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# Wage bargaining and turnover costs with heterogeneous labor: <br> <br> The no-screening case ${ }^{1}$ 

 <br> <br> The no-screening case ${ }^{1}$}

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#### Abstract

We study the effects of mobility costs in a model of wage bargaining between heterogeneous workers and firms, where there is instantaneous matching, free firm entry, and workers' individual productivities are discovered by firms only after being hired.We derive the employment level and the minimum quality standard, in the market solution and in the socially efficient solution. We show that the minimum quality standard chosen by firms is always overoptimal. The rate of hiring among wanted workers is also overoptimal when workers have low bargaining power, but suboptimal when this bargainig power is high. In the latter case overall employment is also suboptimal. The composition of the employed labor force is always inefficient, with a too high average quality of labor. Hiring standards increase when dissipative firing costs for tenured workers increase, and drop when costs of firing unwanted worker increase. An increase in the latter may raise overall employment.


## 1. Introduction

The purpose of this paper is to study effects of hiring and firing costs on employment when workers bargain individually with their employers over the wage. We extend the standard bargaining/matching model (e.g. Pissarides (1985, 1987, 1990)) in two new directions. First, workers are no longer identical but instead have different but given productivities, known to each worker but not to the firm at the time the worker is hired. With heterogeneous labor the issue of firing becomes relevant even when there are no (idiosyncratic or general) shocks, since firms may wish to replace their initially hired workers with other, more productive, ones. Secondly, we assume both hiring costs (paid for by the firm and corresponding to recruiting costs in Pissarides), and costs of getting rid of workers once engaged by firms. While hiring costs do not directly affect firing decisions, firing costs do. We distinguish between three types of firing costs: a) the cost to the firm of immediately getting rid of an unwanted worker; b) a pure (dissipative) cost paid by the firm, which vanishes to the firm-worker relationship, when a "tenured" (or initially wanted) worker leaves the firm; and c) a redundancy payment from the firm to a tenured worker upon separation. Workers' productivities are discovered upon hiring, and firms wish to retain those workers who have the highest productivities, and may wish to fire immediately those with productivities below some minimum level. Since firms are identical, all choose the same cutoff level for productivity, $\mathrm{z}_{1}$, beyond which workers are retained. A simplification relative to the standard matching model is that our process of matching workers and firms involves no frictions, and that operating firms suffer no vacancies. This simplification makes the analysis tractable and has few disadvantages in terms of lack of generality. Secondly, with our approach a standard competitive solution now arises when workers' relative bargaining strength goes to zero, making it possible to investigate the issue of market efficiency in this important special case.

The paper integrates a modified version of the Pissarides-Mortensen matching/bargaining theory for the labor market, with recent literature on turnover costs. It makes a first step in the direction of endogenizing simultaneously worker hiring standards and overall employment when workers have unobservable productivity differences, and points out the implications of turnover costs on labor market performance in such a context. Several of our results are novel, in particular those describing how the efficiency of hiring standards and employment depend on workers' bargaining strength, and how employment may depend on the costs of firing workers immediately.

We present the basic model in section 3, and in section 4 we derive the solution chosen by a social planner subject to the same technological and cost conditions as market agents, and compare this to the market solutions in section 3 . The conclusions are summed up in the final section 5 , where we also point out some potential directions for future research.

## 2. The basic model.

Consider an economy with a large exogenous number of workers, normalized to one, and a large (endogenous) number F of active firms, each employing exactly one worker. All firms and workers are risk neutral. Since the number of active firms equals the number of employed workers, $\mathrm{L}, \mathrm{F}=\mathrm{L}$. All jobs are identical and have fixed productivities over time. There are no capital costs. ${ }^{2}$

Labor is heterogeneous, and workers' productivities denoted by z , distributed according to a continuous distribution $\mathrm{G}(\mathrm{z})$, with support $\left[0, \mathrm{z}_{\mathrm{m}}\right]$. z is known to the worker but not to the firm at the time he is hired. When the worker is hired, the firm incurs a hiring cost H , after which the worker's productivity is immediately revealed to the firm. ${ }^{3}$ The firm chooses to retain the worker given that his productivity falls in the domain $\left[\mathrm{z}_{1}, \mathrm{z}_{\mathrm{m}}\right]$, where $\mathrm{z}_{\mathrm{l}} \geq \mathrm{z}$. Workers with $\mathrm{z}<\mathrm{z}$ are consequently separated immediately. ${ }^{5}$ There is free firm entry and, apart from H , no establishment costs for firms. Assume that all workers remain in the market for an infinite period of time. For a wanted worker (with $\mathrm{z} \geq \mathrm{z}_{1}$ ) who is currently unemployed, his lifetime discounted value of labor market participation, $\mathrm{U}(\mathrm{z})$, is given by

[^1]\[

$$
\begin{equation*}
\mathrm{r} \mathrm{U}(\mathrm{z})=\mathrm{b}+\mathrm{h}[\mathrm{~W}(\mathrm{z})-\mathrm{U}(\mathrm{z})] \tag{1}
\end{equation*}
$$

\]

where $b$ is the level of income (or income-equivalent utility) in the unemployed state, $h$ is the continuous rate of transition from unemployment to employment for qualified workers, and $\mathrm{W}(\mathrm{z})$ is the expected discounted lifetime utility in the employed state.

