a 12-month horizon for the out-of-sample period 1994:1 - 2003:12. See Table 3 for further details.

Table 7: 1994:1 - 1998:12, h=1

Maturity	TRMSPE	RMSPE							MPE						
	all	3m	6m	1y	2y	5y	7y	10y	3m	6m	1y	2y	5y	7y	10y
RW	95.45	17.79	19.05	25.15	28.18	28.70	27.53	26.60	-2.28	-2.05	-1.56	-0.52	1.04	1.47	2.40
FREQUENTIST INFERENCE															
Panel A: Model without macrofactors															
AR	1.00	0.97	0.98	0.99	1.00	1.01	1.01	1.02	3.63	3.75	4.53	4.62	5.03	5.45	6.00
VAR	1.02	1.02	1.31	1.05	1.03	1.02	1.05	1.15	5.94	12.98	6.29	-0.69	5.96	6.94	8.06
NS2-AR	0.96	0.90	0.98	0.92	0.97	1.03	0.98	0.98	-0.67	-1.23	-3.46	4.29	12.15	5.64	2.60
NS2-VAR	1.02	0.97	1.06	0.98	1.03	1.06	1.02	1.04	4.36	2.21	-2.30	3.14	10.33	4.30	1.82
NS1	1.02	1.08	1.04	0.95	1.01	1.02	1.01	1.05	10.63	6.46	-0.92	1.61	7.30	1.42	-0.77
ATSM	1.05	0.87	1.04	1.00	1.08	1.05	1.12	1.06	2.33	9.31	9.78	4.37	-2.19	12.14	7.17
Panel B: Models with macrofactors															
AR-X	0.96	0.88	0.88	0.93	0.95*	0.97	0.97	0.97	-1.31	-1.15	-0.67	-1.08	-0.65	-0.33	0.18
VAR-X	0.97	0.94	0.98	0.93	1.05	0.96	0.98	1.03	-6.24	-7.13	-7.59	-13.72	-5.89	-3.10	-0.28
NS2-AR-X	1.02	1.32	1.33	1.20	1.03	0.94	0.99	1.02	-18.04	-18.91	-21.30	-12.86	-1.58	-6.51	-8.11
NS2-VAR-X	1.03	1.17	1.27	1.22	1.08	0.97	1.01	1.04	-14.27	-16.61	-21.01	-14.28	-2.27	-6.18	-6.75
NS1-X	1.01	0.87	0.99	1.09	1.05	0.97	1.03	1.08	-5.30	-9.14	-15.68	-11.30	-1.89	-6.60	-7.84
ATSM-X	1.16	1.38	1.37	1.31	1.52	0.93	0.91	1.05	-17.50	-17.33	-21.54	-29.50	-3.74	2.78	-4.08
Panel C: Forecast combinations															
FC-EW-X	0.98	0.98	1.04	1.03	1.05	0.95	0.96	1.00	-2.94	-10.33	-12.76	-11.89	-2.14	-2.64	-3.50
FC-MSPE-X	0.97	0.96	1.01	1.00	1.03	0.95	0.97	1.00	-9.28	-9.09	-10.90	-10.29	-1.94	-2.73	-3.16
FC-EW-ALL	0.96	0.87	0.94	0.95	1.00	0.97	0.98	1.00	-2.98	-2.99	-5.80	-5.07	1.81	1.34	0.03
FC-MSPE-ALL	0.96	0.87	0.94	0.95	1.00	0.97	0.98	1.00	-2.94	-3.03	-5.29	-4.52	1.56	1.11	0.26
BAYESIAN INFERENCE															
Panel D: Models without macrofactors															
AR	1.00	0.97	0.98	0.99	1.00	1.00	1.01	1.01	2.67	2.74	3.46	3.42	3.55	3.87	4.34
VAR	1.02	1.19	1.45	1.14	1.07	0.98	0.99	1.03	6.24	12.71	6.33	-1.64	1.17	0.43	-0.82
NS1	1.03	0.99	1.06	0.97	1.03	1.05	1.03	1.09	7.07	4.77	0.01	4.95	10.02	3.21	0.20
ATSM	1.05	0.87	1.04	1.01	1.08	1.06	1.12	1.07	2.37	9.36	9.76	4.35	-2.18	12.05	6.99
Panel E: Models with macrofactors															
AR-X	0.97	0.90	0.91	0.95	0.97	0.98	0.98	0.98	-2.07	-2.15	-2.20	-2.54	-1.35	-1.07	-0.32
VAR-X	0.97	0.97	1.01	0.95	1.05	0.97	0.98	1.03	-6.75	-7.53	-7.72	-13.35	-5.98	-3.93	-2.00
NS1-X	1.30	1.82	1.89	1.71	1.39	1.09	1.20	1.23	-28.64	-32.11	-36.70	-28.48	-14.50	-18.19	-18.43
ATSM-X	1.16	1.41	1.40	1.31	1.52	0.94	0.94	1.03	-17.17	-17.90	-22.16	-29.46	-4.18	3.01	-4.47
Panel F: Forecast Combinations															
FC-EW-X	1.00	1.06	1.10	1.07	1.10	0.96	0.97	0.99	-11.38	-12.35	-14.07	-14.87	-5.00	-3.74	-4.56
FC-MSPE-X	0.99	0.99	1.02	1.00	1.06	0.96	0.97	0.99	-8.17	-8.81	-10.27	-12.15	-4.60	-3.40	-3.56
BMA-X	1.00	1.08	1.10	1.06	1.11	0.94	0.94	0.98	-11.30	-11.95	-13.78	-15.61	-4.34	-0.70	-4.35
FC-EW-ALL	0.97	0.91	0.96	0.97	1.02	0.97	0.98	1.00	-4.29	-3.57	-5.64	-7.03	-1.38	0.09	-1.35
FC-MSPE-ALL	0.97	0.91	0.96	0.96	1.02	0.97	0.98	1.00	-2.57	-2.38	-3.95	-5.69	-1.32	0.20	-0.61
BMA-ALL	0.97	0.91	0.95	0.95	1.01	0.97	0.99	1.00	-3.20	-2.17	-4.24	-5.51	-0.18	3.03	-0.14

Notes: The table reports forecast results for a 1-month horizon for the out-of-sample period 1994:1 - 1998:12. See Table 3 for further details.

Table 8: 1994:1 - 1998:12, h=3

Maturity	TRMSPE	RMSPE							MPE						
	all	3m	6m	1y	2y	5y	7y	10y	3m	6m	1y	2y	5y	7y	10y
RW	194.55	37.86	44.46	54.74	61.30	58.86	56.01	51.56	-6.52	-5.76	-4.81	-1.41	2.58	4.46	6.32
FREQUENTIST INFERENCE															
Panel A: Model without macrofactors															
AR	1.00	0.95	0.96	0.97	0.99	1.01	1.02	1.03	11.15	11.56	13.33	13.83	14.41	16.25	16.97
VAR	1.02	1.02	1.14	1.00	0.97	1.02	1.05	1.17	20.60	25.99	17.60	11.06	19.38	21.35	23.09
NS2-AR	0.88	0.74	0.82	0.81	0.86	0.93	0.91**	0.95	2.00	2.22	1.32	10.97	21.57	15.83	13.41
NS2-VAR	0.99	0.97	1.01	0.95	0.98	1.02	1.00	1.06	14.33	10.86	4.62	8.83	17.64	12.92	11.73
NS1	0.98	1.06	1.02	0.93	0.95	0.97	0.97	1.04	23.81	16.93	6.11	6.12	12.71	7.75	6.38
ATSM	1.01	0.89	1.01	0.98	1.01	1.02	1.07	1.08	12.86	18.56	18.05	12.47	6.88	21.61	16.71
Panel B: Models with macrofactors															
AR-X	0.96	0.89	0.90**	0.93***	0.96**	0.98	0.97	0.98	-6.89	-6.54	-5.41	-5.44	-4.02	-2.19	-1.67
VAR-X	0.95	0.90	0.96	0.95*	0.97	0.95	0.95	1.02	-10.26	-10.65	-10.53	-14.51	-2.98	0.71	4.44
NS2-AR-X	1.01	1.39	1.28	1.16	1.01	0.90	0.93	0.97	-47.03	-47.28	-48.12	-36.14	-16.35	-18.00	-16.74
NS2-VAR-X	1.04	1.24	1.24	1.19	1.08	0.96	0.98	1.02	-38.65	-42.07	-47.22	-38.78	-17.04	-16.35	-12.72
NS1-X	0.97	0.88	1.00	1.04	1.01	0.94	0.97	1.04	-21.26	-27.01	-35.22	-29.95	-13.42	-15.29	-14.20
ATSM-X	1.01	1.12	1.13	1.10	1.16	0.92	0.90	0.98	-32.94	-35.02	-38.60	-44.01	-11.02	-2.14	-6.62
Panel C: Forecast combinations															
FC-EW-X	0.95	0.96	1.00	0.99	0.99	0.94	0.94	0.98	-7.44	-24.91	-27.13	-24.32	-8.89	-6.97	-5.88
FC-MSPE-X	0.95	0.94	0.98	0.97	0.98	0.94	0.94	0.98	-23.36	-22.23	-23.57	-21.95	-8.28	-6.51	-5.33
FC-EW-ALL	0.92	0.80	0.88	0.90	0.93	0.94	0.94	0.99	-6.06	-6.79	-9.91	-8.23	2.33	3.61	3.62
FC-MSPE-ALL	0.92	0.82	0.90	0.91	0.94	0.94	0.95	0.99	-7.44	-8.15	-10.29	-8.55	1.73	3.16	3.63
BAYESIAN INFERENCE															
Panel D: Models without macrofactors															
AR	1.00	0.93	0.94	0.96	0.99	1.01	1.04	1.02	9.45	7.54	9.43	10.49	10.50	9.65	10.25
VAR	1.05	1.33	1.37	1.16	1.05	1.00	0.99	1.04	19.40	25.17	16.52	6.93	8.01	7.10	5.87
NS1	0.98	0.92	1.00	0.97	0.98	0.99	0.99	1.05	3.22	1.31	-2.44	4.07	11.77	5.81	3.23
ATSM	1.01	0.88	1.00	0.97	1.00	1.02	1.07	1.08	12.71	18.43	17.90	12.31	6.76	21.61	16.77
Panel E: Models with macrofactors															
AR-X	0.98	0.96	0.95*	0.97	0.98	0.99	0.99	0.99	-11.03	-12.06	-11.31	-11.63	-6.56	-5.58	-4.11
VAR-X	0.94	0.90	0.97	0.96	0.99	0.93	0.93	1.01	-19.37	-22.63	-24.83	-27.75	-12.66	-6.93	-1.77
NS1-X	1.05	1.21	1.23	1.18	1.07	0.98	1.01	1.06	-34.42	-37.60	-41.44	-31.16	-13.45	-16.20	-15.53
ATSM-X	1.00	1.12	1.13	1.09	1.15	0.91	0.89	0.93	-32.50	-35.62	-39.25	-44.26	-11.56	-2.33	-7.09
Panel F: Forecast Combinations															
FC-EW-X	0.96	0.96	1.00	1.00	1.00	0.95	0.95	0.98	-20.77	-22.73	-24.33	-23.24	-8.33	-5.32	-4.44
FC-MSPE-X	0.96	0.96	1.00	0.99	1.00	0.95	0.95	0.98	-20.34	-21.85	-23.04	-21.89	-8.15	-5.40	-4.27
BMA-X	0.94	0.96	0.99	0.98	0.98	0.93	0.93	0.95	-19.87	-21.27	-22.46	-21.02	-7.62	-3.84	-4.07
FC-EW-ALL	0.95	0.87	0.93	0.94	0.96	0.96	0.96	0.99	-6.56	-6.80	-8.91	-9.16	-0.51	1.95	1.55
FC-MSPE-ALL	0.95	0.88	0.94	0.95	0.97	0.96	0.97	0.99	-8.63	-9.18	-9.89	-9.05	-0.48	1.46	1.45
BMA-ALL	0.93	0.85	0.91*	0.92**	0.94	0.93	0.94	0.97	-7.11	-7.70	-9.66	-9.14	-1.20	2.32	1.18s

Notes: The table reports forecast results for a 3-month horizon for the out-of-sample period 1994:1 - 1998:12. See Table 3 for further details.

