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**Working Paper**

## Variable prizes in forced-distribution-systems: A sabotage-reducing approach?

Tübinger Diskussionsbeiträge, No. 336

**Provided in Cooperation with:**

University of Tuebingen, Faculty of Economics and Social Sciences, School of Business and Economics

*Suggested Citation:* Glökler, Thomas (2014) : Variable prizes in forced-distribution-systems: A sabotage-reducing approach?, Tübinger Diskussionsbeiträge, No. 336, Eberhard Karls Universität Tübingen, Wirtschaftswissenschaftliche Fakultät, Tübingen, <https://nbn-resolving.de/urn:nbn:de:bsz:21-opus-72371>

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<https://hdl.handle.net/10419/94199>

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**Variable Prizes in Forced-Distribution-Systems:  
A Sabotage-Reducing Approach?**

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Tübinger Diskussionsbeitrag Nr. 336  
Februar 2014

Wirtschafts- und Sozialwissenschaftliche Fakultät



# Variable Prizes in Forced-Distribution-Systems:

A sabotage-reducing approach?

Thomas Glökler\*

2014

## Abstract

Forced-Distribution-Systems (FDS) have many indisputable benefits (such as identification of high potential and low performers or incentive effects to exert higher efforts). However, many companies take a critical stance toward FDS, one of the main reasons being the agents' incentive to execute sabotage activities.

While a large number of tournament studies deal with the problem of sabotage, to be best of my knowledge none of the studies investigates the impact of variable tournament prizes on sabotage activities.

Variable prizes are a special tournament design where prizes are not fixed in advance, but are a function of a target variable set by the principal (see GÜTH et al. 2010).

In this study, I theoretically analyze if variable tournament prizes can help in reducing sabotage activities in FDS. Two versions of variable prizes are considered for this study: variable prize levels and variable prize distributions. In the former version, prize levels depend on the cumulative output (higher the output, higher the prize levels), and in the latter version, prize distribution depends on the cumulative output (higher the output, higher the portion of prizes for the winner and lower the portion of prizes for the loser).

The findings of the model are as follows: Variable tournament prizes not only reduce sabotage activities effectively, but also incentivize agents to exert helping activities. Accordingly, variable tournament prizes could be of high importance in organizational practice.

## 1 Introduction

In recent years, no other aspect of performance management has attracted more public attention than Forced-Distribution-Systems (FDS) (see DOMINICK 2009). FDS are a special form of performance appraisal in which the judge is bound to a

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predetermined distribution of single performance categories (see SCHLEICHER/BULL/GREEN 2009). Proponents of FDS particularly argue with the existence of selection as well as incentive functions: forced distribution would prevent lenient appraisals, leading to better identification and selection of low performers as well as high potential (see SCHLEICHER/BULL/GREEN 2009). In addition, FDS promotes a culture of excellence that especially motivates the top performers (see SCULLEN/BERGEY/AIMAN-SMITH 2005).

In spite of these arguments, many firms have recently abandoned their FDS (see MACLENNAN 2007). One of the main reasons for this phenomena appears to be that FDS cause rivalry among team members and colleagues for better appraisals, thus generating a culture of “back-biting” (ALSEVER 2007), which favors unethical behavior (e.g., sabotage and mobbing) and prevents cooperation among team members (see HARBRING/IRLENBUSCH 2008). Thus, the prospect of destruction of teamwork and cooperation seems to outweigh the positive effects of FDS.

A number of studies deal with sabotage activities in tournaments and contests. LAZEAR (1989) was the first such study to address this problem. One of his findings was to reduce the spread in rewards between winning and losing, which, in turn, would reduce the effort level since both effort and sabotage increase with the prize spread (see VANDEGRIFT/YAVAS 2010).

This study is based on the model of HARBRING/IRLENBUSCH (2008), where the authors investigate theoretically and empirically the influence of the number of agents as well as the fraction of prizes for the winner on effort and sabotage activities in tournaments.

While HARBRING/IRLENBUSCH (2008), as well as a vast majority of tournament studies, focus on tournaments with ex-ante determined fixed prizes, prizes in variable-prize tournaments depend on a command variable of the principal (e.g., profit) (see GÜTH et al. 2010). This aspect was first introduced by GÜTH et al. (2010), who showed that variable-prize tournaments outperform fixed-prize tournaments and piece rates with regard to cost-effectiveness. However, they neglect the possibility of the agents to execute sabotage activities. This study investigates theoretically whether variable-prize tournaments are able to reduce or destroy sabotage incentives in FDS.

Two versions of variable prizes are considered: variable prize levels and variable prize distributions. In the former version, prize levels depend on the cumulative output (higher the output, higher the prize levels), and in the latter version, prize distribution depends on the cumulative output (higher the output, higher the portion of prizes for the winner and lower the portion of prizes for the loser).

Both versions differ not only with regard to the dependent variable: in the former version (variable prize levels), all agents can obtain the same prize. With an appropriate high (low) accumulated output, all agents obtain the winner’s prize (loser’s prize). In the latter version (variable prize distribution), there is always a differentiation between winners and losers (regardless of the cumulative output). Hence, the team component and the tournament component have a higher emphasis in the former and the latter versions, respectively. This could

lead to differences between both the versions with respect to effort and sabotage activities.

As a result, the model shows that there are no differences between both the versions with respect to effort and sabotage activities. Even more importantly, variable tournament prizes, in contrast to fixed tournament prizes, not only reduce sabotage activities effectively, but also incentivize agents to exert helping activities. Accordingly, variable tournament prizes could be of high importance in organizational practice.

The remainder of this paper is organized as follows: In section 2, three different tournament designs are compared with respect to the agents' incentive to execute sabotage activities. First, we analyze a standard fixed-prize tournament. Second, we investigate two different forms of variable-prize tournaments: variable prize-distribution and variable prize-levels. Section 3 concludes.