For an employed worker (with $\mathrm{z} \geq \mathrm{Z}_{1}$ ) the equivalent discounted lifetime value is determined by

$$
\begin{equation*}
\mathrm{rW}(\mathrm{z})=\mathrm{w}(\mathrm{z})+\mathrm{s}\left[\mathrm{U}(\mathrm{z})+\mathrm{F}_{2}-\mathrm{W}(\mathrm{z})\right] \tag{2}
\end{equation*}
$$

Here $w(z)$ is the wage earned by a worker of ability $z$, and $s$ is an exogenous rate of job exit. We consequently assume, throughout, that the only reason why a worker at equilibrium can lose his job, is because his job ceases to exist. Assume (apart from immediate separations) that the cost to the firm of separating a worker at the firm's initiative is $F=F_{1}+F_{2}$, where $F_{1}$ represents real dissipative costs incurred by the firm, while $\mathrm{F}_{2}$ is a required redundancy payment from the firm to the worker. From (1)-(2) we now find

$$
\begin{equation*}
W(z)=\frac{1}{r+s}\left[w(z)+s F_{2}+s U(z)\right]=\frac{(r+h)\left[w(z)+s F_{2}+s U(z)\right]}{r(r+s+h)} \tag{3}
\end{equation*}
$$

$$
\begin{equation*}
U(z)=\frac{h\left[w(z)+s F_{2}\right]+(r+s) b}{r(r+s+h)} . \tag{4}
\end{equation*}
$$

Denote the present discounted value to the firms, of having a job filled with a worker of quality $z$, by $J(z)$. Consider a position filled with a worker of quality $z$, where the hiring cost $H$ is sunk and the worker screened. Given that the worker is not fired immediately (i.e., $\mathrm{z} \geq \mathrm{Z}_{1}$ ), $\mathrm{J}(\mathrm{z})$ is given by ${ }^{6}$

$$
\begin{equation*}
\mathrm{rJ}(\mathrm{z})=\mathrm{z}-\mathrm{w}(\mathrm{z})+\mathrm{s}[-\mathrm{F}-\mathrm{J}(\mathrm{z})] \tag{5}
\end{equation*}
$$

[^2]yielding
\[

$$
\begin{equation*}
J(z)=\frac{1}{r+s}[z-w(z)-s F] \tag{5a}
\end{equation*}
$$

\]

Denote by $\phi$ the probability that the firm samples a desirable worker, when drawing among the mass of unemployed workers. This is given by (from the appendix)

$$
\begin{equation*}
\phi=\frac{s\left[1-G\left(z_{1}\right)\right]}{(s+h) G\left(z_{1}\right)+s\left[1-G\left(z_{1}\right)\right]} . \tag{6}
\end{equation*}
$$

The density in the firm's sampling distribution over z levels among desirable unemployed workers, $\mathrm{g}_{\mathrm{s}}(\mathrm{z})$, is given by (also from the appendix)

$$
\begin{equation*}
g_{s}(z)=\frac{s}{(s+h) G\left(z_{1}\right)+s\left[1-G\left(z_{1}\right)\right]} g(z), \quad z \in\left[z_{1}, z_{m}\right], \tag{7}
\end{equation*}
$$

and where $\mathrm{g}_{\mathrm{s}}(\mathrm{z}) / \phi$ is the conditional density for workers who are not immediately fired. Calling the cumulative sampling distribution $\mathrm{G}_{\mathrm{s}}(\mathrm{z})$, note also that by definition $\phi=1-\mathrm{G}_{\mathrm{s}}(\mathrm{z})$. Note here that $\phi<1-\mathrm{G}\left(\mathrm{z}_{1}\right)$. This implies that the probability of sampling an acceptable worker from the pool of the unemployed is lower than the fraction of acceptable workers in the entire labor force, since the unemployment rate of course is lower among the former.

The cost of the firm's first sampling given a vacancy is H. Provided that this worker does not have the required quality (i.e., $\mathrm{z}<\mathrm{z}_{1}$ ), the cost of the next (and possible following) sampling(s) is $\mathrm{H}+\mathrm{F}_{0}$, where $\mathrm{F}_{0}$ is the cost to the firm of firing a worker immediately, after being hired. $\mathrm{F}_{0}$ may contain a mandatory redundancy payment, and a dissipative component. ${ }^{7} \mathrm{~F}_{0}$ may be small, and will be assumed smaller than $\mathrm{F}_{1}$. The total expected cost of filling the job with a worker of ability $\mathrm{z} \geq$ $\mathrm{Z}_{1}$ is now given by

$$
\begin{equation*}
\mathrm{C}=\mathrm{H}+(1-\phi)\left(\mathrm{H}+\mathrm{F}_{0}\right)+(1-\phi)^{2}\left(\mathrm{H}+\mathrm{F}_{0}\right)+\ldots=\left[\mathrm{H}+(1-\phi) \mathrm{F}_{0}\right] / \phi . \tag{8}
\end{equation*}
$$

[^3]Define EJ as the expected value to the firm of a filled job (with a worker of productivity $\mathrm{z} \geq \mathrm{z}_{1}$ ). We then find, integrating (5a) over z ,

$$
\begin{equation*}
E J=\frac{1}{\phi} \int_{z=0}^{z_{m}} J(z) g_{s}(z) d z=\frac{1}{r+s} \frac{1}{\phi} \int_{z=0}^{z_{m}}[z-w(z)] g_{s}(z) d z-\frac{s}{r+s} F . \tag{9}
\end{equation*}
$$

At equilibrium $\mathrm{EJ}=\mathrm{C}$, implying the condition ${ }^{8}$

$$
\begin{equation*}
\frac{1}{r+s} \int_{z=0}^{z_{m}}[z-w(z)] g_{s}(z) d z=H+\phi \frac{s}{r+s} F+(1-\phi) F_{0} . \tag{10}
\end{equation*}
$$

We assume that each firm can unilaterally select the level of $z\left(=z_{1}\right)$ beyond which a worker is retained, and below which he is immediately fired. The wage for each retained worker is determined in an asymmetric Nash bargain between the worker and the firm, with relative bargaining strengths $\beta$ and $1-\beta$, where $\beta \in(0,1)$.

