

Hoel, Michael; de Zeeuw, Aart

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## Technology agreements with heterogeneous countries

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# MEMORANDUM

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## Technology Agreements with Heterogeneous Countries

Michael Hoel and Aart de Zeeuw

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P. O.Box 1095 Blindern  
N-0317 OSLO Norway  
Telephone: + 47 22855127  
Fax: + 47 22855035  
Internet: <http://www.sv.uio.no/econ>  
e-mail: [econdep@econ.uio.no](mailto:econdep@econ.uio.no)

In co-operation with  
**The Frisch Centre for Economic  
Research**

Gaustadalleén 21  
N-0371 OSLO Norway  
Telephone: +47 22 95 88 20  
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# Technology Agreements with Heterogeneous Countries<sup>1</sup>

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**Memo 02/2013-v1**

*Michael Hoel<sup>2</sup> and Aart de Zeeuw<sup>3</sup>*

*January 7, 2013*

## **Abstract**

For sufficiently low abatement costs many countries might undertake significant emission reductions even without any international agreement on emission reductions. We consider a situation where a coalition of countries does not cooperate on emission reductions but cooperates on the development of new, climate friendly technologies that reduce the costs of abatement. The equilibrium size of such a coalition, as well as equilibrium emissions, depends on the distribution across countries of their willingness to pay for emission reductions. Increased willingness to pay for emissions reductions for any group of countries will reduce (or leave unchanged) the equilibrium coalition size. However, the effect of such an increase in aggregate willingness to pay on equilibrium emissions is ambiguous.

*Keywords:* technology agreement, coalition stability, climate, international agreement

*JEL classification:* F42, O32, Q2, C72

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<sup>2</sup> University of Oslo. Email: mihoel@econ.uio.no

<sup>3</sup> Tilburg University. Email: A.J.deZeeuw@uvt.nl

## 1. Introduction

There is a large literature showing that international environmental agreements focusing only on reducing emissions, such as the Kyoto Protocol on climate change, cannot be expected to achieve much (for example, Barrett, 1994, Finus, 2003). One alternative to such a comprehensive international environmental agreement is to instead focus on technological improvements in order to reduce abatement costs. A sufficiently large reduction in abatement costs might induce countries to undertake significant emission reductions. Even without an explicit general agreement on emission reductions, some agreement leading to lower abatement costs as a consequence of the R&D agreed upon might result in a broad reduction of emissions. This is the background for proposals of a climate agreement on technology development (for example, Barrett, 2006, and Hoel and de Zeeuw, 2011). The present paper discusses this issue in more detail, emphasizing the fact that countries differ with respect to their valuation (or willingness to pay) for reducing greenhouse gas emissions.

The previous literature on the relationship between technological development and international environmental agreements considers different aspects. De Coninck c.s. (2008) argue that agreements should focus on technology because technology is essential for handling the problems, technology is already part of environmental policies anyway, and some important countries only want to discuss this type of agreement. Moreover, hold-up problems may arise if technology choice precedes agreements on emission reductions. Buchholz and Conrad (1995) put out a warning that countries have incentives to choose and commit themselves to bad technologies before they enter the negotiations for an agreement, because they may then be able to shift the burden of emission reductions to other countries that have lower costs. However, Battaglini and Harstad (2012) show that in a dynamic context where both the size and length of the agreement are endogenous, this hold-up problem may actually be beneficial. The idea is that the hold-up problem generated by a short-term agreement is a credible threat off

the equilibrium path and reduces the incentives to free ride. Another hold-up problem arises in Goeschl and Perino (2012) who connect a regime of international property rights to an international environmental agreement. They show a hold-up effect from the anticipation of rent extraction by the innovator which induces a reduction in abatement commitments in an agreement.

The main reason international environmental agreements are not expected to achieve much is that large agreements with large possible gains of cooperation are not stable in the sense that free-rider incentives dominate the incentives to cooperate. Benchechrout and Ray Chauduri (2012) show that eco-innovations can reduce the stability, using a farsighted stability concept. Buchner and Carraro (2005) use the FEEM RICE model to assess whether technology agreements perform better than agreements on emission reductions. They show that technology agreements are usually more stable but not necessarily more environmentally effective. Nagashima and Dellink (2008) use the STACO model to show the effects of spillovers of existing technology on international environmental agreements: global emission reductions increase, of course, but the stability of the agreement hardly changes.

To focus on the technology aspect, we assume that there is no cooperation on emission reductions. However, countries may in various ways cooperate on the development of new, climate friendly technology that reduces the costs of abatement. This means that the agreement is on R&D expenditures, for example in the form of joint ventures, and not on emission reductions. We model this very crudely, by assuming that abatement costs are a decreasing function of the total amount of R&D expenditures by a group of cooperating countries. Formally, we consider a three-stage game. In the first stage, each country decides whether or not it wants to belong to a coalition of countries that is undertaking R&D aiming to reduce abatement costs. In the second stage, the coalition decides on its amount of R&D (and how to share this cost among its members). Finally, in stage three all countries (coalition members and outsiders) decide on how much to abate. The decisions at this final stage are made non-cooperatively but the decisions are of course influenced by the previous decision of the coalition. Note that it is possible

that a group of countries decides in stage two to lower the costs of abatement so much that all countries decide to abate in the final stage. Moreover, in the model we assume marginal costs over the relevant range of abatement to be constant. Hence, at this stage each country either chooses zero abatement or some fixed amount of abatement. This decision may differ across countries, since they are assumed to have different valuations of emission reductions: Each country abates if and only if the cost of abatement does not exceed the country's valuation of emission reductions. The basic question is how far the coalition wants to go in its investments in R&D. The higher the investments, the lower the costs of abatement and the higher the number of countries that switch to the climate friendly technology.

## 2. The model

We consider a world consisting of  $N$  countries each having the same abatement potential, normalized to 1. This abatement potential could for instance be all the emissions within a specific sector, e.g. the production of electricity. Abatement decisions are made non-cooperatively, with each country choosing to abate if and only if the cost of doing so does not exceed the country's valuation of the corresponding emission reduction. Countries are assumed to be heterogeneous with respect to these valuations denoted by  $v_i$  for country  $i$ . Countries are indexed so that  $v_1 \geq v_2 \dots \geq v_N \geq 0$ .

The cost of abating (at the amount 1) in each country is given by  $c(M)$ , where  $M$  is the amount of total R&D expenditures by all countries. Knowledge created by R&D is hence considered to be a perfect public good. We make the following assumptions on  $c(M)$ :

$$c(0) > v_1, 0 > c'(M) > -1.$$

The inequality  $c(0) > v_1$  means that without any R&D, no country will abate. Abatement costs are assumed to be declining in total R&D  $M$ , but  $c(M) + M$  is increasing in  $M$ .

This last condition implies that no country will undertake R&D unilaterally in order to reduce its abatement costs.

Consider a coalition of  $k$  countries investing  $M$  in the development of new technology. Define  $m(M)$  as the number of countries satisfying  $v_i \geq c(M)$ . Clearly,  $m(M)$  is (non-strictly) increasing in  $M$ .

If all coalition members abate once the technology is developed, we have  $m(M) \geq k$ . However, as we will see in the next section, there may be equilibria where  $m(M) < k$ , i.e. only some of the coalition members abate, although they all participate in the financing of the new technology. The reason they participate in the coalition is that they obtain benefits from other countries' abating due to the developed technology.