Table 9: 1994:1 - 1998:12, h=6

Maturity	TRMSPE	RMSPE							MPE						
	all	3m	6m	1y	2y	5y	7y	10y	3m	6m	1y	2y	5y	7y	10y
RW	293.03	62.98	70.70	82.43	91.58	88.63	83.33	78.96	-15.03	-13.94	-12.81	-5.97	2.67	6.41	10.27
FREQUENTIST INFERENCE															
Panel A: Model without macrofactors															
AR	0.99	0.90	0.92	0.93	0.96	1.00	1.02	1.04	20.43	20.80	23.43	24.26	26.04	29.64	31.25
VAR	1.03	1.03	1.08	0.97	0.93	1.03	1.09	1.19	44.27	49.17	40.33	35.27	45.62	48.15	50.05
NS2-AR	0.82	0.64*	0.70**	0.72***	0.77***	0.87***	0.88***	0.93	7.61	8.80	9.54	21.55	35.47	30.65	28.98
NS2-VAR	0.98	0.95	0.96	0.92	0.93	0.99	1.01	1.07	25.31	21.24	14.32	18.45	29.46	25.95	25.90
NS1	0.95	1.03	0.97	0.89	0.88	0.93	0.95	1.01	39.84	30.64	16.95	15.12	23.24	19.33	18.85
ATSM	1.00	0.90	0.97	0.96	0.96	1.00	1.07	1.08	24.90	30.14	29.44	24.24	20.10	35.30	30.37
Panel B: Models with macrofactors															
AR-X	0.99	0.93	0.96	0.96	0.99	1.01	1.00	1.01	-19.76	-19.83	-17.68	-16.80	-12.36	-8.53	-7.34
VAR-X	0.94	0.90**	0.95**	0.95**	0.94**	0.93*	0.95**	1.00	-15.47	-15.83	-15.17	-16.26	0.79	5.56	10.43
NS2-AR-X	1.04	1.44	1.33	1.20	1.02	0.90*	0.92*	0.95	-84.24	-83.13	-81.22	-64.07	-33.80	-31.69	-27.20
NS2-VAR-X	1.08	1.35	1.32	1.26	1.12	0.97	0.98	1.01	-74.23	-77.25	-80.87	-68.03	-34.23	-28.67	-20.73
NS1-X	0.94	0.92	1.00	1.04	0.99	0.91	0.93	0.98	-41.63	-48.76	-57.80	-50.48	-25.39	-24.08	-20.37
ATSM-X	1.00	1.11	1.14	1.09	1.09	0.90	0.90	0.98	-56.03	-59.29	-61.34	-61.86	-19.17	-6.95	-8.89
Panel C: Forecast combinations															
FC-EW-X	0.96	1.00	1.02	1.01	0.98	0.93	0.93*	0.97	-16.92	-45.43	-46.70	-40.50	-17.36	-12.56	-9.12
FC-MSPE-X	0.95	0.98	1.01	0.99	0.97	0.93*	0.94*	0.97	-43.77	-41.71	-41.86	-36.87	-16.23	-11.66	-8.13
FC-EW-ALL	0.88	0.76**	0.83***	0.85***	0.87***	0.90*	0.92	0.97	-11.08	-12.10	-14.84	-11.12	4.49	7.77	9.35
FC-MSPE-ALL	0.90	0.81**	0.87***	0.89***	0.90***	0.91	0.93*	0.97	-16.92	-17.58	-18.64	-13.98	2.52	6.28	8.75
BAYESIAN INFERENCE															
Panel D: Models without macrofactors															
AR	1.00	0.88	0.90	0.94	0.98	1.02	1.04	1.07	16.34	16.71	16.84	19.82	15.95	20.62	20.59
VAR	1.09	1.49	1.39	1.10	0.92	0.96	1.05	1.18	58.24	55.78	37.80	24.25	27.97	31.20	34.02
NS1	0.97	0.93	0.99	0.99	0.97	0.97	0.99	1.03	-5.47	-7.20	-10.05	-0.74	11.97	7.56	6.33
ATSM	1.01	0.91	0.98	0.96	0.97	1.00	1.07	1.08	24.68	30.04	29.43	24.32	20.26	35.35	30.44
Panel E: Models with macrofactors															
AR-X	1.01	1.02	1.01	1.02	1.01	1.02	1.00	1.00	-30.40	-31.23	-30.11	-28.82	-19.10	-15.34	-12.35
VAR-X	0.88	0.81	0.88	0.90	0.90	0.86	0.89	0.97	-28.56	-32.77	-35.02	-33.27	-8.86	-0.31	7.14
NS1-X	1.01	1.11	1.14	1.12	1.02	0.96	0.98	1.01	-46.42	-49.24	-51.76	-38.20	-14.64	-15.19	-13.50
ATSM-X	0.99	1.11	1.13	1.09	1.08	0.90	0.90	0.96	-54.90	-58.93	-60.91	-61.54	-19.10	-6.87	-8.07
Panel F: Forecast Combinations															
FC-EW-X	0.95	0.96	1.00	0.99	0.98	0.93	0.94	0.97	-35.06	-37.22	-38.12	-33.56	-11.81	-6.26	-3.30
FC-MSPE-X	0.96	0.98	1.01	1.00	0.98	0.94	0.95	0.97	-36.14	-37.27	-37.68	-32.83	-12.04	-6.57	-3.16
BMA-X	0.95	0.97	0.99	0.99	0.97*	0.93*	0.93*	0.95	-31.44	-32.47	-32.96	-27.66	-9.85	-4.58	-2.53
FC-EW-ALL	0.93	0.85	0.90*	0.91	0.92*	0.94	0.96	1.00	-9.06	-10.09	-12.95	-11.13	1.90	7.05	8.32
FC-MSPE-ALL	0.94	0.88*	0.93**	0.94	0.94*	0.94	0.96	1.00	-15.85	-17.20	-18.86	-14.34	0.77	5.88	7.90
BMA-ALL	0.94	0.89**	0.92**	0.92***	0.93***	0.95	0.95	0.97	-11.13	-10.78	-11.76	-6.81	2.56	6.55	7.93

Notes: The table reports forecast results for a 6-month horizon for the out-of-sample period 1994:1 - 1998:12. See Table 3 for further details.

Table 10: 1994:1 - 1998:12, h=12

Maturity	TRMSPE	RMSPE							MPE						
	all	3m	6m	1y	2y	5y	7y	10y	3m	6m	1y	2y	5y	7y	10y
RW	423.60	104.32	113.44	126.05	133.67	124.01	114.05	109.81	-35.86	-34.57	-32.03	-20.50	0.76	9.15	18.70
FREQUENTIST INFERENCE															
Panel A: Model without macrofactors															
AR	0.96	0.82	0.83	0.85	0.91	1.00	1.04	1.07	36.64	36.42	41.40	40.36	46.74	54.33	59.05
VAR	1.14	1.15	1.13	1.00	0.97	1.13	1.24	1.31	104.26	110.56	102.73	100.03	110.86	112.49	113.02
NS2-AR	0.82	0.66***	0.67***	0.66***	0.73***	0.89***	0.93**	0.97	26.82	29.65	33.05	48.71	67.32	63.70	62.93
NS2-VAR	0.94	0.87	0.86	0.82	0.85	0.97	1.01	1.08	40.75	37.52	32.09	38.54	53.34	51.02	51.95
NS1	0.93	0.98	0.91	0.82	0.83*	0.92	0.96	1.02	66.46	56.37	41.92	40.54	51.80	49.07	49.57
ATSM	0.96	0.86	0.89	0.87	0.89	0.97	1.07	1.08	42.83	49.07	49.65	45.95	44.40	60.22	54.94
Panel B: Models with macrofactors															
AR-X	1.01	0.93***	0.97***	0.96***	1.00	1.05	1.02	1.04	-50.63	-52.23	-46.08	-44.37	-30.34	-20.44	-17.28
VAR-X	0.90	0.86***	0.89***	0.89***	0.89***	0.90***	0.93***	0.97	-27.07	-27.80	-26.37	-20.80	8.36	15.69	22.54
NS2-AR-X	1.09	1.50	1.38	1.23	1.06	0.92**	0.94**	0.94**	-134.84	-130.74	-123.38	-97.34	-51.69	-44.56	-35.86
NS2-VAR-X	1.15	1.49	1.42	1.31	1.16	1.00*	1.00**	1.00	-133.10	-133.27	-131.46	-109.16	-57.64	-45.99	-32.89
NS1-X	0.91	0.96	0.99	0.99	0.93**	0.85***	0.87***	0.90**	-71.77	-78.77	-86.03	-72.93	-34.35	-28.61	-21.35
ATSM-X	0.98	1.13	1.13	1.06	1.02	0.87***	0.88***	0.94**	-94.59	-96.24	-94.20	-87.62	-30.65	-13.17	-11.33
Panel C: Forecast combinations															
FC-EW-X	0.95	1.04	1.04	1.00	0.96**	0.91***	0.92***	0.94**	-33.44	-79.09	-77.08	-64.67	-27.93	-18.28	-11.07
FC-MSPE-X	0.96	1.03	1.03	0.99	0.96**	0.92***	0.92***	0.95**	-78.27	-76.29	-73.40	-61.88	-27.49	-17.52	-10.34
FC-EW-ALL	0.93	0.71***	0.75***	0.76***	0.79***	0.85***	0.89***	0.94	-17.70	-18.00	-18.36	-10.66	13.76	20.22	24.15
FC-MSPE-ALL	0.86	0.80***	0.82***	0.82***	0.83***	0.87***	0.90***	0.94*	-33.44	-32.78	-30.50	-20.69	6.74	14.57	20.19
BAYESIAN INFERENCE															
Panel D: Models without macrofactors															
AR	0.96	0.80	0.85	0.86	0.93	0.99	1.02	1.10	27.54	26.81	29.36	26.60	26.53	39.01	42.58
VAR	1.32	1.69	1.58	1.30	1.13	1.19	1.29	1.36	161.27	163.47	146.45	129.57	120.95	117.37	113.38
NS1	0.95	0.94**	0.98	0.96***	0.94**	0.95***	0.97***	1.00	-25.97	-27.82	-28.51	-13.14	11.98	11.35	13.26
ATSM	0.96	0.86	0.90	0.87	0.89	0.97	1.07	1.08	43.14	49.41	49.77	46.04	44.26	59.98	54.65
Panel E: Models with macrofactors															
AR-X	1.05	1.09	1.10	1.07	1.06	1.06*	1.03**	1.02*	-73.04	-75.62	-73.14	-67.84	-44.11	-33.73	-26.11
VAR-X	0.74	0.59***	0.58***	0.58***	0.62***	0.77***	0.86*	0.95	16.50	12.81	11.45	16.71	45.69	52.38	58.44
NS1-X	0.99	1.07	1.08	1.06	1.00	0.94***	0.96***	0.97*	-69.71	-72.66	-73.03	-53.26	-16.30	-12.64	-7.04
ATSM-X	0.98	1.13	1.13	1.06	1.02	0.87***	0.88***	0.96**	-94.29	-96.06	-94.25	-87.45	-31.13	-13.65	-10.12
Panel F: Forecast Combinations															
FC-EW-X	0.89	0.87***	0.90***	0.89***	0.89***	0.88***	0.90***	0.93**	-51.28	-53.22	-52.20	-42.47	-9.02	0.30	6.77
FC-MSPE-X	0.92	0.94***	0.96***	0.94***	0.93***	0.91***	0.91***	0.93**	-58.90	-59.51	-57.57	-47.24	-12.64	-1.42	7.58
BMA-X	0.93	0.96***	0.97***	0.96***	0.94***	0.92***	0.92***	0.94***	-51.23	-51.79	-50.77	-38.94	-9.99	-0.93	6.01
FC-EW-ALL	0.85	0.72***	0.76***	0.77***	0.81***	0.88***	0.92**	0.97	-5.60	-6.02	-7.10	-2.58	17.63	25.47	28.64
FC-MSPE-ALL	0.89	0.81***	0.85***	0.86***	0.87***	0.91***	0.93***	0.97	-28.23	-29.21	-28.13	-19.19	7.94	17.81	24.74
BMA-ALL	0.91	0.88***	0.89***	0.89***	0.90***	0.92***	0.94***	0.95	-27.78	-27.01	-26.38	-13.88	7.95	14.76	20.32

Notes: The table reports forecast results for a 12-month horizon for the out-of-sample period 1994:1 - 1998:12. See Table 3 for further details.