## 3. The bargaining solution

We will now assume that the threat point of the bargaining solution, relevant both to workers and firms, involves no redundancy payments to a worker who leaves the firm while the firm is operating. This implies for one thing, that if a worker were to quit voluntarily, he would receive no final payment $\mathrm{F}_{2}$ from the firm. It also implies that the firm can in effect force a worker to quit, e.g. by committing to a stream of zero wage payments making it optimal for the worker to utilize the option of leaving the firm. ${ }^{9}$

Consider now an ongoing relationship, i.e., one that was not broken up immediately. For such a relationship $\mathrm{W}(\mathrm{z})$ and $\mathrm{U}(\mathrm{z})$ are still given by (3)-(4). The net utility of the worker from the match,

[^4]$\mathrm{S}(\mathrm{z})$, is given by $\mathrm{W}(\mathrm{z})-\mathrm{U}(\mathrm{z})$, i.e. by
\[

$$
\begin{equation*}
S(z)=\frac{1}{r+s}\left[w(z)+s F_{2}-r U(z)\right]=\frac{w(z)-b+s F_{2}}{r+s+h} . \tag{11}
\end{equation*}
$$

\]

Likewise, the net surplus, $\mathrm{Q}(\mathrm{z})$, of the firm is given by the surplus over the default utility in the case of a worker quit, $\mathrm{J}(\mathrm{z})+\mathrm{F}_{1}$ :

$$
\begin{equation*}
Q(z)=\frac{1}{r+s}\left[z-w(z)+r F_{1}-s F_{2}\right] . \tag{12}
\end{equation*}
$$

Defining $\mathrm{E}_{\mathrm{c}} \mathrm{z}$ as the conditional expectation of z given $\mathrm{z} \geq \mathrm{z}$, we may now characterize market equilibrium, as follows.

Proposition 1: Given a Nash bargaining solution, market equilibrium is characterized by the following equations:

$$
\begin{equation*}
E_{c} z=b+\frac{1}{1-\beta}\left\{[s+\beta(r+h)] F_{1}+(r+s+\beta h)\left(\frac{H}{\phi}+\frac{1-\phi}{\phi} F_{0}\right)\right\} \tag{15}
\end{equation*}
$$

$$
\begin{equation*}
L=\frac{h}{s+h}\left[1-G\left(z_{1}\right)\right], \tag{16}
\end{equation*}
$$

provided that $\mathrm{z}_{1} \in\left[0, \mathrm{z}_{\mathrm{m}}\right]$.

Proof: (13) is derived directly, setting $\mathrm{w}(\mathrm{z})$ to maximize the Nash product $\mathrm{S}(\mathrm{z})^{\beta} \mathrm{Q}(\mathrm{z})^{1-\beta}$, where $\mathrm{U}(\mathrm{z})$ is taken as given in (11). (14) is then found, using that $\mathrm{J}\left(\mathrm{z}_{1}\right)=-\mathrm{F}_{0}$, and recognizing that firms
set $\mathrm{z}_{1}$ unilaterally, given an internal solution for $\mathrm{z}_{1}$. (15) can then easily be derived, using (10). (16) defines L. Q.E.D.

We see from (13) that $\mathrm{w}(\mathrm{z})$ (apart from the term $-\mathrm{sF}_{2}$ ) is a weighted sum of the terms $\mathrm{z}+\mathrm{rF}_{1}$ (the net current value to the firm of continuing rather than ending the employment relationship) and $b$ (the current utility of the worker's outside option), with weights that tend to $\beta$ and $1-\beta$ as $h$ tends to zero, and (for given $\beta>0$ ) to 1 and 0 as h tends to infinity. In addition, an increase in $F_{2}$ leads to a drop in $\mathrm{w}(\mathrm{z})$, by $\mathrm{sF}_{2}$. An increase in $\mathrm{F}_{2}$ here implies no improvement in the worker's attachment value, only that more of this value takes the form of a final redundancy payment, and less the form of wages.

As a condition for Proposition 1, we assume that $\mathrm{z}_{1} \in\left[0, \mathrm{z}_{\mathrm{m}}\right]$. A sufficient condition for $\mathrm{z}_{1}>0$ is $\mathrm{rF}_{0}<\mathrm{b}$, which will be assumed here and in the following. With regard to $\mathrm{z}_{\mathrm{m}}$, a sufficient condition for $\mathrm{z}_{1}<\mathrm{z}_{\mathrm{m}}$, both here and in case 2 below, is

$$
\begin{equation*}
z_{m}>b+\frac{1}{1-\beta}\left[(\beta r+s) F_{1}-(r+s) F_{0}\right], \tag{17}
\end{equation*}
$$

for all relevant values of $\beta$ and $\mathrm{F}_{\mathrm{i}}$, which will also be assumed to hold.

Consider a marginal worker as viewed by firms, i.e., $\mathrm{z}=\mathrm{z}_{1}$. For such a worker,

$$
\begin{equation*}
S\left(z_{1}\right)=\frac{\beta}{1-\beta}\left(F_{1}-F_{0}\right) . \tag{18}
\end{equation*}
$$

Since $F_{0}<F_{1}$, there is a positive net surplus associated with, and going to, a marginal employed worker. Thus a worker with productivity slightly below $\mathrm{Z}_{1}$ would now have been able to enjoy a positive net surplus from not having his match broken immediately but rather continuing it. The reason why the match is still immediately broken (unilaterally by the firm) is that when $F_{0}<F_{1}$, then for a marginally profitable worker it is advantageous to break up the match immediately
rather than later, since this reduces dissipative firing costs. ${ }^{10}$

An important and interesting question is whether the solution for $\mathrm{Z}_{1}$ from (18) is unique. To address this issue, define $\theta\left(\mathrm{z}_{1}\right)=\mathrm{E}_{\mathrm{c}} \mathrm{z}-\mathrm{z}_{1}$, given by

$$
\begin{equation*}
E_{c} z-z_{1}=\frac{1}{1-\beta} \frac{r+s+\beta h}{\phi}\left(H+F_{0}\right) . \tag{19}
\end{equation*}
$$

We may then formulate the following result.

Proposition 2: Equilibrium as characterized in Proposition 1 is unique given that

$$
\begin{equation*}
\frac{1-G\left(z_{1}\right)}{g\left(z_{1}\right)}>E_{c z}-z_{1}-\frac{(r+s)(s+h)}{(1-\beta) s} \frac{H+F_{0}}{1-G\left(z_{1}\right)} \tag{20}
\end{equation*}
$$

holds everywhere.

