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Disentangling economic recessions and depressions

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Non-technical summary

The business cycle is traditionally described as a sequence of recessions and expansions in aggregate economic activity. However, from a historical perspective it is well-known that some recessions are much longer, deeper or more abrupt than others. Thus, the question arises whether the binary characterization of the cyclical economic development is not overly simplistic. For example, a semantic distinction between economic recessions and “depressions” is commonly made in the economic policy discourse. Indeed, vast contractions of real output have stimulated interest in rare “economic disasters” among academic economists. The binary framework does not allow for considering such rare and severe events by construction.

Questioning the validity of the binary approach constitutes the starting point of this paper. We distinguish economic depressions and booms from ordinary recessions and expansions using a novel nonparametric test. Given the peak and trough dates of the U.S. business cycle between 1919 and 2009 provided by the National Bureau of Economic Research (NBER), we compute characteristics which capture the length, depth and shape of each recession and expansion phase. Business cycle phases are then classified into four regimes according to the magnitude of these characteristics, such that we can also distinguish between recessions vs. depressions and expansions vs. booms. Our method identifies three economic depressions and one economic boom in the U.S. business cycle, which coincide with important historical episodes, including the Great Depression and the industrial production boom during the Second World War. Interestingly, despite all comparisons between the Great Depression and the recent financial and economic crisis, the so-called “Great Recession”, dated between December 2007 and June 2009 by the NBER, does not qualify as extraordinary in comparison with previous economic downturns. Even though it is the most severe postwar recession, it is dwarfed along every dimension by the prewar depressions.

In the second part of the paper, we analyze by means of logistic regressions whether the four business cycle regimes can be predicted using macroeconomic and financial variables. Numerous studies have shown that the slope of the yield curve exhibits good predictive power for future recessions. The related literature also attributes some predictive power to stock market returns, the growth rate of real output and the inflation rate. Is it possible to tell whether the economy is heading toward a recession or a depression using these leading indicators? Our results show that although the slope of the yield curve outperforms other leading indicators in predicting recessions, it does not have anything to say about future economic depressions. In contrast, stock market returns, real output growth, and the inflation rate convey statistically relevant information for predicting economic depressions: a drop in these variables signals a higher likelihood of a future depression.

Nicht-technische Zusammenfassung

Der Konjunkturzyklus wird traditionell als Abfolge von Rezessionen und Expansionen der gesamtwirtschaftlichen Aktivität beschrieben. Aus der historischen Perspektive gemeinhin bekannt ist jedoch die Tatsache, dass einige Rezessionen viel länger, tiefgreifender und abrupter sind als andere. Daraus erwächst die Frage, ob die binäre Charakterisierung der Schwankungen in der Wirtschaftsentwicklung nicht allzu simplistisch ist. Semantisch wird beispielsweise im wirtschaftspolitischen Diskurs gewöhnlich zwischen wirtschaftlicher Rezession und „Depression“ unterschieden. In der Tat haben schwerwiegende Kontraktionen der realen gesamtwirtschaftlichen Produktion das Interesse der Wirtschaftswissenschaftler an seltenen „wirtschaftlichen Desastern“ erregt. Der binäre Rahmen lässt es aufgrund seiner Konstruktionsweise nicht zu, solch seltene und einschneidende Ereignisse gesondert unter die Lupe zu nehmen.

Die Hinterfragung der Validität des binären Ansatzes stellt den Ausgangspunkt dieses Forschungspapiers dar. Wir unterscheiden große wirtschaftliche Depressionen und Boomphasen von gewöhnlichen Rezessionen und Aufschwüngen im Konjunkturzyklus mithilfe eines neuen nichtparametrischen Testverfahrens. Anhand von Daten des National Bureau of Economic Research (NBER) zu den Höhe- und Tiefpunkten der Wirtschaftszyklen in den Vereinigten Staaten zwischen 1919 und 2009 berechnen wir Kenngrößen, welche Länge, Tiefe und Ausformung jeder Rezessions- und Aufschwungsphase erfassen. Die Phasen im Wirtschaftszyklus werden dann entsprechend der Dimension dieser Kenngrößen in vier Ausprägungen unterteilt, um zwischen Rezession und Depression bzw. Aufschwung und Boom zu differenzieren. Mit unserer Verfahrensweise identifizieren wir drei wirtschaftliche Depressionen und einen Boom im US-Konjunkturzyklus, die mit wichtigen historischen Zeitabschnitten, so der Großen Depression und dem Industrieboom während des zweiten Weltkriegs, zusammenfallen. Interessant dabei ist die Beobachtung, dass sich ungeachtet aller Vergleiche der Großen Depression mit der letzten Finanz- und Wirtschaftskrise die sogenannte „Große Rezession“, die das NBER auf die Zeitspanne von Dezember 2007 bis Juni 2009 datiert, in der Gegenüberstellung mit vorangegangenen Konjunkturabschwüngen nicht als außergewöhnlich beweist. Obgleich es sich dabei um die schwerste Rezession der Nachkriegszeit handelt, wird sie von den Depressionen der Vorkriegszeit in jeder Dimension um Längen überragt.

Im zweiten Teil des Aufsatzes analysieren wir mittels logistischer Regressionen, ob die bei einer solchen Charakterisierung der wirtschaftlichen Schwankungen identifizierten vier Regime unter Zuhilfenahme von Makroökonomischen- und Finanzvariablen vorhersagbar sind. Zahlreiche Studien haben gezeigt, dass das Gefälle der Zinsstrukturkurve gute Prognoseeigenschaften für kommende Rezessionen aufweist. Eine gewisse Prognosekraft misst die einschlägige Fachliteratur auch den Aktienmarktrenditen, der Wachstumsrate der realen Wirtschaftsleistung und der Inflationsrate bei. Ist es mithilfe dieser Frühindikatoren möglich zu erkennen, ob die Wirtschaft auf eine Rezession oder eine Depression zusteuert? Gemäß unseren Ergebnissen besitzt der Verlauf der Zinsstrukturkurve - trotz deren Überlegenheit gegenüber anderen Frühindikatoren bei der Vorhersage von Rezessionen - keinerlei Aussagekraft mit Blick auf heraufziehende Wirtschaftsdepressionen. Demgegenüber liefern Aktienmarktrenditen, das reale Wirtschaftswachstum und die Inflationsrate statistisch relevante Informationen für die Prognose wirtschaftlicher Depressionen: ein Abfall dieser Variablen signalisiert eine höhere Wahrscheinlichkeit, dass in näherer Zukunft eine schwerwiegende Depression im Anmarsch ist.

Disentangling Economic Recessions and Depressions*

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Abstract

We propose a nonparametric test that distinguishes “depressions” and “booms” from ordinary recessions and expansions. Depressions and booms are defined as coming from another underlying process than recessions and expansions. We find four depressions and booms in the NBER business cycle between 1919 and 2009, including the Great Depression and the World War II boom. Our results suggest that the recent Great Recession does not qualify as a depression. Multinomial logistic regressions show that stock returns, output growth, and inflation exhibit predictive power for depressions. Surprisingly, the term spread is not a leading indicator of depressions, in contrast to recessions.

Keywords: Business cycles, Depression, Leading indicators, Multinomial logistic regression, Nonparametric statistics, Outlier.

JEL classification: C14, C35, E32.

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1 Introduction

The business cycle is traditionally modeled as a sequence of recessions and expansions in aggregate economic activity. This binary approach characterizes the regime-switching models of, *inter alia*, [Hamilton \(1989\)](#) and [Potter \(1995\)](#), as well as the turning-points methodology of [Bry and Boschan \(1971\)](#), [King and Plosser \(1994\)](#) and [Harding and Pagan \(2002\)](#). Moreover, the prediction of business cycles has typically also been approached via binary models, see, e.g., [Estrella and Mishkin \(1998\)](#) and [Kauppi and Saikkonen \(2008\)](#). However, some recessions and expansions have been much stronger than others – e.g., in terms of duration or output growth – throughout the history of the U.S. business cycle. This notion is also reflected by a semantic difference between recessions vs. “depressions” and expansions vs. “booms” often made by policymakers and the popular press. One possibility is that such extreme episodes are tail realizations from the same data generating process (DGP) as ordinary recessions or expansions, which would imply that a two-regime approach delivers correctly specified business cycle models. Alternatively, depressions and booms may be considered as “outliers” arising from a different DGP than recessions and expansions. Neglecting this multi-phase character of the business cycle may distort model specifications of the true DGP.

The objective of this paper is therefore to investigate the possibility that the DGP of the business cycle is characterized by more than two regimes. We propose a novel non-parametric outlier detection framework to distinguish economic depressions and booms from ordinary recessions and expansions based on the occurrence of outliers in some popular business cycle characteristics.¹ The considered characteristics include the duration, amplitude, cumulated movements, and excess cumulated movements suggested by [Harding and Pagan \(2002\)](#). We classify a phase as depression or boom if at least one of its characteristics exhibits an outlier for that particular time frame relative to other historical episodes. Our outlier detection algorithm results in a four-regime classification of the U.S. business cycle. This enables us to refine some of the conclusions drawn from previous recession prediction exercises conducted within the two-regime framework.

The amount of U.S. economic recessions and expansions over the past century is relatively small and the closed form of the statistical distribution of business cycle characteristics is unknown. We therefore propose a distribution-free outlier detection test that combines two results from nonparametric statistics and that performs well in small samples. Our method involves bootstrapping the empirical distribution of the business cycle characteristics and computing the difference between the arithmetic mean and the trimmed mean of each bootstrap sample. This yields a measure of central tendency termed “mean-trimmed mean”. [Singh and Xie \(2003\)](#) show that the mean-trimmed mean displays a multimodal histogram if the original sample contains one or more outliers. We employ the [Silverman \(1981\)](#) test to assess the null hypothesis of the histogram’s unimodality. If the histogram is multimodal, we sequentially remove the most extreme observations from the sample until we end up with a unimodal histogram that is free of outliers. The omitted observations correspond to the outlying business cycle characteristics. Applying this statistical procedure, we identify four depressions and booms of the U.S. business

¹A multitude of outlier definitions co-exist. We adopt the definition that an outlier constitutes an observation that does not arise from the same DGP as the majority of observations in the data set, conform to [Barnett and Lewis \(1994\)](#).

cycle over the time horizon 1919 to 2009. These coincide with economically meaningful episodes, like the Great Depression and the industrial production boom during the Second World War. However, despite all comparisons between the Great Depression and the recent financial and economic crisis, our results suggest that the 2007-2009 Great Recession does not qualify as a depression compared to previous historical episodes.

There is a growing interest in severe economic contractions in the macroeconomic literature (see, e.g., [Kehoe and Prescott, 2002](#)). Large and infrequent economic slumps may require a bolder set of policy interventions than ordinary recessions (see [Eggertsson and Krugman, 2012](#)). At the same time, rare “economic disasters”, such as depressions or wars, have been shown to play a role in determining asset risk premia, see, e.g., [Barro \(2006, 2009\)](#), [Gabaix \(2012\)](#), and [Wachter \(2013\)](#). A handful of papers have tried to incorporate such severe episodes into non-linear regime-switching time series models with more than two possible business cycle states, including [Tiao and Tsay \(1994\)](#), [Sichel \(1994\)](#), and [Cakmakli, Paap, and van Dijk \(2013\)](#). Meanwhile, substantial effort has been invested in testing for the actual number of regimes within Markov-switching models, see, e.g., [Cho and White \(2007\)](#), [Carter and Steigerwald \(2012\)](#), and [Carrasco, Hu, and Ploberger \(2013\)](#). To the best of our knowledge, we are the first to identify rare and severe recessions by means of nonparametric outlier detection techniques. Previous studies that deal with outliers in macroeconomic time series include, e.g., [Balke and Fomby \(1994\)](#) and [Giordani, Kohn, and van Dijk \(2007\)](#), but these are parametric in nature.