Table 11: 1999:1 - 2003:12, h=1

Maturity	TRMSPE	RMSPE							MPE						
	all	3m	6m	1y	2y	5y	7y	10y	3m	6m	1y	2y	5y	7y	10y
RW	107.37	24.09	24.29	26.26	30.02	32.17	30.97	29.24	5.85	5.81	5.59	4.47	2.21	1.36	0.30
FREQUENTIST INFERENCE															
Panel A: Model without macrofactors															
AR	1.04	1.12	1.11	1.10	1.05	1.01	1.01	1.01	12.71	12.34	12.59	10.08	6.01	5.09	3.56
VAR	1.08	1.04	1.18	1.22	1.22	1.05	1.04	1.07	10.53	13.57	16.53	14.77	7.95	6.46	10.98
NS2-AR	1.19	1.24	1.42	1.48	1.36	1.16	1.11	1.14	11.24	15.04	18.90	21.34	15.65	8.16	13.61
NS2-VAR	1.06	0.96	1.13	1.20	1.18	1.06	1.04	1.07	6.56	9.64	12.59	14.51	10.12	3.52	9.82
NS1	1.09	1.10	1.11	1.13	1.17	1.10	1.05	1.06	13.02	12.31	11.79	13.71	11.57	3.53	7.51
ATSM	1.09	0.96	1.21	1.41	1.27	1.03	1.05	1.07	7.27	15.18	23.14	18.21	-3.55	8.20	12.28
Panel B: Models with macrofactors															
AR-X	1.01	0.99	1.00	1.02	1.00	1.01	1.01	1.01	5.51	5.67	6.30	5.04	2.06	1.66	-0.04
VAR-X	1.06	1.02	1.06	1.07	1.18	1.06	1.06	1.04	3.02	2.50	6.02	7.18	3.80	1.50	5.19
NS2-AR-X	1.14	1.17	1.29	1.34	1.28	1.13	1.12	1.10	0.25	4.19	8.46	11.95	8.91	2.46	8.82
NS2-VAR-X	1.07	0.98	1.10	1.18	1.17	1.07	1.07	1.06	-1.55	1.72	5.12	8.04	6.04	0.34	7.44
NS1-X	1.07	1.08	1.06	1.08	1.14	1.10	1.06	1.05	5.80	5.48	5.70	8.92	8.97	1.59	6.09
ATSM-X	1.10	0.96	1.04	1.26	1.33	1.12	1.05	1.07	-7.19	-0.33	8.91	8.00	6.87	6.02	7.97
Panel C: Forecast combinations															
FC-EW-X	1.03	0.96	1.03	1.09	1.13	1.05	1.04	1.03	5.66	3.58	6.58	7.66	5.55	2.13	5.11
FC-MSPE-X	1.03	0.96	1.02	1.07	1.11	1.04	1.04	1.02	1.67	3.87	6.50	7.33	5.31	2.04	4.87
FC-EW-ALL	1.04	0.97	1.06	1.14	1.15	1.04	1.04	1.03	5.62	7.93	10.89	11.25	6.66	3.84	7.19
FC-MSPE-ALL	1.03	0.97	1.05	1.11	1.13	1.04	1.03	1.03	5.66	7.61	10.31	10.70	6.23	3.69	6.95
BAYESIAN INFERENCE															
Panel D: Models without macrofactors															
AR	1.03	1.10	1.08	1.08	1.03	1.01	1.01	1.00	11.69	11.26	11.40	8.78	4.54	3.58	2.00
VAR	1.05	1.05	1.19	1.16	1.14	1.03	1.04	1.04	10.43	13.19	15.45	12.54	4.89	3.46	7.86
NS1	1.11	1.08	1.12	1.14	1.25	1.13	1.05	1.08	12.02	10.46	11.73	17.77	13.49	2.98	5.00
ATSM	1.09	0.96	1.20	1.41	1.27	1.03	1.06	1.08	7.12	15.08	22.84	18.05	-3.57	8.29	12.37
Panel E: Models with macrofactors															
AR-X	1.00	0.97	0.99	1.01	1.00	1.00	1.01	1.01	4.95	5.05	5.57	4.31	1.77	1.34	-0.02
VAR-X	1.06	1.01	1.05	1.06	1.17	1.06	1.07	1.04	2.73	2.25	5.75	6.90	3.57	1.17	4.81
NS1-X	1.25	1.57	1.58	1.46	1.23	1.12	1.23	1.18	-30.33	-32.23	-29.85	-19.76	-14.54	-21.58	-16.39
ATSM-X	1.08	0.98	0.98	1.22	1.27	1.10	1.05	1.18	-6.11	-1.11	7.10	6.64	6.71	6.91	9.34
Panel F: Forecast Combinations															
FC-EW-X	0.99	0.93	0.93	0.99	1.05	1.00	1.01	0.99	-4.58	-4.05	-1.17	0.51	-0.06	-2.16	-0.39
FC-MSPE-X	0.99	0.92	0.94	0.99	1.04	1.00	1.01	0.99	-0.11	0.29	2.23	1.69	0.16	-1.33	0.07
BMA-X	1.01	0.92	0.93	1.02	1.10	1.03	1.03	1.02	-3.64	-2.04	1.91	2.77	2.00	1.18	3.63
FC-EW-ALL	0.99	0.91	0.95	1.01	1.07	1.01	1.01	1.00	2.04	3.31	6.18	6.63	2.12	0.83	2.81
FC-MSPE-ALL	1.00	0.94	0.97	1.03	1.07	1.00	1.01	1.00	4.59	5.26	7.03	7.03	2.06	1.26	3.04
BMA-ALL	1.01	0.90	0.95	1.05	1.11	1.02	1.03	1.03	3.08	4.99	8.50	9.07	3.26	3.36	6.13

Notes: The table reports forecast results for a 1-month horizon for the out-of-sample period 1999:1 - 2003:12. See Table 3 for further details.

Table 12: 1999:1 - 2003:12, h=3

Maturity	TRMSPE	RMSPE							MPE						
	all	3m	6m	1y	2y	5y	7y	10y	3m	6m	1y	2y	5y	7y	10y
RW	197.06	56.75	56.26	55.97	58.38	55.60	50.80	47.81	17.43	17.08	16.16	12.45	6.14	3.32	0.94
FREQUENTIST INFERENCE															
Panel A: Model without macrofactors															
AR	1.09	1.17	1.16	1.17	1.10	1.04	1.03	1.02	37.37	36.16	36.65	29.06	17.48	14.48	10.72
VAR	1.18	1.10	1.25	1.37	1.35	1.16	1.11	1.08	38.94	45.28	49.80	45.38	30.61	25.36	26.76
NS2-AR	1.34	1.30	1.45	1.58	1.53	1.32	1.23	1.17	35.49	41.77	48.48	51.62	37.92	25.87	27.04
NS2-VAR	1.10	1.00	1.13	1.24	1.24	1.11	1.06	1.03	27.04	31.19	35.09	36.21	24.97	14.96	18.15
NS1	1.13	1.09	1.17	1.24	1.25	1.14	1.07	1.02	36.32	36.25	35.78	35.51	25.88	15.02	16.65
ATSM	1.10	0.98	1.17	1.35	1.28	1.03	1.08	1.04	27.83	37.81	46.79	40.26	11.82	20.22	20.84
Panel B: Models with macrofactors															
AR-X	1.01	0.98	0.99	1.02	1.00	1.01	1.02	1.01	16.98	17.20	18.72	14.42	5.75	4.15	-0.17
VAR-X	1.02	1.01	1.03	1.05	1.09	1.03	1.02	0.96	18.09	17.36	20.49	19.33	11.14	7.00	8.97
NS2-AR-X	1.23	1.16	1.27	1.39	1.38	1.24	1.21	1.12	3.00	9.69	17.58	23.70	17.59	8.54	12.33
NS2-VAR-X	1.10	0.94	1.06	1.19	1.23	1.14	1.12	1.04	0.65	5.51	10.99	15.41	11.52	4.27	9.84
NS1-X	1.09	0.99	1.06	1.15	1.20	1.15	1.11	1.03	13.91	15.05	16.89	20.55	17.43	8.47	11.62
ATSM-X	1.07	0.85	0.98	1.18	1.23	1.14	1.10	1.04	1.11	9.55	18.80	17.38	13.63	12.26	13.03
Panel C: Forecast combinations															
FC-EW-X	1.05	0.96	1.02	1.10	1.13	1.07	1.06	1.01	20.26	13.06	17.09	17.61	11.89	6.86	8.08
FC-MSPE-X	1.04	0.95	1.01	1.08	1.11	1.07	1.06	1.01	10.17	14.01	17.59	17.26	11.53	6.76	7.92
FC-EW-ALL	1.07	0.99	1.08	1.18	1.18	1.08	1.06	1.01	21.09	24.61	28.63	27.79	17.84	12.61	13.59
FC-MSPE-ALL	1.06	0.98	1.06	1.15	1.16	1.07	1.06	1.01	20.26	23.37	27.22	26.30	16.85	12.08	13.07
BAYESIAN INFERENCE															
Panel D: Models without macrofactors															
AR	1.07	1.14	1.14	1.14	1.07	1.05	1.02	1.00	34.87	34.23	33.30	25.79	13.71	10.18	6.63
VAR	1.14	1.17	1.27	1.29	1.24	1.09	1.06	1.02	38.44	43.42	45.99	39.31	22.71	17.54	18.78
NS1	1.04	1.02	1.05	1.07	1.10	1.03	1.01	0.99**	22.79	21.16	21.54	25.49	17.12	5.23	6.25
ATSM	1.10	0.99	1.18	1.36	1.29	1.04	1.09	1.04	28.04	38.01	46.88	40.37	11.86	20.15	20.71
Panel E: Models with macrofactors															
AR-X	0.98	0.95	0.95	0.98	0.99	0.99	1.01	1.00	12.15	12.75	14.17	10.28	3.00	1.76	-0.97
VAR-X	1.10	0.98	1.07	1.16	1.23	1.14	1.09	0.99	11.91	13.81	20.24	22.02	15.82	11.24	12.80
NS1-X	0.99	0.90	0.94	0.98	1.00	1.00	1.06	1.03	-18.81	-20.66	-19.00	-11.23	-10.61	-19.13	-15.26
ATSM-X	1.08	0.86	0.95	1.18	1.24	1.15	1.11	1.12	2.04	9.23	17.51	16.14	13.53	12.80	14.02
Panel F: Forecast Combinations															
FC-EW-X	0.98	0.89	0.93	1.00	1.05	1.02	1.01	0.98	4.94	6.44	9.82	9.93	5.58	2.00	2.31
FC-MSPE-X	0.98	0.89	0.93	1.00	1.03	1.01	1.01	0.98	5.66	7.01	9.87	9.27	5.07	1.87	2.35
BMA-X	0.99	0.90	0.94	1.02	1.05	1.03	1.03	0.99	5.98	7.47	10.54	10.05	6.03	3.73	4.50
FC-EW-ALL	1.00	0.94	0.99	1.06	1.08	1.02	1.01	0.98	16.54	18.78	21.87	20.07	10.37	7.01	7.10
FC-MSPE-ALL	1.00	0.93	0.98	1.04	1.06	1.02	1.01	0.98	15.19	16.89	20.36	19.12	10.01	6.59	6.83
BMA-ALL	0.99	0.91	0.95	1.03	1.06	1.02	1.02	0.98	14.17	15.59	18.56	18.22	9.78	7.11	8.13

Notes: The table reports forecast results for a 3-month horizon for the out-of-sample period 1999:1 - 2003:12. See Table 3 for further details.