Proof: We have from (6) that

$$
\begin{equation*}
\frac{1}{\phi}=1+\frac{s+h}{s} \frac{G\left(z_{1}\right)}{1-G\left(z_{1}\right)} \tag{6a}
\end{equation*}
$$

Inserting from (6a) in (19) and differentiating (19) with respect to $\mathrm{z}_{1}$ yields $\mathrm{d} \theta / \mathrm{dz}_{1}<0$ everywhere if and only if (20) holds everywhere. Moreover, for a given $\mathrm{z}_{1}$ we have from (14) that there is a unique equilibrium value of h , implying that $\mathrm{w}(\mathrm{z})$ and L are given uniquely from (13) and (16). Thus (20) is a sufficient condition for equilibrium to be unique. Q.E.D.

The possibility of multiple solutions is here in principle open. In such cases (14) implies that high levels of $\mathrm{z}_{1}$ go together with high levels of h . Since higher z for given h implies lower employment, and higher h higher employment, employment levels cannot generally be ranked among such equilibria. In the following discussion we will however generally assume that

[^5]equilibrium is unique.

The effects of changes in the cost variables $\mathrm{H}, \mathrm{F}_{0}$ and $\mathrm{F}_{1}$ on the key variables h and $\mathrm{z}_{1}$ can be found from differentiating (14) and (15) with respect to h and $\mathrm{z}_{1}$ and the cost variables (inserting for L from (16)). Note initially that changes in $\mathrm{F}_{2}$ have no effects on h and $\mathrm{z}_{1}$. One may readily show that $\mathrm{z}_{1}$ is increased when H and $\mathrm{F}_{1}$ increase. The reason is partly that the general cost level then increases. Moreover, higher $\mathrm{F}_{1}$ raises workers' bargaining threat point and thus the wage. Firms then become more selective with respect to what workers to keep at the time of recruiting. Note that from (16),

$$
\begin{equation*}
d L=\frac{s}{(s+h)^{2}}\left[1-G\left(z_{1}\right)\right] d h-\frac{h}{s+h} g\left(z_{1}\right) d z_{1} . \tag{21}
\end{equation*}
$$

Total employment must then drop when H and $\mathrm{F}_{1}$ increase. We find, in appendix B , that ${ }_{1} \mathrm{Z}$ decreases with $\mathrm{F}_{0}$, and h most likely decreases as well. A lowering of z however in addition reduces screening costs in individual firms since fewer worker types are immediately dismissed, and reduces the pool of undesirable workers, thus lowering average screening costs. This raises firm entry and thus also possibly h , and tend to make the effect on employment of a higher $\mathrm{F}_{0}$ more positive when screening is imperfect.

## 5. Efficient solutions

Because of the mechanism for determining the wage (bilateral bargaining) and aggregate employment (firm entry to make net profits equal to zero), there is no reason to expect either of the derived market solutions to be efficient. We will in this section derive the constrained efficient solution, and compare it to the two market solutions derived above. Our procedure is to find the levels of $\mathrm{z}_{1}$ and h , and consequently L , that would be set by a social planner who could set these directly, given that such a planner faces real hiring costs $\mathrm{H}_{0}$, real dissipative firing costs $\mathrm{F}_{00}$ for workers to be dismissed and $\mathrm{F}_{10}$ for workers to be kept, and is subject to the same screening technology as that facing firms.

Define the total match value for a given z , once the worker is employed, H sunk and the worker

$$
\begin{equation*}
M(z)=\frac{1}{r+s}\left(z-b-s F_{10}\right) . \tag{22}
\end{equation*}
$$

retained (i.e., $\mathrm{z} \geq \mathrm{z}_{1}$ ), by $\mathrm{M}(\mathrm{z})$, where $\mathrm{rM}(\mathrm{z})=\mathrm{z}-\mathrm{b}-\mathrm{sF}_{10}+\mathrm{s}(-\mathrm{M}(\mathrm{z})$ ), implying A match breakup involves a social firing cost of $\mathrm{F}_{10}$ and loss of the match value $\mathrm{M}(\mathrm{z})$. Denote by EM the expected ex ante value of a successful match (i.e., a match where $z \geq Z_{1}$ is realized), including costs sunk in order to accomplish such a match. This is given by $\mathrm{EM}=\mathrm{E}[\mathrm{M}(\mathrm{z})]-\left[\mathrm{H}_{0}+\mathrm{G}_{\mathrm{s}}\left(\mathrm{z}_{1}\right) \mathrm{F}_{00}\right] /\left[1-\mathrm{G}_{\mathrm{s}}\left(\mathrm{z}_{1}\right)\right]$, where as before $\mathrm{G}_{\mathrm{s}}(\mathrm{z})$ is the sampling distribution over z for hiring firms, from the pool of unemployed workers. Define T as the ex ante value of all successful matches in existence at a given time. Since the rate of employment among wanted workers (with $\mathrm{z} \geq \mathrm{z}_{1}$ ) is $\mathrm{h} /(\mathrm{s}+\mathrm{h})$, total employment $L$ is $\left[1-G\left(z_{1}\right)\right][h /(s+h)]$. T may be expressed by:

$$
\begin{align*}
T\left(z_{1}, h\right) & =\frac{h}{s+h}\left\{\int_{z=z_{1}}^{z_{m}} \frac{z-b-s F_{10}}{r+s} g(z) d z\right.  \tag{23}\\
& \left.-\left\{\frac{(s+h)}{s} G\left(z_{1}\right)+\left[1-G\left(z_{1}\right)\right]\right\} H_{0}-\frac{(s+h)}{s} G\left(z_{1}\right) F_{00}\right\} .
\end{align*}
$$

The government's objective is to maximize (23) directly with respect to $\mathrm{z}_{1}$ and $\mathrm{h} .{ }^{11}$
The solution to this problem can be formulated in the following proposition.