From the preceding literature it is well-known that financial and macroeconomic variables, such as the slope of the yield curve, stock market returns, or real output growth, exhibit some predictive power for future recessions and expansions (see, e.g., [Harvey, 1988](#); [Estrella and Hardouvelis, 1991](#); [Estrella and Mishkin, 1998](#); [Birchenhall, Jessen, Osborn, and Simpson, 1999](#); [Hamilton and Kim, 2002](#); [Kauppi and Saikkonen, 2008](#); [Rudebusch and Williams, 2009](#); [Christiansen, 2013](#)). The question arises to what extent these leading indicator properties carry over to the four-regime business cycle classification that emerges from applying our outlier detection test. The predictive ability of financial and macroeconomic variables is of potential importance for policymakers who would like to distinguish between an impending recession and a rare economic disaster. Thus, one would like to determine whether traditional leading indicators of recessions and expansions exhibit a different information content for depressions and booms. In fact, upon applying a multinomial logistic regression model to the four-regime business cycle, we find that the slope of the yield curve preserves its predictive power towards recessions but its leading indicator property vanishes for economic depressions. However, we are able to show that variables like past real output growth, inflation, and stock market returns help to predict extraordinary business cycle fluctuations.²

The remainder of the paper is organized as follows. Section 2 starts with a definition of business cycles and their characteristics. Subsequently, we introduce the nonparametric outlier test. Finally, we report estimated cycle characteristics across the NBER business cycle between 1919 and 2009, and we present the results from the outlier detection procedure. Section 3 provides a short theoretical digression on multinomial logit regressions. Next, the section compares empirical results of binomial with multinomial logit specifica-

²Throughout the paper “prediction” refers to *in-sample* predictive ability, since out-of-sample forecasts of depressions and booms would not make sense given the scant amount of genuine depression and boom phases within the historical sample.

tions and univariate with multivariate logit specifications, respectively. Finally, Section 4 provides a summary and conclusions.

2 The four-regime business cycle

2.1 Characterizing business cycles

Classical studies define business cycle fluctuations by means of turning points (peaks and troughs) in the level of real economic activity (see, e.g., [Burns and Mitchell, 1946](#); [Bry and Boschan, 1971](#); [King and Plosser, 1994](#); [Harding and Pagan, 2002](#)). A complete cycle in logarithmic real output y_t consists of a recession phase from a peak to the subsequent trough and an expansion phase from a trough to the subsequent peak. We use the National Bureau of Economic Research (NBER) peaks and troughs in economic activity to construct a binary variable S_t^{bin} that either reflects a recession phase ($S_t^{bin} = 1$) or an expansion phase ($S_t^{bin} = 0$). The turning point dates published by the NBER represent a consensus chronology of the U.S. business cycle.

To assess the relative strength of recessions and expansions, we focus on the unconditional frequency distribution of four commonly used business cycle characteristics: the duration (D), amplitude (A), cumulative movements (C) and excess cumulative movements (E) computed for each business cycle phase. Formal definitions of these characteristics are presented in the appendix. Previous work that characterizes the cyclical behavior of real economic activity by means of these business cycle characteristics include, e.g., [Harding and Pagan \(2002\)](#) and [Camacho, Perez-Quiros, and Saiz \(2008\)](#). [Bordo and Haubrich \(2010\)](#) have employed these characteristics to analyze cycles in money, credit, and output, while others have used the same measures to investigate financial cycles (see, e.g., [Pagan and Sossounov, 2003](#); [Claessens, Kose, and Terrones, 2012](#)).

2.2 Testing for depressions and booms

In order to distinguish depressions and booms from ordinary business cycle phases, we apply an outlier testing procedure to the previously described business cycle characteristics. A given recession (expansion) episode is classified as an economic depression (boom) if at least one of its four characteristics is an outlier for that period. More precisely, a depression (boom) is identified as follows. Provided that n recessions (expansions) are observed, we have a sample for each recession (expansion) characteristic denoted by X_1, \dots, X_n (X_i stands for either D_i , A_i , C_i , or E_i). The observation X_i that corresponds to phase $i = 1, \dots, n$ can be seen as a random draw from an unknown cumulative distribution function $F_X(\cdot)$. We define phase i as a depression (boom) if X_i is an outlier with respect to the distribution $F_X(\cdot)$, i.e., X_i is *not* distributed according to $F_X(\cdot)$.

We propose a distribution-free test for the null hypothesis that the sample X_1, \dots, X_n is free of outliers. The test combines two established results from nonparametric statistics. First, [Singh and Xie \(2003\)](#) introduce a “Bootlier Plot” to graphically detect the presence of outliers in a data set. The Bootlier Plot is the bootstrap density plot of the sample mean-trimmed mean statistic and its multimodality reflects the presence of outliers in the sample. Second, [Silverman \(1981\)](#) proposes a distribution-free test for the unimodality of a probability density function. Hence, we apply Silverman’s test to Bootlier Plots of phase

characteristics in order to detect outliers in these characteristics. If the null hypothesis of unimodality is rejected for the full sample, we proceed with ordering the observations X_1, \dots, X_n into ascending order, sequentially dropping observations from the tails of the ordered sample, and repeating Silverman's test on Bootlier Plots of these shrinking subsamples. We continue the iterative omission of the most extreme observations until the (subsample) Bootlier Plot becomes unimodal. The deleted observations can be identified as outliers.³

Let us now discuss this nonparametric outlier detection procedure in somewhat more detail. The Bootlier Plot is obtained as follows. Let X_1^b, \dots, X_n^b ($b = 1, 2, \dots, B$) denote bootstrapped samples of size n based on the original sample X_1, \dots, X_n of a particular business cycle characteristic. The resampling is repeated $B = 10,000$ times. The mean-trimmed mean (MTM) statistic is defined as the difference between the arithmetic mean and the k -trimmed mean of the b th bootstrap sample:

$$MTM^b = \frac{1}{n} \sum_{i=1}^n X_i^b - \frac{1}{n-2k} \sum_{i=k+1}^{n-k} X_{(i)}^b, \quad (1)$$

where $X_{(i)}^b$ are the ascending order statistics and k is a trimming value. Upon assuming that $F_X(\cdot)$ exhibits finite first and second moments, the central limit theorem applies and the pdf of the mean-trimmed mean $f_{MTM}(\cdot)$ converges asymptotically to a standard normal distribution in the absence of outliers. [Singh and Xie \(2003\)](#) show that $f_{MTM}(\cdot)$ can be expressed as a multimodal mixture of normal densities if the original sample X_1, \dots, X_n contains at least one outlier. The separation between the normal mixing components arises because only some of the bootstrap samples contain the outliers. As a result, $f_{MTM}(\cdot)$ exhibits one mode associated with the distribution $F_X(\cdot)$ and at least another mode corresponding to one or more outliers.⁴

In order to test the null hypothesis of unimodality of $f_{MTM}(\cdot)$ (absence of outliers in X_1, \dots, X_n) against the alternative hypothesis of multimodality (presence of one or more outliers), we apply the test proposed by [Silverman \(1981\)](#). The test uses as an input the kernel density estimate of the density function $f_{MTM}(\cdot)$. The kernel density estimator of the MTM statistic at any point x can be expressed as:

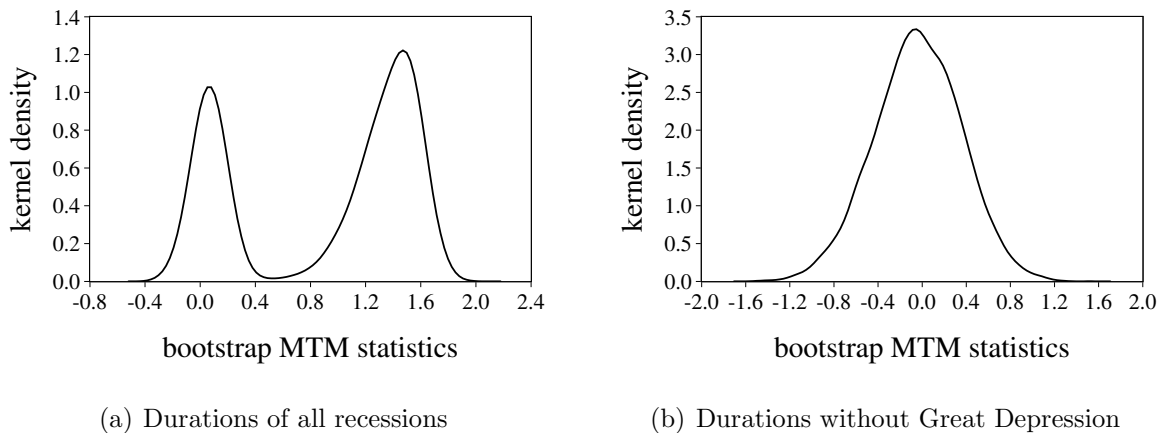
$$f(x, h) = \frac{1}{Bh} \sum_{b=1}^B K\left(\frac{x - MTM^b}{h}\right), \quad (2)$$

where h is a bandwidth and $K(\cdot)$ is a kernel function chosen to be the standard normal density function following [Silverman \(1981\)](#). For a large class of kernel functions including the standard normal, the number of modes of $f(x, h)$ decreases as the bandwidth h is increased. Thus, a sufficiently large h exists, for which the kernel density $f(x, h)$ has a single mode in the interior of a given closed interval \mathfrak{S} . The narrowest bandwidth for which

³By definition, outliers must be located in the upper or lower tails of the ascending order statistics $X_{(1)}, X_{(2)}, \dots, X_{(n-1)}, X_{(n)}$. We sequentially cancel the most extreme observations by considering the subsamples: $(X_{(1)}, \dots, X_{(n-1)})$, $(X_{(2)}, \dots, X_{(n)})$, $(X_{(1)}, \dots, X_{(n-2)})$, $(X_{(2)}, \dots, X_{(n-1)})$, $(X_{(3)}, \dots, X_{(n)})$, $(X_{(1)}, \dots, X_{(n-3)})$, etc. We stop this process once unimodality can no longer be rejected.

⁴As recommended by [Singh and Xie \(2003\)](#), we compute the MTM statistic with a trimming value of $k = 2$. [Singh and Xie \(2003\)](#) show that in the presence of an outlier the separation between the modes of the bootstrap density $f_{MTM}(\cdot)$ is approximately proportional to $1/k$ independent of the sample size.

Figure 1: Bootlier Plots of NBER Recession Durations



Note: Bootlier Plots of durations for NBER business cycle recessions between March 1919 and June 2009 (full sample) and for the subsample that excludes the duration of the Great Depression (40 months).

the kernel density estimate is unimodal is called the “critical bandwidth”. Intuitively, the critical bandwidth of a multimodal density should be larger than that of a unimodal density because a larger bandwidth is required to smooth out the multiple modes. This provides a rationale for using the critical bandwidth as test statistic.

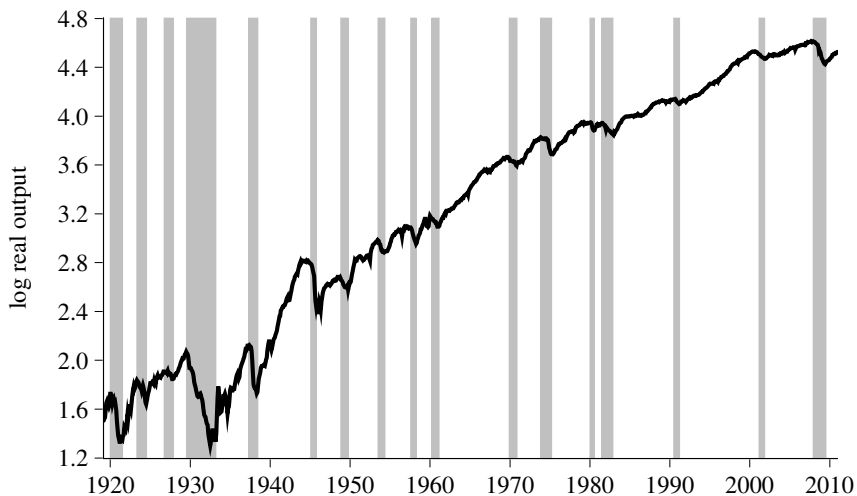
To implement Silverman’s test, we start with estimating the density of the MTM statistic in Equation (1) using the kernel density estimator in Equation (2). Next, we estimate the critical bandwidth \hat{h}_{crit} . Let us denote the kernel density estimated with this bandwidth as $\hat{f}(\cdot, \hat{h}_{crit})$. In order to determine the small sample distribution of the critical bandwidth \hat{h}_{crit} , we draw 1,000 bootstrap samples from the kernel density $\hat{f}(\cdot, \hat{h}_{crit})$. For each draw, the density of the bootstrapped MTM statistic is again estimated using the kernel density estimator defined in Equation (2). Let \hat{h}_{crit}^* denote the critical bandwidth of the kernel density obtained for one bootstrap draw. The null hypothesis of unimodality is rejected if $Pr(\hat{h}_{crit}^* \leq \hat{h}_{crit}) \geq 1 - \alpha$, where α is the nominal size.