## 2 The model

This section begins with the analysis of a fixed-prize tournament. This serves as a benchmark to compare the results of variable tournament prizes. In the second step, we investigate how efforts, sabotage activities, and utility of the principal change, if fixed prizes are replaced by variable prizes. Two versions of variable prizes are considered: variable prize levels and variable prize distributions. In the former version, prize levels depend on the cumulative output (higher the output, higher the prize levels). In the latter version, prize distribution depends on the cumulative output (higher the output, higher the portion of prizes for the winner and lower the portion of prizes for the loser).

### 2.1 Model assumptions

$n$  homogenous and risk-neutral agents compete in an FDS ( $n > 2$ ). The assumption of homogenous and risk-neutral agents is used in numerous tournament models (see e.g. LAZEAR/ROSEN 1981; AKERLOF/HOLDEN 2012; HARBRING/IRLENBUSCH 2008). This assumption (as well as the following assumptions regarding cost functions and random variable) has no influence on the fundamental model conclusions, since in this study different tournament designs are compared by keeping these assumptions constant. In organizational practice, FDS consist of at least three ranking categories (see WELCH/WELCH 2005; GROTE 2005). Hence, in this model, we assume that there are three prize levels:  $W_1$ ,  $W_2$ , and  $W_3$  ( $W_1, W_2 \geq 0, W_3 = 0$ ). Each agent is attributed to one prize level, depending on the realized outputs: The agents with the highest outputs (a) obtain  $W_1$ , the agents with the lowest outputs (d) obtain  $W_3$ , and all other agents (b) obtain  $W_2$  (a, b, d  $> 0$  and a + b + d = n). Agent  $i$ 's output  $q_i$  is defined as follows (in line with HARBRING/IRLENBUSCH (2008) or HARBRING/IRLENBUSCH (2005)):  $q_i = e_i - \sum_{-i} s_{-i} + \varepsilon_i$ . Agent  $i$ 's output depends on his effort level  $e_i$ , the sum of the sabotage activities carried out by the other agents  $s_{-i}$ , and the exogenous given random variable  $\varepsilon_i$  ( $\varepsilon_i$  is expected to be identically and

independently distributed (i.i.d) on the interval  $[-\bar{\varepsilon}; \bar{\varepsilon}]$  (see MÜNSTER 2007; ORRISON/SCHOTTER/WEIGELT 2004). As with GÜRTLER/MÜNSTER (2010) and HARBRING/IRLENBUSCH (2011), the agents' cost for effort and sabotage activities are  $c(e_i) = \frac{e_i^2}{c_e}$  and  $c(s_i) = \frac{s_i^2}{c_s}$ , respectively. Furthermore, it is assumed that  $c_e = c_s = c$  even though a predominant part of the existing literature assumes that  $c_e > c_s$  (see GARICANO/PALACIOS-HUERTA 2005). However, the aim of this analysis is to compare fixed and variable prizes in FDS with respect to their effect on effort and sabotage activities. This is why it is assumed that sabotage activities are relatively cheap, unlike in other models.

Agent  $i$ 's expected utility is expressed as follows:

$$E(U_i) = P_1W_1 + P_2W_2 + P_3W_3 - c(e_i) - c(s_i). \quad (1)$$

$P_1$ ,  $P_2$ , and  $P_3$  are the probabilities to obtain the prize levels  $W_1$ ,  $W_2$ , and  $W_3$ , where  $P_1 + P_2 + P_3 = 1$ .

## 2.2 Benchmark: Fixed-prize FDS

Agent  $i$ 's expected utility was already defined in equation 1 as follows:

$$\begin{aligned} E(U_i) &= P_1W_1 + P_2W_2 + P_3W_3 - c(e_i) - c(s_i) \\ &= P_1(W_1 - W_2) + P_3(W_3 - W_2) + W_2 - \frac{e_i^2}{c} - \frac{s_i^2}{c}. \end{aligned}$$

Agent  $i$  maximizes his expected utility by choosing optimal levels of  $e_i$  and  $s_i$ :

$$\begin{aligned} \frac{\partial E(U_i)}{\partial e_i} = 0 &\Leftrightarrow \frac{\partial P_1}{\partial e_i}(W_1 - W_2) + \frac{\partial P_3}{\partial e_i}(W_3 - W_2) - \frac{2}{c}e_i = 0 \\ \frac{\partial E(U_i)}{\partial s_i} = 0 &\Leftrightarrow \frac{\partial P_1}{\partial s_i}(W_1 - W_2) + \frac{\partial P_3}{\partial s_i}(W_3 - W_2) - \frac{2}{c}s_i = 0. \end{aligned}$$

Given the i.i.d assumption of the random component, at a pure symmetric Nash equilibrium, the marginal probabilities of winning are constant and depend only on the size of the interval from which the random components are drawn (see HARBRING/IRLENBUSCH (2005) and ORRISON/SCHOTTER/WEIGELT (2004)):

$$\frac{\partial P_1}{\partial e_i} = \frac{1}{2\bar{\varepsilon}}.$$

$P_1$  is agent  $i$ 's probability to be one of the agents with the highest outputs (a).  $P_3$  is agent  $i$ 's probability to be one of the agents with the lowest outputs (d). If  $P_1$  increases because of a marginal increase in  $e_i$  by  $\frac{1}{2\bar{\varepsilon}}$ , c.p.  $P_3$  decreases because of the marginal increase in  $e_i$  by  $-\frac{1}{2\bar{\varepsilon}}$ :

$$\frac{\partial P_3}{\partial e_i} = -\frac{1}{2\bar{\varepsilon}}.$$

The same applies to the marginal winning probabilities if the sabotage activities are increased:  $\frac{\partial P_1}{\partial s_i} = \frac{1}{2\bar{\varepsilon}}$  and  $\frac{\partial P_3}{\partial s_i} = -\frac{1}{2\bar{\varepsilon}}$ .