Proposition 3: The government's constrained optimal solution for $\mathrm{z}_{1}$ and h is given by

$$
\begin{equation*}
z_{1}=b-\frac{(r+s) h}{s} H_{0}+s F_{10}-\frac{(r+s)(s+h)}{s} F_{00} \tag{24}
\end{equation*}
$$

[^6]\[

$$
\begin{equation*}
E_{c} z=b+s F_{10}+(r+s) H_{0}+\frac{(s+h)^{2}(r+s)}{s^{2}} \frac{G\left(z_{1}\right)}{1-G\left(z_{1}\right)}\left(H_{0}+F_{00}\right), \tag{25}
\end{equation*}
$$

\]

provided that $\mathrm{z}_{1} \in\left[0, \mathrm{z}_{\mathrm{m}}\right)$, and $\mathrm{G}\left(\mathrm{z}_{1}\right)$ strictly positive.

Proof: Maximizing (23) with respect to $\mathrm{z}_{1}$ and h yields

$$
\begin{gather*}
\frac{d T}{d z_{1}}=\frac{h}{s+h} g\left(z_{1}\right)\left\{\frac{-z_{1}+b+s F_{10}}{r+s}+H_{0}-\frac{s+h}{s}\left(H_{0}+F_{00}\right)\right\} \leq 0  \tag{26}\\
\frac{d T}{d h}=\frac{s}{h(s+h)} T-\frac{h}{s(s+h)} G\left(z_{1}\right)\left(H_{0}+F_{00}\right) \geq 0 . \tag{27}
\end{gather*}
$$

(26) here holds with equality if and only if $\mathrm{z}_{1} \in\left[0, \mathrm{z}_{\mathrm{m}}\right]$. (27) always holds with equality. Inserting for T from (23) in (27) then yields (24)-(25). Q.E.D.

In (24)-(25), the costs of sorting out unwanted workers, in the form of increased firing costs $\mathrm{F}_{00}$ and subsequent hiring costs $\mathrm{H}_{0}$, affect the (constrained and second-best) optimal solution. The effect in (24) of increased $\mathrm{H}_{0}$ and $\mathrm{F}_{00}$ is to reduce ${ }_{1} \mathrm{z}$ below its unconstrained optimal level, reducing overall sorting costs when fewer worker types are screened out at equilibrium. The second-best optimum trades off this saving in sorting costs against the efficiency loss from retaining workers with too low productivities. Note also that it can never be efficient to have full employment among desirable workers, i.e., $h$ must be finite. To see this, consider $h \rightarrow \infty$. But then the market equilibrium would imply (from appendix a) that $\mathrm{G}_{\mathrm{s}}\left(\mathrm{z}_{1}\right)$ is very close to 1 , i.e., (almost) all workers in firms' sampling distribution over unemployed workers would be unwanted. Clearly this cannot be efficient, since the marginal hiring and firing costs, associated with hiring one additional qualified worker, would go to infinity.

It may seem surprising that an increase in $\mathrm{H}_{0}$ reduces the minimum hiring standard z . When interpreting the effect of an increase in $\mathrm{H}_{0}$ on $\mathrm{z}_{1}$ in (24), note that ( $\mathrm{s}+\mathrm{h}$ )/s expresses the rate at which "unqualified" workers are sampled versus "qualified" ones, in terms of the original
distribution $\mathrm{G}(\mathrm{z})$. Since $(\mathrm{s}+\mathrm{h}) / \mathrm{s}>1$, the overall burden of hiring costs associated with unqualified workers is greater than that associated with qualified ones. This implies that there is an overall efficiency gain to be had from grouping more workers in the qualified category, when $\mathrm{H}_{0}$ increases.
(24)-(25) are constructed to facilitate a comparison with the market solutions. We are here in particular interested in deriving the conditions for a constrained efficient solution to be implementable by the market in these two cases. We assume that the government can freely tax or subsidize hiring and firing costs, and that $\mathrm{H}-\mathrm{H}_{0}$ and $\mathrm{F}_{\mathrm{i}}-\mathrm{F}_{\mathrm{i} 0}, \mathrm{i}=1,2$, represent net government subsidy rates (or tax rates when negative). There are no net costs to the government associated with positive net subsidies to firms, nor are there gains due to net taxes. We impose no prior constraints on H or $\mathrm{F}_{\mathrm{i}}$, i.e., either of these could be negative as part of an implemented efficient solution.

An efficient solution can then in principle always be implemented in both cases 1 and 2 above, by only setting H and the $\mathrm{F}_{\mathrm{i}}$ at appropriate levels. ${ }^{12}$ The interesting issue in our context is what properties distinguish the efficient from the unregulated market solutions. To shed light on this issue we consider the following two examples.

Example a: $\beta \rightarrow 0$. In this case, note that (14)-(15) now can be written as

$$
\begin{gather*}
\mathrm{z}_{1}=\mathrm{b}+\mathrm{sF}_{1}-(\mathrm{r}+\mathrm{s}) \mathrm{F}_{0}  \tag{14a}\\
E_{c} z=b+s F_{1}+(r+s) H+\frac{(s+h)(r+s)}{s} \frac{G\left(z_{1}\right)}{1-G\left(z_{1}\right)}\left(H+F_{0}\right) . \tag{15a}
\end{gather*}
$$

Given that $\mathrm{H}=\mathrm{H}_{0}$ and $\mathrm{F}_{\mathrm{i}}=\mathrm{F}$ io, $\mathrm{i}=0,1$, comparing (14a) to (24) reveals that $\mathrm{z}_{1}$ is unambiguously higher in the market solution than in the efficient solution. As a result, minimum hiring standards are inefficiently high in the unregulated market solution. The intuitive reason for this is again the negative externality related to dismissing a worker that is initially engaged, since such a dismissal

[^7]leads to a "contamination" of the unemployment pool, and increases the hiring (and subsequent firing) costs of other firms.

Comparing (15a) to (25) similarly reveals that h is overoptimal in the market solution (since $\mathrm{E}_{\mathrm{c}} \mathrm{z}$ increases strictly in $\mathrm{z}_{1}$ for given h , and must consequently be greater in (15) than in (27))). This is due to a negative externality related to firm establishment and subsequent hiring in the market. Absorbing a high-quality worker from the unemployment pool namely also "contaminates" the pool of the unemployed, in a similar way as when a low-quality worker is fired.