We assume that a coalition of  $k$  countries consists of the countries that benefit most from the coalition. These are the countries with the highest valuations of abatement, i.e. countries  $1, 2, \dots, k$ . The benefit to the coalition of  $k$  countries of one unit of abatement is hence

$$(1) \quad W(k) = \sum_{i \leq k} v_i$$

which is larger the larger is  $k$  (and only defined for integer values of  $k$ ).

Using the definitions above, it is clear that the benefit to the coalition members of  $m(M)$  countries abating is  $m(M)W(k)$ . The investment cost of the coalition is  $M$ . The abatement cost of the coalition is  $kc(M)$  if all members abate, and  $m(M)c(M)$  otherwise. The payoff to a coalition of  $k$  countries that optimizes its amount of R&D is hence

$$(2) \quad V(k) = \max_M \left\{ m(M)W(k) - \left( \min[k, m(M)] \right) c(M) - M \right\}$$



Notice that  $V(k) \geq 0$  for all  $k$ , since the coalition always has the option of setting  $M = 0$  and obtaining  $m(0) = 0$  (from our assumptions about the valuations  $v_i$  and the cost function  $c(M)$ ) and hence  $V(k) = 0$ .

To have a non-trivial equilibrium we assume that  $V(1) = 0$  and  $V(k) > 0$  for sufficiently high values of  $k \leq N$ . For these values of  $k$   $V(k)$  is strictly increasing in  $k$ . This is easiest to see by treating  $k$  as a continuous variable instead of an integer. Applying the envelope theorem to (2) then gives us

$$(3) \quad V'(k) = m(M)W'(k) \text{ for } m(M) < k$$

$$(4) \quad V'(k) = m(M)W'(k) - c'(M) \text{ for } m(M) > k$$

Since  $W'(k) > 0$  it is immediately clear that  $V'(k) > 0$  for  $1 < m(M) < k$ . For the case  $m(M) > k$  we must have  $v_k \geq c'(M)$ , which together with  $W'(k) = v_k$  and  $m(M) > 1$  implies  $V'(k) > 0$ .

The optimization problem defined by (2) gives  $M$  as a function of  $k$ . From our assumptions and the discussion above it follows that  $M(k) = 0$  for sufficiently low values of  $k$  but  $M(k) > 0$  for  $k \geq k^*$ , where  $k^*$  is some threshold not exceeding  $N$ . The coalition size  $k^*$  is a coalition size satisfying the conditions for internal stability (see e.g. d'Aspremont et al., 1983; Barrett, 1994): No country will want to leave a coalition of size  $k^*$ , since members of this coalition receive a positive payoff while members of a coalition of size  $k^* - 1$  will receive a payoff of zero (due to  $M(k^* - 1) = 0$ ).

For  $k^*$  to be the largest possible stable coalition, it must be true that for any coalition larger than  $k^*$  at least one country will benefit from leaving the coalition. Consider a coalition  $k$  larger than  $k^*$ . The total payoff to the coalition may be written as

$$(5) \quad V(k) = \left\{ m(M(k-1))W(k) - \left( \min[k, m(M(k-1))] \right) c(M(k-1)) - M(k-1) \right\} + \varepsilon(k)$$

where  $\varepsilon(k)$  is the loss in payoff from choosing  $M(k-1)$  instead of the optimal value  $M(k)$ . This will typically be a small number as the payoff function is flat at the top.

The abatement decision of each county is independent of whether or not the country is a member of the coalition. The gain from leaving the coalition for country  $i$  is therefore simply its saved investment costs. However, by leaving the coalition it also obtains a loss in the form of its share of  $\varepsilon(k)$ . Formally country  $i$  is hence better off in the coalition than outside if

$$(6) \quad \alpha_i M(k-1) < \varepsilon_i(k)$$

where  $\alpha_i$  is county  $i$ 's share of the investment costs (with  $\sum_i \alpha_i = 1$ ) and  $\varepsilon_i(k)$  being some numbers satisfying  $\sum_i \varepsilon_i(k) = \varepsilon(k)$ . To have a stable coalition no country must be able to gain from leaving. Hence, inequalities of the type (6) must hold for all members. Summing over these inequalities we obtain the following condition for coalition stability:

$$(7) \quad M(k-1) < \varepsilon(k)$$

If this inequality holds, it is possible to find  $\alpha_i$ 's satisfying  $\sum_i \alpha_i = 1$  that make (7) hold for all  $i$ , hence making the coalition stable.

Clearly, the inequality (7) holds for  $k^*$  defined above, since  $M(k^*-1) = 0$  by the definition of  $k^*$  and  $\varepsilon(k^*) = V(k^*) > 0$ . Can we have a stable coalition for values of  $k$  above  $k^*$ ? We cannot rule out this possibility. However, typically  $\varepsilon(k)$  will be "small", implying that this will only occur if  $M(k-1)$  is sufficiently small. In the rest of this paper we assume that the valuations  $v_i$  and the cost function  $c(M)$  have properties implying that the only stable equilibrium is  $k^*$  as defined above (i.e. the lowest integer giving  $M(k) > 0$ ).

The equilibrium  $\{k^*, M(k^*), m(M(k^*))\}$  will of course depend on all valuations  $v_i$  and on the cost function  $c(M)$ . We start by considering how the equilibrium coalition size depends on the cost function.

## 2.1 Coalition size and the cost function

Let the cost function be given by  $c(M) + \alpha g(M)$  where  $g(M) \geq 0$  with a strict inequality for some  $M$ , and the parameter  $\alpha$  is equal to 0 initially. An increase in  $\alpha$  is thus equivalent to some positive shift in the cost function.

Inserting  $c(M) + \alpha g(M)$  into (2) and differentiating with respect to  $\alpha$  gives (using the envelope theorem)

$$(8) \quad \frac{dV(k)}{d\alpha} = -(\min[k, m(M)]) g(M) \leq 0$$

In other words, any positive shift in the cost function will either leave the function  $V(k)$  unchanged (if  $g(M) = 0$ ) or it will decline (if  $g(M) > 0$ ). If  $V(k)$  is unchanged there will be no change in the coalition size. If, however, it is reduced, the maximal value of  $k$  giving  $V(k) = 0$  will increase. In this change is sufficiently large, the equilibrium coalition size will increase. We can thus conclude that to the extent that the equilibrium coalition size is affected by the cost function  $c(M)$ , it is larger the higher is the position of the cost function.

The result above is quite intuitive. A higher investment cost required to achieve some level of abatement (i.e. some countries abating) reduces the benefits of a coalition trying to achieve this level of abatement, since there will be more investment costs to cover. The coalition hence loses from such a cost increase. The coalition can of course adjust its ambition with respect to abatement, but this will only reduce the loss, not eliminate it. The loss to the coalition means that more countries are needed in the

coalition in order for the members to have a positive net benefit of being coalition members.

## 2.2 Coalition size and valuations of abatement

An increase in some or all the valuations  $v_i$  can be represented by a positive shift in the function  $W(k)$  defined in (1). This shift is introduced by replacing  $W(k)$  by  $W(k) + \beta f(k)$ , where  $f(k) \geq 0$  with a strict inequality for some  $k$ , and the parameter  $\beta$  is equal to 0 initially. A shift in the valuations will generally also affect the number of countries who want to abate at any given cost  $c(M)$ . Hence the function  $m(M)$  also gets a positive shift to  $m(M) + \beta r(M)$ , where  $r(M) \geq 0$ . Inserting  $W(k) + \beta f(k)$  and  $m(M) + \beta r(M)$  into (2) and differentiating with respect to  $\beta$  gives (using the envelope theorem)

$$(9) \quad \begin{aligned} \frac{dV(k)}{d\beta} &= m(M)f(k) + W(k)r(M) \text{ for } k < m(M) \\ \frac{dV(k)}{d\beta} &= m(M)f(k) + [W(k)r(M) - c(M)r(M)]r(M) \text{ for } k > m(M) \end{aligned}$$

which is non-negative for all  $k$ , since  $W(k) > c(M) = v_{m(M)}$  for  $k > m(M)$ .