That leads to

$$\begin{aligned} e^* &= \frac{c}{4\bar{\varepsilon}} (W_1 - W_3) \\ s^* &= \frac{c}{4\bar{\varepsilon}} (W_1 - W_3). \end{aligned}$$

The expected principal utility is given by

$$\begin{aligned} E(U_P) &= V \left( \sum_{i=1}^n e_i - s_{-i} \right) - aW_1 - bW_2 \\ &= Vn(e^* - (n-1)s^*) - aW_1 - bW_2. \end{aligned}$$

The expected principal utility for each agent is

$$E \left( \frac{U_P}{n} \right) = V(e^* - (n-1)s^*) - \left( \frac{aW_1 + bW_2}{n} \right). \quad (2)$$

Agent  $i$ 's expected utility is given by

$$E(U_i) = P_1W_1 + P_2W_2 - c(e_i) - c(s_i).$$

The principal has to make sure that  $E(U_i) \geq 0$  (participation constraint of the agents). From this, it follows that

$$P_1W_1 + P_2W_2 = c(e_i) + c(s_i).$$

Since  $P_1 = \frac{a}{n}$  and  $P_2 = \frac{b}{n}$ , it follows that

$$\left( \frac{aW_1 + bW_2}{n} \right) = c(e_i) + c(s_i). \quad (3)$$

By inserting equation 3 in equation 2, it follows that

$$E \left( \frac{U_P}{n} \right) = V(e^* - (n-1)s^*) - c(e_i) - c(s_i). \quad (4)$$

The principal chooses  $W_1$  to maximize equation 4

$$\frac{\partial E \left( \frac{U_P}{n} \right)}{\partial W_1} = 0 \Leftrightarrow V \left( \frac{\partial e^*}{\partial W_1} - (n-1) \frac{\partial s^*}{\partial W_1} \right) - c'(e_i) \frac{\partial e^*}{\partial W_1} - c'(s_i) \frac{\partial s^*}{\partial W_1} = 0.$$

Since  $\frac{\partial e^*}{\partial W_1} = \frac{c}{4\bar{\varepsilon}}$  and  $\frac{\partial s^*}{\partial W_1} = \frac{c}{4\bar{\varepsilon}}$ , it follows that

$$V \left( \frac{c}{4\bar{\varepsilon}} - (n-1) \frac{c}{4\bar{\varepsilon}} \right) - c'(e_i) \frac{c}{4\bar{\varepsilon}} - c'(s_i) \frac{c}{4\bar{\varepsilon}} = 0$$

$$\Leftrightarrow (2-n) - c'(s_i) = c'(e_i).$$

Since  $c'(e_i) = \frac{1}{2\bar{\varepsilon}}(W_1 - W_3)$  and  $c'(s_i) = \frac{1}{2\bar{\varepsilon}}(W_1 - W_3)$ , it follows that

$$V(2-n) - \frac{1}{2\bar{\varepsilon}}(W_1 - W_3) = \frac{1}{2\bar{\varepsilon}}(W_1 - W_3)$$

$$V(2-n) = \frac{1}{\bar{\varepsilon}}(W_1 - W_3).$$

The utility maximizing prize spread is

$$(W_1 - W_3) = \bar{\varepsilon}V(2-n).$$

From  $W_3 = 0$ , it follows that  $W_1 = \bar{\varepsilon}V(2-n)$ .

**Proposition 1:** Since  $n > 2$ , it follows that  $W_1 < 0$ . Since according to our assumptions,  $W_1 \geq 0$ , it follows that  $W_1^* = 0$ . That leads to  $W_1^* = W_2^* = W_3^* = 0$ ,  $e^* = 0$ ,  $s^* = 0$ , and  $EU_P = 0$ .

A fixed-prize FDS incentivizes agents to exert effort as well as sabotage activities. As a result, FDS is not worthwhile for the principal.

This analysis supports the critics of FDS. Although the agents are incentivized to exert positive effort levels, they also have strong incentives to sabotage each other.

### 2.3 Version 1: FDS with variable prize distributions

In this section, the number of winner resp. loser prizes (a, b, d) depends on the cumulative output of all agents.

$$\begin{aligned} E(U_i) &= P_1W_1 + P_2W_2 + P_3W_3 - c(e_i) - c(s_i) \\ &= P_1(W_1 - W_2) + P_3(W_3 - W_2) + W_2 - \frac{e_i^2}{c} - \frac{s_i^2}{c}. \end{aligned}$$

The probability functions for each prize level are assumed to be

$$\begin{aligned} P_1 &= \frac{a}{n}, \text{ where} \\ a &= \begin{cases} 0 & \text{für } \sum_{i=1}^n q_i \leq z \Rightarrow P_1 = 0 \\ n - b & \text{für } \sum_{i=1}^n q_i > z \Rightarrow n - n \left( 2 - \frac{\sum_{i=1}^n q_i}{z} \right) \Rightarrow P_1 = \left( \frac{\sum_{i=1}^n q_i - z}{z} \right) \end{cases} \\ P_2 &= \frac{b}{n}, \text{ where} \\ b &= \begin{cases} \frac{n \sum_{i=1}^n q_i}{z} & \text{für } \sum_{i=1}^n q_i \leq z \Rightarrow P_2 = \frac{\sum_{i=1}^n q_i}{z} \\ n \left( 2 - \frac{\sum_{i=1}^n q_i}{z} \right) & \text{für } \sum_{i=1}^n q_i > z \Rightarrow P_2 = \left( 2 - \frac{\sum_{i=1}^n q_i}{z} \right) \end{cases} \\ P_3 &= \frac{d}{n}, \text{ where} \\ d &= \begin{cases} n - b & \text{für } \sum_{i=1}^n q_i \leq z \Rightarrow n - \frac{n \sum_{i=1}^n q_i}{z} \Rightarrow P_3 = \left( \frac{z - \sum_{i=1}^n q_i}{z} \right) \\ 0 & \text{für } \sum_{i=1}^n q_i > z \Rightarrow P_3 = 0. \end{cases} \end{aligned}$$



The variable  $z$  can be interpreted as the “yardstick” of cumulative output.