If unimodality of the full-sample Bootlier Plot is rejected, we calculate Bootlier Plots and perform Silverman’s test on subsamples by eliminating the most extreme observations from the tails of the full sample X_1, \dots, X_n . We continue shrinking the sample until the null of unimodality can no longer be rejected.

Further details on the nonparametric outlier detection procedure, including its finite sample behavior, are reported in a companion paper (see [Candelon and Metiu, 2013](#)). The latter paper, *inter alia*, shows that the size and power properties of the outlier test are satisfactory in small samples like the ones encountered in this paper.

As a simple example of how our outlier testing procedure works, consider the durations of NBER business cycle recessions. Figure 1 (a) and Figure 1 (b) report the Bootlier Plots for the full sample and the subsample excluding the duration of the Great Depression phase, respectively. The multimodal plot implies that the Great Depression’s duration is an outlier of the duration’s empirical distribution. This suggests that the 1929-33 down-

Figure 2: The U.S. Business Cycle



Note: Log levels of U.S. real industrial production, March 1919 - December 2010. NBER recessions are shaded in grey.

turn was a genuine depression episode. Upon removing this data point from the historical duration sample, the subsample Bootlier Plot becomes unimodal and Silverman’s test does no longer reject the null of unimodality (p-value of 0.22). This indicates that there are no other outliers left in the empirical distribution of recession durations.

2.3 Depressions and booms in the U.S. business cycle

We compute the duration, amplitude, cumulated movements, and excess cumulated movements of the U.S. business cycle using the NBER turning points. Figure 2 plots the log of monthly U.S. real industrial production output between March 1919 and December 2010 with NBER recessions shaded in grey. Table 1 reports values of the characteristics computed for each recession and expansion phase. While on average recessions last about 10 months, the longest recession lasted 40 months from the peak in August 1929 till the trough in March 1933. The severity of recessions also varies significantly across different episodes: the average peak-to-trough amplitude of a recession is -17.6%, while the difference between the amplitude of the most severe (August 1929 - March 1933) and the mildest (March 2001 - November 2001) recession is 68.6 percentage points. Similar patterns emerge for expansions: the difference between the shortest and longest expansion is roughly 9 years, while the trough-to-peak output gains vary between 5.9% and 99.5% over the analyzed period. The varying strength of contractions and expansions suggests that one may gain more insight into the nature of the business cycle by further refining its traditional two-regime classification.⁵

Figure 3 shows the kernel density estimates of the Bootlier Plots corresponding to each

⁵The varying strength of U.S. contractions and expansions has also been studied by, e.g., [Neftci \(1984\)](#), [DeLong and Summers \(1986\)](#), [McQueen and Thorley \(1993\)](#), [McKay and Reis \(2008\)](#), and [Morley and Piger \(2012\)](#).

Table 1: U.S. Business Cycle Characteristics

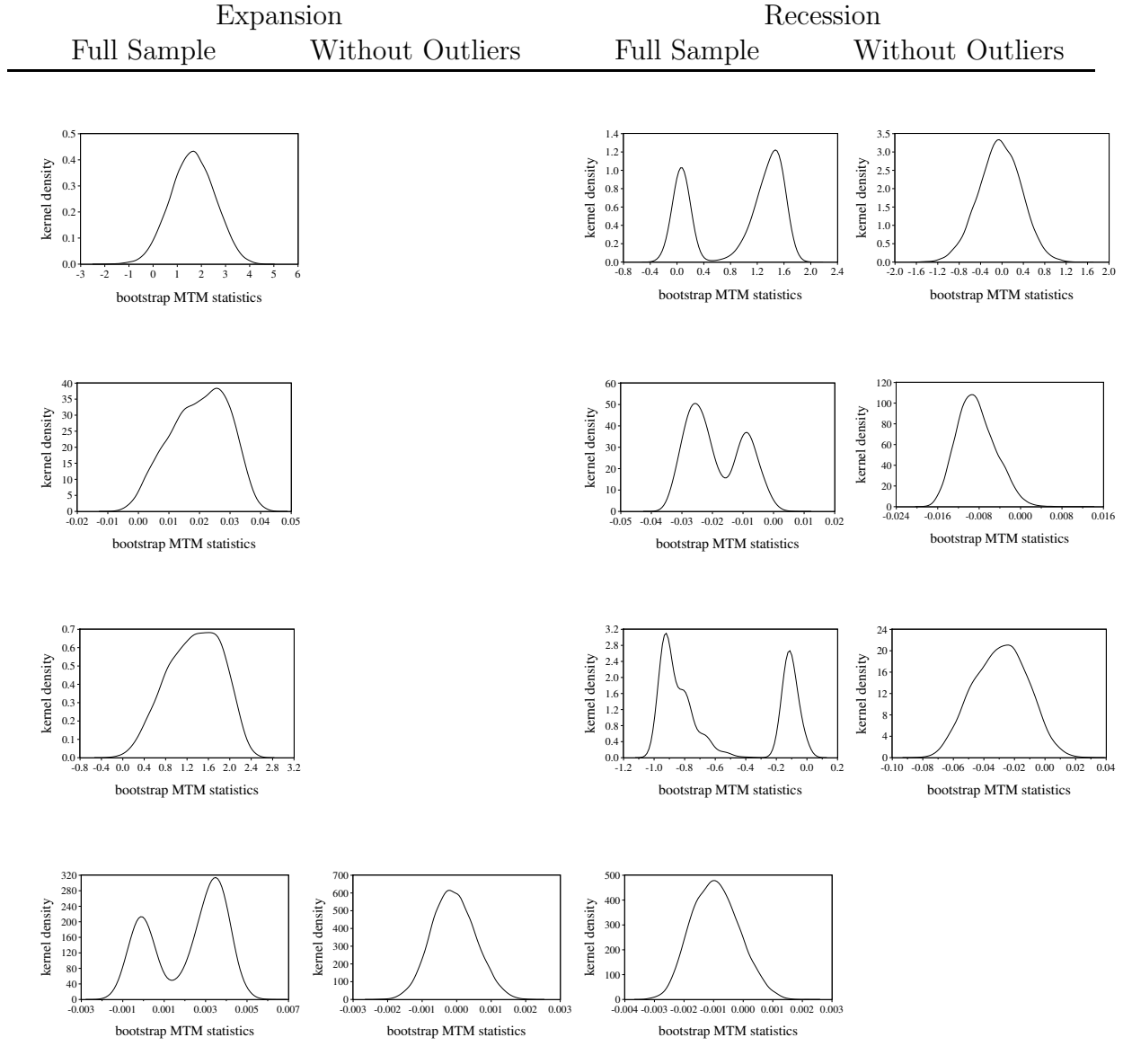
Turning points		Duration (months)		Amplitude (%)		Cumulation (%)		Excess (%)	
Peak date	Trough date	T-to-P	P-to-T	T-to-P	P-to-T	T-to-P	P-to-T	T-to-P	P-to-T
	Mar 1919								
Jan 1920	Jul 1921	10	15	20.07	-37.95	97.90	-366.46	-1.25	-4.19
May 1923	Jul 1924	22	11	45.81	-18.92	509.09	-98.40	-0.81	1.38
Oct 1926	Nov 1927	27	10	23.21	-5.76	410.78	-21.65	3.18	1.00
Aug 1929	Mar 1933	21	40	20.87	-72.11	235.09	-1742.67	0.26	-6.61
May 1937	Jun 1938	50	10	72.25	-37.03	2023.92	-276.69	3.63	-7.30
Feb 1945	Oct 1945	80	5	99.52	-33.65	4964.97	-105.90	11.68	-0.99
Nov 1948	Oct 1949	37	8	18.38	-8.04	407.15	-46.65	1.56	-1.31
Jul 1953	May 1954	45	7	38.06	-8.85	1049.35	-62.08	3.86	-3.81
Aug 1957	Apr 1958	39	5	19.54	-12.76	519.24	-52.63	3.29	-2.87
Apr 1960	Feb 1961	24	7	19.61	-6.25	325.38	-33.23	3.34	-1.18
Dec 1969	Nov 1970	106	8	55.29	-4.15	3518.74	-12.86	5.29	0.73
Nov 1973	Mar 1975	36	13	21.05	-13.57	366.10	-61.14	-0.65	2.60
Jan 1980	Jul 1980	58	3	25.95	-6.75	943.87	-20.29	3.07	-2.26
Jul 1981	Nov 1982	12	13	5.91	-8.91	47.82	-77.69	0.78	-1.18
Jul 1990	Mar 1991	92	5	29.00	-4.08	1731.95	-15.17	4.17	-0.58
Mar 2001	Nov 2001	120	5	40.58	-3.47	2633.43	-14.91	1.48	-0.90
Dec 2007	Jun 2009	73	15	14.37	-18.52	545.13	-147.67	0.18	0.03
P-value without outliers		0.50	0.22	0.30	0.29	0.18	0.19	0.07	0.24

Note: NBER peak (P) and trough (T) dates of the U.S. business cycle and corresponding business cycle characteristics (duration, amplitude, cumulated movements and excess cumulated movements). The outlier phase characteristics are shaded in grey. We report the p-values of Silverman's modality test for the subsamples without outliers.

of the four business cycle characteristics. Recall that multimodality of the Bootlier Plot indicates the presence of one or more outlying observations. The Bootlier Plots of the excess measure of expansions and of the duration, amplitude and cumulated movements of recessions exhibit more than one mode. Outlier detection tests performed on the basis of these plots lead to p-values smaller than 1%, indicating a rejection of the null hypothesis of unimodality (no outliers). We iteratively run outlier detection tests to determine the subsamples that are free of outliers. Business cycle phases with outlying characteristics are shaded in grey in Table 1. We detect six outliers via our iterative procedure, one in the excess cumulated movements of expansions, one in recession durations, one in recession amplitudes, and three in the cumulated movements of recessions. At the bottom of Table 1 we also report the p-values of the outlier detection test for the subsamples with unimodal Bootlier Plots. As Figure 3 shows, each Bootlier Plot becomes unimodal once the outliers are removed from the samples.

