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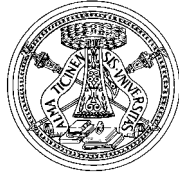
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Quaderni di Dipartimento

**Environmental Externality and Capital
Accumulation in an Overlapping Generations
Model with Heterogeneous Preference**

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Environmental Externality and Capital Accumulation in an Overlapping Generations Model with Heterogeneous Preferences*

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Abstract

We study an overlapping generations model *a' la* Diamond in which two types of agents are alive at the same time. One type of agents cares about its own level of consumption and an index of environmental quality, while the other type of agents care only about its own consumption. At any period, the environmental quality is negatively affected by the saving decision of the previous generation, and this creates an intergenerational externality over time. Young agents that care about environmental quality can invest in productive capital and/or in environmental preservation, while the other agents can invest only in productive capital. We show that in such a framework, capital and environmental quality accumulation over time depend crucially on the characteristics of the technology. Furthermore, we derive a general condition for the local stability of a steady state implying an high level of capital and a high level of environmental quality. This stability condition depends on the relative effects of the capital and environmental quality on the saving functions of the individuals. Finally, we show that "saddle-node" bifurcations may arise in our model.

Keywords: overlapping generations, environmental externality.

JEL:D62, D91

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1 Introduction

A vast amount of economic literature has dealt with the relation between economic activity and environmental externalities over the years. The traditional neoclassical point of view explains the root of environmental problems as a result from market failures. In the case of collective goods these failures are due to difficulties in establishing markets, while in the presence of negative externalities, the failures are due to a lack of well-defined property rights. The solution proposed by Coase (1960) to environmental problems would be to determine property rights as a basis for negotiations between involved parties, but because of transaction costs and several other real world problems this would only rarely be applicable. Suggestions have thus mainly concentrated on the design of environmental regulation able to realized a socially optimal level of pollution through the use of either tariffs or tradeable permits. The main feature of this traditional analysis is the evaluation of the costs and benefits of existing and proposed regulations¹. Following John et al.(1995) we can say that such analysis, being implicitly static, ignores two important aspects related to environmental problems. First, since environment is an asset which is passed on to future generations, environmental externalities are intra- as well as intergenerational: actions taken today affect the welfare of future generations. Such external effects are difficult to internalise and their existence alters the set of policies that are socially desirable. Second, the macroeconomic perspective is missing. Actions that affect the environment both influence and respond to macroeconomic variables, and environmental policy decisions have implications for economic growth and capital accumulation as showed by John and Pecchenino (1994) and Stokey (1998) among others.

In recent years, researchers have started to investigate the conflict between environmental preservation and economic growth in a dynamic setting. Examples of such analysis are John and Pecchenino (1994), Ono (1996), Bovemberg and De Mooj (1997) among the others.² A common result that arises in these models is that, from a welfare perspective, there is too much environmental degradation in the early of economic growth. Thus government, evaluating the effect of negative externality in the optimization process, can design an optimal environmental policy using fiscal instruments in order to reach a better intertemporal allocation of resources. However, the trade-off between environmental quality and economic growth needs not to follow a monotonic path. John and Pecchenino (1994) and Stokey (1998), among the others, have shown that the relation between output growth and environmental quality can follow a “U”-shaped curve, also known as the “Environmental Kuznet’s Curve”. This implies that pollution tends to decrease once output is higher enough.

¹See Van Der Straaten (1998) for a critical analysis of the traditional theory of environmental policy

²For example, Bovemberg and De Mooj (1997) consider the effects of environmental taxes on growth in a model with pre-existing distortionary taxes.

While John and Pecchenino (1994) use an overlapping generations framework, the analysis of Stokey (1998) is based on the assumption of a representative agent economy. However, by assuming that the life span of individuals and the economy are the same, all these models restricted themselves to the analysis of the intragenerational conflict given by the existence of the well-understood free-rider problems within a generation. However, once dynamics is introduced in models of environmental policy, intergenerational issues become a predominant part of the analysis as pointed out by Solow (1986) which considers these problems in the field of economics of exhaustible natural resources. In fact, the overlapping generations approach allows intertemporal aspects to be disentangled from intergenerational considerations. In this paper we consider a discrete time overlapping generations model in which there are two types of agents alive at the same time in the economy. One type of agents cares about its own consumption as well as an index of the environmental quality, while the other type of agents cares only about its own consumption level. Following the work of John and Pecchenino (1994), we assume that when young, individuals cannot consume. Thus, they divide their income between investment in productive capital and, if they care about environmental quality, they can also invest in environmental maintenance. Our main goal is to analyse the effects of the presence of an environmental externality coupled with the heterogeneity of agents, on capital accumulation and thus on long-run growth. Our model is closely related with the model of John and Pecchenino (1994). However, our analysis differs from their one in several aspects. First of all, we consider a model in which there is heterogeneity of the agents that have preferences over environmental quality and thus, the dynamic properties of our economy will differ from the one considered by John and Pecchenino (1994). Second of all, in our model, environmental quality is negatively affected by production, and thus, by the saving decisions of previous generations. This creates an intergenerational externality that has important effects on the capital accumulation of the economy under analysis. Moreover, these two elements affect the design of an optimal environmental policy, since a policy derived in our framework will have different features with respect the one derived from the structure of John and Pecchenino's model.³ We show that in a model with an environmental externality and heterogeneity in preferences, the dynamics of capital accumulation becomes richer and more complex than in standard overlapping generations models without externalities. In particular, a positive capital accumulation depends on the characteristics of the technology available in the economy. Furthermore, we show that the accumulation of capital and environmental quality are positively related if the technology exhibits particular curvature properties, and thus, an increase in the proportion of agents that care about environmental quality may decrease capital accumulation and this can have a negative impact on environmental quality. The paper is organised as follows. In Section 2 we present and

³For an example of such a policy in the framework of John and Pecchenino's model, see Ono (1996), while Ono (2003) derive an optimal environmental policy using a model in which is production that deteriorates the environment.

discuss the main features of the model. In Section 3, we derive the main properties of a competitive equilibrium of our economy. In Section 4 we analyse in detail the steady state equilibrium and we derive the conditions for its stability. In Section 5 we discuss the possibility that complex dynamics may arise in our model. Section 6 concludes.