In other words, any positive shift increase in some of all valuation parameters  $v_i$  will either leave the function  $V(k)$  unchanged or it will increase. If  $V(k)$  is unchanged there will be no change in the coalition size. If, however, it is increased, the maximal value of  $k$  giving  $V(k) = 0$  will decline. If this change is sufficiently large, the equilibrium coalition size will decline. We can thus conclude that to the extent that the equilibrium coalition size is affected by a valuation parameter  $v_i$ , it is smaller the larger is this valuation parameter.

The result above is quite intuitive. Higher valuations of abatement among coalition members increase the benefits of the coalition for any given amount of abatement. Moreover, higher valuations of abatement may induce more countries to abate for any

given abatement cost, this will also be beneficial to the coalition. The increased benefits to the coalition countries mean that fewer countries will be needed in the coalition in order for the members to have a positive net benefit of being coalition members.

### 2.3 Investment and abatement

The number of countries abating will be lower the higher is the cost  $c(M)$ , and for any given abatement cost  $c(M)$  the number of countries abating will be higher the higher are the valuations of the countries. However, changes in either the cost function  $c(M)$  or the countries' valuations of abatement will generally change the equilibrium value of  $M$ . As we shall see in the next section, it is not obvious in which direction  $M$  moves, and it is therefore not possible to say how the equilibrium abatement depends on the valuations and the cost function for the general case. To be able to shed some light on this issue we therefore proceed by considering a special case of the general model used so far.

## 3. Model with two types

In the rest of this paper we consider the special case of only two types of countries, one with "high" valuation  $h$  of abatement and one with "low" valuation  $l(< h)$  of abatement. There are  $n \in [0, N]$  of the  $h$ -types. Compared with the notation in section 2 we hence have  $v_1 = v_2 = \dots = v_n = h$  and  $v_{n+1} = v_{n+2} = \dots = v_N = l$ .

There are two critical levels of investment:  $M_1$  making only  $h$ -types abate and  $M_2(> M_1)$  making all countries abate. These values are defined by  $c(M_1) = h$  and  $c(M_2) = l$ , as illustrated in Figure 1.

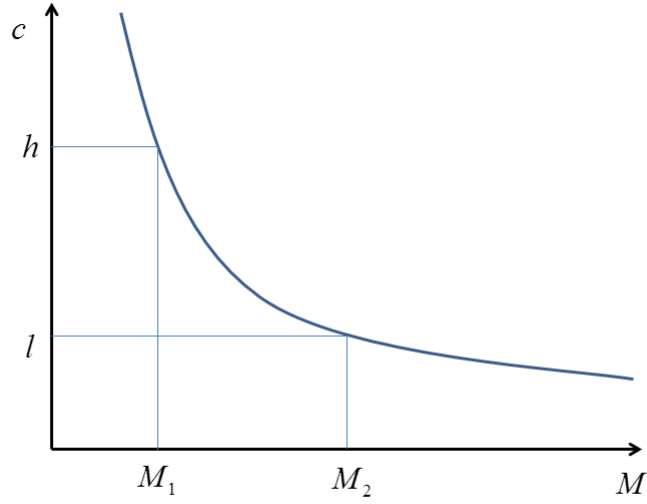


Figure 1

A coalition of  $k$  countries has three relevant options: The first option is trivial; it is characterized by zero investment and hence no abatement. The two non-trivial options are to invest  $M_1$  and achieve abatement by the  $n$   $h$ -countries (henceforth called partial abatement) or to invest  $M_2$  and achieve abatement by all countries (henceforth called full abatement).

Consider first the case of investing  $M_2$ , and hence achieving full abatement. The payoff to the coalition depends on whether  $k$  is smaller or larger than  $n$ , and is given by

$$(10) \quad V^F(n, k) = k[Nh - l] - M_2 \text{ for } k \leq n$$

$$(11) \quad V^F(n, k) = n[Nh - l] + (k - n)[Nl - l] - M_2 \text{ for } k > n$$

The curve for  $V^F(n, k)$  is increasing in  $k$  in the  $(k, V)$  space, with a kink at  $k = n$ . At  $k = n$  the slope of the piecewise linear curve drops from  $Nh - l$  to  $Nl - l$ .<sup>4</sup>

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<sup>4</sup> Notice that if our previous assumption that  $V(N) > 0$  is to hold for all values of  $n$  we must have  $V^F(0, N) > 0$  i.e.  $N(N-1)l > M_2$ .

If the coalition instead invests only  $M_1$  it only achieves partial abatement. As in the case above the payoff depends on whether  $k$  is smaller or larger than  $n$ , and is given by

$$(12) \quad V^P(n, k) = k[nh - h] - M_1 \text{ for } k \leq n$$

$$(13) \quad V^P(n, k) = n[nh - h] + (k - n)nl - M_1 \text{ for } k > n$$

The curve for  $V^P(n, k)$  is increasing in  $k$  in the  $(k, V)$  space, with a kink at  $k = n$ . At  $k = n$  the slope of the piecewise linear curve changes from  $nh - h$  to  $nl$ .<sup>5</sup>

Given that a coalition maximizes its payoff, we get (ignoring the possibility of achieving 0 by not investing)<sup>6</sup>

$$(14) \quad \tilde{V}(n, k) = \max[V^F(n, k), V^P(n, k)]$$

This payoff is piecewise linear and increasing in  $k$ , and typically has two kinks; one at  $k = n$  and one at  $V^F = V^P$ . The stable coalition  $k^*$  is the smallest integer satisfying  $\tilde{V}(n, k) \geq 0$ .

---

<sup>5</sup> In the figures it is implicitly assumed that  $nl < (n - 1)h$ .

<sup>6</sup> Including the option of not investing gives us the value function defined in section 2, i.e.