The overall consequence of these conclusions is that when workers' bargaining power is very low, too few worker types are retained by firms, but the rate of employment among those retained is inefficiently high. The overall effect of this on employment is difficult to judge in general.

Example $\mathrm{b}: \mathrm{F}_{\mathrm{i}}=\mathrm{F}_{\mathrm{i}}=0, \mathrm{i}=1$,2. In this case there are no firing costs. Now $\mathrm{z}=\mathrm{b}$ from (14). Consequently the market-determined hiring standard is still above the constrained optimal level. To study the effect for h , note that (15) now can be written as

$$
\begin{equation*}
E_{c} z=b+\frac{r+s+\beta h}{1-\beta}\left[1+\frac{s+h}{s} \frac{G\left(z_{1}\right)}{1-G\left(z_{1}\right)}\right] H . \tag{15b}
\end{equation*}
$$

We again find that when $\beta$ is low, h must be higher at the market solution than at the constrained efficient solution. When $\beta$ grows higher and approaches one, by contrast, we see that $\mathrm{E}_{\mathrm{c}} \mathrm{z}$ for given $h$ is increases and approaches infinity in the limit. This implies that $h$ must approach zero, and in fact hit zero at a level of $\beta$ below one (since $\mathrm{z}_{1}$ is a constant, from (14), and thus $\mathrm{E}_{\mathrm{c}} \mathrm{z}$ a constant). The implication of this is that as $\beta$ increases from zero, $h$ is reduced, from an overoptimal level (as exposed in example a) to a suboptimal one. For a sufficiently high $\beta$, of course, there can be no market solution as $h$ in our model is below zero. The interpretation of this case is that with sufficiently high $\beta$, no firms find it profitable to enter the market, when there are positive hiring costs.

An overall conclusion from these two examples is that the minimum worker hiring standard is
always higher than the constrained optimal standard chosen by a social planner. The rate of hiring and thus employment among those above the minimum standard is also overoptimal when workers have very low bargaining power, but falls (to a suboptimal level) when this bargaining power increases. As far as the overall level of employment, this could be higher or lower than the constrained optimal level when workers have low bargaining power, but is always suboptimal when workers' bargaining power is high.

## 5. Conclusions

We have studied a model of the labor market where there is instantaneous matching and subsequent wage bargaining between individual workers and firms, workers differ in their productivities, and it is costly for firms to hire and fire workers, and firms cannot observe workers' individual productivities prior to hiring them. In section 3 above we have derived the minimum quality standard beyond which workers are retained by firms, and the equilibrium wage levels of workers as a function of their productivities, among those retained. In the model, regular firing costs are incurred only when workers lose their jobs because firms (exogenously) close down. Severance payments to such workers then have no allocation effects, and only affect the distribution of the worker's attachment value between wage and redundancy payments. Dissipative firing costs (that are lost to the worker-firm relationship) however reduce employment and increase the minimum hiring standard. An increased cost of immediately disposing of newly hired workers always reduces this standard and has an ambiguous effect on employment. The reason why employment then may increase is two fold. First, lower hiring standards implies that the rate of hiring among wanted workers would drop given that overall employment is constant, and this drop results in a lower wage, through a weakening of workers' bargaining position. Secondly, the lowering of the quality standard implies a positive externality for the labor market as a whole, whereby overall costs of recruiting and testing new workers are reduced. Both these effects reduce the overall costs of firms, spurring firm entry and thus employment.

In section 4, we then derived the constrained efficient solution, implemented by a social planner facing the same technological constraints as those facing firms in the market. To understand the results derived here, note that there are social recruitment costs associated with sorting out
unwanted workers, in the form of immediate firing costs and costs of subsequent necessary hirings. These recruitment costs are reduced when the group of unwanted workers is small, relative to the group of wanted workers. Recruitment costs are reduced when the minimum worker quality standard is reduced (thus leaving fewer workers unwanted) ; and when fewer among the wanted group are hired (thus leaving more of these in the pool of the unemployed, and increasing the probability that a searching firm will find one of these when recruiting in the market).

In section 4 we also demonstrate that the market solution always entails a too high quality standard. We then show that the rate of hiring among wanted workers is too high when workers have very low bargaining power, but is reduced, to a suboptimal level, when workers' bargaining power increases. This implies that overall employment may be too low or too high when workers' bargaining power is low, but is always too low when this power is high. In all cases, the composition of the employed labor force is inefficient, in the direction of firms being too selective and thus the average quality of employed workers too high.

These results can be contrasted to those obtained in a related model, where we instead assume "perfect history screening", implying that firms do not incur sorting costs (i.e., at the time a worker is first hired, the firm already knows whether the worker is in the desirable group or not; although the firm does not know the worker's actual productivity). Such a case is explored in an accompanying paper, Strand (1997). I then show that the market-determined minimum productivity level for workers is inefficiently low, i.e., the diametrically opposite case to that derived here. The reason is that in this case the hiring cost becomes a net burden associated with recruiting a worker, which firms in the market have no incentive to consider once hiring already is done. With perfect history screening we also find, in contrast to the current case, that the rate of hiring among desirable workers is always too low, while it is always too high in the current model given that workers have a low bargaining power. The implication is that many of the results derived from the current and presumably related models, are sensitive to detailed assumptions with respect to the ability of firms to distinguish worker types prior to hiring them. Clearly, these conclusions call for more research along related lines.