2 The Model

We consider a discrete time overlapping generations model where at each period t a new generation is born and lives for two periods. We assume no population growth and we normalize the size of the economy in each period to be the unit interval $[0, 1]^4$. Following the analysis of John and Pacchenino (1994) we assume that preferences of each generation are defined only in the second period of its life. In the economy, in each period, there are two types of agents, one is environmental friendly (type E) while the other has no preference over environmental quality. Preferences of the environmental friendly type are defined over consumption in old age, c_{t+1}^E , and over an index of environmental quality in old age, E_{t+1} . These preferences are given by the following utility function: $U^E(c_{t+1}^E, E_{t+1})$.

A possible justification for the fact that agents care about environment when they are old could be found in the possible relationship between pollution and health costs, as in Williams (2002) and Gutierrez (2003). In their models, an increase in pollution will deteriorate consumers' health. Thus, in their models, consumers care indirectly about the environmental quality through the health costs that enter in the consumers' budget constraints. Preferences of the non-environmental friendly (type N) are defined only on consumption in old age c_{t+1}^N and are given by the following utility function $U^N(c_{t+1}^N, \cdot)$.

Agents born in period t are endowed with one unit of labour that they supply inelastically to firms and they receive a competitive wage w_t . Type E agents divide the wage into saving, s_t^E and investment in environmental maintenance m_t , while type N agents saves only (s_t^N). In period $t + 1$ each type of agents retires and supply their savings (s_t^E and s_t^N) to firms and earn the gross return $(1 + r_{t+1})^5$.

We model the heterogeneity of the agents as in Nourry and Venditti (2001), that is, in our economy, there are a proportion $0 \leq p \leq 1$ of type E agents and a proportion $(1 - p)$ of type N agents. The proportion is exogenously given.

Assumption 1. *The utility functions $U^E(c_{t+1}^E, E_{t+1})$ and $U^N(c_{t+1}^N, \cdot)$ are twice continuously differentiable with: $U_c^E(\cdot) > 0, U_E^E(\cdot) > 0, U_{cc}^E(\cdot) < 0, U_{EE}^E(\cdot) < 0, U_{cE}^E \geq 0$, and $U_c^N > 0, U_{cc}^N < 0$. Furthermore, we assume that $\lim_{c \rightarrow 0} [U^E(c, E)] = \infty$ and $\lim_{E \rightarrow 0} [U^E(c, E)] = \infty$;*

⁴Since we are interested in intergenerational issues, as in John and Pecchenino (1994), we abstract from the well-known intragenerational free-rider problem.

⁵In our model we are implicitly assuming that there exists a old generation of each type in period 0 that is endowed with a total capital of k_0 units of capital that is supplied inelastically to firms.

The firms are perfectly competitive and have access at the same technology given by the following production function: $F(K_t, L_t) = Y_t$, where K_t is the stock of capital, L_t is the labour supply and Y is the level of output at time t . We assume that $F(\cdot)$ displays constant return to scale, thus, we can rewrite it as $y_t = f(k_t)$, where k_t is the usual capital/labour ratio.⁶

Assumption 2. *The production function $f(k)$ is twice continuously differentiable with: $f' > 0, f'' \leq 0$. Furthermore, we impose that $\lim_{k \rightarrow \infty} f'(k) = 0, \lim_{k \rightarrow 0} f'(k) = \infty$ and $f(0) = 0$;*

Environmental quality is a public good that is affected negatively by human activity. We assume that the environmental quality is a decreasing function of the production activity.⁷ However, each generation of type E can decide to invest in maintenance or improvement of the environment when they are young and this affects positively the environmental quality.

The environmental quality is assumed to evolve according to the following law of motion:⁸

$$E_{t+1} = \alpha E_t - \beta y + \gamma p m_t \quad (1)$$

where $\beta, \gamma > 0$ are exogenous parameters that measure the effects on the environmental quality of the production activity and of the investment in maintenance respectively. While $\alpha \in (0, 1)$ measures the degree of persistence of the environmental quality.⁹ The initial level $E_0 > 0$ is given. If $\beta < \gamma$, then the investment in maintenance is efficient. Possible interpretations of E_t can be the quality of soil, the quantity of national parks or the inverse of the concentration in the atmosphere of greenhouse gases.

2.1 Saving Decisions and Profit Maximization

The saving decision of type N agents is quite simple, since they save all their wage when young and they consume all that saving when old. Formally we have $w_t = s_t^N$ and $c_{2t+1}^N = (1+r_{t+1})s_t^N$. Type E agents maximise their utility function $U^E(c_{t+1}^E, E_{t+1})$, subject to the evolution of environmental quality given by 1) and of the following constraints:

⁶We assume that $F(K_t, L_t)$ is a net production function, that is it incorporate the fact that capital depreciate. This is the same approach used in Blanchard and Fisher (1989).

⁷See John and Pecchenino (1994) for the case where environmental quality is negatively affected by consumption activity.

⁸This linear specification for the evolution of the environmental quality, introduced by John and Pacchenino (1994) and has been widely used in the recent literature. See for example Ono (1996, 2003), Jouvét *et al.* (2000) and Gutierrez (2003) among the others.