$$V(k) = \max[0, \tilde{V}(n, k)]$$

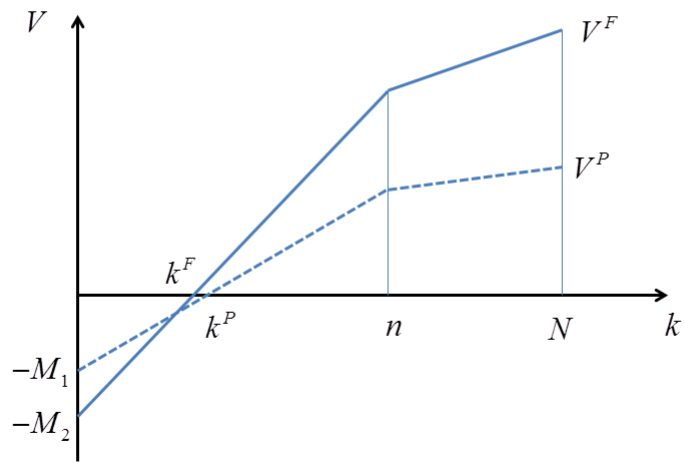


Figure 2

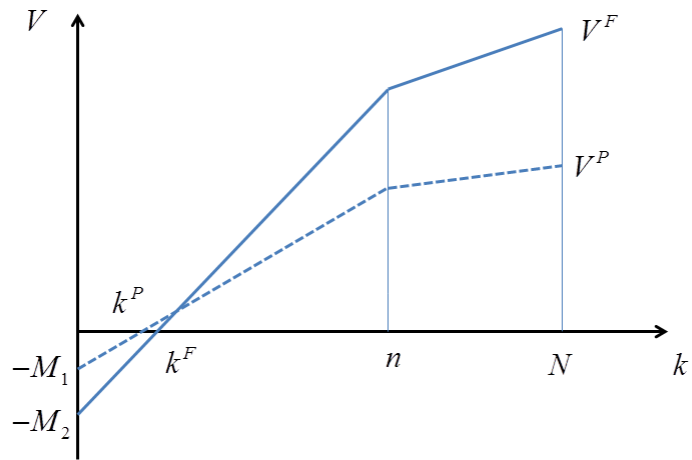


Figure 3



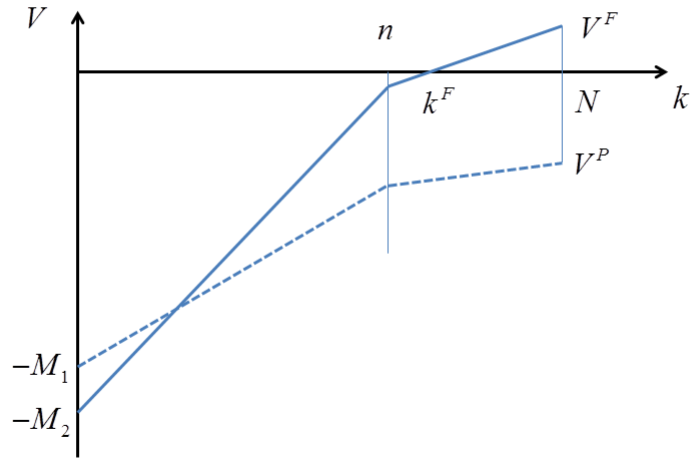


Figure 4

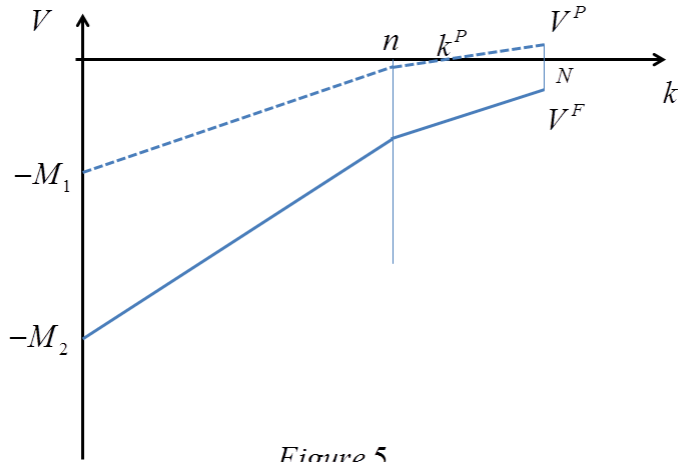


Figure 5

The possible equilibria are illustrated in Figures 2 – 5. In all figures the  $V^F$ -curve starts at  $-M_2$  and increases with  $k$ , with a kink at  $k = n$ . The  $V^P$ -curve starts at  $-M_1$  and increases with  $k$ , with a kink at  $k = n$ . For all values of  $k$  the  $V^F$ -curve is steeper than

the  $V^P$ -curve. The value function  $\tilde{V}(n, k)$  is the piecewise linear curve equal to the maximum of  $V^F$  and  $V^P$ . Our assumption  $V(N) > 0$  implies that at least one of the two curves must intersect the horizontal axis at some  $k$ . We define the values  $k^F$  and  $k^P$  by  $V^F(n, k^F) = 0$  and  $V^P(n, k^P) = 0$ , respectively. From (10)-(13) we hence have

$$(15) \quad k^F = \frac{M_2}{Nh-l} \text{ for } k \leq n$$

$$(16) \quad k^F = n + \frac{M_2 - n(Nh-l)}{(N-1)l} \text{ for } k > n$$

$$(17) \quad k^P = \frac{M_1}{(n-1)h} \text{ for } k \leq n$$

$$(18) \quad k^P = n + \frac{M_1 - n(n-1)h}{nl} \text{ for } k > n$$

From the previous section we know that an internally stable coalition is given by the smallest integer  $k^*$  satisfying  $k^* \geq \min[k^F, k^P]$ . In Appendix A we show that this is the only possible stable equilibrium under reasonable conditions.

Consider first Figure 2. In this figure we have  $k^F < k^P$  and  $k^F < n$ . The stable coalition  $k^*$  in this case hence consists only of  $h$ -countries, and they invest so much that full abatement is achieved.

In Figure 3 we have  $k^P < k^F$  and  $k^P < n$ . Also in this case the stable coalition  $k^*$  therefore consists only of  $h$ -countries. However, in this case the coalition invests only  $M_1$ , so that only  $h$ -countries abate in equilibrium.

In Figure 4  $V^P(n, k) < 0$  for all  $k$ , and  $k^F > n$ . (The properties of the equilibrium would be the same if we instead had assumed  $V^P(n, k^P) = 0$  for some  $k^P \in (k^F, N)$ .) The

stable coalition  $k^*$  in this case hence consists of all  $h$ -countries and some  $l$ -countries, and they invest so much that full abatement is achieved.

In Figure 5  $V^F(n, k) < 0$  for all  $k$ , and  $k^F > n$ . (The properties of the equilibrium would be the same if we instead had assumed  $V^F(n, k^F) = 0$  for some  $k^F \in (k^P, N)$ .) The stable coalition  $k^*$  in this case hence consists of all  $h$ -countries and some  $l$ -countries. In this case the coalition invests only  $M_1$ , so that only  $h$ -countries abate in equilibrium.

The size of the coalition, and, more importantly, the equilibrium amount of abatement depend on both the properties of the cost function  $c(M)$  and the preference parameters  $(h, l, n)$ . The next section discusses how properties of  $c(M)$  and the preference parameters affect the equilibrium coalition size, while the determinants of the amount of abatement are discussed in section 5.

### 3.1 Determinants of the coalition size

As explained in the previous section, the size of the coalition is determined by the intersection point between  $\tilde{V}(n, k)$  and the horizontal axis, i.e. by the lowest of the values  $k^F$  and  $k^P$ . We start by considering how  $k^F$  and  $k^P$  are affected by a shift in the cost function.

From the analysis in section 2.1 we know that this will increase the equilibrium coalition size. This can also be seen directly from Figures 2-5: From the definitions of  $M_1$  and  $M_2$  it is clear that a positive shift in the cost function  $c(M)$  will generally increase both  $M_1$  and  $M_2$ . This affects the starting points of the curves for  $V^F$  and  $V^P$ , but not their slopes. It therefore immediately follows from Figures 2-5 that the equilibrium size  $k^*$  of the coalition must increase. Such a cost increase may therefore also move us from an equilibrium where only  $h$ -countries cooperate to an equilibrium where all  $h$ -countries and some  $l$ -countries cooperate.