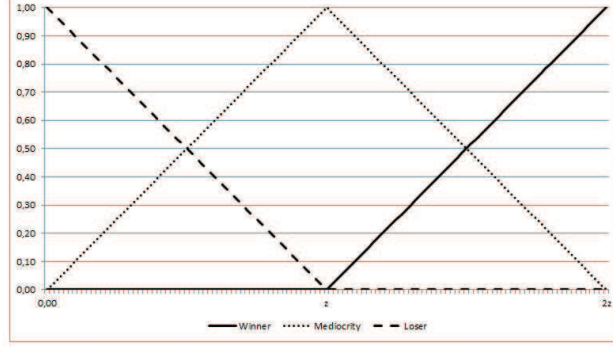


Figure 1: Relation between prize distribution and cumulative output

Cumulative output values are entered on the x-axis and the percentage of the prizes on the y-axis. If  $\sum_{i=1}^n q_i = 0$ , all agents obtain the loser’s prize  $W_3$  ( $d=1$ ). If  $\sum_{i=1}^n q_i$  achieves the yardstick  $z$ , all agents obtain  $W_2$  ( $b=1$ ). If  $\sum_{i=1}^n q_i = 2z$ , all agents obtain the winner’s prize  $W_1$  ( $a=1$ ). If the cumulative output is for instance  $0,5z$ , the best 50% of all agents receive  $W_2$ , and all other agents  $W_3$ . In contrast, if the cumulative output is  $1,3z$ , the best 30% of all agents obtain  $W_1$ , and all other agents  $W_2$ .

In the following, effort, sabotage levels, and principal utility are determined for  $q_i > z$ . Identical results are obtained for  $q_i \leq z$  (see appendix A.1).

$$\begin{aligned} E(U_i) &= \left( \frac{\sum_{i=1}^n q_i}{z} - 1 \right) (W_1 - W_2) + W_2 - \frac{e_i^2}{c} - \frac{s_i^2}{c} \\ &= \left( \frac{n(e^* - (n-1)s^*)}{z} - 1 \right) (W_1 - W_2) + W_2 - \frac{e_i^2}{c} - \frac{s_i^2}{c}. \end{aligned} \quad (5)$$

In order to determine the optimal effort and sabotage levels, equation 5 is derived with respect to  $e_i$  and  $s_i$ :

$$\begin{aligned} \frac{\partial E(U_i)}{\partial e_i} &= \frac{n}{z} (W_1 - W_2) - \frac{2e^*}{c} = 0 \\ &\Rightarrow e^* = \frac{nc}{2z} (W_1 - W_2) \\ \frac{\partial E(U_i)}{\partial s_i} &= \left( -\frac{n(n-1)}{z} \right) (W_1 - W_2) - \frac{2s^*}{c} = 0 \\ &\Rightarrow s^* = -\frac{nc(n-1)}{2z} (W_1 - W_2). \end{aligned}$$

The principal anticipates the agents’ choice and determines the optimal prize levels  $W_1$ ,  $W_2$ , and  $W_3$ :

$$E(U_P) = Vn(e^* - (n-1)s^*) - aW_1 - bW_2 - dW_3.$$

The principal utility per agent is

$$\begin{aligned} E\left(\frac{U_P}{n}\right) &= V(e^* - (n-1)s^*) - \frac{a}{n}W_1 - \frac{b}{n}W_2 - \frac{d}{n}W_3 \\ &= V(e^* - (n-1)s^*) - P_1W_1 - P_2W_2 - P_3W_3. \end{aligned} \quad (6)$$

By  $E(U_i) \geq 0$ , the principal makes sure that the agents' participation constraint is fulfilled:

$$\begin{aligned} E(U_i) \geq 0 &\Rightarrow P_1W_1 + P_2W_2 + P_3W_3 - c(e_i) - c(s_i) \geq 0 \\ &\Rightarrow P_1W_1 + P_2W_2 + P_3W_3 \geq c(e_i) + c(s_i). \end{aligned}$$

By inserting the participation constraint in equation 6, we obtain

$$E\left(\frac{U_P}{n}\right) = V(e^* - (n-1)s^*) - c(e_i) - c(s_i). \quad (7)$$

Equation 7 is derived with respect to  $W_1$ - $W_2$ :

$$\begin{aligned} \frac{\partial E(U_P)}{\partial (W_1 - W_2)} &= V\left(\frac{\partial e^*}{\partial (W_1 - W_2)} - (n-1)\frac{\partial s^*}{\partial (W_1 - W_2)}\right) \\ &\quad - c'(e_i)\frac{\partial e^*}{\partial (W_1 - W_2)} - c'(s_i)\frac{\partial s^*}{\partial (W_1 - W_2)} = 0. \end{aligned}$$

Since  $\frac{\partial s^*}{\partial (W_1 - W_2)} = -\frac{nc(n-1)}{2z}$ ,  $\frac{\partial e^*}{\partial (W_1 - W_2)} = \frac{nc}{2z}$ ,  $c'(e_i) = \frac{2}{c}e^* = \frac{n}{z}[W_1 - W_2]$ , and  $c'(s_i) = \frac{2}{c}s^* = -\frac{n(n-1)}{z}(W_1 - W_2)$ :

$$\begin{aligned} \frac{Vnc}{2z}\left(1 + (n-1)^2\right) - (W_1 - W_2)\frac{n^2c\left(1 + (n-1)^2\right)}{2z^2} &= 0 \\ \Leftrightarrow (W_1 - W_2) &= \frac{Vz}{n}. \end{aligned} \quad (8)$$

**Proposition 2:** From equation 8, it follows that  $e^* = \frac{Vc}{2}$  and  $s^* = -\frac{Vc(n-1)}{2}$ . From the assumptions  $V, c > 0$  and  $n > 2$ , it follows that  $e^* > 0$  and  $s^* < 0$ . This result shows a positive effort level and a negative sabotage level ("helpfulness") of the agents.