⁹Since $\alpha \in (0, 1)$ if there is no human activity, the environmental quality tends to a an autonomous level in which $E = 0$, and α measures the speed of this natural process.

$$\begin{aligned}
w_t &= s_t^E - m_t & (2) \\
c_{t+1}^E &= (1 + r_{t+1})s_t^E \\
c_{t+1}^E, s_t^E, m_t &\geq 0
\end{aligned}$$

Since there is no uncertainty in our framework, we assume that agents are endowed with perfect foresight. This implies that $r_{t+1}^e = r_{t+1}$, where r_{t+1}^e is the expected value at time t of the interest rate at time $t + 1$. Given assumption 1, the problem above admits a solution and the first order condition is:

$$(1 + r_{t+1})U_c^E(c_{t+1}^E, E_{t+1}) - \gamma p U_E^E(c_{t+1}^E, E_{t+1}) = 0 \quad (3)$$

Suppose that at the solution we have $s_t^E, m_t > 0$, condition 3) gives us a simple arbitrage condition between the rate of return on the private saving, $(1 + r_{t+1})$, and the rate of return on the investment in environmental improvement, γ , that is $U_c^E(\cdot)/U_E^E(\cdot) = \gamma p/(1 + r_{t+1})$.¹⁰ This condition says that consumers of type E choose s_t^E and m_t in order to equate the marginal rate of substitution between consumption and environmental quality with the marginal rate of transformation.¹¹

Condition 3) implicitly defines a saving function $s_t^E(w_t, r_{t+1}, E_t, y)$. The following proposition states the main properties of that saving function:

Proposition 1 *Using Assumption 1, the main static comparative properties of the saving function $s_t^E(w_t, r_{t+1}, E_t, y)$ are:*

$$s_w^E \equiv -\frac{-\gamma p [\gamma p U_{EE} + (1 + r_{t+1})U_{cE}]}{(1 + r_{t+1})^2 U_{cc} + \gamma p [\gamma p U_{EE} - 2(1 + r_{t+1})U_{cE}]} > 0; \quad (4)$$

$$s_r^E \equiv -\frac{U_{cc} s^E (1 + r_{t+1}) + U_c - \gamma p s^E U_{cE}}{(1 + r_{t+1})^2 U_{cc} + \gamma p [\gamma p U_{EE} - 2(1 + r_{t+1})U_{cE}]} \leq 0; \quad (1)$$

$$s_E^E \equiv -\frac{\alpha [(1 + r_{t+1})U_{cE} - \gamma p U_{EE}]}{(1 + r_{t+1})^2 U_{cc} + \gamma p [\gamma p U_{EE} - 2(1 + r_{t+1})U_{cE}]} > 0;$$

$$s_y^E \equiv -\frac{\beta [-(1 + r_{t+1})U_{cE} + \gamma p U_{EE}]}{(1 + r_{t+1})^2 U_{cc} + \gamma p [\gamma p U_{EE} - 2(1 + r_{t+1})U_{cE}]} < 0; \quad (2)$$

A sufficient (not necessary) condition for $s_r^E \geq 0$ to hold is that the intertemporal elasticity of substitution ($-\frac{U_{cc}c}{U_c}$) is greater or equal than 1.

Proposition 1 gives us the signs of the main derivatives of the saving function of type E agents implied by the implicit differentiation of condition 3). The sign of

¹⁰Notice that there is nothing in the model that precludes the possibility that $m_t = 0$.

¹¹This is the same condition as in John and Pecchenino (1994). Note that the Samuelson condition for the optimal provision of public good is satisfied in our framework, since the size of each generation has been normalised to one.

the derivative of the saving with respect the interest is ambiguous as in the classic Diamond's model. However, differently from that model, the fact that the elasticity of substitution is greater than one is only sufficient in order to have a positive relationship between saving and interest rate.

The saving function is increasing in the wage rate, and furthermore $s_w^E \in (0, 1)$, it depends positively on the environmental quality, while it depends negatively on the production level in period t . The saving of type E agents depends positively on the environmental quality because higher is the environmental quality and lower will be the investment in maintenance, and this implies an higher level for the private saving. On the other hand, private saving depends negatively on the production in period t because an higher production implies a degradation of the environment and thus an higher investment in maintenance. Furthermore, it can be shown that $\frac{\partial s^E}{\partial p} > 0$, that is an increase in the proportion of agents of type E increases the saving of those agents in productive capital. The intuition is that if there are more agents that can invest in environmental maintenance, each type E agent, for a given level of environmental quality, can reduce its level of investment m increasing the level of s^E . Given the properties of s^E the properties of the investment maintenance m follow, since the wage rate is given.

Firms are identical and perfectly competitive. Maximization of the profits, together with Assumption 2, imply the following first order conditions for the representative firm:

$$\begin{aligned} r_t &= f'(k_t) \\ w_t &= f(k_t) - f'(k_t)k_t \end{aligned} \tag{5}$$

Conditions 5) are the standard conditions stating that firms hire capital and labour until their marginal products equal their factor prices.

3 The Competitive Equilibrium

In the previous section we have analysed the behaviour of firms and consumers and we have derived the optimal saving functions and the conditions for profit maximization. In this section we define the competitive equilibrium of our model and we shall analyse in more details the dynamic of capital accumulation and environmental quality along an equilibrium path. The goods market clears when the capital stock at time $t + 1$, k_{t+1} , equals the private saving decisions taken at time t by the two types of agent, that is:

$$k_{t+1} = ps_t^E(w_t, r_{t+1}, E_t, y_t) + (1 - p)s_t^N \tag{6}$$

While equations 5) give us the equilibrium conditions for the market of productive factors.

Given these market clearing conditions, we can define a competitive equilibrium for our economy as follows

Definition 1 *A competitive equilibrium for the economy under analysis is a sequence $\{c_t^{E*}, c_t^{N*}, r_t^*, w_t^*, s_t^{E*}, s_t^{N*}, m_t^*, k_t^*, E_t^*\}_{t=0}^{\infty}$ such that, given the initial conditions of the state variables $\{k_0, E_0\}$: i) firms maximize profits; ii) consumers maximize their utility function; iii) markets clear.*

If we know the equilibrium paths for the capital level (k) and the environmental quality (E), then we can obtain all the equilibrium sequences of our model. Thus, we can limit ourselves in study the dynamics of the capital accumulation and of the environmental quality.