The result above is quite intuitive. A higher investment cost required to achieve either partial or full abatement reduces the benefits of a coalition of any given size, since there will be more investment costs to cover. Hence, more countries are needed in the coalition in order for the members to have a positive net benefit of being coalition members.

Consider next a change in the preference parameters. Increasing  $n$ ,  $h$  or  $l$  are equivalent to an increase in some  $v_i$ 's in the general case. It therefore follows from the analysis in section 2.2. that the equilibrium coalition size either remains unchanged or declines. To see whether the coalition size is independent of or increasing in  $n$ ,  $h$  or  $l$  we can use equation (9) from section 2. We only study small changes that do not induce a switch from partial to full abatement or vice versa. The next section discusses switches between abatement regimes in more detail.

Consider first an increase in  $n$ . For a small coalition ( $k < n$ ) this will increase the number of abating countries, hence  $r(k)$  in (9) is positive. This means that  $dV(k)/dn > 0$ , so that the equilibrium coalition size goes down. For a large coalition all countries abate, so  $r(k) = 0$ . However, in this case the change in the valuation from  $l$  to  $h$  for one or more countries increases  $W(k)$ , i.e.  $f(k) > 0$ . Therefore  $dV(k)/dn > 0$  also in this case, so that the equilibrium coalition size goes down.

Consider next an increase in  $h$ . Whatever the size of the coalition this increases  $W(k)$ , since there are always some  $h$ -countries in the coalition. This means that  $f(k) > 0$ , implying  $dV(k)/dh > 0$ , so that the equilibrium coalition size goes down.

Finally, consider an increase in  $l$ . For a large coalition ( $k > n$ ) this leads to a higher value of  $W(k)$ , i.e.  $f(k) > 0$ . This means that  $dV(k)/dl > 0$ , so that the equilibrium coalition size goes down. If the coalition is small ( $k < n$ ),  $W(k)$  is unaffected by an increase in  $l$ , so  $f(k) = 0$ . Since both partial and full abatement are independent of the values of  $l$  under consideration, the number of countries abating is independent of  $l$ , hence

$r(k) = 0$ . According to (9) this leaves  $V(k)$  unchanged. However, the analysis leading to (9) only gives first-order effects. If the small coalition is investing  $M_2$  in order to induce full abatement, its value function  $V(k)$  will be unaffected by  $l$  if the coalition leaves its investment unchanged and higher than necessary. However, the increase in  $l$  implies that the investment needed for full abatement goes down. This gives the coalition a benefit, so that  $V(k)$  in fact increases in this case.

To conclude, the optimal coalition size declines as a response to an increase in  $n$ ,  $h$  or  $l$  with one exception. The exception is that an increase in  $l$  has no effect on a small coalition which is only investing so much that partial abatement is achieved.

### 3.2. Determinants of abatement

This section discusses how properties of  $c(M)$  and the preference parameters affect abatement. We start by considering the cost function.

The properties of the cost function  $c(M)$  will obviously generally affect whether we get an equilibrium with full or only partial abatement. From Figures 2-5 or equations (15)-(18) we immediately see the following:

- If the cost function  $c(M)$  changes so that  $M_1$  increases while  $M_2$  remains unchanged,  $k^P$  will increase while  $k^F$  will remain unchanged. Hence such a shift in the cost function may move us from an equilibrium with partial abatement to an equilibrium with full abatement.
- If the cost function  $c(M)$  changes so that  $M_2$  increases while  $M_1$  remains unchanged,  $k^F$  will increase while  $k^P$  will remain unchanged. Hence such a shift in the cost function may move us from an equilibrium with full abatement to an equilibrium with partial abatement.
- If the cost function  $c(M)$  changes so that  $M_1$  and  $M_2$  increase by the same amount  $k^F$  and  $k^P$  will both increase, but  $k^P$  will increase most since the  $V^P$ -

curve is flatter than the  $V^F$ -curve. Hence such a shift in the cost function may move us from an equilibrium with partial abatement to an equilibrium with full abatement.

We next turn to the preference parameters: we start by considering  $n$  (the number of  $h$ -countries). First consider the situation that the stable coalition consists of only  $h$ -countries. Partial abatement can only be an option for a coalition of  $h$ -countries if the number  $n$  of  $h$ -countries is sufficiently large so that the total benefits (net of abatement costs)  $n(n-1)h$  are higher than the investment costs  $M_1$ . This implies that in this situation only values of  $n$  have to be considered that are larger than  $n^*$ , where  $n^* > 0$  satisfies  $n^*(n^*-1)h = M_1$  or

$$n^* = \frac{1 + \sqrt{1 + 4M_1 / h}}{2}.$$

The size of the stable coalition  $k^P$  is the number of  $h$ -countries that yields coalitional net benefits just covering the investment costs, so that  $k^P = M_1 / (n-1)h$  as given by (17). As shown in Section 3.1, a larger number  $n$  of  $h$ -countries reduces the size of the stable coalition. The reason is that the net benefits of partial abatement per country  $(n-1)h$  increase so that fewer  $h$ -countries are needed in the stable coalition to just cover the investment costs  $M_1$ .

Full abatement provides net benefits  $(Nh-l)$  to each of the  $h$ -countries. This implies that the size of the stable coalition  $k^F$  is the number of  $h$ -countries that yields coalitional net benefits just covering the investment costs  $M_2$ , so that  $k^F = M_2 / (Nh-l)$  as given by (15). Note that  $k^F$  is independent of the total number  $n$  of  $h$ -countries. The switch from partial abatement to full abatement occurs at the value of  $n$  where  $k^F = k^P$ . It is clear that this happens when the investment costs  $M_2$  are sufficiently small, or when the valuation  $l$  is sufficiently large. Interesting is, however, that it will be harder to achieve this switch by lowering  $M_2$  when the number  $n$  of  $h$ -

countries gets larger. The reason is that the coalition has a lower incentive to induce full abatement because it receives more net benefits from partial abatement. The situation is depicted in Figure 6, where  $k^P$  and  $k^F$  are drawn as functions of  $n$ , and where  $n^*$  denotes the minimal number of  $h$ -countries needed to have partial abatement as an option for a coalition consisting of only  $h$ -countries:

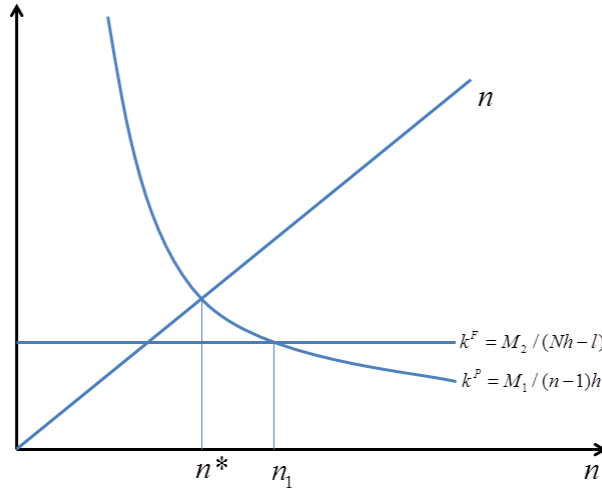


Figure 6

The curves  $k^P$  and  $k^F$  only intersect for  $n > n^*$  when

$$\frac{M_2}{Nh-l} < n^*.$$

In fact we are employing the condition  $k^P = k^F < n$ . The point  $n_1$  in Figure 6 is defined by the intersection of the curves  $k^P$  and  $k^F$ . From (15), (17) and  $k^P = k^F$ , we get

$$(19) \quad n_1 = 1 + \frac{(Nh-l)M_1}{hM_2} \Leftrightarrow M_2 = \frac{(Nh-l)M_1}{h(n_1-1)},$$

so that an inverse relationship between  $M_2$  and  $n$  results. The switch point  $n_1$  lies further to the right the lower is  $M_2$ . For  $n > n_1$  we have that  $k^P < k^F$  so that only

partial abatement occurs. For  $n^* < n < n_1$  we have that  $k^F < k^P$  so that full abatement occurs.