An important feature of our model is that the implementation of an efficient solution requires the correct determination of two independent variables (namely the hiring standard and the rate of employment). An efficient solution thus cannot in general be implemented just by requiring $\beta$ to take a certain value (as in the model discussed by Pissarides (1990)). In addition at least one cost variable facing firms must be set by the government. ${ }^{13}$

In focussing on the effects of turnover costs and labor heterogeneity, we have deliberately disregarded a number of potentially important features. In our model all actual firings among desirable ("tenured") workers are fully exogenous. Our analysis is thus quite distinct from other recent contributions where worker turnover and turnover costs are central, such as Bentolila and Bertola (1990), Bertola (1990) and Lazear (1990) dealing with turnover costs in more partialequilibrium settings, and Mortensen and Pissarides (1994), Bertola and Caballero (1994) and Saint-Paul (1995) who consider bargaining models with turnover and turnover costs but where labor is assumed to be homogeneous. Extensions of our framework, which are possible avenues for future research, may be to incorporate productivity variations (as e.g. in MortensenPissarides), on-the-job search (as in Pissarides (1994)), stochastic match values (as in BertolaCaballero), and the possibility of rehiring laid-off workers. Incorporating such alternative assumptions could also further serve to integrate the theories of contracts and matching and clarify their relationships when there are positive turnover costs.

[^8]
## Appendix A: Derivation of the firm's sampling distribution over worker qualities

We here wish to derive the sampling distribution $\mathrm{G}_{\mathrm{s}}(\mathrm{z})$, for firms' sampling of workers from the pool of unemployed. First define the distribution over qualities for the entire set of unemployed workers, $G_{u}(z)$. The fraction of workers in the entire labor force that has $z<z_{1}$ is $G\left(z_{1}\right)$, and the equivalent fraction among the unemployed is denoted by $\mathrm{G}_{\mathrm{u}}\left(\mathrm{z}_{1}\right)$. Note that

$$
\begin{equation*}
\mathrm{NG}\left(\mathrm{z}_{1}\right)=\mathrm{UG}_{\mathrm{u}}\left(\mathrm{z}_{1}\right)=\mathrm{U}_{1}, \tag{A1}
\end{equation*}
$$

where $U$ is the total number of unemployed workers, and $U_{1}$ is the number of unemployed workers with $\mathrm{z}<\mathrm{z}_{1}$, since all workers with $\mathrm{z}<\mathrm{z}$ are unemployed at equilibrium (when each is employed by firms only for an infinitely short period of time after being hired).

For a worker with $\mathrm{z} \geq \mathrm{z}_{1}$, the unemployment rate $\mathrm{u}_{2}$ equals the average fraction of the time spent in unemployment, $\mathrm{s} /(\mathrm{s}+\mathrm{h})$, while the employment rate equals the fraction of the time spent in employment, $\mathrm{h} /(\mathrm{s}+\mathrm{h})$. Denoting by $\mathrm{U}_{2}$ and $\mathrm{N}_{2}$ the number of workers with $\mathrm{z} \geq \mathrm{z}_{1}$ in unemployment and employment respectively. We then have

$$
\begin{align*}
& U_{2}=N\left[1-G\left(z_{1}\right)\right] \frac{s}{s+h}  \tag{A2}\\
& N_{2}=N\left[1-G\left(z_{1}\right)\right] \frac{h}{s+h} .
\end{align*}
$$

Since $U_{1}=N G\left(z_{1}\right), N_{1}=0$, we then find

$$
\begin{equation*}
G_{u}\left(z_{1}\right)=\frac{(s+h) G\left(z_{1}\right)}{s+h G\left(z_{1}\right)} \tag{A4}
\end{equation*}
$$

Since $G_{u}(z)$ has density proportional to $G(z)$, piecewise on $\left[z_{0}, z_{1}\right)$ and on $\left[z_{1}, z_{m}\right]$, we find

$$
G_{u}(z)=\frac{s+h}{s+h G\left(z_{1}\right)} G(z), \quad z<z_{1}
$$

(A5)

$$
=\frac{(s+h) G\left(z_{1}\right)}{s+h G\left(z_{1}\right)}+\frac{s}{s+h G\left(z_{1}\right)}\left[G(z)-G\left(z_{1}\right)\right], \quad z \geq z_{1} .
$$

Since the relative frequency with which a worker with $\mathrm{z}<\mathrm{z}_{1}$ will be sampled by the firm is one, the probability that the firm samples such a worker is

$$
\begin{equation*}
G_{s}\left(z_{1}\right)=\frac{G_{u}\left(z_{1}\right)}{G_{u}\left(z_{1}\right)+1-G_{u}\left(z_{1}\right)}=1-\phi, \tag{A6}
\end{equation*}
$$

which yields, using (A4),

$$
\begin{equation*}
1-\phi=\frac{(s+h) G\left(z_{1}\right)}{(s+h) G\left(z_{1}\right)+s\left[1-G\left(z_{1}\right)\right]} . \tag{A7}
\end{equation*}
$$

The firm's sampling distribution is then given by

$$
G_{s}(z)=\frac{(s+h)}{(s+h) G\left(z_{1}\right)+s\left[1-G\left(z_{1}\right)\right]} G(z), \quad z<z_{1}
$$

(A8)

$$
\begin{aligned}
& =\frac{(s+h) G\left(z_{1}\right)}{(s+h) G\left(z_{1}\right)+s\left[1-G\left(z_{1}\right)\right]} \\
& +\frac{s}{(s+h) G\left(z_{1}\right)+s\left[1-G\left(z_{1}\right)\right]}\left[G(z)-G\left(z_{1}\right)\right], \quad z \geq z_{1} .
\end{aligned}
$$

## Appendix B: Comparative-static results of changes in $\mathbf{F}_{0}$.

The effects of changes in $\mathrm{F}_{0}$ on h and $\mathrm{z}_{1}$ are:
(B1)

$$
\frac{d h}{d F_{0}}=-\frac{1}{D}(r+s+\beta h)\left[\frac{D\left(z_{1}\right)}{1-\beta}-\frac{(s+h)}{s} \frac{r+s+\beta h}{1-\beta} \frac{1-2 G\left(z_{1}\right)}{\left(1-G\left(z_{1}\right)\right)^{2}} g\left(z_{1}\right)\left(H+F_{0}\right)+\frac{1-\phi}{\phi}\right]
$$