Substituting conditions 5) into equation 6) we obtain that the capital stock in period $t + 1$ evolves according to:

$$k_{t+1} = ps_t^E \left(f(k_t) - f'(k_t)k_t, f'(k_{t+1}), E_t, f(k_t) \right) + (1 - p) \left(f(k_t) - f'(k_t)k_t \right) \quad (7)$$

which is a non-linear first-order difference equation that define implicitly k_{t+1} as a function of k_t and E_t . The evolution of the environmental quality is found in a similar way, and is given by:

$$E_{t+1} = \alpha E_t - \beta f(k_t) + \gamma p \left(f(k_t) - f'(k_t)k_t - s_t^E \left(f(k_t) - f'(k_t)k_t, f'(k_{t+1}), E_t, f(k_t) \right) \right) \quad (8)$$

Equations 7)-8) form a system of non-linear first order difference equations that describes the dynamics of the capital accumulation and the evolution of the environmental quality along the competitive equilibrium path of the model. We first analyse the behaviour of equations 7)-8) separately. Thus, the results we are going to obtain are only partial results, since the system 7)-8) should be analysed jointly. However, those partial results will be useful once we will consider the linearized version of system 7)-8). With this in mind, we start with the analysis of the function that describes the capital accumulation.

The law of capital accumulation 7) can be written as:

$$\begin{aligned} & k_{t+1} - ps_t^E \left(f(k_t) - f'(k_t)k_t, f'(k_{t+1}), E_t, f(k_t) \right) - (1 - p) \left(f(k_t) - f'(k_t)k_t \right) \\ \equiv & \Psi(E_t, k_t) = 0 \end{aligned} \quad (9)$$

The properties of the function $\Psi(\cdot)$ are stated in the following proposition:

Proposition 2 Suppose that $s_r^E \geq 0$, then the function $\Psi(\cdot)$ has the following properties: i) $\Psi_{k_t} \geq 0$ if and only if $-\frac{k_t f''(k_t)}{f'(k_t)} \geq \frac{\beta p}{\gamma p - A(p-1)}$; ii) $\Psi_{E_t} > 0$ always. Where $A = -(1 + r_{t+1})^2 U_{cc} - \gamma p [\gamma p U_{EE} - 2(1 + r_{t+1}) U_{cE}] > 0$.

Proof. To prove result i) we need to differentiate implicitly equation 9) and then, using Assumption 1 and 2 and the properties of the saving function s_t^E . The derivative we are looking for is (where we have dropped the sup-script E):

$\partial k_{t+1} / \partial k_t = \frac{-p s_w k_t f''(k_t) + p s_y f'(k_t) - (1-p) f''(k_t) k_t}{1 - s_r f''(k_{t+1}) p}$. The denominator is positive if $s_r \geq 0$, while manipulating the numerator, we can find that is positive only if $-\frac{k_t f''(k_t)}{f'(k_t)} > \frac{\beta p}{\gamma p - A(p-1)}$. To prove the result ii) we follow the same steps, and we obtain $\partial k_{t+1} / \partial E_t = p s_E / (1 - s_r f''(k_{t+1}))$ that is clearly positive. ■

The first result in Proposition 2 says under which conditions the level of capital next period is increasing in the level of capital today. Given the presence of an environmental externality that depends on the lagged value of the level of production, the capital accumulation equation is increasing in k_t only if $s_r^E \geq 0$ and the curvature of the production function is such that is greater than $\frac{\beta p}{\gamma p - A(p-1)}$. Thus, an important role in our analysis is played by the characteristics of the production function. The ratio $-\frac{k_t f''(k_t)}{f'(k_t)}$ is related to the curvature of the production function, for example, if the production function is linear that ratio is zero, while if the production function is Cobb-Douglas, that is $f(k_t) = k_t^\theta$, that ratio is constant and equal to $-(\theta - 1)$. If this condition does not hold, then it is possible that the economy can display a global contraction where $k_{t+1} = [p s_t^E + (1-p) s_t^N] < k_t \forall t$.¹² For example, if the production function is closely to be linear, then it is likely that the condition does not hold. Notice that for $p = 0$, as in a classical Diamond model, that condition automatically hold, while if $p = 1$, as in the John and Pecchenino's model, the condition becomes $-\frac{k_t f''(k_t)}{f'(k_t)} \geq \frac{\beta}{\gamma}$, that is, positive capital accumulation depends on the relative weight of the effects of production (β) and investment maintenance (γ) on environmental quality. Higher is the effectiveness of the investment maintenance (γ) respect to β and higher is the possibility that the economy will accumulate capital over time. Furthermore, we impose, $|\Psi_{k_t}| < 1$, that is the equilibrium path of capital is not explosive. Notice that this will true if the production function is a Cobb-Douglas like the one described above. The second result of Proposition 2 says that if $s_r^E \geq 0$, environmental quality affects positively the future level of capital. This follows from the fact that higher (lower) is the level of environmental quality that each generation inherits and lower (higher) should be the investment in maintenance by agents of type E . The result will be a higher (lower) investment in productive capital and consequently a higher (lower) level of future capital. Finally, we can notice that $\frac{\partial k_{t+1}}{\partial p} \leq 0$, that is, higher is the proportion of type E agents in the economy and lower

¹²Galor and Ryder (1989) analyse the conditions for global contraction in a standard Diamond's model with productive capital.

will be the capital level. This is because that a decrease in $(1 - p)$ will lower the saving of type N agents more than it will increase in the saving of the type E agents. The net effect is a decrease in the total saving in the economy.