Note that full abatement with only  $h$ -countries is also achieved for values of  $n$  below  $n^*$  as long as  $M_2 < (Nh - l)n$ . Otherwise, some  $l$ -countries are needed in the stable coalition to cover the investment costs  $M_2$ .

Consider now the situation that the stable coalition consists of all  $h$ -countries and some  $l$ -countries. This situation is more complicated than the previous one. The switch points are determined by:

$$k^P = k^F \geq n, k^P = n + \frac{M_1 - n(n-1)h}{nl}, k^F = n + \frac{M_2 - n(Nh-l)}{(N-1)l}.$$

Again, lowering the investment costs  $M_2$  will move  $k^F$  below  $k^P$  and induce a shift from partial abatement to full abatement. However, the form of the relationship between  $M_2$  and  $n$  that determines the switch points is not immediately clear because both  $k^F$  and  $k^P$  decrease when  $n$  increases, as was seen in the previous section. It is shown in Appendix B that in this situation a decreasing relationship between  $n_1$  and  $M_2$  holds as well and is given by

$$(20) \quad n_1 = \frac{M_2 - (N-1)h - \sqrt{(M_2 - (N-1)h)^2 - 4(h-l)(N-1)M_1}}{2(h-l)}.$$

Summarizing, we can draw a graph in the  $(n, M_2)$ -plane as illustrated in Figure 7



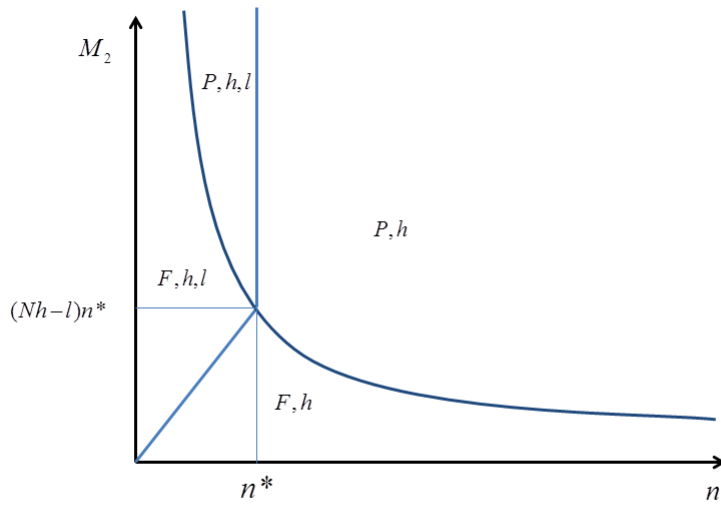


Figure 7

When  $n > n^*$  we get partial agreement with only  $h$ -countries in the stable coalition unless  $M_2$  becomes sufficiently small to induce a shift to full abatement, as we have seen in the first part of the analysis above. Moreover, we have seen in that part of the analysis that fixing  $M_2$  and decreasing  $n$  below  $n^*$  requires at some value of  $n$  to add  $l$ -countries to the stable coalition that achieves full abatement. This value is determined by  $M_2 = (Nh-l)n$ . The upper-left part of the figure shows the switch points from partial abatement to full abatement in the case the stable coalition consists of all  $h$ -countries and some  $l$ -countries. This curve was derived in Appendix B.

Figure 7 shows that we need a sufficiently low investment level  $M_2$  to make it worthwhile to invest to achieve full abatement, given the number  $n$  of  $h$ -countries, which is to be expected. More interesting, however, it also shows that given the investment level  $M_2$ , we need a sufficiently low number  $n$  of  $h$ -countries to achieve full abatement. Otherwise, the stable coalition will prefer partial abatement.

Finally we need to say what happens to Figure 7 when the parameters  $M_1$ ,  $h$  and  $l$  change. It is easy to see that an increase in  $M_1$  only (meaning that the cost function

only shifts out around  $h$ ) moves the whole figure out:  $n^*$  increases and both curves, given by (19) and (20), shift out. This implies that the area where full abatement occurs becomes larger, which is to be expected since the investment costs of partial abatement are larger.

The effect of an increase in  $h$ , and therefore a decrease in  $M_1$ , is more complicated. The direct effect of an increase in  $h$  is that the lower part of the curve in Figure 7 shifts out, because  $n^*$  decreases, the slope of the line  $(Nh-l)n$  increases and from (19)

$$\frac{\partial M_2}{\partial h} = \frac{\partial}{\partial h} \frac{(Nh-l)M_1}{h(n-1)} = \frac{lM_1}{h^2(n-1)} > 0.$$

However, combined with the indirect effect of the decrease in  $M_1$ , the total effect of an increase in  $h$  is not clear.

The effect of an increase in  $l$ , and therefore a decrease in  $M_2$ , is also not clear. The direct effect of an increase in  $l$  is that the lower part of the curve in Figure 7 shifts in,  $n^*$  does not change and the slope of the line  $(Nh-l)n$  decreases. However, this has to be interpreted for a lower  $M_2$  and therefore the total effect of an increase in  $l$  is not clear.

For a given value of  $n$ , we have two possible values of abatement: full ( $= N$ ) or partial ( $= n$ ). When  $n$  varies, there is a larger range of possible values of abatement. This is illustrated in Figure 8, based on a given set of values for  $(M_1, M_2, h, l)$ . For values of  $n$  up to  $n_1$  there is full abatement, i.e. abatement equal to  $N$ . As  $n$  passes  $n_1$  abatement drops to  $n_1$ . As  $n$  increases further toward  $N$ , abatement also increases toward  $N$ .

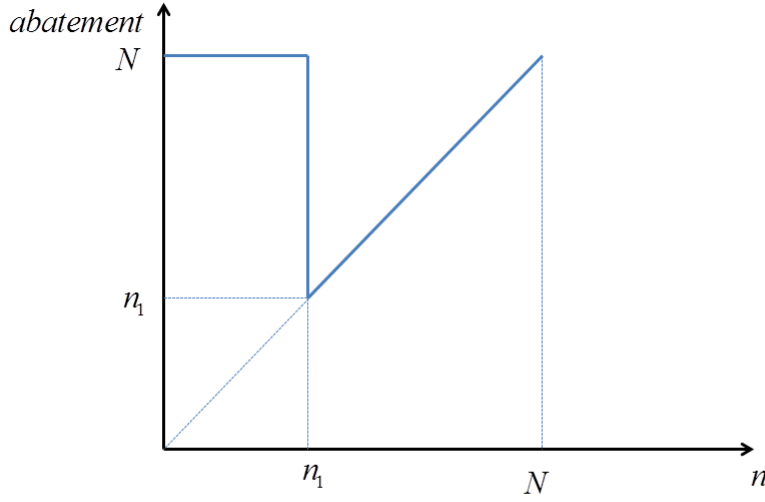


Figure 8

Notice that Figure 8, as the rest of the analysis above, was based on the assumption that the value function for the coalition is positive if the coalition is sufficiently large, i.e.  $V(N) > 0$ . This is an implicit assumption on the sizes of the elements in the vector  $(M_1, M_2, n, h, l)$ . Not all combinations of values will satisfy  $V(N) > 0$ . For Figure 8 to be valid we must have  $V(N) > 0$  even if  $n = 0$ . From (10)-(14) it follows that  $V^P(0, N) < 0$ . To achieve  $V(N) > 0$  even if  $n = 0$  we therefore must have  $V^F(0, N) > 0$ . From (11) we see that this holds if  $M_2 < N(N-1)l$  (see also footnote 4). Since  $M_2$  is higher the lower is  $l$ , this inequality is less likely to hold the lower is  $l$ . If the inequality does not hold, we will have zero abatement for  $n$  sufficiently low.