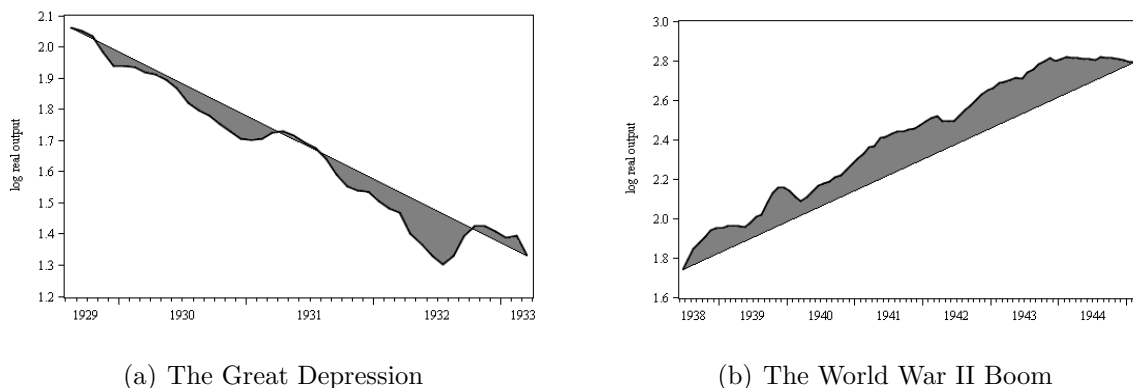
The outlying phase characteristics correspond to four extraordinary episodes in the history of the U.S. business cycle. The expansion that corresponds with an outlying excess measure is relabeled as a boom, while the three recessions that exhibit outlying characteristics are classified as depressions. The first depression occurred between January 1920 - July 1921. [Friedman and Schwartz \(1963\)](#) argue that this deflationary downturn may have been triggered by a negative aggregate demand shock partly caused by restrictive

Figure 3: Bootlier Plots



Note: Bootlier Plots graphically represent kernel estimates of the bootstrap density of MTM statistics for each business cycle characteristic. If the full-sample Bootlier Plot is multimodal, we iteratively remove observations from the tails of the sample until the subsample Bootlier Plot becomes unimodal. The left (right) panel shows full-sample and subsample Bootlier Plots for expansions (recessions). The four rows correspond with duration, amplitude, cumulated movements, and excess cumulated movements, respectively.

Figure 4: Depressions and Booms of the U.S. Business Cycle



Note: Figure (a): NBER peak-to-trough depression between August 1929 and March 1933. Figure (b): NBER trough-to-peak boom between June 1938 and February 1945. The excess cumulative movements are shaded in grey.

monetary policy between 1920 and 1921. The second depression is the August 1929 - March 1933 collapse in real economic activity known as the Great Depression. Figure 4 (a) plots log industrial production from peak to trough. Our results reveal that the Great Depression corresponds to the longest, sharpest, and most abrupt decline in real output, with an amplitude of -72.1% and a cumulative loss of 1742.7% over 40 months. These values are between three to four standard deviations larger than for ordinary recessions. The vast literature on the Great Depression typically describes an adverse interplay between a contraction in aggregate production, systemic banking crises, and tight monetary policy (see e.g., [Bernanke, 1983](#)). The third depression that we detect is dated between May 1937 and June 1938. [Velde \(2009\)](#) argues that the 1937-38 episode is a prime example of a double-dip recession triggered by premature policy tightening in the aftermath of a severe recession. Despite multiple comparisons with the Great Depression, the 2007-09 Great Recession is not classified as a depression by our statistical procedure. However, it is undoubtedly the longest (15 months) postwar recession, with the largest amplitude (-18.5%) and cumulative output loss (147.7%).⁶

The June 1938 - February 1945 wartime expansion constitutes the only genuine boom phase. The WWII boom exhibits by far the largest excess cumulative movements: 11.7% compared to a mean of 2.5% and standard deviation of 3.06% . Figure 4 (b) shows log industrial production from trough to peak. The grey area in the plot corresponds to the measure of excess cumulative movements. This measure reflects the departure of the real output series from a triangular path for which the transition between two consecutive turning points would be linear, it thus conveys information about the shape of the busi-

⁶Notice that the outlier detection procedure does not directly compare the Great Recession with the identified depression episodes. Whether the Great Recession is a depression solely depends on its characteristics compared to other recessions, since the sequential omission of the depression characteristics in the testing algorithm is equivalent to ultimately considering a subsample that does no longer contain the three depression periods. Thus, if the Great Recession was a depression, it should lead to a multimodal Bootlier Plot for this latter subsample, and this is not the case.

ness cycle phase. The concave shape of the upswing reflects a sharp surge in aggregate production related to the arms industry, the expansion of productive capacity through government-owned, privately operated capital, and an increase in labor force participation in durable goods manufacturing during World War II (see [Braun and McGrattan, 1993](#)). As the impetus of the economic boom abates by the end of the war, the economy approaches its peak at a rather subdued pace, giving rise to the strong concavity observed in the figure.

3 Predicting the four-regime business cycle

3.1 Multinomial logistic regression

We examine by means of multinomial logistic regressions whether macroeconomic and financial variables reflect leading information on the four-regime U.S. business cycle. Based on the outlier detection results discussed in the preceding sections, we map the binomial cycle variable into a multinomial variable S_t^{mul} that corresponds to the expansion ($S_t^{mul} = 0$), recession ($S_t^{mul} = 1$), boom ($S_t^{mul} = 2$), and depression ($S_t^{mul} = 3$) regimes of the business cycle. Let I_t represent the information set which contains the past history of an L -dimensional vector of exogenous variables, \mathbf{x}_t . In a four-state multinomial logit model, the probability that the business cycle is in regime $j = 1, 2, 3$ at time $t + m$ conditional on I_t obeys a logistic distribution function:

$$Pr(S_{t+m}^{mul} = j | I_t) = \frac{\exp(\beta_j' \mathbf{x}_t)}{1 + \sum_{h=1}^3 \exp(\beta_h' \mathbf{x}_t)}, \quad (3)$$

where $m = 3, 6, \dots, 24$ stands for the prediction horizon (expressed in months). The model is identified by imposing the condition that the expansion regime ($j = 0$) is the reference state and all coefficients are expressed relative to this regime. Hence, using the fact that the state probabilities must sum to unity, the conditional probability of the expansion regime is given by:

$$Pr(S_{t+m}^{mul} = 0 | I_t) = \frac{1}{1 + \sum_{h=1}^3 \exp(\beta_h' \mathbf{x}_t)}. \quad (4)$$

The multinomial logit model nests the binomial model which has been used in much of the preceding literature (e.g., [Estrella and Hardouvelis, 1991](#); [Estrella and Mishkin, 1998](#); [Kauppi and Saikkonen, 2008](#)).

Estimation is done by maximum likelihood optimization.⁷ The log-likelihood for a

⁷In the business cycle literature probit models are more commonly employed than logit models to predict recessions (e.g., [Estrella and Hardouvelis, 1991](#); [Estrella and Mishkin, 1998](#); [Kauppi and Saikkonen, 2008](#); [Christiansen, 2013](#)). However, this approach is less preferable in the multinomial context because evaluating the state probabilities involves calculating high-dimensional integrals of the multivariate normal distribution, which makes the computation of the maximum likelihood estimates very complex (see [McCulloch and Rossi, 1994](#)). Therefore, we opt for the logistic approach. Nevertheless, binary probit and logit outcomes for our data were found to lie very close to each other, which suggests that it does not matter whether one assumes a normal density or a logistic density for sake of maximum likelihood optimization.

time series consisting of T observations is given by:

$$\log(L(\boldsymbol{\beta})) = \sum_{t=1}^T \sum_{j=0}^3 \mathbf{1}(S_{t+m}^{mul} = j) \log(\Pr(S_{t+m}^{mul} = j | I_t)), \quad (5)$$

where $\mathbf{1}(\cdot)$ is the indicator function. We measure the model’s goodness-of-fit using [McFadden \(1974\)](#)’s pseudo- R^2 . For time horizons of $m > 1$, the prediction horizon exceeds the data frequency, which creates serially correlated logit disturbance terms by construction. We remedy for this overlapping data problem using heteroskedasticity and serial correlation robust standard errors (see also [Estrella and Mishkin, 1998](#)).

We are interested in the effect of a *ceteris paribus* change in one of the right-hand-side (RHS) variables on the response probability defined in Equation (3), i.e., the so-called marginal effects.⁸ The l th marginal effect gives the change in the probability that the business cycle is in state j at time $t + m$ in response to a one unit increase of the l th RHS variable relative to its mean value ($l = 1, \dots, L$):

$$\frac{\partial \Pr(S_{t+m}^{mul} = j | I_t)}{\partial x_{l,t}} = \Pr(S_{t+m}^{mul} = j | I_t) \left(\beta_{j,l} - \frac{\sum_{h=1}^3 \beta_{h,l} \exp(\boldsymbol{\beta}'_h \mathbf{x}_t)}{1 + \sum_{h=1}^3 \exp(\boldsymbol{\beta}'_h \mathbf{x}_t)} \right), \quad (6)$$

where $\beta_{j,l}$ is the l th element of $\boldsymbol{\beta}_h$.

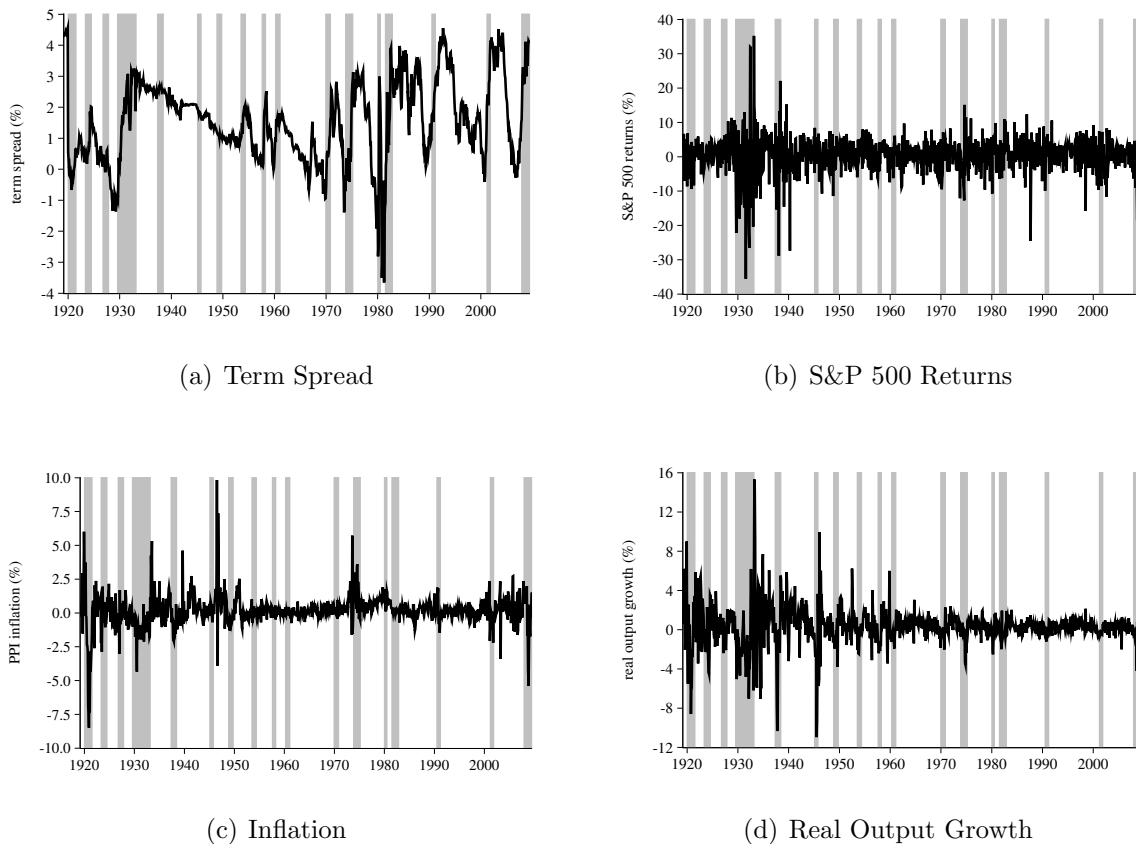
Within this multinomial framework, we would like to assess to what extent macroeconomic and financial variables that traditionally exhibit leading indicator properties for recessions and expansions have a different information content for depressions and booms. [Anderson \(1984\)](#) defines a pair of regimes as “indistinguishable” if the RHS variables \mathbf{x}_t deliver the same prediction for both regimes. If so, the two regimes can be merged into a single regime. More precisely, let $\beta_{1,l}$ denote the coefficients that correspond to the explanatory variables $x_{l,t}$ in the recession regime relative to the reference (expansion) state, and let $\beta_{3,l}$ stand for the coefficients of the depression regime relative to the reference state. The null hypothesis that recessions and depressions are indistinguishable with respect to \mathbf{x}_t is given by $\beta_{1,l} = \beta_{3,l}$ for $l = 1, \dots, L$, which can be tested using conventional likelihood-ratio (LR) and Wald statistics. Notice that a rejection of Anderson’s indistinguishability hypothesis would provide further justification for a four-regime business cycle classification emerging from our outlier detection procedure.