Table 13: 1999:1 - 2003:12, h=6

Maturity	TRMSPE	RMSPE							MPE						
	all	3m	6m	1y	2y	5y	7y	10y	3m	6m	1y	2y	5y	7y	10y
RW	308.65	97.90	97.58	95.57	93.89	84.04	74.90	65.40	36.33	35.95	35.26	28.03	14.35	8.52	3.61
FREQUENTIST INFERENCE															
Panel A: Model without macrofactors															
AR	1.13	1.19	1.18	1.21	1.14	1.06	1.06	1.04	74.01	72.32	74.33	60.30	36.77	30.64	23.08
VAR	1.33	1.30	1.41	1.51	1.47	1.26	1.22	1.25	94.27	103.20	107.23	97.88	71.94	62.14	59.81
NS2-AR	1.34	1.27	1.37	1.49	1.50	1.33	1.25	1.23	64.98	73.31	82.17	85.06	62.21	45.32	42.00
NS2-VAR	1.11	1.07	1.15	1.23	1.24	1.09	1.04	1.06	61.92	66.21	69.51	67.59	46.32	32.04	31.40
NS1	1.15	1.16	1.21	1.25	1.24	1.11	1.05	1.05	73.77	73.71	72.07	67.49	46.75	31.97	30.37
ATSM	1.11	1.06	1.19	1.31	1.26	1.02	1.07	1.06	63.13	73.92	82.38	72.44	34.34	38.10	33.94
Panel B: Models with macrofactors															
AR-X	1.00	0.98	0.98	1.01	1.01	1.00	1.02	1.01	35.39	36.13	39.69	31.27	12.88	9.38	0.63
VAR-X	1.02	1.02	1.02	1.05	1.06	1.01	0.99	0.98	40.65	40.33	43.41	38.80	23.07	16.09	15.51
NS2-AR-X	1.20	1.11	1.19	1.29	1.32	1.22	1.20	1.17	2.74	11.90	23.02	31.42	22.27	10.75	12.14
NS2-VAR-X	1.07	0.93	1.02	1.14	1.18	1.11	1.09	1.07	4.60	10.63	17.55	22.66	16.12	7.19	11.17
NS1-X	1.08	0.97	1.04	1.12	1.17	1.11	1.08	1.06	25.76	28.40	31.77	35.30	27.36	16.26	17.56
ATSM-X	1.04	0.88	0.99	1.12	1.15	1.08	1.07	1.06	14.55	24.21	33.36	29.84	21.87	18.76	18.04
Panel C: Forecast combinations															
FC-EW-X	1.03	0.95	1.00	1.07	1.10	1.05	1.04	1.03	42.80	26.79	32.01	31.04	19.70	12.42	11.24
FC-MSPE-X	1.02	0.94	0.99	1.05	1.08	1.04	1.04	1.03	22.86	29.21	33.98	31.47	19.52	12.47	11.15
FC-EW-ALL	1.07	1.01	1.08	1.16	1.16	1.06	1.04	1.03	45.55	50.02	54.75	51.39	33.56	25.17	23.02
FC-MSPE-ALL	1.05	1.00	1.06	1.13	1.14	1.05	1.04	1.03	42.80	46.87	51.79	48.46	31.52	23.71	21.53
BAYESIAN INFERENCE															
Panel D: Models without macrofactors															
AR	1.12	1.18	1.18	1.17	1.12	1.10	1.05	1.10	69.87	68.75	68.63	54.22	30.37	22.54	16.20
VAR	1.36	1.41	1.51	1.55	1.46	1.24	1.21	1.28	109.31	112.83	109.56	94.53	65.14	56.17	54.56
NS1	1.02	1.02	1.03	1.04	1.06	1.00	0.99	1.02	42.66	41.24	40.83	41.25	25.09	10.91	10.06
ATSM	1.12	1.07	1.20	1.32	1.27	1.03	1.08	1.07	63.02	73.87	82.40	72.45	34.26	37.95	33.94
Panel E: Models with macrofactors															
AR-X	0.96	0.93	0.93	0.96	0.96	0.97	1.00	0.98	25.23	26.29	29.29	21.87	7.52	4.85	-1.39
VAR-X	1.14	1.06	1.14	1.24	1.28	1.16	1.09	1.04	40.50	44.15	52.49	53.22	41.83	34.71	33.90
NS1-X	0.93	0.85	0.87	0.91	0.94	0.96	1.00	1.02	2.14	0.63	1.49	5.42	-1.77	-12.59	-10.89
ATSM-X	1.04	0.89	0.99	1.12	1.15	1.08	1.07	1.07	15.76	24.39	31.80	28.41	21.34	19.54	18.23
Panel F: Forecast Combinations															
FC-EW-X	0.98	0.92	0.95	1.01	1.03	1.00	0.99	0.97	23.99	26.28	30.06	27.39	16.65	11.01	8.69
FC-MSPE-X	0.97	0.91	0.95	1.00	1.02	1.00	0.99	0.97	23.24	25.43	28.75	25.86	15.81	10.71	8.68
BMA-X	0.97	0.94	0.96	1.00	1.00	0.99	0.99	0.98	25.31	26.35	27.98	23.95	13.82	9.46	7.65
FC-EW-ALL	1.02	0.99	1.04	1.09	1.09	1.01	1.00	1.00	44.98	47.56	50.19	44.38	26.46	20.29	17.58
FC-MSPE-ALL	1.01	0.97	1.01	1.06	1.06	1.01	1.00	0.99	38.84	41.03	44.24	40.40	24.98	18.99	16.27
BMA-ALL	0.99	0.98	1.00	1.03	1.02	0.98	0.98	0.97	37.92	38.03	39.50	35.61	19.94	14.57	12.85

Notes: The table reports forecast results for a 6-month horizon for the out-of-sample period 1999:1 - 2003:12. See Table 3 for further details.

Table 14: 1999:1 - 2003:12, h=12

Maturity	TRMSPE	RMSPE							MPE						
	all	3m	6m	1y	2y	5y	7y	10y	3m	6m	1y	2y	5y	7y	10y
RW	479.67	169.29	170.87	165.13	149.43	118.35	102.82	86.76	76.24	76.82	76.64	67.75	41.79	30.62	20.22
FREQUENTIST INFERENCE															
Panel A: Model without macrofactors															
AR	1.20	1.20	1.19	1.23	1.21	1.15	1.16	1.15	142.06	141.45	146.34	127.06	85.12	73.44	58.54
VAR	1.62	1.49	1.55	1.62	1.67	1.65	1.70	1.86	205.78	217.30	219.81	206.53	170.05	155.74	149.72
NS2-AR	1.28	1.15	1.20	1.29	1.38	1.36	1.31	1.34	110.34	119.88	129.74	131.47	100.07	79.01	71.87
NS2-VAR	1.18	1.14	1.16	1.21	1.25	1.18	1.14	1.20	133.05	136.55	137.70	130.53	96.17	76.75	71.62
NS1	1.21	1.21	1.21	1.23	1.25	1.18	1.13	1.17	135.27	145.89	152.60	137.86	87.28	84.81	73.72
ATSM	1.19	1.14	1.20	1.27	1.28	1.12	1.19	1.21	135.27	145.89	152.60	137.86	87.28	84.81	73.72
Panel B: Models with macrofactors															
AR-X	1.03	0.96	0.98	1.02	1.05	1.06	1.09	1.10	72.77	75.45	81.53	69.05	34.19	26.74	9.98
VAR-X	1.03	1.01	1.02	1.04	1.06	1.02	1.02	1.04	84.98	86.47	90.77	81.53	54.22	43.08	38.86
NS2-AR-X	1.17	1.05	1.08	1.16	1.24	1.25	1.26	1.26	-0.98	10.19	24.03	34.61	23.99	10.89	10.69
NS2-VAR-X	1.07	0.95	0.99	1.07	1.15	1.14	1.14	1.13	13.50	20.58	28.83	34.71	26.25	16.02	18.74
NS1-X	1.08	0.96	1.00	1.08	1.15	1.16	1.15	1.16	49.38	53.45	58.04	60.71	46.54	32.89	32.06
ATSM-X	1.05	0.94	1.00	1.07	1.12	1.11	1.13	1.13	42.10	52.69	60.51	55.12	42.80	37.83	35.17
Panel C: Forecast combinations															
FC-EW-X	1.03	0.94	0.97	1.03	1.08	1.08	1.09	1.09	86.41	53.67	60.05	57.64	38.54	28.30	23.67
FC-MSPE-X	1.03	0.94	0.97	1.02	1.07	1.08	1.09	1.09	48.28	58.35	64.39	60.09	39.07	28.91	24.09
FC-EW-ALL	1.10	1.03	1.06	1.12	1.16	1.12	1.11	1.12	93.31	98.77	103.76	97.47	69.41	57.06	50.73
FC-MSPE-ALL	1.07	1.01	1.04	1.10	1.13	1.10	1.10	1.10	86.41	91.81	97.74	91.73	63.86	51.64	45.02
BAYESIAN INFERENCE															
Panel D: Models without macrofactors															
AR	1.15	1.19	1.17	1.19	1.15	1.14	1.10	1.10	134.71	132.59	135.48	113.94	71.18	58.16	41.43
VAR	1.85	1.80	1.83	1.86	1.87	1.82	1.86	2.03	269.72	276.96	269.69	243.22	190.66	172.85	163.91
NS1	1.02	1.01	1.01	1.02	1.03	1.01	0.99	1.02	82.24	82.79	82.67	78.68	50.72	32.49	28.02
ATSM	1.20	1.15	1.20	1.28	1.28	1.13	1.19	1.21	135.34	145.94	152.65	137.87	87.39	84.96	73.90
Panel E: Models with macrofactors															
AR-X	0.97	0.90	0.93	0.97	1.00	1.01	1.04	1.03	53.42	55.41	62.04	49.26	23.18	18.15	6.12
VAR-X	1.37	1.23	1.27	1.35	1.43	1.43	1.44	1.51	132.22	137.47	147.04	145.37	124.85	114.18	109.69
NS1-X	0.93	0.89	0.90	0.92	0.94	0.95	0.97	0.99	43.54	44.02	45.32	45.07	25.19	9.85	7.80
ATSM-X	1.05	0.94	0.99	1.07	1.12	1.11	1.13	1.17	42.71	51.02	58.27	53.05	42.20	38.76	34.57
Panel F: Forecast Combinations															
FC-EW-X	1.02	0.96	0.98	1.02	1.06	1.05	1.05	1.05	69.63	72.95	77.86	72.10	51.44	42.31	35.68
FC-MSPE-X	1.01	0.95	0.97	1.01	1.04	1.04	1.04	1.04	66.70	70.41	75.29	69.55	49.27	40.74	34.01
BMA-X	0.98	0.96	0.97	0.99	1.00	1.00	1.00	1.00	67.08	68.13	69.34	62.31	41.08	32.33	25.98
FC-EW-ALL	1.10	1.06	1.08	1.13	1.14	1.10	1.10	1.11	107.79	111.45	114.42	103.80	73.02	62.23	53.96
FC-MSPE-ALL	1.05	1.01	1.03	1.07	1.09	1.07	1.06	1.07	90.06	93.32	97.82	90.42	63.68	53.29	45.54
BMA-ALL	1.01	1.01	1.01	1.02	1.03	1.00	0.99	0.99	83.69	84.65	86.10	79.53	53.30	42.11	34.96

Notes: The table reports forecast results for a 12-month horizon for the out-of-sample period 1999:1 - 2003:12. See Table 3 for further details.

Table 15: Hit Rate, 1994:1 - 2003:12

Maturity	h=1							h=3						
	3m	6m	1y	2y	5y	7y	10y	3m	6m	1y	2y	5y	7y	10y
FREQUENTIST INFERENCE														
Panel A: Model without macrofactors														
AR	0.55	0.54	0.51	0.61	0.59	0.64	0.54	0.69	0.70	0.68	0.68	0.59	0.59	0.61
VAR	0.51	0.55	0.51	0.62	0.61	0.62	0.58	0.70	0.69	0.68	0.69	0.64	0.61	0.60
NS2-AR	0.49	0.57	0.53	0.60	0.59	0.61	0.58	0.66	0.69	0.69	0.70	0.62	0.61	0.60
NS2-VAR	0.51	0.56	0.53	0.61	0.60	0.61	0.58	0.68	0.69	0.69	0.70	0.63	0.61	0.58
NS1	0.53	0.53	0.53	0.61	0.60	0.59	0.57	0.63	0.68	0.70	0.69	0.64	0.62	0.59
ATSM	0.53	0.59	0.50	0.62	0.62	0.63	0.58	0.69	0.68	0.69	0.69	0.63	0.60	0.60
Panel B: Models with macrofactors														
AR-X	0.57	0.57	0.53	0.64	0.59	0.64	0.55	0.64	0.64	0.63	0.64	0.59	0.57	0.59
VAR-X	0.53	0.57	0.53	0.61	0.62	0.62	0.58	0.67	0.70	0.71	0.65	0.62	0.60	0.59
NS2-AR-X	0.52	0.59	0.54	0.59	0.62	0.62	0.59	0.62	0.65	0.68	0.66	0.62	0.60	0.59
NS2-VAR-X	0.52	0.59	0.54	0.60	0.62	0.62	0.58	0.64	0.67	0.66	0.66	0.58	0.58	0.59
NS1-X	0.55	0.56	0.55	0.63	0.61	0.62	0.58	0.65	0.65	0.66	0.63	0.62	0.58	0.59
ATSM-X	0.49	0.59	0.54	0.59	0.62	0.66	0.57	0.62	0.64	0.68	0.65	0.61	0.59	0.57
Panel C: Forecast combinations														
FC-EW-X	0.54	0.56	0.53	0.63	0.62	0.63	0.59	0.67	0.68	0.68	0.67	0.59	0.58	0.58
FC-MSPE-X	0.54	0.56	0.51	0.63	0.62	0.63	0.59	0.65	0.69	0.68	0.67	0.60	0.58	0.58
FC-EW-ALL	0.53	0.56	0.50	0.63	0.61	0.63	0.59	0.67	0.70	0.69	0.71	0.64	0.59	0.59
FC-MSPE-ALL	0.51	0.56	0.50	0.63	0.61	0.63	0.59	0.67	0.70	0.68	0.71	0.64	0.59	0.59
BAYESIAN INFERENCE														
Panel D: Model without macrofactors														
AR	0.55	0.54	0.51	0.61	0.59	0.64	0.54	0.67	0.68	0.64	0.66	0.58	0.56	0.58
VAR	0.53	0.53	0.51	0.62	0.59	0.62	0.58	0.59	0.58	0.62	0.61	0.58	0.56	0.56
NS1	0.55	0.57	0.51	0.60	0.60	0.60	0.59	0.67	0.69	0.69	0.70	0.62	0.60	0.58
ATSM	0.54	0.58	0.53	0.59	0.61	0.63	0.61	0.71	0.68	0.69	0.69	0.64	0.61	0.60
Panel E: Models with macrofactors														
AR-X	0.60	0.58	0.53	0.64	0.59	0.62	0.55	0.64	0.65	0.64	0.65	0.58	0.58	0.58
VAR-X	0.53	0.59	0.53	0.62	0.61	0.62	0.58	0.64	0.67	0.69	0.66	0.62	0.61	0.58
NS1-X	0.56	0.58	0.53	0.64	0.61	0.63	0.60	0.62	0.67	0.69	0.65	0.60	0.58	0.57
ATSM-X	0.54	0.61	0.55	0.59	0.63	0.64	0.53	0.64	0.68	0.66	0.66	0.58	0.59	0.60
Panel F: Forecast Combinations														
FC-EW-X	0.57	0.58	0.53	0.64	0.63	0.64	0.58	0.65	0.67	0.68	0.66	0.61	0.58	0.58
FC-MSPE-X	0.57	0.57	0.54	0.64	0.63	0.64	0.58	0.65	0.66	0.69	0.67	0.61	0.58	0.58
BMA-X	0.56	0.57	0.53	0.64	0.61	0.64	0.57	0.66	0.65	0.64	0.64	0.58	0.58	0.57
FC-EW-ALL	0.55	0.56	0.51	0.63	0.61	0.63	0.60	0.67	0.70	0.67	0.69	0.61	0.59	0.58
FC-MSPE-ALL	0.53	0.57	0.52	0.64	0.59	0.63	0.59	0.66	0.72	0.69	0.68	0.61	0.58	0.58
BMA-ALL	0.54	0.57	0.53	0.60	0.62	0.60	0.59	0.67	0.68	0.69	0.68	0.60	0.59	0.58