An equilibrium with positive abatement will occur when  $n$  is large enough to make either  $V^F(n, N) > 0$  or  $V^P(n, N) > 0$ . The critical value of  $n$  for positive abatement, denoted  $n_0$ , is hence given by  $n_0 = \min[n_0^F, n_0^P]$ , where  $n_0^F$  and  $n_0^P$  are defined by  $V^F(n_0^F, N) = 0$  and  $V^P(n_0^P, N) = 0$ . From (11) and (13) it follows that

$$\begin{aligned} n_0^F [Nh - l] + (N - n_0^F)[N - 1]l - M_2 &= 0 \\ n_0^P [n_0^P h - h] + (N - n_0^P)n_0^P l - M_1 &= 0 \end{aligned}$$

Solving, we obtain

$$n_0 = \min \left[ \frac{M_2 - N(N-1)l}{N(h-l)}, \frac{-(Nl-h) + \sqrt{(Nl-h)^2 + (h-l)M_1}}{2(h-l)} \right]$$

Figure 9 illustrates the case when the first of the two numbers in square brackets is the smaller of the two, while Figure 10 illustrates the opposite case.

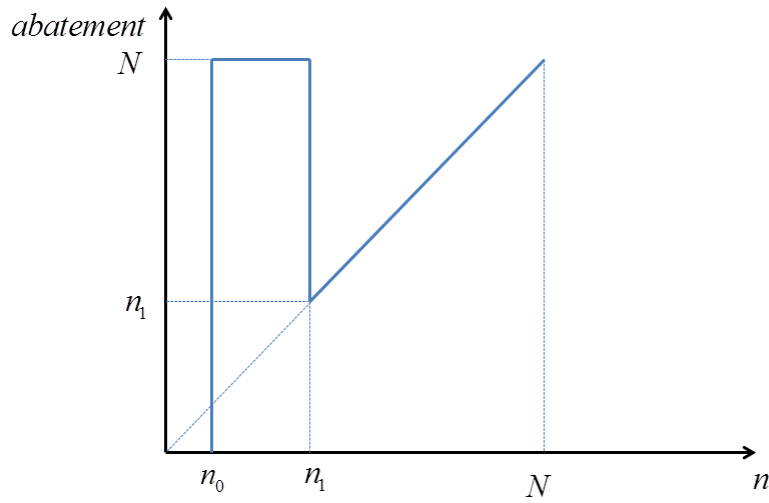


Figure 9

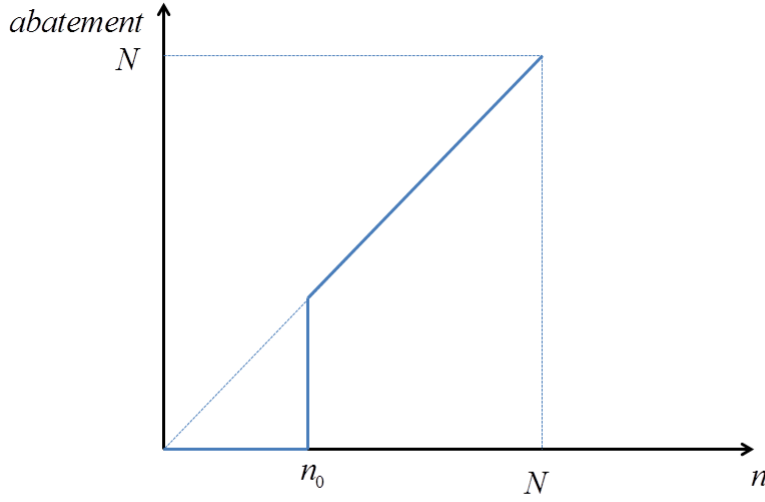


Figure 10

### 3.3 A numerical illustration

To take a numerical example, suppose that  $h = 2$ ,  $l = 1$ ,  $N = 20$ , and the cost function is  $c(M) = \gamma / M$ . With this cost function we have  $M_1 = \gamma / h$  and  $M_2 = \gamma / l$ . From the previous section we know that Figure 8 is valid for  $M_2 < N(N-1)l$ ; inserting the numerical values for  $(N, h, l)$  from above gives  $\gamma < 380$ . If e.g.  $\gamma = 250$  it follows from (20) that  $n_1$  in Figure 8 is equal to 10.8. In other words, abatement is equal to 20 for  $n \in [0, 10]$  and equal to  $n$  for  $n \in [11, 20]$ . For higher values of  $\gamma$  we get either Figure 9 or Figure 10. If e.g.  $\gamma = 500$  we have Figure 9 with  $n_0 = 6$  and  $n_1 = 10.5$ , and if  $\gamma = 1000$  we have Figure 10 with  $n_0 = 15.1$ .

Consider the case of  $\gamma = 500$  in more detail: For  $n < 6$  there is no investment in R&D and no abatement. For  $n \in [6, 10]$  there is a coalition of the  $n$   $h$ -countries and some  $l$ -countries investing  $M_2$ , hence giving full abatement. For  $n = 11$  the coalition consists of the 11  $h$ -countries and 3  $l$ -countries, investing  $M_1$  so that abatement is 11. For

$n \in [12, 20]$  the coalition will consist only of  $h$ -countries, investing  $M_1$ , giving abatement equal to  $n$ .

The effect on the critical values  $n_0$  and  $n_1$  of changes in the valuations  $h$  and  $l$  for the case of  $\gamma = 500$  is illustrated in table 1. We immediately see that increasing the valuation for the  $l$ -countries increases the range of  $n$  giving full abatement, while increasing the valuation of the  $h$ -countries reduces this range.

Table 1:  $n_0 / n_1$  for different values of  $h$  and  $l$

$l \downarrow, h \rightarrow$	2	3
1	5/10,8	3/7,4
1,5	0/15,4	0/10,8

#### 4. Concluding remarks

Even without any international agreement on emission reductions, significant emission reductions are possible if abatement costs are sufficiently low. In principle, future abatement “costs” could be negative, i.e., reducing emissions could give benefits even when the effect on the climate is ignored. This would be the case if a form of carbon-free energy with costs lower than the costs of fossil energy is discovered. A more likely scenario is that some future technology will give abatement costs that are positive, but sufficiently low that countries with a valuation of emission reductions exceeding some threshold will use this technology to reduce emissions. This is the situation we have analyzed in this paper, with an emphasis on heterogeneity across countries with regard to their valuations of emission reductions.