3.2 Logistic regression results

Potential leading indicator variables are selected in line with previous empirical literature on business cycle prediction (see, e.g., the seminal paper by [Estrella and Mishkin, 1998](#)). Taking into account that our RHS variables need to be available from 1919 onwards, we end up with the term spread ($SPREAD$), the S&P 500 stock index returns ($\Delta \log SP500$), the inflation rate ($\Delta \log PPI$), and the growth rate of log industrial production (Δy_t),

⁸The majority of related business cycle studies only report coefficient levels β in Equation (3) because the sign of the coefficients and the marginal effects are identical in the binary framework. However, in the multinomial model an explanatory variable’s marginal effect depends on all coefficients, which implies that it does not necessarily exhibit the same sign as the coefficient on the corresponding independent variable.

Figure 5: Time Series and the U.S. Business Cycle



Note: Time series are expressed as monthly percentages between March 1919 and June 2009. NBER recessions are shaded in grey.

all at the monthly frequency.⁹ Data are taken from the St. Louis Fed and from [Goyal and Welch \(2008\)](#). In line with the business cycle data, the sample covers the period from March 1919 until June 2009. Figure 5 shows the data together with the NBER recessions. The figure already provides some casual evidence that the business cycle and the macroeconomic and financial variables are related. The term spread seems to display counter-cyclical dynamics. Although stock returns, inflation, and output growth do not exhibit a clear-cut pattern in the wake of recessions, they tend to decline during the three depressions identified.

We distinguish between binomial and multinomial models with either a single RHS variable (univariate model) or multiple RHS variables (multivariate model). Tables 2-8 report estimation results for these four possible logit model specifications. The tables have a comparable structure and information content. Each table considers prediction

⁹The term spread is defined as the difference between 10-year U.S. Treasury bonds and 3-month Treasury bills. As inflation variable, we consider the growth rate of the producer price index (PPI), which is a potential leading indicator for real economic activity because it measures the change in selling prices received by domestic producers. We also ran a robustness check with CPI inflation and our results were nearly identical.

time horizons of $m = 3$ up to $m = 24$ months and reports marginal effects together with robust standard errors and accompanying Z -statistics. Since all regressors are expressed in percentages, the marginal effects can be interpreted as changes in the m -month ahead probability of a recession, depression, expansion or boom, in response to a one-percent rise of a RHS variable. The tables also report log-likelihoods and pseudo- R^2 values.¹⁰

Table 2 reports estimation results for univariate binomial logistic regressions as a benchmark for comparison with the multinomial results, as well as a robustness check of earlier binomial business cycle studies. In line with previous evidence, we find that the slope of the yield curve is an accurate predictor of future recessions for all considered time horizons. Rising term spreads reduce future recession likelihoods in a statistically significant way. Different theoretical explanations have been launched for this empirical observation. The most popular ones are related to the stance of monetary policy and the information content on expectations regarding future economic prospects reflected in the term spread. Current monetary policy can have a simultaneous impact on both the yield curve and future real activity. For example, an expansionary monetary policy can jointly induce a decline in the short rate – leading to a steeper yield curve – and stimulate future economic activity. Furthermore, the Rational Expectations Hypothesis (REH) of the term structure of interest rates can also contribute to understanding the empirical relation between the yield curve and the business cycle. According to the REH, an upward (downward) sloping yield curve indicates that future short-term interest rates are expected to rise (fall). Hence, given that short-term interest rates are typically pro-cyclical, a positive (negative) term spread signals a future business cycle expansion (recession).

We also consider S&P 500 stock index returns in the univariate logit model. The Efficient Markets Hypothesis implies that current stock prices equal the present value of the expected future dividend stream, which in turn reflects expectations about future real economic activity. Thus, in line with what one would expect, rising stock index returns significantly reduce recession likelihoods for time horizons up to one year ahead.

Turning to the significance of the macroeconomic variables in the univariate logit regressions, rising real output growth significantly reduces the probability of future recessions for time horizons up to three quarters ahead. The significant outcomes reflect the temporal persistence of economic activity. However, the sign of the coefficient reverses for a time horizon of two years ahead, which suggests a boom-bust cycle in the data. Finally, inflation seems to predict the business cycle only up to six months ahead. Interestingly, a marginal rise in inflation leads to a significant drop in the recession probability up to two quarters in the future. This result is in line with, e.g., [Estrella and Hardouvelis \(1991\)](#) but it contradicts certain structural macroeconomic studies, e.g., [Smets and Wouters \(2007\)](#) who find a negative correlation between current inflation and future output growth.

Summarizing the univariate binomial model outcomes, both financial and macro variables impact the recession likelihoods, but the term spread is by far the best (in-sample) predictor of future real economic activity. First of all, the term spread is the only variable that exhibits significant predictive power over all time horizons. Moreover, the absolute

¹⁰The considered models are estimated over the entire sample period. We do not perform out-of-sample prediction exercises for the depression and boom phases of the business cycle because the number of in-sample depressions and booms (four in total) is too low to make a credible out-of-sample prediction assessment.

Table 2: Univariate Binomial Logistic Regressions

$Pr(S_{t+m} = 1 I_t) = F(\beta_0 + \beta_1 SPREAD_t)$								
$m = \text{months ahead}$								
	3	6	9	12	15	18	21	24
$\frac{\partial Pr(S_{t+m}=1)}{\partial SPREAD_t}$	-0.059 (0.010)	-0.079 (0.010)	-0.093 (0.010)	-0.094 (0.010)	-0.089 (0.010)	-0.081 (0.010)	-0.077 (0.010)	-0.069 (0.009)
Z-statistic	-5.98 ^a	-7.58 ^a	-8.86 ^a	-8.97 ^a	-8.59 ^a	-8.21 ^a	-8.09 ^a	-7.37 ^a
Pseudo R^2	0.034	0.061	0.085	0.088	0.079	0.067	0.062	0.050
Log-likelihood	- 541.493	- 525.523	- 511.739	- 506.967	- 507.791	- 509.912	- 508.471	- 509.971
$Pr(S_{t+m} = 1 I_t) = F(\beta_0 + \beta_1 \Delta \log SP500_t)$								
$m = \text{months ahead}$								
	3	6	9	12	15	18	21	24
$\frac{\partial Pr(S_{t+m}=1)}{\partial \Delta \log SP500_t}$	-0.014 (0.003)	-0.013 (0.002)	-0.012 (0.002)	-0.008 (0.002)	-0.003 (0.002)	-0.000 (0.002)	0.002 (0.002)	0.005 (0.002)
Z-statistic	-4.97 ^a	-4.72 ^a	-4.86 ^a	-3.43 ^a	-1.190	-0.080	0.970	2.37 ^b
Pseudo R^2	0.035	0.031	0.023	0.010	0.001	0.000	0.001	0.005
Log-likelihood	- 541.144	- 542.547	- 546.257	- 550.145	- 550.493	- 546.498	- 541.378	- 534.386
$Pr(S_{t+m} = 1 I_t) = F(\beta_0 + \beta_1 \Delta \log PPI_t)$								
$m = \text{months ahead}$								
	3	6	9	12	15	18	21	24
$\frac{\partial Pr(S_{t+m}=1)}{\partial \Delta \log PPI_t}$	-0.0623 (0.014)	-0.033 (0.013)	-0.002 (0.012)	0.017 (0.012)	0.016 (0.011)	0.010 (0.011)	0.005 (0.010)	0.0001 (0.010)
Z-statistic	-4.37 ^a	-2.50 ^b	-0.14	1.46	1.43	0.94	0.53	0.09
Pseudo R^2	0.030	0.009	0.000	0.002	0.002	0.001	0.000	0.000
Log-likelihood	- 544.064	- 555.090	- 559.128	- 554.639	- 550.097	- 546.068	- 541.691	- 537.053
$Pr(S_{t+m} = 1 I_t) = F(\beta_0 + \beta_1 \Delta y_t)$								
$m = \text{months ahead}$								
	3	6	9	12	15	18	21	24
$\frac{\partial Pr(S_{t+m}=1)}{\partial \Delta y_t}$	-0.075 (0.011)	-0.052 (0.009)	-0.023 (0.007)	-0.003 (0.006)	0.004 (0.006)	0.008 (0.006)	0.011 (0.006)	0.014 (0.006)
Z-statistic	-6.66 ^a	-6.03 ^a	-3.30 ^a	-0.460	0.660	1.250	1.86 ^c	2.48 ^b
Pseudo R^2	0.106	0.056	0.012	0.000	0.000	0.001	0.003	0.005
Log-likelihood	- 500.941	- 528.805	- 552.622	- 555.723	- 550.963	- 545.739	- 540.248	- 534.297

Note: Estimation results of univariate binomial logistic regressions with $SPREAD_t$, $\Delta \log SP500_t$, $\Delta \log PPI_t$, or Δy_t on the right-hand side. The marginal effect ($\partial Pr(\cdot)/\partial x_{i,t}$) is the partial derivative of the predicted probability with respect to the RHS variable evaluated at its mean. Robust standard errors are in brackets and the superscripts ^a, ^b, and ^c denote significance at the 1%, 5% and 10% level, respectively. Identification is achieved by normalizing with the coefficients of the expansion state. We provide the model log-likelihood, and goodness-of-fit is measured by McFadden's pseudo R-squared.

Table 3: Multivariate Binomial Logistic Regression

	$Pr(S_{t+m} = 1 I_t) = F(\beta_0 + \beta_1 SPREAD_t + \beta_2 SP500_t + \beta_3 INFL_t + \beta_4 \Delta y_t)$							
	$m = \text{months ahead}$							
	3	6	9	12	15	18	21	24
$\frac{\partial Pr(S_{t+m}=1)}{\partial SPREAD_t}$	-0.058 (0.009)	-0.077 (0.010)	-0.091 (0.010)	-0.094 (0.011)	-0.090 (0.010)	-0.083 (0.010)	-0.080 (0.010)	-0.074 (0.009)
Z-statistic	-6.21 ^a	-7.50 ^a	-8.77 ^a	-8.91 ^a	-8.63 ^a	-8.37 ^a	-8.41 ^a	-7.92 ^a
$\frac{\partial Pr(S_{t+m}=1)}{\partial \Delta \log SP500_t}$	-0.012 (0.003)	-0.011 (0.003)	-0.011 (0.002)	-0.008 (0.002)	-0.003 (0.003)	0.000 (0.003)	0.003 (0.002)	0.006 (0.002)
Z-statistic	-4.10 ^a	-4.29 ^a	-4.94 ^a	-3.40 ^a	-0.95	0.05	1.08	2.52 ^b
$\frac{\partial Pr(S_{t+m}=1)}{\partial \Delta \log PPI_t}$	-0.032 (0.014)	-0.008 (0.013)	0.010 (0.011)	0.017 (0.011)	0.014 (0.011)	0.007 (0.010)	0.000 (0.010)	-0.005 (0.010)
Z-statistic	-2.36 ^b	-0.66	0.98	1.64	1.31	0.64	0.04	-0.48
$\frac{\partial Pr(S_{t+m}=1)}{\partial \Delta y_t}$	-0.065 (0.011)	-0.044 (0.008)	-0.019 (0.007)	0.001 (0.007)	0.008 (0.007)	0.012 (0.007)	0.016 (0.006)	0.018 (0.006)
Z-statistic	-5.83 ^a	-5.37 ^a	-2.72 ^a	0.16	1.07	1.68 ^c	2.33 ^b	2.80 ^a
Pseudo R^2	0.175	0.142	0.117	0.101	0.083	0.071	0.069	0.065
Log-likelihood	-462.707	-480.348	-493.654	-499.453	-505.234	-507.683	-504.684	-502.060

Note: Estimation results of the multivariate binomial logistic regression with $SPREAD_t$, $\Delta \log SP500_t$, $\Delta \log PPI_t$, and Δy_t on the right-hand side. The marginal effect ($\partial Pr(\cdot)/\partial x_{i,t}$) is the partial derivative of the predicted probability with respect to the RHS variable evaluated at its mean. Robust standard errors are in brackets and the superscripts ^a, ^b, and ^c denote significance at the 1%, 5% and 10% level, respectively. Identification is achieved by normalizing with the coefficients of the expansion state. We provide the model log-likelihood, and goodness-of-fit is measured by McFadden's pseudo R-squared.