Notes: The table reports the Hit Rate (HR) for *individual* yield models, with and without macrofactors, estimated using the frequentist approach (Panels A and B) and using Bayesian inference (Panels D and E). Panels C and F show results for different forecast *combination* methods for the frequentist and Bayesian estimated models. Results are for 1-month and 3-month horizons for the out-of-sample period 1994:1 - 2003:12. The following model abbreviations are used: RW stands for the Random Walk, (V)AR for the first-order (Vector) Autoregressive Model, NS2-(V)AR for two-step Nelson-Siegel model with a (V)AR specification for the factors, NS1 for the one-step Nelson-Siegel model, ATSM for the affine model, FC-EW and FC-MSPE for the forecast combination based on equal weights and MSPE-based weights respectively, BMA for the Bayesian Model Averaging forecast. The affix 'X' indicates that macrofactors have been added as additional variables.

Table 16: Hit Rate, 1994:1 - 2003:12

Maturity	h=6							h=12						
	3m	6m	1y	2y	5y	7y	10y	3m	6m	1y	2y	5y	7y	10y
FREQUENTIST INFERENCE														
Panel A: Model without macrofactors														
AR	0.66	0.67	0.53	0.57	0.54	0.54	0.50	0.52	0.53	0.52	0.43	0.39	0.40	0.34
VAR	0.68	0.65	0.57	0.56	0.55	0.56	0.51	0.56	0.53	0.56	0.50	0.42	0.43	0.39
NS2-AR	0.69	0.69	0.56	0.59	0.56	0.56	0.55	0.60	0.59	0.56	0.47	0.41	0.41	0.34
NS2-VAR	0.68	0.64	0.56	0.57	0.53	0.54	0.50	0.51	0.52	0.57	0.49	0.43	0.44	0.43
NS1	0.66	0.65	0.57	0.55	0.50	0.52	0.49	0.48	0.50	0.55	0.50	0.45	0.46	0.42
ATSM	0.68	0.64	0.57	0.57	0.52	0.53	0.50	0.55	0.53	0.56	0.49	0.45	0.42	0.43
Panel B: Models with macrofactors														
AR-X	0.59	0.58	0.58	0.56	0.50	0.50	0.49	0.51	0.50	0.53	0.47	0.40	0.43	0.38
VAR-X	0.68	0.67	0.57	0.56	0.53	0.51	0.48	0.54	0.54	0.53	0.50	0.44	0.44	0.42
NS2-AR-X	0.66	0.63	0.58	0.57	0.55	0.53	0.50	0.59	0.57	0.51	0.48	0.42	0.40	0.35
NS2-VAR-X	0.64	0.59	0.53	0.54	0.51	0.52	0.48	0.56	0.53	0.48	0.45	0.36	0.37	0.32
NS1-X	0.65	0.61	0.55	0.56	0.53	0.50	0.50	0.61	0.57	0.48	0.47	0.38	0.39	0.37
ATSM-X	0.63	0.62	0.56	0.53	0.51	0.52	0.49	0.57	0.52	0.49	0.46	0.40	0.39	0.37
Panel C: Forecast combinations														
FC-EW-X	0.64	0.65	0.53	0.53	0.51	0.52	0.49	0.61	0.52	0.49	0.46	0.39	0.42	0.38
FC-MSPE-X	0.63	0.63	0.53	0.53	0.51	0.52	0.49	0.56	0.52	0.48	0.46	0.39	0.42	0.38
FC-EW-ALL	0.68	0.68	0.56	0.57	0.54	0.53	0.50	0.61	0.59	0.54	0.51	0.43	0.43	0.40
FC-MSPE-ALL	0.69	0.66	0.56	0.57	0.54	0.52	0.50	0.61	0.58	0.54	0.50	0.42	0.42	0.40
BAYESIAN INFERENCE														
Panel D: Model without macrofactors														
AR	0.65	0.69	0.54	0.61	0.50	0.53	0.50	0.53	0.51	0.51	0.46	0.41	0.39	0.37
VAR	0.59	0.56	0.52	0.53	0.55	0.54	0.49	0.44	0.48	0.52	0.55	0.40	0.42	0.36
NS1	0.70	0.64	0.53	0.56	0.54	0.51	0.46	0.52	0.52	0.50	0.49	0.41	0.43	0.38
ATSM	0.69	0.65	0.57	0.54	0.52	0.54	0.50	0.52	0.51	0.55	0.50	0.45	0.44	0.42
Panel E: Models with macrofactors														
AR-X	0.60	0.55	0.57	0.55	0.50	0.51	0.50	0.45	0.45	0.50	0.48	0.41	0.43	0.37
VAR-X	0.65	0.63	0.57	0.57	0.51	0.51	0.48	0.58	0.60	0.59	0.55	0.50	0.50	0.46
NS1-X	0.59	0.56	0.51	0.51	0.50	0.49	0.47	0.38	0.40	0.47	0.45	0.41	0.41	0.38
ATSM-X	0.62	0.62	0.57	0.53	0.52	0.52	0.53	0.57	0.55	0.47	0.46	0.40	0.39	0.34
Panel F: Forecast Combinations														
FC-EW-X	0.63	0.59	0.56	0.54	0.51	0.50	0.50	0.61	0.53	0.53	0.48	0.40	0.45	0.39
FC-MSPE-X	0.61	0.58	0.55	0.54	0.51	0.50	0.50	0.57	0.52	0.53	0.47	0.39	0.44	0.40
BMA-X	0.63	0.60	0.54	0.54	0.50	0.50	0.48	0.44	0.46	0.51	0.47	0.46	0.45	0.39
FC-EW-ALL	0.68	0.65	0.55	0.57	0.53	0.51	0.50	0.53	0.55	0.55	0.50	0.44	0.44	0.43
FC-MSPE-ALL	0.69	0.64	0.56	0.56	0.53	0.51	0.50	0.57	0.59	0.54	0.50	0.43	0.46	0.44
BMA-ALL	0.67	0.64	0.53	0.55	0.50	0.50	0.49	0.52	0.51	0.53	0.52	0.49	0.47	0.41

Notes: The table reports Hit Rate results for 6-month and 12-month horizons for the out-of-sample period 1994:1 - 2003:12. See Table 15 for further details.

Table 17: Hit Rate, 1994:1 - 1998:12

Maturity	h=1							h=3						
	3m	6m	1y	2y	5y	7y	10y	3m	6m	1y	2y	5y	7y	10y
FREQUENTIST INFERENCE														
Panel A: Model without macrofactors														
AR	0.58	0.58	0.55	0.60	0.60	0.63	0.58	0.59	0.64	0.59	0.66	0.66	0.66	0.69
VAR	0.47	0.52	0.52	0.63	0.62	0.63	0.62	0.64	0.64	0.62	0.64	0.67	0.67	0.71
NS2-AR	0.48	0.57	0.58	0.62	0.60	0.63	0.63	0.59	0.67	0.64	0.66	0.69	0.69	0.71
NS2-VAR	0.52	0.55	0.57	0.62	0.62	0.63	0.65	0.64	0.66	0.64	0.66	0.67	0.67	0.67
NS1	0.57	0.60	0.53	0.62	0.62	0.62	0.63	0.55	0.60	0.66	0.66	0.71	0.69	0.67
ATSM	0.52	0.55	0.50	0.63	0.65	0.63	0.62	0.67	0.62	0.62	0.64	0.66	0.67	0.71
Panel B: Models with macrofactors														
AR-X	0.55	0.58	0.57	0.63	0.62	0.65	0.60	0.62	0.62	0.59	0.60	0.62	0.62	0.67
VAR-X	0.50	0.53	0.53	0.62	0.67	0.65	0.63	0.60	0.67	0.67	0.62	0.69	0.67	0.69
NS2-AR-X	0.50	0.60	0.60	0.62	0.62	0.63	0.65	0.57	0.62	0.64	0.66	0.71	0.69	0.67
NS2-VAR-X	0.50	0.60	0.60	0.63	0.62	0.63	0.65	0.55	0.62	0.60	0.62	0.64	0.66	0.67
NS1-X	0.57	0.60	0.55	0.63	0.63	0.63	0.63	0.57	0.64	0.62	0.59	0.67	0.64	0.67
ATSM-X	0.45	0.55	0.58	0.62	0.65	0.65	0.55	0.55	0.62	0.60	0.60	0.67	0.67	0.64
Panel C: Forecast combinations														
FC-EW-X	0.53	0.55	0.57	0.63	0.63	0.63	0.63	0.62	0.67	0.60	0.62	0.66	0.66	0.67
FC-MSPE-X	0.53	0.55	0.55	0.63	0.63	0.63	0.63	0.59	0.67	0.62	0.62	0.66	0.66	0.67
FC-EW-ALL	0.53	0.55	0.52	0.62	0.62	0.65	0.63	0.60	0.67	0.66	0.67	0.71	0.67	0.69
FC-MSPE-ALL	0.50	0.55	0.52	0.62	0.62	0.65	0.63	0.62	0.67	0.66	0.67	0.71	0.67	0.69
BAYESIAN INFERENCE														
Panel D: Model without macrofactors														
AR	0.58	0.58	0.55	0.60	0.60	0.63	0.58	0.55	0.59	0.55	0.62	0.64	0.62	0.67
VAR	0.50	0.52	0.52	0.62	0.60	0.63	0.63	0.48	0.53	0.57	0.57	0.60	0.60	0.64
NS1	0.58	0.62	0.53	0.58	0.63	0.63	0.65	0.60	0.62	0.64	0.66	0.69	0.67	0.66
ATSM	0.53	0.57	0.55	0.60	0.65	0.65	0.65	0.67	0.60	0.62	0.64	0.67	0.67	0.71
Panel E: Models with macrofactors														
AR-X	0.58	0.58	0.58	0.63	0.62	0.62	0.60	0.62	0.62	0.60	0.62	0.62	0.62	0.64
VAR-X	0.50	0.58	0.55	0.63	0.65	0.65	0.63	0.64	0.67	0.67	0.66	0.71	0.71	0.71
NS1-X	0.60	0.60	0.55	0.65	0.63	0.67	0.63	0.57	0.62	0.60	0.60	0.64	0.60	0.62
ATSM-X	0.52	0.58	0.62	0.62	0.70	0.65	0.55	0.60	0.66	0.60	0.60	0.66	0.67	0.69
Panel F: Forecast Combinations														
FC-EW-X	0.52	0.57	0.57	0.65	0.67	0.67	0.63	0.62	0.64	0.60	0.62	0.66	0.62	0.66
FC-MSPE-X	0.52	0.55	0.57	0.65	0.67	0.67	0.63	0.62	0.62	0.60	0.62	0.66	0.62	0.66
BMA-X	0.52	0.57	0.57	0.65	0.63	0.65	0.63	0.57	0.55	0.55	0.59	0.64	0.66	0.66
FC-EW-ALL	0.52	0.55	0.53	0.63	0.65	0.65	0.65	0.64	0.66	0.62	0.64	0.67	0.66	0.67
FC-MSPE-ALL	0.48	0.57	0.55	0.65	0.62	0.65	0.65	0.64	0.67	0.66	0.62	0.67	0.64	0.67
BMA-ALL	0.57	0.60	0.53	0.58	0.63	0.63	0.65	0.60	0.62	0.64	0.66	0.67	0.66	0.66

Notes: The table reports Hit Rate results for 1-month and 3-month horizons for the out-of-sample period 1994:1 - 1998:12. See Table 15 for further details.