(B2)

$$
\begin{gathered}
\frac{d z_{1}}{d F_{0}}=-\frac{1}{D} \frac{r+s+\beta h}{1-\beta}\left[\frac{\beta}{(1-\beta) \phi}\left((1-\beta+\beta \phi) F_{1}+H+\beta(1-\phi) F_{0}\right)\right. \\
\left.+=\frac{1}{s}(r+s+\beta h)\left(H+F_{0}\right) \frac{G\left(z_{1}\right)}{1-G\left(z_{1}\right)}\right]
\end{gathered}
$$

where

$$
\begin{gather*}
D=\frac{\beta}{(1-\beta) \phi}\left[H+\left(1-\phi+\beta \phi D\left(z_{1}\right)\right) F_{0}+\left(1-D\left(z_{1}\right)\right) F_{1}\right] \\
+\frac{1}{s} \frac{1}{1-G}(r+s+\beta h)\left[G+\frac{\beta}{1-\beta}\left(\left(1-2 G\left(z_{1}\right)\right)(s+h)\left(F_{1}-F_{0}\right)\right]\left(H+F_{0}\right) .\right. \tag{B3}
\end{gather*}
$$

which is positive by virtue of the second-order conditions for an internal optimal solution being fulfilled.

## References

Bentolila, S. and Bertola, G. (1990), Firing costs and labour demand: How bad is Eurosclerosis? Review of Economic Studies, 57, 381-402.

Bertola, G. (1990), Job security, employment and wages. European Economic Review, 34, 851886.

Bertola, G. (1992), Labor turnover costs and average labor demand. Journal of Labor Economics, 10, 389-411.

Bertola, G. and Caballero, R. J. (1994), Cross-sectional efficiency and labour hoarding in a matching model of unemployment. Review of Economic Studies, 61, 435-456.

Hosios, A. On the efficiency of matching and related models of search and unemployment. Review of Economic Studies, 57, 279-298.

Lazear, E. P. (1990), Job security provisions and employment. Quarterly Journal of Economics, 105, 699-726.

Mortensen, D. T. and Pissarides, C. (1994), Job creation and destruction in the theory of unemployment. Review of Economic Studies, 61, 397-415.

Pissarides, C. (1985), Short-run equilibrium dynamics of unemployment, vacancies and real wages. American Economic Review, 75, 676-690.

Pissarides, C. (1987), Search, wage bargains, and cycles. Review of Economic Studies, 54, 473483.

Pissarides, C. (1990), Equilibrium unemployment theory. London: Basil Blackwell.
Pissarides. C. (1994), Search unemployment with on-the-job search. Review of Economic Studies, 61, 457-475.

Saint-Paul, G. (1995), The high unemployment trap. Qaurterly Journal of Economics, 110, 527550.

Strand, J. (1987), Unemployment as a discipline device with heterogeneous labor. American Economic Review, 77, 489-493.

Strand, J. (1997), Wage bargaining with heterogeneous labor and turnover costs: Perfect history screening. Working paper, Department of Economics, University of Oslo.


[^0]:    ${ }^{1}$ This paper is part of the research project "Employment and the labor market" at SNF-Oslo.

[^1]:    ${ }^{2}$ A trivial extension would be to assume a given rental cost of capital per job, as in Pissarides (1990), with no consequences for the main conclusions in the following.
    ${ }^{3} \mathrm{H}$ may also include any recruiting costs expended by the firm. Note that we assume that no part of H is paid by the worker, e.g. due to limitations on workers' assets or access to credit markets, by legal restrictions on such worker payments, or by an assumption that H must be incurred before the worker is actually engaged by the firm.
    ${ }^{4}$ The assumption that workers' productivities are discovered immediately can easily be relaxed without any of the main results being altered. One alternative would be to assume that workers have to go through a traineeship or test period, whose length is stochastic and exponentially distributed.
    ${ }^{5}$ We will demonstrate below that the profitability of employing workers in our model always increases strictly in $z$, for all firms.

[^2]:    ${ }^{6}$ This requires that $\mathrm{z}-\mathrm{w}(\mathrm{z})$ be strictly increasing in z , which holds in all cases studied below.

[^3]:    ${ }^{7}$ A transfer component of $\mathrm{F}_{0}$ may represent salary to the worker in an initial test period over which the worker's productivity is discovered, cf. also footnote 3 above.

[^4]:    ${ }^{8}$ We here assume that such a condition can be fulfilled for $\mathrm{h} \in(0, \infty)$ and with $\mathrm{z}_{1}$ given from (15) below. This implies that H and the $\mathrm{F}_{\mathrm{i}}$ are all not in excess of certain levels. In the opposite case no firms could profitably enter the market, and there would be no employment.
    ${ }^{9}$ The viability of such a solution requires an assumption that an operating firm is in principle allowed to make zero wage payments to workers and still escape the final redundancy payment, provided that the worker actually quits; and is the Stackelberg leader in the game initiated by a zero wage payment by the firm. Then the worker quitting option will define the default utility of the firm, relevant in the bargaining solution described below.

[^5]:    ${ }^{10} \mathrm{~A}$ consequence of this is that some workers with z in some range below but close to $\mathrm{z}_{1}$ will have an incentive to make an up-front payment to the firm upon joining, in order for the firm not to fire them immediately. We here rule out such up-front payments.

[^6]:    ${ }^{11}$ As opposed to in e.g. Hosios (1990) and Pissarides (1990), such a maximization is meaningful here even when there is positive discounting $(r>0)$, since there is instantaneous matching of firms. This implies that we can in principle view the market solution as resulting from the optimal stock of workers being hired at a given instant of time. (25) expresses the social value of such hirings provided that employment is kept at a constant level over time, after the initial hirings.

[^7]:    ${ }^{12}$ The issue of implementation of efficient solutions in the market, starting from a nonefficient solution, is however more complex than the discussion here indicates, since it may then be necessary to also describe the optimal paths to a new stationary equilibrium. This will not be discussed further in the following.

[^8]:    ${ }^{13}$ The basic property that efficiency requires the efficient determination of other variables than the share parameter $\beta$, is common with other recent related work which takes on a more complicated structure than the basic Pissarides model, e.g. Bertola and Caballero (1994).