Table 4: Multinomial Logistic Regression with Term Spread

	$Pr(S_{t+m} = j I_t) = F(\beta_0 + \beta_1 SPREAD_t)$							
	$m = \text{months ahead}$							
	3	6	9	12	15	18	21	24
$\frac{\partial Pr(S_{t+m}=0)}{\partial SPREAD_t}$	0.031	0.047	0.055	0.056	0.052	0.047	0.046	0.039
	(0.010)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.010)	(0.010)
Z-statistic	3.13 ^a	4.49 ^a	4.97 ^a	4.97 ^a	4.65 ^a	4.31 ^a	4.50 ^a	3.99 ^a
$\frac{\partial Pr(S_{t+m}=1)}{\partial SPREAD_t}$	-0.055	-0.072	-0.082	-0.082	-0.079	-0.072	-0.062	-0.049
	(0.008)	(0.008)	(0.007)	(0.007)	(0.007)	(0.007)	(0.007)	(0.007)
Z-statistic	-7.34 ^a	-9.39 ^a	-11.05 ^a	-10.88 ^a	-10.58 ^a	-10.46 ^a	-9.04 ^a	-6.90 ^a
$\frac{\partial Pr(S_{t+m}=2)}{\partial SPREAD_t}$	0.026	0.027	0.029	0.030	0.030	0.030	0.031	0.031
	(0.003)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.005)
Z-statistic	7.35 ^a	7.42 ^a	7.49 ^a	7.54 ^a	7.58 ^a	7.61 ^a	7.63 ^a	7.66 ^a
$\frac{\partial Pr(S_{t+m}=3)}{\partial SPREAD_t}$	-0.002	-0.003	-0.002	-0.004	-0.004	-0.005	-0.015	-0.021
	(0.007)	(0.008)	(0.009)	(0.010)	(0.009)	(0.010)	(0.008)	(0.006)
Z-statistic	-0.25	-0.32	-0.22	-0.41	-0.40	-0.52	-1.92 ^c	-3.59 ^a
Pseudo R^2	0.033	0.054	0.075	0.076	0.068	0.057	0.047	0.040
Log-likelihood	-938.930	-917.079	-895.691	-890.054	-889.984	-892.180	-894.103	-892.523
LR: $Bo \equiv Ex$	12.520 ^a	12.121 ^a	12.435 ^a	13.462 ^a	14.892 ^a	16.450 ^a	17.596 ^a	19.322 ^a
Wald: $Bo \equiv Ex$	12.190 ^a	11.840 ^a	12.167 ^a	13.150 ^a	14.502 ^a	15.950 ^a	16.975 ^a	18.545 ^a
LR: $De \equiv Re$	12.603 ^a	24.255 ^a	38.657 ^a	34.557 ^a	27.860 ^a	18.881 ^a	3.428 ^c	0.216
Wald: $De \equiv Re$	12.256 ^a	22.995 ^a	35.618 ^a	32.033 ^a	26.188 ^a	18.059 ^a	3.378 ^c	0.216

Note: Estimation results of the univariate multinomial logistic regression with $SPREAD_t$ on the right-hand side. The marginal effect ($\partial Pr(\cdot)/\partial x_{l,t}$) is the partial derivative of the predicted probability with respect to the RHS variable evaluated at its mean. Robust standard errors are in brackets and the superscripts ^a, ^b, and ^c denote significance at the 1%, 5% and 10% level, respectively. Identification is achieved by normalizing with the coefficients of the expansion state. We provide the model log-likelihood, and goodness-of-fit is measured by McFadden's pseudo R-squared. We also report the LR and Wald test for the null hypothesis that all coefficients (except intercepts) associated with a given pair of business cycle regimes (boom (Bo) and expansion (Ex) or depression (De) and recession (Re)) are equal (indistinguishability). The test statistics are $\chi^2(1)$ distributed under the null.

Table 5: Multinomial Logistic Regression with Stock Returns

	$Pr(S_{t+m} = j I_t) = F(\beta_0 + \beta_1 \Delta \log SP500_t)$							
	$m = \text{months ahead}$							
	3	6	9	12	15	18	21	24
$\frac{\partial Pr(S_{t+m}=0)}{\partial \Delta \log SP500_t}$	0.014 (0.003)	0.014 (0.003)	0.013 (0.003)	0.010 (0.003)	0.005 (0.002)	0.002 (0.003)	-0.000 (0.003)	-0.003 (0.003)
Z-statistic	4.50 ^a	4.58 ^a	4.75 ^a	3.71 ^a	2.11 ^b	0.99	-0.02	-1.24
$\frac{\partial Pr(S_{t+m}=1)}{\partial \Delta \log SP500_t}$	-0.007 (0.002)	-0.007 (0.002)	-0.006 (0.002)	-0.004 (0.002)	0.000 (0.001)	0.002 (0.002)	0.002 (0.001)	0.004 (0.002)
Z-statistic	-3.61 ^a	-4.01 ^a	-3.26 ^a	-2.56 ^a	0.20	1.02	1.37	2.80 ^a
$\frac{\partial Pr(S_{t+m}=2)}{\partial \Delta \log SP500_t}$	-0.000 (0.002)	-0.001 (0.002)	-0.002 (0.002)	-0.002 (0.002)	-0.003 (0.002)	-0.002 (0.002)	-0.002 (0.002)	-0.002 (0.002)
Z-statistic	-0.05	-0.40	-0.95	-1.15	-1.56	-1.35	-1.15	-1.20
$\frac{\partial Pr(S_{t+m}=3)}{\partial \Delta \log SP500_t}$	-0.006 (0.001)	-0.006 (0.001)	-0.006 (0.001)	-0.004 (0.001)	-0.003 (0.002)	-0.002 (0.002)	0.000 (0.002)	0.001 (0.002)
Z-statistic	-4.75 ^a	-3.95 ^a	-4.72 ^a	-2.71 ^a	-1.83 ^c	-0.94	0.06	0.61
Pseudo R^2	0.024	0.021	0.018	0.009	0.005	0.003	0.002	0.004
Log-likelihood	-947.329	-949.444	-950.831	-954.210	-950.140	-943.983	-936.551	-925.964
LR: $Bo \equiv Ex$	0.725	1.753	3.737 ^b	3.852 ^b	4.244 ^b	2.764 ^c	1.722	1.341
Wald: $Bo \equiv Ex$	0.725	1.770	3.846 ^b	4.025 ^b	4.493 ^b	2.903 ^c	1.793	1.395
LR: $De \equiv Re$	6.787 ^a	3.745 ^b	5.856 ^b	1.935	3.384 ^c	1.940	0.180	0.143
Wald: $De \equiv Re$	6.689 ^a	3.772 ^b	5.839 ^b	1.978	3.49 ^c	1.990	0.181	0.142

Note: Estimation results of the univariate multinomial logistic regression with $\Delta \log SP500_t$ on the right-hand side. The marginal effect ($\partial Pr(\cdot)/\partial x_{i,t}$) is the partial derivative of the predicted probability with respect to the RHS variable evaluated at its mean. Robust standard errors are in brackets and the superscripts ^a, ^b, and ^c denote significance at the 1%, 5% and 10% level, respectively. Identification is achieved by normalizing with the coefficients of the expansion state. We provide the model log-likelihood, and goodness-of-fit is measured by McFadden's pseudo R-squared. We also report the LR and Wald test for the null hypothesis that all coefficients (except intercepts) associated with a given pair of business cycle regimes (boom (Bo) and expansion (Ex) or depression (De) and recession (Re)) are equal (indistinguishability). The test statistics are $\chi^2(1)$ distributed under the null.

Table 6: Multinomial Logistic Regression with Inflation

	$Pr(S_{t+m} = j I_t) = F(\beta_0 + \beta_1 \Delta \log PPI_t)$							
	$m = \text{months ahead}$							
	3	6	9	12	15	18	21	24
$\frac{\partial Pr(S_{t+m}=0)}{\partial \Delta \log PPI_t}$	0.039 (0.015)	0.018 (0.013)	-0.003 (0.012)	-0.018 (0.012)	-0.020 (0.012)	-0.019 (0.012)	-0.018 (0.013)	-0.014 (0.013)
Z-statistic	2.55 ^b	1.36	-0.21	-1.44	-1.59	-1.52	-1.43	-1.13
$\frac{\partial Pr(S_{t+m}=1)}{\partial \Delta \log PPI_t}$	-0.006 (0.015)	0.007 (0.012)	0.020 (0.009)	0.031 (0.010)	0.032 (0.010)	0.029 (0.009)	0.029 (0.009)	0.024 (0.009)
Z-statistic	-0.40	0.61	2.20 ^b	3.24 ^a	3.31 ^a	3.15 ^a	3.31 ^a	2.65 ^a
$\frac{\partial Pr(S_{t+m}=2)}{\partial \Delta \log PPI_t}$	0.007 (0.007)	0.004 (0.007)	0.000 (0.007)	-0.000 (0.007)	0.002 (0.007)	0.006 (0.008)	0.008 (0.008)	0.007 (0.007)
Z-statistic	0.99	0.55	0.06	-0.01	0.28	0.74	1.01	1.03
$\frac{\partial Pr(S_{t+m}=3)}{\partial \Delta \log PPI_t}$	-0.040 (0.007)	-0.029 (0.007)	-0.018 (0.007)	-0.013 (0.007)	-0.014 (0.007)	-0.016 (0.006)	-0.018 (0.005)	-0.018 (0.005)
Z-statistic	-5.82 ^a	-4.42 ^a	-2.63 ^a	-1.80 ^c	-2.09 ^b	-2.75 ^a	-3.73 ^a	-3.89 ^a
Pseudo R^2	0.038	0.017	0.007	0.007	0.008	0.009	0.011	0.010
Log-likelihood	-933.459	-952.812	-962.034	-955.796	-938.015	-947.039	-927.815	-920.492
LR: $Bo \equiv Ex$	0.104	0.054	0.008	0.047	0.255	0.832	1.315	1.212
Wald: $Bo \equiv Ex$	0.107	0.054	0.008	0.047	0.254	0.839	1.345	1.242
LR: $De \equiv Re$	40.986 ^a	23.785 ^a	12.862 ^a	11.534 ^a	12.745 ^a	15.016 ^a	18.858 ^a	16.693 ^a
Wald: $De \equiv Re$	30.370 ^a	21.312 ^a	13.446 ^a	12.197 ^a	13.599 ^a	16.118 ^a	20.050 ^a	17.792 ^a

Note: Estimation results of the univariate multinomial logistic regression with $\Delta \log PPI_t$ on the right-hand side. The marginal effect ($\partial Pr(\cdot)/\partial x_{l,t}$) is the partial derivative of the predicted probability with respect to the RHS variable evaluated at its mean. Robust standard errors are in brackets and the superscripts ^a, ^b, and ^c denote significance at the 1%, 5% and 10% level, respectively. Identification is achieved by normalizing with the coefficients of the expansion state. We provide the model log-likelihood, and goodness-of-fit is measured by McFadden's pseudo R-squared. We also report the LR and Wald test for the null hypothesis that all coefficients (except intercepts) associated with a given pair of business cycle regimes (boom (Bo) and expansion (Ex) or depression (De) and recession (Re)) are equal (indistinguishability). The test statistics are $\chi^2(1)$ distributed under the null.