**Proposition 3:** The principal utility is as follows<sup>1</sup>:

<sup>1</sup>It seems strange that the principal utility increases exponentially with a growth in the number of agents ( $n$ ). However, this effect also appears in other tournament models (see appendix A.2)

$$\begin{aligned}
E\left(\frac{U_P}{n}\right) &= V(e^* - (n-1)s^*) - c(e_i) - c(s_i) \\
&= V\left(\frac{Vc}{2} + \frac{Vc(n-1)^2}{2}\right) - \frac{1}{c}e^2 - \frac{1}{c}s^2 \\
E(U_P) &= \frac{V^2nc(n^2 - 2n + 2)}{4}.
\end{aligned}$$

From the assumptions  $V, c > 0$  and  $n > 2$ , it follows that  $E(U_P) > 0$ .

## 2.4 Version 2: FDS with variable prize levels

The dependencies between cumulative output and the single prize levels are assumed to be as follows:

$$\begin{aligned}
W_1 &= \alpha \sum_{i=1}^n q_i, \\
W_2 &= \beta \sum_{i=1}^n q_i, \\
W_3 &= 0,
\end{aligned}$$

where  $\alpha, \beta = 0$ .

The expected utility of agent  $i$  is obtained by substituting these expressions in equation 1:

$$\begin{aligned}
E(U_i) &= P_1 \left( \alpha \sum_{i=1}^n q_i \right) + P_2 \left( \beta \sum_{i=1}^n q_i \right) - \frac{e_i^2}{c} - \frac{s_i^2}{c} \\
&= P_1 \left( \alpha \sum_{i=1}^n e_i + \varepsilon_i - s_{-i} \right) + P_2 \left( \beta \sum_{i=1}^n e_i + \varepsilon_i - s_{-i} \right) - \frac{e_i^2}{c} - \frac{s_i^2}{c} \\
&= (P_1\alpha + P_2\beta) \left( \sum_{i=1}^n e_i + \varepsilon_i - s_{-i} \right) - \frac{e_i^2}{c} - \frac{s_i^2}{c}.
\end{aligned}$$

Assumably,  $E(\varepsilon_i) = 0$ . In a symmetric Nash equilibrium, all  $n$  agents choose the same levels of  $e^*$  and  $s^*$ .

This leads to  $\sum_{i=1}^n (e_i + \varepsilon_i - s_{-i}) = n(e^* - (n-1)s^*)$

$$\begin{aligned}
E(U_i) &= n(e^* - (n-1)s^*) (P_1\alpha + P_2\beta) - \frac{e_i^2}{c} - \frac{s_i^2}{c} \\
&= n(e^* - (n-1)s^*) (P_1(\alpha - \beta) + (1 - P_3)\beta) - \frac{e_i^2}{c} - \frac{s_i^2}{c}.
\end{aligned}$$

Agent i maximizes his expected utility by choosing  $e^*$  and  $s^*$ :

$$\begin{aligned}\frac{\partial E(U_i)}{\partial e_i} &= n [P_1 (\alpha - \beta) + (1 - P_3) \beta] \\ &\quad + n(e^* - (n-1)s^*) \left[ \frac{1}{2\bar{\varepsilon}} (\alpha - \beta) + \frac{1}{2\bar{\varepsilon}} \beta \right] - \frac{2}{c} e^* = 0 \\ \Rightarrow e^* &= \frac{c [2(a\alpha + \beta b) - n(n-1)s^* \alpha]}{4\bar{\varepsilon} - n c \alpha}\end{aligned}$$

$$\begin{aligned}\frac{\partial E(U_i)}{\partial s_i} &= -(n-1) [P_1 (\alpha - \beta) + (1 - P_3) \beta] \\ &\quad + n(e^* - (n-1)s^*) \left[ \frac{1}{2\bar{\varepsilon}} (\alpha - \beta) + \frac{1}{2\bar{\varepsilon}} \beta \right] - \frac{2}{c} s^* = 0 \\ \Rightarrow s^* &= \frac{c [2\bar{\varepsilon}(\frac{1-n}{n}) [a\alpha - a\beta + a\beta + b\beta] + n e^* \alpha]}{4\bar{\varepsilon} + c n (n-1) \alpha}.\end{aligned}$$

By substituting  $s^*$  into  $e^*$  we obtain

$$\begin{aligned}e^* &= \frac{c \left[ 2\bar{\varepsilon} (a\alpha + \beta b) - n(n-1) \left[ \frac{c [2\bar{\varepsilon}(\frac{1-n}{n}) [a\alpha + b\beta] + n e^* \alpha]}{4\bar{\varepsilon} + c n (n-1) \alpha} \right] \alpha \right]}{4\bar{\varepsilon} - n c \alpha} \\ \Rightarrow e^* &= \frac{c (a\alpha + \beta b) [4\bar{\varepsilon} + 2c\alpha (n-1) (n - \frac{1}{2})]}{8\bar{\varepsilon} + 2n c \alpha (n-2)}.\end{aligned}\quad (9)$$

By substituting  $e^*$  into  $s^*$  we obtain

$$\begin{aligned}s^* &= \frac{c \left[ 2\bar{\varepsilon}(\frac{1-n}{n}) [a\alpha + b\beta] + n \alpha \left[ \frac{c [2\bar{\varepsilon} (a\alpha + \beta b) - n(n-1)s^* \alpha]}{4\bar{\varepsilon} - n c \alpha} \right] \right]}{4\bar{\varepsilon} + c n (n-1) \alpha} \\ \Rightarrow s^* &= \frac{c [a\alpha + b\beta] [4\bar{\varepsilon} (1-n) + 2 (n - \frac{1}{2}) n c \alpha]}{2n [4\bar{\varepsilon} + n c \alpha (n-2)]}.\end{aligned}\quad (10)$$