Table 18: Hit Rate, 1994:1 - 1998:12

Maturity	h=6							h=12						
	3m	6m	1y	2y	5y	7y	10y	3m	6m	1y	2y	5y	7y	10y
FREQUENTIST INFERENCE														
Panel A: Model without macrofactors														
AR	0.53	0.58	0.44	0.55	0.55	0.55	0.56	0.45	0.45	0.45	0.27	0.29	0.27	0.29
VAR	0.56	0.58	0.51	0.53	0.53	0.53	0.51	0.49	0.45	0.53	0.47	0.45	0.45	0.43
NS2-AR	0.60	0.60	0.49	0.56	0.55	0.58	0.58	0.55	0.53	0.47	0.33	0.35	0.35	0.33
NS2-VAR	0.56	0.56	0.51	0.53	0.49	0.49	0.49	0.45	0.43	0.55	0.45	0.45	0.43	0.49
NS1	0.56	0.58	0.51	0.55	0.49	0.49	0.49	0.41	0.41	0.51	0.47	0.47	0.47	0.47
ATSM	0.58	0.58	0.51	0.53	0.47	0.47	0.49	0.47	0.45	0.53	0.45	0.49	0.39	0.49
Panel B: Models with macrofactors														
AR-X	0.45	0.45	0.47	0.47	0.44	0.44	0.47	0.45	0.41	0.41	0.35	0.35	0.35	0.37
VAR-X	0.55	0.58	0.47	0.51	0.51	0.47	0.45	0.47	0.47	0.45	0.41	0.41	0.37	0.41
NS2-AR-X	0.62	0.56	0.49	0.53	0.51	0.51	0.49	0.53	0.47	0.33	0.35	0.35	0.37	0.39
NS2-VAR-X	0.58	0.49	0.44	0.49	0.45	0.45	0.45	0.47	0.41	0.29	0.33	0.33	0.33	0.35
NS1-X	0.58	0.55	0.47	0.51	0.47	0.45	0.45	0.61	0.49	0.31	0.35	0.33	0.33	0.35
ATSM-X	0.51	0.45	0.44	0.47	0.47	0.49	0.47	0.49	0.43	0.31	0.33	0.37	0.33	0.35
Panel C: Forecast combinations														
FC-EW-X	0.53	0.56	0.44	0.45	0.44	0.45	0.47	0.55	0.39	0.31	0.33	0.33	0.35	0.37
FC-MSPE-X	0.51	0.51	0.42	0.45	0.44	0.45	0.47	0.45	0.39	0.29	0.33	0.33	0.35	0.37
FC-EW-ALL	0.58	0.64	0.47	0.51	0.51	0.49	0.47	0.55	0.53	0.45	0.45	0.41	0.41	0.41
FC-MSPE-ALL	0.60	0.58	0.47	0.51	0.51	0.47	0.47	0.55	0.51	0.45	0.43	0.41	0.39	0.41
BAYESIAN INFERENCE														
Panel D: Model without macrofactors														
AR	0.55	0.60	0.47	0.58	0.53	0.51	0.53	0.47	0.45	0.43	0.31	0.33	0.29	0.31
VAR	0.51	0.47	0.51	0.51	0.55	0.53	0.53	0.43	0.49	0.57	0.55	0.33	0.33	0.27
NS1	0.60	0.56	0.44	0.51	0.53	0.47	0.44	0.47	0.43	0.41	0.41	0.37	0.35	0.33
ATSM	0.60	0.58	0.51	0.51	0.47	0.49	0.49	0.43	0.41	0.51	0.47	0.49	0.45	0.45
Panel E: Models with macrofactors														
AR-X	0.47	0.42	0.45	0.45	0.42	0.44	0.47	0.31	0.31	0.35	0.37	0.35	0.35	0.35
VAR-X	0.56	0.51	0.44	0.49	0.47	0.47	0.45	0.53	0.59	0.55	0.55	0.59	0.55	0.55
NS1-X	0.45	0.40	0.36	0.42	0.44	0.44	0.45	0.16	0.20	0.33	0.31	0.33	0.33	0.35
ATSM-X	0.47	0.53	0.51	0.47	0.47	0.47	0.49	0.49	0.45	0.29	0.33	0.37	0.33	0.33
Panel F: Forecast Combinations														
FC-EW-X	0.47	0.44	0.42	0.45	0.45	0.44	0.47	0.55	0.41	0.43	0.37	0.37	0.39	0.37
FC-MSPE-X	0.44	0.42	0.40	0.45	0.45	0.44	0.47	0.49	0.39	0.41	0.35	0.35	0.37	0.37
BMA-X	0.49	0.44	0.38	0.45	0.44	0.44	0.45	0.27	0.29	0.37	0.35	0.41	0.37	0.37
FC-EW-ALL	0.58	0.55	0.45	0.51	0.49	0.47	0.45	0.49	0.49	0.49	0.43	0.43	0.39	0.45
FC-MSPE-ALL	0.60	0.55	0.45	0.49	0.49	0.47	0.45	0.49	0.53	0.45	0.43	0.41	0.43	0.45
BMA-ALL	0.60	0.53	0.40	0.47	0.44	0.44	0.47	0.45	0.41	0.43	0.47	0.47	0.41	0.43

Notes: The table reports Hit Rate results for 6-month and 12-month horizons for the out-of-sample period 1994:1 - 1998:12. See Table 15 for further details.

Table 19: Hit Rate, 1999:1 - 2003:12

Maturity	h=1							h=3						
	3m	6m	1y	2y	5y	7y	10y	3m	6m	1y	2y	5y	7y	10y
FREQUENTIST INFERENCE														
Panel A: Model without macrofactors														
AR	0.53	0.48	0.45	0.62	0.58	0.65	0.52	0.81	0.79	0.79	0.72	0.55	0.55	0.55
VAR	0.57	0.57	0.50	0.60	0.58	0.62	0.57	0.79	0.76	0.76	0.76	0.62	0.57	0.50
NS2-AR	0.50	0.58	0.48	0.58	0.57	0.60	0.55	0.74	0.74	0.76	0.78	0.57	0.53	0.52
NS2-VAR	0.52	0.55	0.50	0.60	0.57	0.60	0.53	0.74	0.74	0.78	0.78	0.60	0.57	0.48
NS1	0.52	0.45	0.52	0.60	0.57	0.58	0.53	0.72	0.78	0.78	0.76	0.60	0.57	0.53
ATSM	0.57	0.62	0.50	0.60	0.57	0.63	0.57	0.74	0.76	0.78	0.76	0.62	0.55	0.50
Panel B: Models with macrofactors														
AR-X	0.60	0.55	0.47	0.63	0.58	0.62	0.50	0.71	0.67	0.67	0.67	0.53	0.48	0.47
VAR-X	0.57	0.60	0.52	0.60	0.57	0.58	0.55	0.74	0.74	0.74	0.66	0.55	0.50	0.50
NS2-AR-X	0.53	0.58	0.48	0.57	0.62	0.60	0.53	0.67	0.69	0.69	0.64	0.50	0.48	0.48
NS2-VAR-X	0.53	0.58	0.48	0.55	0.62	0.60	0.53	0.72	0.72	0.69	0.67	0.50	0.48	0.47
NS1-X	0.53	0.52	0.55	0.60	0.57	0.60	0.55	0.72	0.67	0.67	0.64	0.53	0.50	0.48
ATSM-X	0.53	0.63	0.50	0.57	0.60	0.67	0.60	0.69	0.67	0.72	0.67	0.53	0.50	0.50
Panel C: Forecast combinations														
FC-EW-X	0.55	0.57	0.48	0.60	0.60	0.62	0.57	0.72	0.69	0.72	0.69	0.50	0.48	0.45
FC-MSPE-X	0.55	0.57	0.47	0.62	0.60	0.62	0.57	0.72	0.71	0.71	0.69	0.52	0.48	0.45
FC-EW-ALL	0.52	0.57	0.48	0.63	0.58	0.62	0.57	0.74	0.74	0.71	0.72	0.57	0.52	0.50
FC-MSPE-ALL	0.52	0.57	0.48	0.63	0.58	0.60	0.57	0.72	0.74	0.67	0.72	0.57	0.52	0.50
BAYESIAN INFERENCE														
Panel D: Model without macrofactors														
AR	0.53	0.48	0.45	0.62	0.58	0.65	0.52	0.79	0.81	0.78	0.72	0.52	0.50	0.48
VAR	0.58	0.53	0.50	0.62	0.57	0.60	0.55	0.71	0.66	0.69	0.67	0.57	0.53	0.48
NS1	0.53	0.52	0.47	0.62	0.55	0.58	0.55	0.76	0.78	0.76	0.78	0.57	0.53	0.52
ATSM	0.57	0.58	0.48	0.58	0.55	0.62	0.58	0.78	0.78	0.78	0.78	0.62	0.57	0.50
Panel E: Models with macrofactors														
AR-X	0.62	0.58	0.45	0.63	0.58	0.60	0.50	0.69	0.69	0.69	0.66	0.52	0.50	0.48
VAR-X	0.57	0.60	0.52	0.60	0.57	0.58	0.55	0.64	0.67	0.67	0.64	0.52	0.50	0.47
NS1-X	0.52	0.57	0.53	0.62	0.60	0.58	0.55	0.67	0.72	0.74	0.67	0.53	0.52	0.47
ATSM-X	0.57	0.63	0.48	0.57	0.58	0.63	0.53	0.69	0.69	0.69	0.69	0.48	0.50	0.52
Panel F: Forecast Combinations														
FC-EW-X	0.62	0.60	0.50	0.62	0.62	0.62	0.55	0.69	0.71	0.72	0.67	0.53	0.50	0.50
FC-MSPE-X	0.62	0.58	0.50	0.60	0.62	0.62	0.55	0.69	0.71	0.74	0.67	0.53	0.50	0.52
BMA-X	0.60	0.57	0.50	0.62	0.60	0.62	0.52	0.76	0.76	0.72	0.67	0.50	0.48	0.47
FC-EW-ALL	0.58	0.57	0.48	0.62	0.58	0.62	0.57	0.71	0.79	0.72	0.72	0.53	0.52	0.50
FC-MSPE-ALL	0.60	0.57	0.48	0.62	0.58	0.62	0.55	0.69	0.79	0.74	0.72	0.53	0.52	0.50
BMA-ALL	0.53	0.53	0.50	0.62	0.58	0.58	0.55	0.76	0.78	0.76	0.69	0.52	0.50	0.50

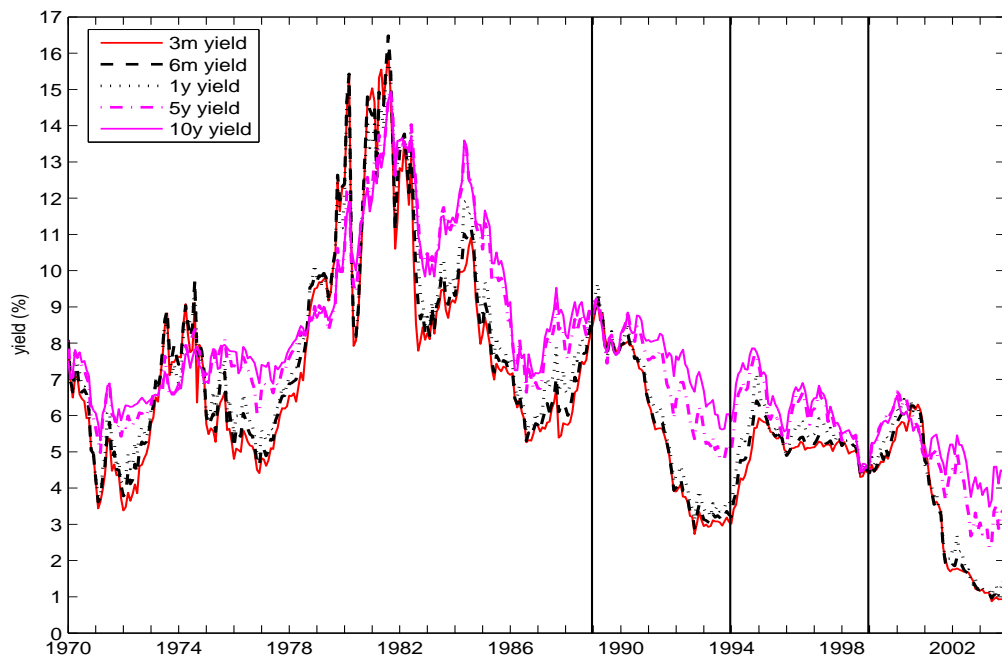
Notes: The table reports Hit Rate results for 1-month and 3-month horizons for the out-of-sample period 1999:1 - 2003:12. See Table 15 for further details.