We now look at Equation 8) that defines E_{t+1} as a function of E_t, k_{t+1} and k_t . Using the fact that $k_{t+1} = \Psi(k_t, E_t)$, the law of motion for E can be written as:

$$E_{t+1} = \Phi(E_t, k_t) \quad (13)$$

The following proposition states the main properties of the function $\Phi(\cdot)$:

Proposition 3 *Suppose that $s_r \geq 0$, then the function $\Phi(\cdot)$ has the following properties: i) $\Phi_{E_t} \gtrless 0$ if and only if $\alpha \gtrless -(\gamma p \Delta)$; ii) $\Phi_{k_t} \gtrless 0$ if and only if $\gamma p [s_y f'(k_t) - f''(k_t)k_t(1 - s_w) - s_r f''(k_{t+1})\Psi_{k_t}] \gtrless \beta f'(k_t)$.*

$$\text{Where } \Delta = \left[\frac{-s_r f''(k_{t+1})p}{1 - s_r f''(k_{t+1})p} - 1 \right] \text{ and } -1 < \Delta < 0.$$

Proof. We simply differentiate equation 13), taking into account Assumption 2, results in Proposition 2 and the properties of the function s^E derived previously. ■

Let's analyse in detail the result i) in Proposition 3. The level of environmental quality in t has three effects on the evolution of E_{t+1} . First there are two positive effects, one is given by $\alpha > 0$, and the other is given by $\gamma p s_r f''(k_{t+1})\Psi_{E_t}$. The first effect is a direct effect of E_t on E_{t+1} . The second is an indirect effect through the saving function s^E . In particular it measures the effect that E_t has on k_{t+1} , and thus on the interest rate at time $t + 1$. From Proposition 2), we know that an increase in the environmental quality today will increase the level of capital tomorrow. Given the properties of the production function, this will result in a reduction of the interest rate tomorrow and this will have a positive effect on the investment in maintenance m . The negative effect of E_t on E_{t+1} is given by $\gamma p s_E$. An increase in the environmental quality today will decrease the investment in maintenance since it increases s^E . In order to have $\frac{\partial E_{t+1}}{\partial E_t} > 0$, the net effect has to be positive. As far as result ii) is concerned, it says that E_{t+1} increases in k_t only if the negative effect of k_t on E_{t+1} , given by $\beta f'(k_t)$, is less than the benefit that k_t has on E_{t+1} given by $\gamma p f''(k_t)k_t(1 - s_w) - \gamma p s_r f''(k_{t+1})\Psi_{k_t}$. The first term of that benefit function $-\gamma p f''(k_t)k_t(1 - s_w)$ is positive, since higher is the wage rate w_t and thus higher is the investment in maintenance. The second term, $-s_r f''(k_{t+1})\Psi_{k_t}$ captures the effect of k_t , through the interest rate, on the s^E and thus on m . Thus effect is clearly positive. The third term $\gamma p s_y f'(k_t)$ captures the positive effect of the production level of last period on the environmental quality next period through a decrease in s^E . Furthermore, we have that if $\frac{\partial E_{t+1}}{\partial E_t} > 0$, then $0 < \Phi_{E_t} < 1$, that is equation 13) is not explosive in E_t . Condition $0 < \Phi_{E_t} < 1$ implies that $\frac{\alpha}{1 - \gamma p \Delta} < 1$, and given the assumptions on the parameters involved, is always satisfied. Finally, it can be shown that $\frac{\partial E_{t+1}}{\partial p} \geq 0$, that is, higher is the proportion of agents of type E and higher is the investment in maintenance of environmental quality, thus, everything else constant, environmental quality increases.

4 Properties of the Steady State

In this section we shall analyse in detail the properties of the dynamic system given by 7) and 8). We start with the derivation of the steady state equilibrium of our model, that is an allocation in which capital and environmental quality remain constant over time. Substituting the facts that $E_{t+1} = E_t = E$ and $k_{t+1} = k_t = k$ into equations 7) and 8) give us the following system:

$$k - ps^E \left(f(k) - f'(k)k, f'(k), E, f(k) \right) + (1-p) \left(f(k) - f'(k)k \right) = 0 \quad (14)$$

$$E = \frac{p\gamma}{(1-\alpha)} \left(f(k) - f'(k)k - s^E \left(f(k) - f'(k)k, f'(k), E, f(k) \right) \right) - \frac{\beta f(k)}{(1-\alpha)} \quad (15)$$

Define with $\{\bar{k}, \bar{E}\}$ the steady state levels of the capital and the environmental quality that result as a solution of the above system. Assume that a steady state with $\bar{E} > 0$ and $\bar{k} > 0$ exists. In order to derive the local stability properties of system 7)-8), we linearize that system around that positive steady state. The linearized system is given by:

$$\begin{bmatrix} k_{t+1} - \bar{k} \\ E_{t+1} - \bar{E} \end{bmatrix} = \begin{bmatrix} \Psi_{\bar{k}} & \Psi_{\bar{E}} \\ \Phi_{\bar{k}} & \Phi_{\bar{E}} \end{bmatrix} \begin{bmatrix} k_t - \bar{k} \\ E_t - \bar{E} \end{bmatrix} \quad (16)$$

where the partial derivative in the Jacobian matrix of system 16) are given by:

$$\Psi_{\bar{k}} = \frac{-\sigma_{\bar{k}} [p(1-s_w) - 1] + ps_{\bar{y}}}{1 - s_r f''(\bar{k})p}$$

$$\Psi_{\bar{E}} = \frac{ps_{\bar{E}}}{1 - s_r f''(\bar{k})p}$$

$$\Phi_{\bar{k}} = -\gamma p f''(\bar{k}) [\bar{k}(1-s_w) - s_r \Psi_{\bar{k}}] + f'(\bar{k}) [\gamma ps_{\bar{y}} - \beta]$$

$$\Phi_{\bar{E}} = \alpha - \gamma p (s_{\bar{E}} + s_r f''(\bar{k}) \Psi_{\bar{E}})$$

where $\sigma_{\bar{k}} = -\frac{\bar{k} f''(\bar{k})}{f'(\bar{k})}$. An equilibrium $\{\bar{k}, \bar{E}\}$ is locally stable if the eigenvalues of the Jacobian matrix in 16) are both less than 1. In particular, if both eigenvalues are positive and less than one, the convergence towards that steady-state, starting from any initial condition, is also monotonic.