The principal anticipates  $e^*$  and  $s^*$ , and maximizes his utility by choosing the utility maximizing  $\alpha$  and  $\beta$ :

$$\begin{aligned}E\left(\frac{U_P}{n}\right) &= [V - a\alpha - b\beta] (e^* - (n-1)s^*) \\ &= V (e^* - (n-1)s^*) - [(a\alpha + b\beta) (e^* - (n-1)s^*)].\end{aligned}\quad (11)$$

The utility of agent i is as follows:

$$\begin{aligned}E(U_i) &= n (e^* - (n-1)s^*) [P_1 (\alpha - \beta) + (1 - P_3) \beta] - c(e) - c(s) \\ &= (e^* - (n-1)s^*) [a\alpha + b\beta] - c(e) - c(s).\end{aligned}$$

By  $E(U_i) \geq 0$ , the principal makes sure that the agents' participation constraint is fulfilled:

$$\begin{aligned} E(U_i) = 0 &\Leftrightarrow (e^* - (n-1)s^*)[a\alpha + b\beta] - c(e) - c(s) = 0 \\ (e^* - (n-1)s^*)[a\alpha + b\beta] &= c(e) + c(s). \end{aligned} \quad (12)$$

By substituting equation 12 into equation 11, it follows that

$$E\left(\frac{U_P}{n}\right) = V(e^* - (n-1)s^*) - c(e) - c(s).$$

After inserting  $e^*$  and  $s^*$ , we obtain

$$\begin{aligned} E\left(\frac{U_P}{n}\right) &= V\left(\frac{c(a\alpha + \beta b)[4\bar{\varepsilon} + 2c\alpha(n-1)(n - \frac{1}{2})]}{8\bar{\varepsilon} + 2nc\alpha(n-2)}\right. \\ &\quad \left. - (n-1)\frac{c[a\alpha + b\beta][4\bar{\varepsilon}(1-n) + 2(n - \frac{1}{2})nc\alpha]}{2n[4\bar{\varepsilon} + nc\alpha(n-2)]}\right) \\ &\quad - c(e) - c(s). \end{aligned}$$

Since  $(e_i) = \frac{e_i^2}{c_e}$  and  $c(s_i) = \frac{s_i^2}{c_s}$ , it follows that

$$\begin{aligned} E\left(\frac{U_P}{n}\right) &= V\left(\frac{c(a\alpha + \beta b)[4\bar{\varepsilon} + 2c\alpha(n-1)(n - \frac{1}{2})]}{8\bar{\varepsilon} + 2nc\alpha(n-2)}\right. \\ &\quad \left. - (n-1)\frac{c[a\alpha + b\beta][4\bar{\varepsilon}(1-n) + 2(n - \frac{1}{2})nc\alpha]}{2n[4\bar{\varepsilon} + nc\alpha(n-2)]}\right) \\ &\quad - \frac{1}{c}\left(\frac{c(a\alpha + \beta b)[4\bar{\varepsilon} + 2c\alpha(n-1)(n - \frac{1}{2})]}{8\bar{\varepsilon} + 2nc\alpha(n-2)}\right)^2 \\ &\quad - \frac{1}{c}\left(\frac{c[a\alpha + b\beta][4\bar{\varepsilon}(1-n) + 2(n - \frac{1}{2})nc\alpha]}{2n[4\bar{\varepsilon} + nc\alpha(n-2)]}\right)^2 \\ &\quad - \left(\frac{c(a\alpha + \beta b)^2 n^2 [4\bar{\varepsilon} + 2c\alpha(n-1)(n - \frac{1}{2})]^2}{4n^2 (4\bar{\varepsilon} + nc\alpha(n-2))^2}\right) \\ &\quad - \left(\frac{c[a\alpha + b\beta]^2 [4\bar{\varepsilon}(1-n) + 2(n - \frac{1}{2})nc\alpha]^2}{4n^2 (4\bar{\varepsilon} + nc\alpha(n-2))^2}\right) \\ &= \frac{2V\bar{\varepsilon}c(n^2 - n + 1)(a\alpha + \beta b)}{n(4\bar{\varepsilon} + nc\alpha(n-2))} \\ &\quad - \frac{c(a\alpha + \beta b)^2}{4n^2(4\bar{\varepsilon} + nc\alpha(n-2))^2} \left(16\bar{\varepsilon}^2(2n^2 - 2n + 1)\right. \\ &\quad \left. + 8n\bar{\varepsilon}c\alpha(2n^3 - 5n^2 + 4n - 1)\right. \\ &\quad \left. + n^2 c^2 \alpha^2 (4n^4 - 12n^3 + 17n^2 - 10n + 2)\right). \end{aligned}$$

This expression is derived with respect to  $\alpha$  and  $\beta$ :

$$\begin{aligned} \frac{\partial E\left(\frac{U_P}{n}\right)}{\partial \alpha} &= 0 \\ \Leftrightarrow \frac{2V\bar{\epsilon}c(n^2 - n + 1)an(4\bar{\epsilon} + n\alpha(n - 2)) - 2V\bar{\epsilon}c(n^2 - n + 1)(a\alpha + \beta b)n(nc(n - 2))}{n^2(4\bar{\epsilon} + n\alpha(n - 2))^2} \\ &- \frac{4n^2(4\bar{\epsilon} + n\alpha(n - 2))^2}{16n^4(4\bar{\epsilon} + n\alpha(n - 2))^4}c \left[ (2a^2\alpha + 2a\beta b)(16\bar{\epsilon}^2(2n^2 - 2n + 1)) \right. \\ &+ 8n\bar{\epsilon}c\alpha(2n^3 - 5n^2 + 4n - 1) + n^2c^2\alpha^2(4n^4 - 12n^3 + 17n^2 - 10n + 2) \\ &\left. + (a\alpha + \beta b)^2(8n\bar{\epsilon}c(2n^3 - 5n^2 + 4n - 1) + 2n^2c^2\alpha(4n^4 - 12n^3 + 17n^2 - 10n + 2)) \right] \\ &+ \frac{4n^2(8\bar{\epsilon}nc(n - 2) + 2n^2c^2\alpha(n - 2)^2)c(a\alpha + \beta b)^2}{16n^4(4\bar{\epsilon} + n\alpha(n - 2))^4} \left( 16\bar{\epsilon}^2(2n^2 - 2n + 1) \right. \\ &\left. + 8n\bar{\epsilon}c\alpha(2n^3 - 5n^2 + 4n - 1) + n^2c^2\alpha^2(4n^4 - 12n^3 + 17n^2 - 10n + 2) \right) = 0. \end{aligned}$$