Table 20: Hit Rate, 1999:1 - 2003:12

Maturity	h=6							h=12						
	3m	6m	1y	2y	5y	7y	10y	3m	6m	1y	2y	5y	7y	10y
FREQUENTIST INFERENCE														
Panel A: Model without macrofactors														
AR	0.87	0.84	0.71	0.69	0.62	0.62	0.51	0.76	0.76	0.73	0.71	0.61	0.65	0.49
VAR	0.87	0.82	0.71	0.67	0.64	0.65	0.58	0.78	0.76	0.73	0.69	0.57	0.59	0.51
NS2-AR	0.78	0.80	0.67	0.67	0.64	0.58	0.58	0.67	0.65	0.63	0.71	0.61	0.61	0.45
NS2-VAR	0.87	0.80	0.67	0.67	0.64	0.65	0.58	0.76	0.76	0.73	0.69	0.57	0.59	0.51
NS1	0.87	0.82	0.69	0.64	0.56	0.58	0.55	0.76	0.73	0.73	0.69	0.57	0.57	0.51
ATSM	0.87	0.80	0.71	0.67	0.64	0.65	0.56	0.78	0.76	0.73	0.69	0.57	0.59	0.51
Panel B: Models with macrofactors														
AR-X	0.73	0.73	0.71	0.62	0.51	0.53	0.45	0.55	0.55	0.57	0.53	0.51	0.57	0.43
VAR-X	0.80	0.75	0.71	0.58	0.56	0.56	0.56	0.63	0.65	0.65	0.55	0.61	0.65	0.55
NS2-AR-X	0.67	0.64	0.62	0.58	0.55	0.51	0.45	0.47	0.49	0.51	0.49	0.51	0.45	0.31
NS2-VAR-X	0.67	0.64	0.56	0.55	0.53	0.55	0.45	0.49	0.49	0.49	0.47	0.41	0.43	0.31
NS1-X	0.69	0.64	0.58	0.56	0.55	0.51	0.51	0.51	0.53	0.49	0.51	0.47	0.49	0.43
ATSM-X	0.71	0.73	0.62	0.55	0.53	0.53	0.55	0.55	0.53	0.51	0.51	0.49	0.51	0.49
Panel C: Forecast combinations														
FC-EW-X	0.73	0.71	0.58	0.56	0.55	0.55	0.45	0.63	0.51	0.51	0.51	0.47	0.53	0.43
FC-MSPE-X	0.73	0.71	0.60	0.56	0.55	0.55	0.45	0.53	0.53	0.51	0.51	0.47	0.53	0.43
FC-EW-ALL	0.75	0.73	0.65	0.60	0.58	0.58	0.58	0.71	0.65	0.61	0.61	0.63	0.61	0.51
FC-MSPE-ALL	0.75	0.73	0.64	0.58	0.58	0.58	0.58	0.63	0.61	0.59	0.59	0.57	0.61	0.51
BAYESIAN INFERENCE														
Panel D: Model without macrofactors														
AR	0.85	0.85	0.71	0.71	0.53	0.62	0.55	0.76	0.73	0.73	0.73	0.59	0.63	0.53
VAR	0.80	0.76	0.62	0.64	0.64	0.64	0.53	0.76	0.73	0.71	0.71	0.59	0.63	0.55
NS1	0.87	0.80	0.69	0.69	0.62	0.56	0.55	0.73	0.71	0.63	0.67	0.57	0.59	0.53
ATSM	0.87	0.82	0.71	0.67	0.64	0.65	0.56	0.78	0.76	0.73	0.69	0.57	0.59	0.51
Panel E: Models with macrofactors														
AR-X	0.71	0.65	0.67	0.60	0.53	0.55	0.49	0.53	0.51	0.53	0.53	0.51	0.55	0.43
VAR-X	0.71	0.69	0.64	0.60	0.58	0.58	0.56	0.69	0.67	0.67	0.69	0.55	0.59	0.51
NS1-X	0.73	0.67	0.58	0.55	0.55	0.55	0.60	0.51	0.51	0.49	0.51	0.53	0.53	0.43
ATSM-X	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.55	0.55	0.49	0.51	0.49	0.51	0.47
Panel F: Forecast Combinations														
FC-EW-X	0.75	0.71	0.64	0.58	0.55	0.53	0.55	0.55	0.53	0.49	0.51	0.51	0.59	0.55
FC-MSPE-X	0.75	0.71	0.64	0.58	0.55	0.53	0.55	0.55	0.53	0.49	0.51	0.51	0.59	0.57
BMA-X	0.76	0.73	0.65	0.60	0.53	0.51	0.45	0.55	0.53	0.53	0.53	0.55	0.57	0.51
FC-EW-ALL	0.82	0.82	0.71	0.62	0.62	0.56	0.60	0.76	0.76	0.71	0.69	0.57	0.61	0.53
FC-MSPE-ALL	0.75	0.75	0.64	0.62	0.60	0.56	0.60	0.69	0.71	0.65	0.67	0.59	0.65	0.55
BMA-ALL	0.82	0.82	0.69	0.64	0.53	0.51	0.45	0.67	0.59	0.53	0.53	0.55	0.59	0.45

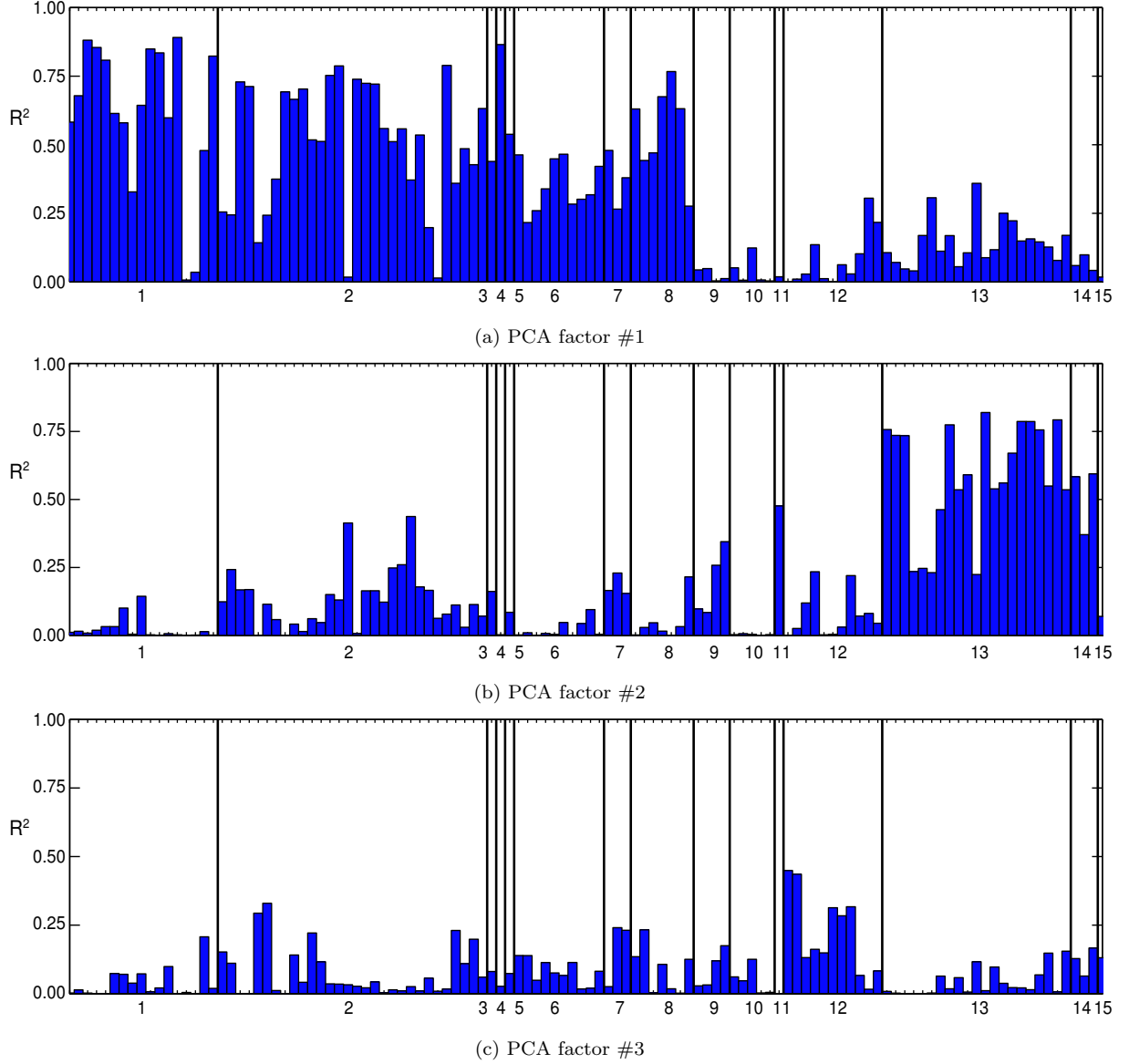
Notes: The table reports Hit Rate results for 6-month and 12-month horizons for the out-of-sample period 1999:1 - 2003:12. See Table 15 for further details.

Figure 1: US zero-coupon yields



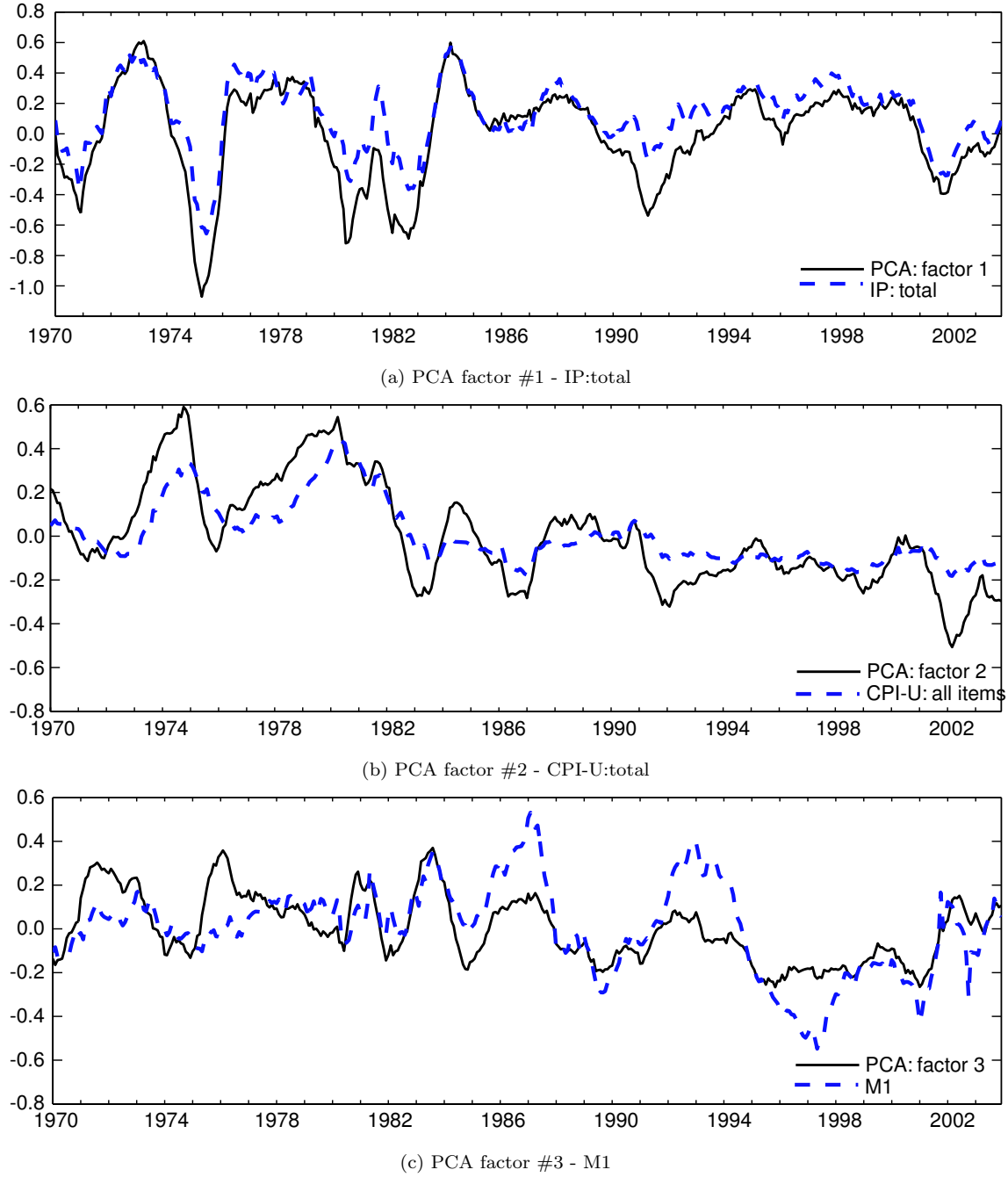
Notes: The figure shows time series plots for a subset of maturities of end-of-month unsmoothed US zero coupon yields constructed using the Fama and Bliss (1987) bootstrap method. Sample period is January 1970 - December 2003 (408 observations). The vertical lines bound the three forecasting subsamples.