In the following proposition we state the conditions under which a steady-state $\{\bar{k}, \bar{E}\}$ is locally stable:

Proposition 4 *Assume that $0 < \Psi_{\bar{k}} < 1$ and $0 < \Phi_{\bar{E}} < 1$, then the steady-state $\{\bar{k}, \bar{E}\}$, involving a high level of capital and a high level of environmental quality, sufficient (but not necessary) condition for the steady-state to be locally stable are:*

$$\sigma_k < \beta - \frac{s_y}{s_E} \left((1+p)s_E - \frac{\alpha}{\gamma} \right)$$

Proof. Given the assumptions, the trace of the Jacobian matrix is positive and less than 2. This implies that the sum of the two eigenvalues (at least the real part) is positive and less than 2. If the determinant of the Jacobian matrix is positive, then we have that the two eigenvalues are between zero and one, meaning that the steady-state is clearly stable and the convergence towards that equilibrium is also monotonic. The determinant of the Jacobian matrix may be written as $-\sigma_k \alpha [(1-s_w)p - 1] + \gamma p s_E (\beta - \sigma_k) + p s_y [\alpha - \gamma s_E (1+p)]$. We simply impose that the determinant must be positive and thus, we find the condition stated above. ■

The results in Proposition 4) guarantee the topological equivalence between the non-linear system 7)-8) and the linear system 14)-15) in a neighborhood of a hyperbolic steady state.¹³ The condition for local stability involves the curvature of

the production function. This is not surprising since we saw that the curvature of the production function played an important role in determining the sign of capital accumulation. We can notice that the condition in Proposition 4) depends crucially on the ratio $\frac{s_y}{s_E}$. In particular, anything else given, if $|s_E| < |s_y|$, the term $\beta - \frac{s_y}{s_E} \left((1+p)s_E - \frac{\alpha}{\gamma} \right)$ is negative and thus the stability condition does not hold. On the other hand, if $|s_E| > |s_y|$, that is a necessary (but not sufficient) condition for the stability condition to hold. Thus, we need the saving function of the type E individuals to be quite sensitive to the environmental quality conditions. The intuition is simple. In our economy there are two types of agents and independently of the level of the environmental quality, there are always agents (type N) that will invest in productive capital sustaining capital accumulation (under the assumptions of Proposition 2). However, capital accumulation needs also the saving of the type E agents to be effective and to lead higher environmental quality. This will true if the saving function of type E agents is more sensitive to the environmental quality (that has a positive effect on s^E) than to the output level (that has a negative effect on s^E).

We now consider the effects of a change in the proportion of type E agents on the steady-state equilibrium defined in Proposition 4). The next proposition summarises those effects:

Proposition 5 *Given the steady-state $\{\bar{k}, \bar{E}\}$ defined in Proposition 4) we have that:*

$$\frac{\partial \bar{k}}{\partial p} \geq 0, \frac{\partial \bar{E}}{\partial p} \leq 0.$$

¹³An equilibrium $\{\bar{k}, \bar{E}\}$ is hyperbolic if none of the eigenvalues of the Jacobian matrix has modulus equal to one. See Azariadis (1993) ch. 6.

Proof. We have that $\Psi_p = \frac{\partial k_{t+1}}{\partial p} \leq 0$ and $\Phi_p = \frac{\partial E_{t+1}}{\partial p} \geq 0$. Given the fact that the Jacobian matrix in 16) is not singular under the assumptions of Proposition 4), we can apply the implicit function theorem that gives us:

$$\frac{\partial \bar{k}}{\partial p} = -\frac{\det \begin{bmatrix} \Psi_p & \Psi_{\bar{E}} \\ \Phi_p & \Phi_{\bar{E}} \end{bmatrix}}{\det \begin{bmatrix} \Psi_{\bar{k}} & \Psi_{\bar{E}} \\ \Phi_{\bar{k}} & \Phi_{\bar{E}} \end{bmatrix}} \geq 0 \text{ and } \frac{\partial \bar{E}}{\partial p} = -\frac{\det \begin{bmatrix} \Psi_{\bar{k}} & \Psi_p \\ \Phi_{\bar{k}} & \Phi_p \end{bmatrix}}{\det \begin{bmatrix} \Psi_{\bar{k}} & \Psi_{\bar{E}} \\ \Phi_{\bar{k}} & \Phi_{\bar{E}} \end{bmatrix}} \leq 0$$

■

Proposition 5) gives us an interesting result. It may be worse for the environmental quality (in steady-state) to have an increase in the number of people that are "environmental friendly". There are many reasons for that result. First of all, in our model there is no population growth, thus, increasing the number of type E agents will necessarily reduce the number of type N agents, and this will have a clear effect on capital accumulation. On one hand it will have a negative effect by reducing the saving of type N agents, on the other hand, it will have a positive effect by increasing the saving of type E agents. The results in Proposition 5) says that the latter effect in general equilibrium is stronger than the former one. Second of all, an increase in the number of type E agents will reduce the investments in maintenance. For each type E agent, there is no need to invest the same amount of resources, since there are more individuals that can invest in environmental quality. The results in Proposition 5) says that the effect of reducing the investment in maintenance is stronger than the effect due to the presence of more type E agents investing in environmental preservation.

5 The Rise of Complex Dynamics: an Example

In the previous section we have analysed the local stability properties of the steady-state of our model under the assumption that capital accumulation is positive. We showed that there is a steady-state involving high capital level and high environmental quality level that is locally stable. However, the dynamic system 7) and 8) that describes our economy is in general non-linear, and thus, we can expect that complex dynamic behaviour of the state variables may arise in our model.¹⁴ In order to analyse this issue, we consider the simplest case of logarithmic utility functions and logarithmic production function. The utility function for type E agents is assumed to be $U^E = \ln(c_{t+1}^E) + \ln(E_{t+1})$, while the one of type N agents is simply $U^N = \ln(c_{t+1}^N)$.

¹⁴Zhang (1999), using the same specification of John and Pecchenino (1994), showed that if the maintenance efficiency relative to environmental degradation is not sufficiently high, cyclically or chaotically fluctuating equilibria are more likely to exist. This implies that the transition towards an environmentally sustainable state is not trivial.