By solving for  $\alpha$ , we obtain  $\alpha = -\frac{4\bar{\epsilon}(n^2 - 2n + 1)}{nc(2n^3 - 5n^2 + 6n - 2)}$

$$\begin{aligned} \frac{\partial E\left(\frac{U_P}{n}\right)}{\partial \beta} &= 0 \\ \Leftrightarrow \frac{2V\bar{\epsilon}c(n^2 - n + 1)b}{n(4\bar{\epsilon} + n\alpha(n - 2))} - \frac{c(2a\alpha b + 2\beta b^2)}{4n^2(4\bar{\epsilon} + n\alpha(n - 2))^2} \left( 16\bar{\epsilon}^2(2n^2 - 2n + 1) \right. \\ &\left. + 8n\bar{\epsilon}c\alpha(2n^3 - 5n^2 + 4n - 1) + n^2c^2\alpha^2(4n^4 - 12n^3 + 17n^2 - 10n + 2) \right) = 0. \end{aligned}$$

By solving for  $\beta$ , it follows that  $\beta = \frac{Vn^3c(n^2 - 2n + 2) + 4a\bar{\epsilon}(n^2 - 2n + 1)}{ncb(n^2 - 2n + 2)(2n - 1)}$ .

By substituting  $\alpha$  and  $\beta$  in equation 9 and 10 we obtain

$$\begin{aligned} e^* &= \frac{c \left( a \left( -\frac{4\bar{\epsilon}(2n^3 - 5n^2 + 4n - 1)}{nc(4n^4 - 12n^3 + 17n^2 - 10n + 2)} \right) + \left( \frac{Vn^3c(n^2 - 2n + 2) + 4a\bar{\epsilon}(n^2 - 2n + 1)}{ncb(n^2 - 2n + 2)(2n - 1)} \right) b \right)}{8\bar{\epsilon} + 2nc \left( -\frac{4\bar{\epsilon}(2n^3 - 5n^2 + 4n - 1)}{nc(4n^4 - 12n^3 + 17n^2 - 10n + 2)} \right) (n - 2)} \\ &\quad * \frac{\left[ 4\bar{\epsilon} + 2c \left( -\frac{4\bar{\epsilon}(2n^3 - 5n^2 + 4n - 1)}{nc(4n^4 - 12n^3 + 17n^2 - 10n + 2)} \right) (n - 1) \left( n - \frac{1}{2} \right) \right]}{8\bar{\epsilon} + 2nc \left( -\frac{4\bar{\epsilon}(2n^3 - 5n^2 + 4n - 1)}{nc(4n^4 - 12n^3 + 17n^2 - 10n + 2)} \right) (n - 2)} \\ &= \frac{Vc}{2}. \end{aligned}$$

$$\begin{aligned}
s^* &= \frac{c \left[ a \left( -\frac{4\bar{\varepsilon}(2n^3-5n^2+4n-1)}{nc(4n^4-12n^3+17n^2-10n+2)} \right) + b \left( \frac{Vn^3c(n^2-2n+2)+4a\bar{\varepsilon}(n^2-2n+1)}{ncb(n^2-2n+2)(2n-1)} \right) \right]}{2n \left[ 4\bar{\varepsilon} + nc \left( -\frac{4\bar{\varepsilon}(2n^3-5n^2+4n-1)}{nc(4n^4-12n^3+17n^2-10n+2)} \right) (n-2) \right]} \\
&\quad * \frac{\left[ 4\bar{\varepsilon}(1-n) + 2 \left( n - \frac{1}{2} \right) nc \left( -\frac{4\bar{\varepsilon}(2n^3-5n^2+4n-1)}{nc(4n^4-12n^3+17n^2-10n+2)} \right) \right]}{2n \left[ 4\bar{\varepsilon} + nc \left( -\frac{4\bar{\varepsilon}(2n^3-5n^2+4n-1)}{nc(4n^4-12n^3+17n^2-10n+2)} \right) (n-2) \right]} \\
&= -\frac{Vc(n-1)}{2}.
\end{aligned}$$

**Proposition 4:** From the assumptions  $V, c > 0$  and  $n > 2$ , it follows that  $e^* > 0$  and  $s^* < 0$ . This result shows a positive effort level and a negative sabotage level (“helpfulness”) of the agents.

**Proposition 5:** The principal utility is as follows:

$$\begin{aligned}
E \left( \frac{U_P}{n} \right) &= V(e^* - (n-1)s^*) - c(e_i) - c(s_i) \\
&= V \left( \frac{Vc}{2} + \frac{Vc(n-1)^2}{2} \right) - \frac{1}{c}e^2 - \frac{1}{c}s^2 \\
E(U_P) &= \frac{V^2nc(n^2-2n+2)}{4}.
\end{aligned}$$

From the assumptions  $V, c > 0$  and  $n > 2$ , it follows that  $E(U_P) > 0$ .