Table 7: Multinomial Logistic Regression with Output Growth

	$Pr(S_{t+m} = j I_t) = F(\beta_0 + \beta_1 \Delta y_t)$							
	$m = \text{months ahead}$							
	3	6	9	12	15	18	21	24
$\frac{\partial Pr(S_{t+m}=0)}{\partial \Delta y_t}$	0.055 (0.010)	0.031 (0.008)	0.010 (0.007)	-0.008 (0.007)	-0.015 (0.008)	-0.018 (0.008)	-0.022 (0.008)	-0.027 (0.009)
Z-statistic	5.21 ^a	3.89 ^a	1.590	-1.180	-1.88 ^c	-2.22 ^b	-2.66 ^a	-3.03 ^a
$\frac{\partial Pr(S_{t+m}=1)}{\partial \Delta y_t}$	-0.046 (0.008)	-0.025 (0.006)	-0.007 (0.004)	0.006 (0.004)	0.008 (0.005)	0.010 (0.005)	0.012 (0.005)	0.015 (0.005)
Z-statistic	-5.53 ^a	-4.22 ^a	-1.530	1.142	1.750 ^c	1.99 ^b	2.46 ^b	3.18 ^a
$\frac{\partial Pr(S_{t+m}=2)}{\partial \Delta y_t}$	0.018 (0.004)	0.017 (0.004)	0.011 (0.005)	0.010 (0.005)	0.010 (0.005)	0.010 (0.005)	0.010 (0.005)	0.011 (0.005)
Z-statistic	4.74 ^a	4.35 ^a	2.37 ^b	2.20 ^b	2.12 ^b	2.03 ^b	2.15 ^b	2.26 ^b
$\frac{\partial Pr(S_{t+m}=3)}{\partial \Delta y_t}$	-0.027 (0.004)	-0.023 (0.004)	-0.014 (0.004)	-0.008 (0.004)	-0.004 (0.005)	-0.001 (0.005)	0.000 (0.004)	0.001 (0.004)
Z-statistic	-6.89 ^a	-6.54 ^a	-3.71 ^a	-1.84 ^c	-0.690	-0.250	0.030	0.150
Pseudo R^2	0.074	0.048	0.014	0.007	0.005	0.005	0.007	0.009
Log-likelihood	-898.841	-923.124	-955.277	-956.439	-949.483	-941.182	-931.428	-920.729
LR: $Bo \equiv Ex$	11.425 ^a	12.560 ^a	5.938 ^b	7.378 ^a	7.697 ^a	7.532 ^a	8.924 ^a	10.511 ^a
Wald: $Bo \equiv Ex$	12.304 ^a	13.326 ^a	6.405 ^b	7.899 ^a	8.216 ^a	8.027 ^a	9.509 ^a	11.191 ^a
LR: $De \equiv Re$	12.691 ^a	18.241 ^a	7.543 ^a	5.268 ^b	2.224	1.293	1.022	1.356
Wald: $De \equiv Re$	11.881 ^a	16.590 ^a	7.641 ^a	5.435 ^b	2.239	1.285	1.008	1.324

Note: Estimation results of the univariate multinomial logistic regression with Δy_t on the right-hand side. The marginal effect ($\partial Pr(\cdot)/\partial x_{l,t}$) is the partial derivative of the predicted probability with respect to the RHS variable evaluated at its mean. Robust standard errors are in brackets and the superscripts ^a, ^b, and ^c denote significance at the 1%, 5% and 10% level, respectively. Identification is achieved by normalizing with the coefficients of the expansion state. We provide the model log-likelihood, and goodness-of-fit is measured by McFadden's pseudo R-squared. We also report the LR and Wald test for the null hypothesis that all coefficients (except intercepts) associated with a given pair of business cycle regimes (boom (Bo) and expansion (Ex) or depression (De) and recession (Re)) are equal (indistinguishability). The test statistics are $\chi^2(1)$ distributed under the null.

Table 8: Multivariate Multinomial Logistic Regression

$Pr(S_{t+m} = j I_t) = F(\beta_0 + \beta_1 SPREAD_t + \beta_2 \Delta \log SP500_t + \beta_3 \Delta \log PPI_t + \beta_4 \Delta y_t)$								
$m = \text{months ahead}$								
	3	6	9	12	15	18	21	24
$\frac{\partial Pr(S_{t+m}=0)}{\partial SPREAD_t}$	0.033 (0.009)	0.050 (0.010)	0.057 (0.011)	0.057 (0.011)	0.053 (0.011)	0.048 (0.011)	0.047 (0.010)	0.042 (0.010)
Z-statistic	3.47 ^a	4.67 ^a	5.16 ^a	5.07 ^a	4.80 ^a	4.45 ^a	4.59 ^a	4.32 ^a
$\frac{\partial Pr(S_{t+m}=0)}{\partial \Delta \log SP500_t}$	0.011 (0.003)	0.012 (0.003)	0.012 (0.003)	0.009 (0.002)	0.004 (0.002)	0.002 (0.002)	-0.001 (0.003)	-0.004 (0.003)
Z-statistic	3.76 ^a	4.19 ^a	4.55 ^a	3.67 ^a	1.79 ^c	0.71	-0.23	-1.45
$\frac{\partial Pr(S_{t+m}=0)}{\partial \Delta \log PPI_t}$	0.028 (0.023)	0.012 (0.012)	-0.000 (0.012)	-0.006 (0.012)	-0.006 (0.012)	-0.004 (0.012)	-0.004 (0.012)	-0.002 (0.012)
Z-statistic	2.08 ^b	0.99	0.05	-0.50	-0.50	-0.37	-0.31	-0.14
$\frac{\partial Pr(S_{t+m}=0)}{\partial \Delta y_t}$	0.045 (0.010)	0.024 (0.008)	0.005 (0.007)	-0.013 (0.007)	-0.018 (0.008)	-0.021 (0.008)	-0.025 (0.008)	-0.028 (0.009)
Z-statistic	4.35 ^a	3.07 ^a	0.74	-1.83 ^c	-2.34 ^b	-2.62 ^a	-2.97 ^a	-3.22 ^a
$\frac{\partial Pr(S_{t+m}=1)}{\partial SPREAD_t}$	-0.054 (0.007)	-0.070 (0.008)	-0.081 (0.008)	-0.082 (0.008)	-0.079 (0.008)	-0.073 (0.007)	-0.063 (0.007)	-0.051 (0.007)
Z-statistic	-7.27 ^a	-9.08 ^a	-10.82 ^a	-10.72 ^a	-10.36 ^a	-10.30 ^a	-9.05 ^a	-7.19 ^a
$\frac{\partial Pr(S_{t+m}=1)}{\partial \Delta \log SP500_t}$	-0.007 (0.002)	-0.007 (0.002)	-0.004 (0.002)	-0.003 (0.002)	0.001 (0.002)	0.003 (0.002)	0.003 (0.002)	0.005 (0.002)
Z-statistic	-3.30 ^a	-3.78 ^a	-2.70 ^a	-2.01 ^b	0.85	1.55	1.71 ^c	2.93 ^a
$\frac{\partial Pr(S_{t+m}=1)}{\partial \Delta \log PPI_t}$	0.000 (0.011)	0.008 (0.009)	0.017 (0.007)	0.021 (0.008)	0.024 (0.008)	0.022 (0.008)	0.022 (0.008)	0.018 (0.009)
Z-statistic	0.01	0.96	2.40 ^b	2.83 ^a	3.12 ^a	2.94 ^a	2.71 ^a	1.99 ^b
$\frac{\partial Pr(S_{t+m}=1)}{\partial \Delta y_t}$	-0.044 (0.008)	-0.021 (0.006)	-0.006 (0.005)	0.008 (0.004)	0.008 (0.005)	0.010 (0.005)	0.011 (0.005)	0.014 (0.005)
Z-statistic	-5.41 ^a	-3.91 ^a	-1.16	1.70 ^c	1.71 ^c	1.87 ^c	2.24 ^b	2.78 ^a
$\frac{\partial Pr(S_{t+m}=2)}{\partial SPREAD_t}$	0.024 (0.004)	0.025 (0.004)	0.027 (0.004)	0.028 (0.004)	0.028 (0.004)	0.029 (0.004)	0.029 (0.004)	0.029 (0.004)
Z-statistic	6.70 ^a	6.69 ^a	6.64 ^a	6.63 ^a	6.67 ^a	6.77 ^a	6.79 ^a	6.82 ^a

Note: See next page.

Table 8: Multivariate Multinomial Logistic Regression - *Continued*

$\frac{\partial Pr(S_{t+m}=2)}{\partial \Delta \log SP500_t}$	-0.001 (0.002)	-0.001 (0.002)	-0.002 (0.002)	-0.002 (0.002)	-0.003 (0.001)	-0.002 (0.001)	-0.002 (0.001)	-0.002 (0.001)
<i>z</i> -statistic	-0.44	-0.75	-1.29	-1.52	-2.00 ^b	-1.76 ^c	-1.55	-1.59
$\frac{\partial Pr(S_{t+m}=2)}{\partial \Delta \log PPI_t}$	-0.003 (0.007)	-0.006 (0.007)	-0.007 (0.007)	-0.008 (0.007)	-0.005 (0.007)	-0.001 (0.007)	0.001 (0.008)	0.000 (0.007)
<i>Z</i> -statistic	-0.38	-0.81	-1.04	-1.06	-0.71	-0.16	0.09	0.03
$\frac{\partial Pr(S_{t+m}=2)}{\partial \Delta y_t}$	0.016 (0.004)	0.015 (0.004)	0.010 (0.004)	0.009 (0.004)	0.008 (0.004)	0.008 (0.004)	0.008 (0.004)	0.009 (0.004)
<i>Z</i> -statistic	4.40 ^a	4.24 ^a	2.42 ^b	2.27 ^b	2.16 ^b	1.88 ^c	1.90 ^c	2.02 ^b
$\frac{\partial Pr(S_{t+m}=3)}{\partial SPREAD_t}$	-0.003 (0.005)	-0.003 (0.007)	-0.002 (0.009)	-0.003 (0.009)	-0.002 (0.009)	-0.003 (0.010)	-0.013 (0.008)	-0.020 (0.006)
<i>Z</i> -statistic	-0.50	-0.45	-0.27	-0.36	-0.27	-0.34	-1.65 ^c	-3.40 ^a
$\frac{\partial Pr(S_{t+m}=3)}{\partial \Delta \log SP500_t}$	-0.004 (0.001)	-0.004 (0.001)	-0.005 (0.001)	-0.004 (0.002)	-0.003 (0.002)	-0.002 (0.002)	0.000 (0.001)	0.001 (0.002)
<i>Z</i> -statistic	-2.89 ^a	-2.58 ^a	-3.86 ^a	-2.46 ^b	-1.84 ^c	-1.01	0.04	0.71
$\frac{\partial Pr(S_{t+m}=3)}{\partial \Delta \log PPI_t}$	-0.025 (0.006)	-0.015 (0.006)	-0.009 (0.007)	-0.008 (0.007)	-0.012 (0.007)	-0.017 (0.006)	-0.019 (0.006)	-0.017 (0.005)
<i>Z</i> -statistic	-4.05 ^a	-2.46 ^b	-1.25	-1.05	-1.77 ^c	-2.77 ^a	-3.39 ^a	-3.25 ^a
$\frac{\partial Pr(S_{t+m}=3)}{\partial \Delta y_t}$	-0.018 (0.004)	-0.018 (0.004)	-0.010 (0.004)	-0.004 (0.005)	0.001 (0.005)	0.003 (0.005)	0.005 (0.004)	0.005 (0.004)
<i>Z</i> -statistic	-4.69 ^a	-4.77 ^a	-2.37 ^b	-0.99	0.12	0.70	1.24	1.36
Pseudo R^2	0.140	0.121	0.107	0.097	0.086	0.074	0.066	0.062
Log-likelihood	-834.393	-852.227	-865.332	-869.637	-872.657	-876.007	-876.350	-871.475
LR: $Bo \equiv Ex$	23.996 ^a	26.089 ^a	22.290 ^a	24.612 ^a	26.575 ^a	25.996 ^a	27.228 ^a	29.928 ^a
Wald: $Bo \equiv Ex$	23.963 ^a	26.066 ^a	22.318 ^a	24.618 ^a	26.530 ^a	25.780 ^a	26.845 ^a	29.330 ^a
LR: $De \equiv Re$	52.661 ^a	49.638 ^a	53.525 ^a	49.370 ^a	46.060 ^a	37.487 ^a	22.035 ^a	16.251 ^a
Wald: $De \equiv Re$	39.953 ^a	41.496 ^a	46.066 ^a	44.043 ^a	42.218 ^a	35.203 ^a	21.731 ^a	16.782 ^a