If all countries have a sufficiently low valuation of emission reductions, there will be no emission reductions if such reductions have a positive cost. However, when some countries have a sufficiently high valuation of emission reductions, we have shown that there may be an equilibrium with a coalition of countries undertaking R&D in order to bring down abatement costs, and with some countries non-cooperatively adopting the new technology and hence reducing emissions. This implies that a focus on technology development in international environmental agreements may be successful in terms of emission reductions without the need for a broad participation in the agreement.

One of our results is that the equilibrium size of the coalition will be smaller (or unaffected) the higher is any country's valuation of emission reductions. However, the relationship between aggregate abatement and the countries' valuations of emission reductions is ambiguous. In the numerical illustration in section 3.3 an increase in the number of high-valuation countries could either reduce or increase aggregate abatement. Increased valuation by the high-valuation countries could reduce abatement, while increased valuation by the low-valuation countries could increase abatement.

In our formal analysis we have ignored all types of uncertainty. In reality, the consequences of a given R&D expenditure for abatement costs and hence total abatement will be uncertain. However, introducing uncertainty will not change the analysis. In the general expression for a coalition's payoff (equation (2)), the terms for abatement costs and total abatement must simply be reinterpreted as expected values instead of being deterministic. The analysis for the general case will be unchanged by this reinterpretation. The details of the specific case analyzed in section 3 must be modified, but the main conclusions above will remain valid.

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## Appendix A: Stable coalitions larger than $k^*$ ?

The condition for a coalition to be internally stable was given by (7) in section 2. Consider first an integer  $K$  such that  $V^F(n, k) > V^P(n, k)$  for both  $K$  and  $K-1$ . A coalition of size  $K$  cannot be internally stable, since the optimal investment (and amount of abatement) are identical for  $K$  and for  $K-1$ . Hence  $\varepsilon(K) = 0$ , so that the condition (7) for internal stability is violated.

Consider instead an integer  $K$  such that

$$\begin{aligned} V^F(n, K) &\geq V^P(n, K) \\ V^F(n, K-1) &< V^P(n, K-1) \end{aligned}$$

In this case the value of  $\varepsilon(K)$  is equal to the difference between the actual payoff  $V^F(n, K)$  to the  $K$  countries in the coalition and what they would have gotten by investing  $M_1$  instead of  $M_2$ . The latter is simply  $V^P(n, K)$ . Hence

$$\varepsilon(K) = V^F(n, K) - V^P(n, K)$$

Define  $k^{**}$  by  $V^F(n, k^{**}) = V^P(n, k^{**})$ , implying  $K-1 < k^{**} \leq K$ . It follows that

$$\varepsilon(K) = (K - k^{**}) \left[ \frac{\partial V^F(n, k)}{\partial k} - \frac{\partial V^P(n, k)}{\partial k} \right]$$

From the definitions of  $V^F(n, K)$  and  $V^P(n, K)$  it follows that

$$\begin{aligned}\varepsilon(K) &= (K - k^{**})[(N - n + 1)h - l] \text{ for } k \leq n \\ \varepsilon(K) &= (K - k^{**})[(N - n - 1)l] \text{ for } k > n\end{aligned}$$

The condition for  $K$  to be stable is therefore (from (7))

$$\begin{aligned}M_1 &< (K - k^{**})[(N - n + 1)h - l] \text{ for } k \leq n \\ M_1 &< (K - k^{**})[(N - n - 1)l] \text{ for } k > n\end{aligned}$$

Since  $K - k^{**} < 1$ , a necessary condition for the first inequality to hold is that the square bracket is larger than  $M_1$ . A necessary condition for this is in turn that  $(N - n + 1)h > M_1$ , which means that a single  $h$ -country would be willing to pay  $M_1$  in order to get  $N - n + 1$  countries to abate. A necessary condition for the second inequality to hold is that  $(N - n - 1)l > M_1$ , which means that a single  $l$ -country would be willing to pay  $M_1$  in order to get  $N - n + 1$  countries to abate.

We cannot rule out that a coalition of size  $K > k^*$  is internally stable if  $N$  is sufficiently large. Although a coalition of this size is stable by our formal definition, we believe that such coalition sizes are not very relevant from an economic point of view for the following reason: Assume a coalition of size  $K > k^*$  is stable. Consider a very small change in either the cost function  $c(M)$  or one of the parameters  $(h, l, n)$  such that  $k^{**}$  increases but remains below  $K$ . If such a change makes  $k^{**}$  sufficiently close to  $K$ , (7) will no longer hold, and  $K$  will no longer be a stable coalition. From an economic point of view, a coalition size that depends on integer properties in this manner does not seem to be of particular interest.

## Appendix B: The switch points between regimes

The switch points in the second situation are determined by

$$k^P = k^F \geq n, k^P = n + \frac{M_1 - n(n-1)h}{nl}, k^F = n + \frac{M_2 - n(Nh-l)}{(N-1)l}.$$

This implies that the switch points are determined by the intersections  $n_1$  of the quadratic functions

$$f_1(n) = n(M_2 - (Nh-l)n), f_2(n) = (N-1)(M_1 - n(n-1)h).$$

Note that the function  $f_1$  has roots in 0 and  $M_2 / (Nh-l)$  and the function  $f_2$  has a root in  $n^*$  and is maximal in  $n = 1/2$ . The situation is depicted in Figure 11.

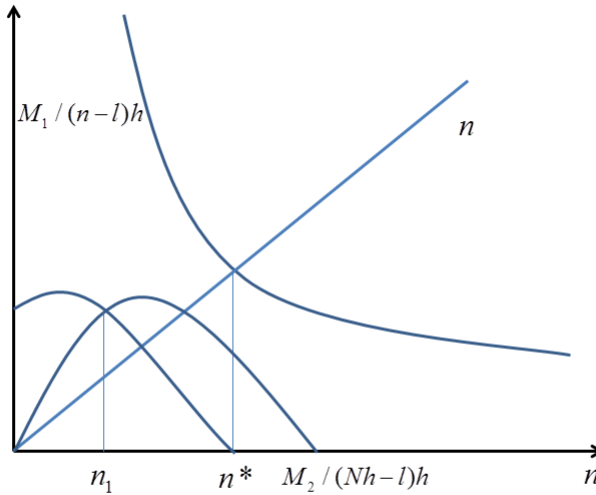


Figure 11

For  $n_1 < n < n^*$  we have that  $k^P < k^F$  so that only partial abatement occurs. For  $n < n_1$  we have that  $k^F < k^P$  so that full abatement occurs.

The difference of the quadratic functions is given by

$$f_2(n) - f_1(n) = (h-l)n^2 - (M_2 - (N-1)h)n + (N-1)M_1.$$

Since this difference is positive at  $n=0$  and the slope is negative at  $n = M_2 / 2(Nh-l)$ , it can only have positive roots. The switch point  $n_1$  is the smallest root and it is given by

$$n_1 = \frac{M_2 - (N-1)h - \sqrt{(M_2 - (N-1)h)^2 - 4(h-l)(N-1)M_1}}{2(h-l)}.$$

Note that for  $M_2 = (Nh-l)n^*$  it follows that  $f_1(n^*) = f_2(n^*) = 0$  so that  $n_1 = n^*$ .

Since

$$\frac{\partial n_1}{\partial M_2} = \frac{\sqrt{(M_2 - (N-1)h)^2 - 4(h-l)(N-1)M_1} - (M_2 - (N-1)h)}{2(h-l)\sqrt{(M_2 - (N-1)h)^2 - 4(h-l)(N-1)M_1}} < 0,$$

$n_1$  is decreasing in  $M_2$ . It approaches zero as  $M_2$  goes to infinity.