The production function is given by: $y_t = a \ln(1+k_t)$, with $a > 0$. It is easy to see that the utility and production function specified satisfy assumptions 1) and 2). Given that specification the saving function of type E agents is simply: $s_t^E = \frac{w_t}{2} + \frac{\alpha E_t - \beta y_t}{2\gamma p}$. The linearised version of system 7) and 8) is given by:

$$\begin{bmatrix} k_{t+1} - \bar{k} \\ E_{t+1} - \bar{E} \end{bmatrix} = \begin{bmatrix} \left(\frac{pa \ln(1+\bar{k})}{2(1+\bar{k})} + \frac{\beta}{2\gamma} \right) & \frac{1}{2\gamma} \alpha \\ \frac{1}{2} \gamma p \left(1 - \frac{a \ln(1+\bar{k})}{(1+\bar{k})} \right) - \frac{\beta}{2} & \frac{1}{2} \alpha \end{bmatrix} \begin{bmatrix} k_t - \bar{k} \\ E_t - \bar{E} \end{bmatrix} \quad (17)$$

The determinant of the Jacobian matrix in 17) is simply $\frac{\alpha[(\bar{k}+1)(2\beta-\gamma p)+2pa \ln(\bar{k}+1)]}{4\gamma(1+\bar{k})}$, and it is surely positive for $\beta \geq \frac{\gamma p}{2}$. The trace of the Jacobian is $\frac{pa \ln(1+\bar{k})}{2(1+\bar{k})} + \frac{\beta}{2\gamma} + \frac{1}{2} \alpha$. Define the eigenvalues of the Jacobian as: $\lambda_i(k, a, \beta, \gamma, \alpha)$ with $i = 1, 2$. Here we focus on a particular case of complex dynamics, that is called "saddle-node" bifurcation. A saddle-node bifurcation arises when a change in the parameters results on eigenvalue equal to one while the other remain in absolute value less than one. When this happens, the implicit function theorem cannot be applied and the steady-state of the linearised version loses its topological equivalence with the non-linear version. Using the classical properties of the determinant and the trace, the critical region in which such a bifurcation can arise is defined as: $\det(J) = \lambda_1 \lambda_2 \in (-1, 1)$ and $tr(J) = \lambda_1 + \lambda_2 \in (0, 2)$.¹⁵

In order to show the existence of saddle-node bifurcation, we need to show that for particular values of the parameters, we have $\lambda_1 = 1$ and $|\lambda_2| < 1$. Furthermore, those values need to respect the various constraints we have assumed, that is, $\alpha \in (0, 1)$, $p \in [0, 1]$ and $\beta, \gamma, k \geq 0$.

We need first to calculate the two eigenvalues of J , after some computation, we find:

$$\lambda = \frac{[p\gamma a \ln(1+\bar{k}) + (k+1)(\beta + \gamma\alpha)]}{4(1+\bar{k})\gamma} \pm \sqrt{\frac{(1+\bar{k})[\alpha\gamma(\alpha\gamma - 6\beta + 4\gamma p) + \beta^2] + \gamma pa \ln(1+\bar{k})[2(1+\bar{k})(\beta - 3\gamma\alpha) + \gamma pa \ln(1+\bar{k})]}{4(1+\bar{k})\gamma}}$$

where λ_1 is the eigenvalue with the positive sign in front of the square root, while λ_2 is the one with the negative sign in front of the square root.

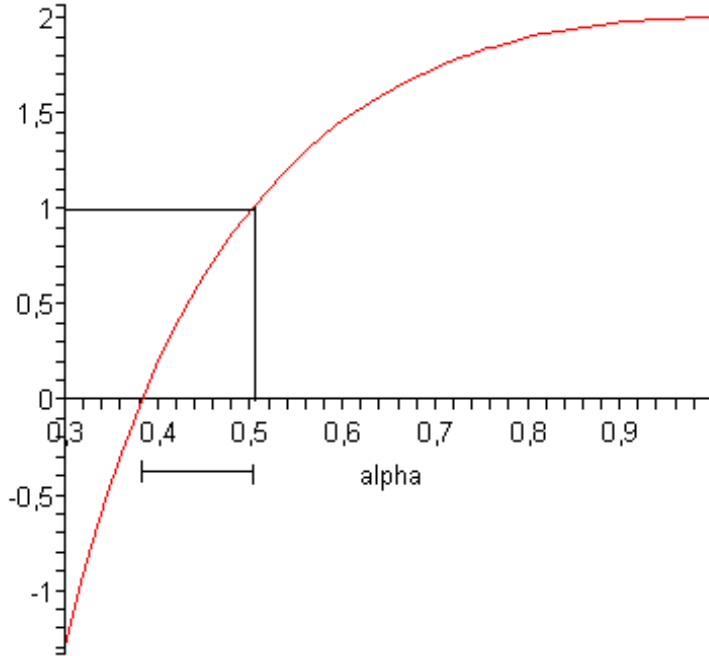
We just need to show that there are some values of the parameters such that $\lambda_1 = 1$ and $|\lambda_2| < 1$. We decide to solve the system implied by the two eigenvalues for β and p such that $\lambda_1 = 1$ and $\lambda_2 = 0.5$. This is enough to show that a saddle-node bifurcation may arise.

Next proposition states the main result:

Proposition 6 *There exists an interval for $\alpha \in (\underline{\alpha}, \bar{\alpha})$ such that a saddle-node bifurcation may arise in system 17) if: $p^* = \frac{2(3\alpha - \alpha - 1)}{\alpha}$ and $\beta^* = \frac{\gamma[2a \ln(1+\bar{k})(1 - 3\alpha + \alpha^2) - (1+\bar{k})\alpha(\alpha - 3)]}{\alpha(1+\bar{k})}$.*

Proof. From Proposition 6) we have for such values of β and p one eigenvalue of the Jacobian matrix is equal to one, while the other is equal to 0.5. In such a

¹⁵See Azariadis (1993) pp.92-95, for a more technical treatment.