Note: Estimation results of the multivariate multinomial logistic regression. The marginal effect ($\partial Pr(\cdot)/\partial x_{l,t}$) is the partial derivative of the predicted probability with respect to the RHS variable evaluated at its mean. Robust standard errors are in brackets and the superscripts ^a, ^b, and ^c denote significance at the 1%, 5% and 10% level, respectively. Identification is achieved by normalizing with the coefficients of the expansion state. We provide the model log-likelihood, and goodness-of-fit is measured by McFadden's pseudo R-squared. We also report the LR and Wald test for the null hypothesis that all coefficients (except intercepts) associated with a given pair of business cycle regimes (boom (*Bo*) and expansion (*Ex*) or depression (*De*) and recession (*Re*)) are equal (indistinguishability). The test statistics are $\chi^2(4)$ distributed under the null.

values of the marginal effects in the term-spread regressions and the pseudo- R^2 exceed the other variables' marginal effects and pseudo- R^2 values for the majority of considered time horizons. Finally, the term spread is the only leading indicator of which (the absolute value of) the marginal effects and the pseudo- R^2 exhibit an inverted U-shape (reaching a maximum value for the annual time horizon); all other marginal effects and pseudo- R^2 's monotonically decline with the time horizon. Hence, the slope of the yield curve seems to have an "optimal" predictive horizon for recessions that lies between three and five quarters.

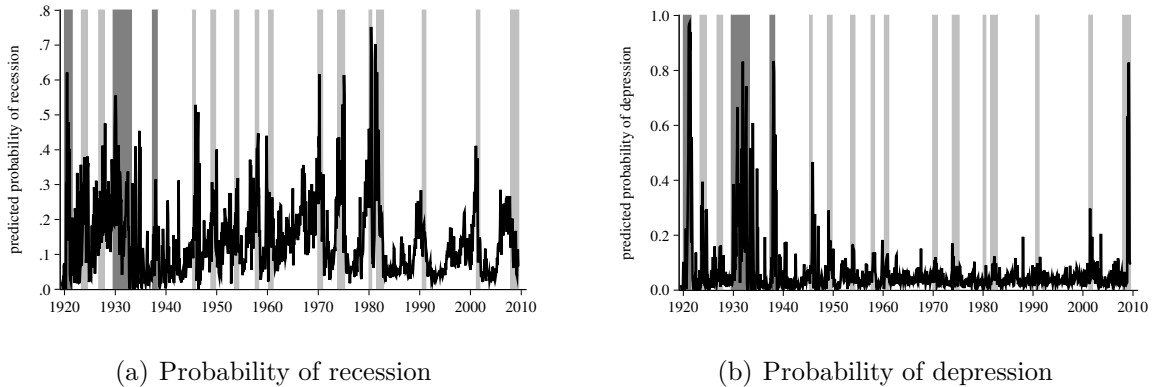
We also run the multivariate version of the binomial logit regression as a robustness check, see Table 3. The outcomes of the univariate regressions are generally confirmed. The term spread continues to have predictive power for future recessions over the whole forecast horizon and its marginal effects are significant up to two years into the future. Similarly, stock market returns and real output growth preserve their predictive power for horizons of three quarters and one year, respectively. However, the marginal inflation effect is no longer significant for the 6-month horizon. The binomial logistic regression results largely confirm the preceding literature on binary business cycle prediction (e.g., Estrella and Hardouvelis, 1991; Estrella and Mishkin, 1998; Hamilton and Kim, 2002; Kauppi and Saikkonen, 2008).

Turning to the multinomial logistic regressions, univariate results are reported in Tables 4-7 for the term spread, S&P 500 returns, the inflation rate, and real output growth, respectively, while multivariate results are summarized in Table 8. The marginal effects are reported across the four business cycle phases and add up to 0 for identification purposes. We also report testing outcomes (LR and Wald) for the null hypothesis that the RHS variables exhibit coefficients that are equal across a regime pair (recession vs. depression or expansion vs. boom). If the null hypothesis cannot be rejected, the two regimes are indistinguishable and can be merged into one regime; in case of rejection, the RHS variables provide different predictions across the two regimes (see Anderson, 1984).

The marginal effects of ordinary recessions and expansions are somewhat smaller (in absolute value) in the multinomial regressions compared to the binomial results but remain statistically significant for the same variables and time horizons. Hence, the binary results are robust to adding regimes to the business cycle. Strikingly, however, the term spread fails to predict depressions despite the fact that it preserves predictive power for recessions and expansions. At the same time, pseudo- R^2 values and marginal recession effects are largest for the term spread across different time horizons. Just as in the binomial case, the term spread is the only early warning indicator whose marginal effects and pseudo- R^2 exhibit an inverted U-shape and a comparable optimal predictive horizon.

In contrast to the term spread, the depression probability significantly diminishes when output growth, S&P 500 returns or inflation rise. Specifically, the univariate outcomes show that output growth and stock returns exhibit predictive power for depressions up to the 1-year time horizon. Remarkably, inflation seems to be the most important leading indicator for economic depressions in terms of marginal effects and across time horizons (up to 24 months). Moreover, the inflation rate has a significantly different information content for recessions and depressions across all forecast horizons (see the outcomes of the LR and Wald tests). A *ceteris paribus* one percent rise in inflation leads to a significant increase in the recession probability between 9 and 24 months ahead, while it leads to a significant drop in the depression probability at virtually every forecast horizon. The ap-

Figure 6: Predicted Probability of Recession and Depression (Multivariate)



Note: Predicted probabilities of a recession (Figure (a)) and depression (Figure (b)) occurring three months ahead. The probabilities are estimated from the multivariate multinomial logit model including all leading indicators ($SPREAD$, $\Delta \log SP500$, $\Delta \log PPI$, and Δy). Recessions are shaded in light grey and depressions are shaded in dark grey.

parently different impact of a rise in inflation on the occurrence of recession vs. depression regimes is somewhat puzzling but the existing literature gives some hints to what may be behind this phenomenon. Using a New Keynesian model on postwar macroeconomic data, [Smets and Wouters \(2007\)](#) show that inflation is primarily driven by price and wage mark-up shocks, inducing a negative correlation between current inflation and future output. This is in line with our positive marginal recession effect for inflation. Notice indeed that most recessions in our sample are situated after World War II. On the other hand, the significantly negative effect of a rise in inflation on the depression probability is in line with the observation that all three (prewar) depressions were deflationary episodes, see Figure 5 (c).

As a robustness exercise, we report estimates of a multinomial logit regression that includes all four explanatory variables in Table 8. The variables' marginal effects preserve the same signs and degrees of statistical significance. Moreover, LR and Wald tests that compare coefficients across regimes lead to the same conclusions as the test results obtained from univariate logit regressions.

Figure 6 complements the regression results with graphs of the three-month-ahead predicted probabilities of a recession and depression for the multivariate multinomial logit model. These probability plots provide a qualitative complement to goodness-of-fit measures like the pseudo- R^2 . The figure reveals that the leading indicators clearly differentiate between ordinary and extraordinary business cycle phases. The peaks in the predicted recession probability coincide with the recessions shaded in light grey, while the peaks of depression probabilities are associated with depressions shaded in dark grey. The central message from the figure is that depression probabilities exceed recession probabilities during depressions: they are higher than 80% in all three cases. Thus, the multinomial

model successfully disentangles economic recessions and depressions.¹¹

4 Conclusions

The business cycle is traditionally approached as a sequence of recessions and expansions in real economic activity. However, it is well-known that some recessions and expansions have been much longer, deeper or more abrupt than others. Thus, the question arises whether the binary characterization of the business cycle is not overly simplistic. Vast contractions of real output have indeed stimulated interest in rare economic disasters among economists. The binary framework may be overly restrictive for modeling such rare and severe events.

Questioning the validity of the binary approach therefore constitutes the starting point of this paper. We distinguish depression and boom episodes from ordinary recessions and expansions by means of a novel nonparametric framework. Using industrial production data and the peak and trough dates of the NBER business cycle between 1919 and 2009, we compute the duration, amplitude, cumulated movements, and excess cumulated movements of each recession and expansion phase. A nonparametric outlier detection test is applied to the empirical distribution of these business cycle characteristics. The outlier test enables one to distinguish tail events in cycle characteristics arising from the same probability distribution from outlying characteristics generated by a different underlying process. We relabel recessions and expansions in U.S. business cycles as depressions and booms provided at least one of the corresponding cycle characteristics exhibits outlying behavior. The presence of outlying business cycle phases justifies the specification of an econometric model that is richer in dynamics than the binary approach. The re-mapping of the binary NBER business cycle into a four-regime cycle constitutes the first contribution of the paper. We identify four extraordinary episodes: three recessions are relabeled as economic depressions and one expansion is classified as an economic boom. Interestingly, the 2007-09 Great Recession is not identified as a depression by our outlier test procedure.

In the second part of the paper, we use multinomial logistic regressions to analyze whether the four-regime business cycle can be predicted using macroeconomic and financial variables. Several key results emerge from our logistic regression analysis. First, we find that recessions become less likely when the term spread, stock market returns, and real output growth increase, regardless of whether one considers binomial or multinomial business cycle phases. Stated otherwise, the predictive power that macroeconomic and financial variables exhibit towards recessions in a binary prediction framework carries over to the multinomial framework. Second, using statistical tests within the multinomial logit framework, depressions are found to be significantly different from recessions. This provides further justification for the four-regime business cycle classification emerging from our outlier detection procedure. Finally, although the slope of the yield curve outperforms other leading indicators in predicting recessions, it does not have anything to say about

¹¹Interestingly, the indicators signal an impending depression with a probability of about 80% in March 2009, which suggests that the U.S. economy was evolving towards a depression regime at that time. However, the depression probability drops to approximately 10% by June 2009; hence, the economy avoided slipping into depression.

future economic depressions. In contrast, a decline in stock market returns, real output growth, or inflation increases the likelihood of a depression.

A Appendix: Business cycle characteristics

Formally, we observe $i = 1, \dots, n$ business cycle phases over a given period $t = 1, \dots, T$ ($n < T$). First, the *duration* of the i th recession (expansion) measures the number of months between a peak (trough) occurring at t_1 and the next trough (peak) occurring at t_2 :

$$D_i = \sum_{t=t_1}^{t_2} Q_t,$$

where $Q_t = S_t^{bin}$ in case of a recession and $Q_t = 1 - S_t^{bin}$ in case of an expansion. Second, the *amplitude* of the i th recession (expansion) measures the change in real output y_t between a peak (trough) at t_1 to the next trough (peak) at t_2 :

$$A_i = y_{t_2} - y_{t_1}.$$

Third, the *cumulative movements* of y_t within the i th phase measure the overall cost (gain) of the recession (expansion):

$$C_i = \sum_{j=t_1+1}^{t_2} (y_j - y_{t_1}).$$

Finally, the *excess cumulative movements* measure the curvature of y_t within the phase defined as:

$$E_i = (C_i - 0.5A_i - 0.5A_iD_i)/D_i.$$

The excess cumulated movements provide an approximation to the second derivative of the time series. If $E_i > 0$, the business cycle phase exhibits a concave shape, i.e., the slope of y_t changes abruptly at the beginning of the phase but the changes in slope become more gradual as the phase approaches its turning point. Conversely if $E_i < 0$, the shape is convex, and the slope changes more gradually at early stages of the phase (see [Camacho et al., 2008](#)).

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