Figure 2: R^2 in regressions of PCA factors on individual macro series



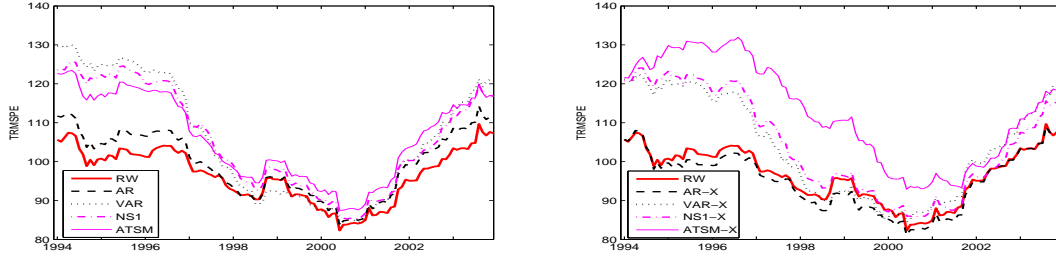
Notes: The figure shows the R^2 when regressing the first three macro factors on each individual series in the macro panel. Panel (a), (b) and (c) show the results, grouped by each of the 15 macro categories, as indicated on the horizontal axis, for the first, second and third macro factor resp. The macro dataset consists of 116 (transformed to ensure stationarity) series and the sample period used is January 1970 - December 2003 (408 monthly observations). The group categories are 1:=real output and income, 2:=employment and hours, 3:=real retail, 4:=manufacturing and trade sales, 5:=consumption, 6:=housing starts and sales, 7:=real inventories, 8:=orders, 9:=stock prices, 10:=exchange rates, 11:=federal funds rate, 12:=money and credit quantity aggregates, 13:=prices indexes, 14:=average hourly earnings and 15:=miscellaneous.

Figure 3: Macro factors and individual macroseries

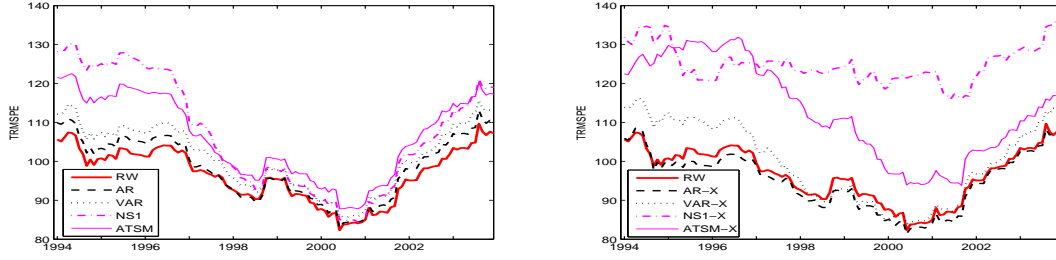


Notes: The figure shows timeseries plots of the first three common macro factors and the main individual (transformed) macroseries from the category with which the factor is most related to. The first factor is plotted together with Industrial Production Index: Total Index (R^2 is 0.88%), the second factor is plotted with the Consumer Price Index: All Items (R^2 is 0.77) and the third factor is plotted with Money Stock: M1 (R^2 is 0.44). The macro dataset consists of 116 (transformed to ensure stationarity) series and the sample period used is January 1970 - December 2003 (408 monthly observations). The group categories are 1:=real output and income, 2:=employment and hours, 3:=real retail, 4:=manufacturing and trade sales, 5:=consumption, 6:=housing starts and sales, 7:=real inventories, 8:=orders, 9:=stock prices, 10:=exchange rates, 11:=federal funds rate, 12:=money and credit quantity aggregates, 13:=prices indexes, 14:=average hourly earnings and 15:=miscellaneous.

Figure 4: $h=1$

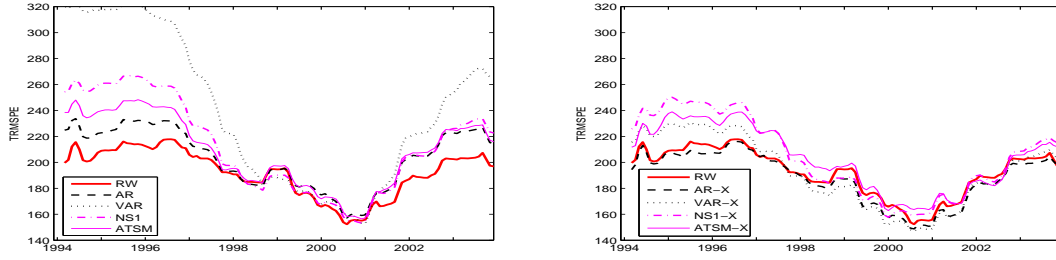


(a) Classical inference

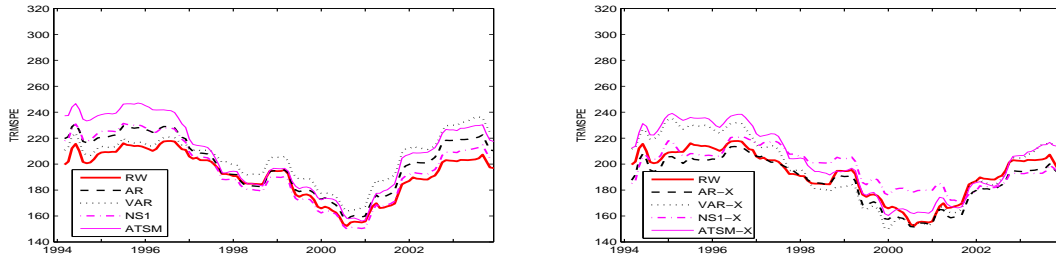


(b) Bayesian inference

Figure 5: $h=3$



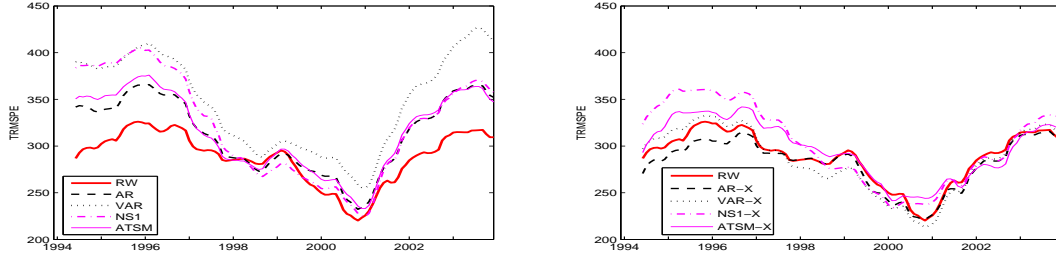
(a) Classical inference



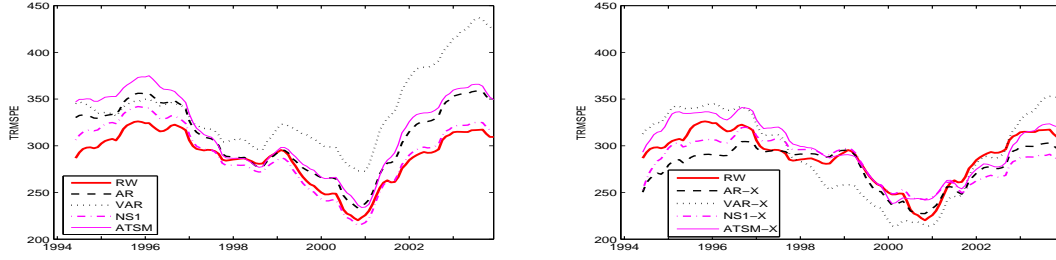
(b) Bayesian inference

Notes: The figure presents the 60-month moving average TRMSPE for individual models in the left panel and for individual models augmented with macrofactors in the right panel. The TRMSPE is shown for the out-of-sample period 1994:1-2003:12 for a 1-month horizon in Figure 4 and a 3-month horizon in Figure 5. The models depicted are the Random Walk [RW], first order (Vector) Autoregressive [(V)AR], State-Space Nelson-Siegel [NS1] and the affine [ATSM] model. The affix 'X' indicates that macrofactors have been added as additional variables.

Figure 6: $h=6$

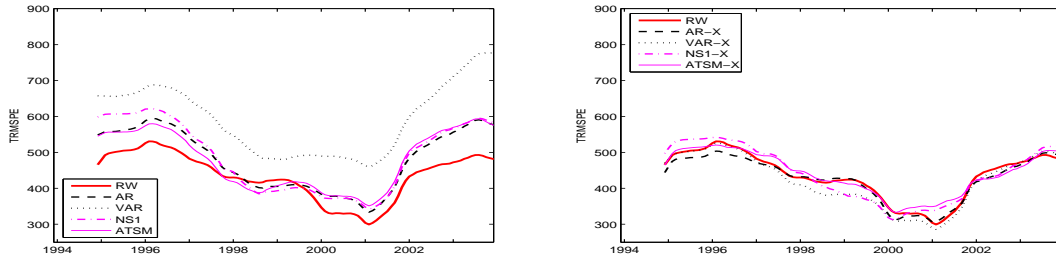


(a) Classical inference

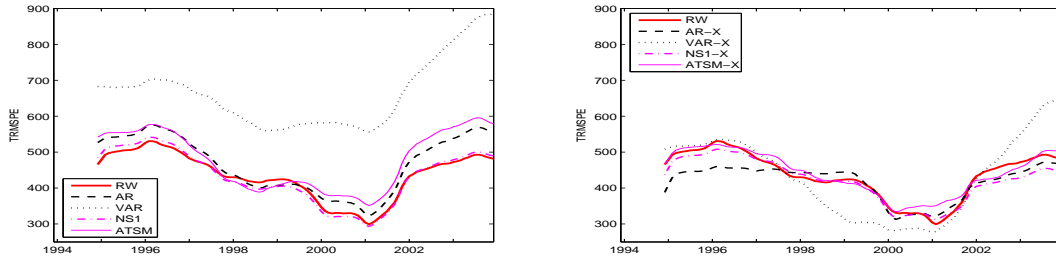


(b) Bayesian inference

Figure 7: $h=12$



(a) Classical inference



(b) Bayesian inference

Notes: The figure presents the 60-month moving average TRMSPE for individual models in the left panel and for individual models augmented with macrofactors in the right panel. The TRMSPE is shown for the out-of-sample period 1994:1-2003:12 for a 6-month horizon in Figure 6 and a 12-month horizon in Figure 7. The models depicted are the Random Walk [RW], first order (Vector) Autoregressive [(V)AR], State-Space Nelson-Siegel [NS1] and the affine [ATSM] model. The affix 'X' indicates that macrofactors have been added as additional variables.