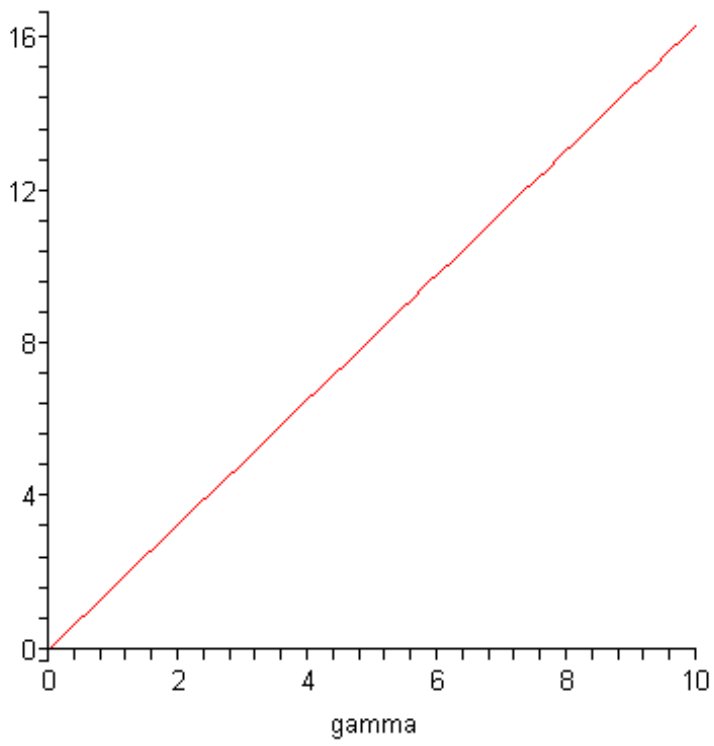
circumstance our system displays a saddle-node bifurcation. However, the critical value of the proportion p depends only on α . The interval for α in Proposition 6) is the interval for which the value of p^* is between 0 and 1. This is because, the range for the p^* stated above is between $(-\infty, 2)$ as $\alpha \in (0, 1)$. Thus, there is a subset of the interval $(0, 1)$ for α such that the value of p^* lie between 0 and 1. The value of β^* instead is always positive if the value of a is not too high. ■

To describe better the relationship between p^* and α , we plot the function $p^*(\alpha)$ as stated in Proposition 6) in the following Figure:

That Figure shows that for values of α that lie in a small interval (for example, between 0.4 and 0.5) there are values of p^* such that a saddle-node bifurcation may arise. Another interesting fact that arises from that figure is that the degree of persistence of the environmental quality (α) should not be very high (close to 1). In the next figure we plot the solution of β^* as a function of γ , the other two parameters of interest for our analysis.

Given the values of the other parameters (with the value of α included in the critical interval discussed above) that relationship is a straight line¹⁶:

¹⁶The figure is drawn with the following parametrisation: $\alpha = 0.5, a = 1$ and $k = 5$. The values of the parameter would affect the slope of the relationship between β and γ but not the qualitative



The important feature of the figure above is that the slope of the straight line is greater than one. Thus, a saddle-node bifurcation may arise only if $\beta > \gamma$, that is, if environmental degradation is higher than the efficiency of environmental maintenance.¹⁷ Furthermore, the degree of persistence of the environmental quality needs to be not very high and should lie in a particular interval.

6 Conclusion

We studied a discrete time overlapping generations model in which, in each period, there are two types of individuals that live for two periods. One type has preferences over its own consumption and also on the level of environmental quality, while the other type cares only on its own consumption. Young individuals of both types can invest in productive capital in order to consume when they are old. Furthermore, agents that care also on environmental quality can invest in environmental preservation as well. The environmental quality at each period is affected negatively by production in the previous period. This creates an intergenerational externality between different generations that affects the capital accumulation process of our economy. Capital accumulation is positive in our framework only if the curvature of the production function satisfies a particular condition that mainly depends on the parameters of the environmental quality evolution law. In particular, if the investment in environmental quality is not very effective, then it may be possible that positive capital accumulation will not occur and thus, our economy will display a global contraction. This is because, "environmental friendly" agents will tend to invest more in environmental quality than in productive capital, this will slow down the capital accumulation process. If capital accumulation slow down, also investment in environmental quality will slow down, since the resources (wages) available to each new generation will be lower than before. Thus, there will be less resources that can be invested in environmental maintenance. In our model we show that there is a local stable steady-state characterised by high capital and high environmental quality if the saving function of the agents that are "environmental friendly" is more sensitive to environmental quality than to the output level experienced in the previous. Since the output level affects negatively the environmental quality, if the saving function of the "environmental friendly" agents is very sensitive to that, those agents will reduce their saving in productive capital to invest in environmental maintenance. However, this fact, will reduce capital accumulation and in the long run also the level of environmental quality (since the two variables move in the same direction). Furthermore, an increase in the proportion of people that care about environmental quality, taking constant the overall population, may be worse for the environment, since in equilibrium there will be a reduction in the investment for environmental

result that β should be greater than γ .

¹⁷This is a similar result as in Zhang (1999).

maintenance.

The existence of an equilibrium involving high capital and high environmental quality level is consistent with observations that relatively poor countries experience higher environmental degradation than developed countries. This result is also consistent with the idea that the relationship between per-capita income and environmental quality follows a “U”-shaped curve, in which there is environmental degradation in the first part of the growing path of the economy, but when capital is sufficiently high, environmental quality tends to increase.

Finally, given the non-linear feature of our model, we showed that complex dynamics can arise. Using a simple specification, with logarithmic utility and logarithmic production function, we found that for some particular values of the parameters of the model, a saddle-node bifurcation may arise. In particular, that can happen if the environmental degradation is higher than the efficiency of environmental maintenance and if the degree of persistence of the environmental quality is not very high and lies in a particular interval. In such a situation, the steady-state equilibrium of our model loses its stability properties and more steady-states may arise. We focused only on a particular case of bifurcation, however, it would be interesting to analyse other kind of bifurcations that can give rise to limit cycles or even chaotic dynamics.

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