Both versions 1 and 2 induce identical effort levels and helpfulness. The theoretical reason being identical principal utility in the consideration of the participation constraints:

$$E \left( \frac{U_P}{n} \right) = V(e - (n-1)s) - \frac{1}{c}e^2 - \frac{1}{c}s^2. \quad (13)$$

The principal utility maximizing  $e$  and  $s$  are as follows:

$$\begin{aligned}
\frac{\partial \left( \frac{EU_P}{n} \right)}{\partial e} &= V - \frac{2e}{c} = 0 \\
\Rightarrow e^* &= \frac{Vc}{2} \\
\frac{\partial \left( \frac{EU_P}{n} \right)}{\partial s} &= -V(n-1) - \frac{2s}{c} = 0 \\
\Rightarrow s^* &= -\frac{Vc(n-1)}{2}.
\end{aligned}$$

The principal has to incentivize the agents to choose  $e^*$  and  $s^*$  by choosing the parameters which can be influenced by him accordingly. These parameters are different in the two versions: In version 1 (variable prize distribution), the principal's parameters are  $W_1$ - $W_2$  and  $z$ , and in version 2 (variable prize levels), the principal's parameters are  $\alpha$ ,  $\beta$  and the number of prizes in each prize level  $a$  and  $b$ . The deciding factor is that in both versions, the principal is able to configure the available parameters so that the agents choose  $e^*$  and  $s^*$ . Regarding the benchmark case (Fixed-prize FDS), the principal utility corresponds to equation 13, but there is no way for the principal to choose  $W_1$ - $W_3$  in a manner that the agents choose  $e^*$  and  $s^*$ .

**Proposition 6:** In both versions 1 and 2, we obtain the same  $e^*$ ,  $s^*$ , and  $E(U_P)$ .

### 3 Conclusion

This study theoretically investigated if the implementation of variable prizes in FDS leads to a reduction in sabotage activities and a higher principal utility compared to a fixed-prize FDS. We showed that a fixed-prize FDS leads to positive effort levels as well as sabotage activities. Several tournament models have shown similar results (see ROSEN 1988; HARBRING/IRLENBUSCH 2008; CHEN 2003). Hence, FDS is not worthwhile for the principal.

While variable prizes lead to identical effort levels similar to a fixed-prize FDS, the former, in contrast to the latter, induce negative sabotage levels, which can be interpreted as helpfulness. As a result, we obtain positive principal utility. Both variable prize versions (variable prize distribution and variable prize levels) lead to identical results regarding effort levels and sabotage activities.

The difference in sabotage activities between fixed prizes and variable prizes are due to the following effect: In a fixed-prize FDS, only relative output is relevant for the agents' payment and cumulative output is irrelevant for the agents. This is why it is rational for the agents to strengthen their relative output with sabotage activities regardless of the cumulative output, which, in turn, is the most important factor for the principal. In contrast, in variable-prize FDS, not only relative output, but also cumulative output is an important factor for the agents' utility. Although sabotage activities still strengthen relative output, they reduce cumulative output in contrast to effort activities. That is why fighting for the highest relative output over effort levels is a rational agent behavior.

Since in this model a unit of sabotage (resp. helpfulness) has a stronger impact on cumulative output than a unit of effort, it makes sense for the agents to increase cumulative output by exerting helping activities.

Assuming heterogeneous agents, variable tournament prizes could still be an effective instrument: The problem of reduced agent efforts (see ROSEN 1988) could be compensated since cumulative output induces agents to exert high efforts, even if the relative position is already determined with the utmost prob-



ability. There is a need for further research on this aspect.

The results of this study provide important findings for both research and organizational practice: Variable tournament prizes can specifically reduce the sabotage problem of FDS, and tournaments, in general. Because of the dependence of tournament prizes (and thereby, agents' utility) on a team variable (in this study, cumulative output), agents are encouraged to exert cooperative instead of destructive activities. Nevertheless, competition between the agents is maintained. However, the agents' effort level is the only adjusting lever in this competition. Hence, variable tournament prizes could be an effective instrument to ensure the positive effects of FDS and tournaments, even in a sabotage-supporting environment.

## References

- AKERLOF, Robert J; Richard T HOLDEN (2012). "The nature of tournaments". In: *Economic Theory* 51.2, pp. 289–313.
- ALSEVER, Jennifer (2007). "What is Forced Ranking?" URL: [http://www.cbsnews.com/8301-505125\\_162-51059306/what-is-forced-ranking/](http://www.cbsnews.com/8301-505125_162-51059306/what-is-forced-ranking/).
- CHEN, Kong-Pin (2003). "Sabotage in promotion tournaments". In: *Journal of Law, Economics, and Organization* 19.1, pp. 119–140.
- DOMINICK, Peter G (2009). "Forced rankings: Pros, cons, and practices". In: *Performance management: Putting research into practice*, pp. 411–443.
- GARICANO, Luis; Ignacio PALACIOS-HUERTA (2005). "Sabotage in Tournaments: Making the Beautiful Game a Bit Less Beautiful". In: *CEPR Discussion Papers* No. 5231.
- GROTE, Dick (2005). *Forced ranking*. Boston: Harvard Business School Press.
- GÜRTLER, Oliver; Johannes MÜNSTER (2010). "Sabotage in dynamic tournaments". In: *Journal of Mathematical Economics* 46.2, pp. 179–190.
- GÜTH, Werner et al. (2010). "Tournaments and Piece Rates Revisited: A Theoretical and Experimental Study of Premium Incentives". In: *Jena Economic Research Papers* 2010, p. 039.
- HARBRING, Christine; Bernd IRLENBUSCH (2005). "Incentives in tournaments with endogenous prize selection". In: *Journal of Institutional and Theoretical Economics JITE* 161.4, pp. 636–663.
- (2008). "How many winners are good to have? On tournaments with sabotage". In: *Journal of Economic Behavior & Organization* 65.3, pp. 682–702.
- (2011). "Sabotage in tournaments: Evidence from a laboratory experiment". In: *Management Science* 57.4, pp. 611–627.
- LAZEAR, Edward P (1989). "Pay equality and industrial politics". In: *Journal of political economy*, pp. 561–580.
- LAZEAR, Edward P; Sherwin ROSEN (1981). "Rank-Order Tournaments as Optimum Labor Contracts". In: *The Journal of Political Economy*, pp. 841–864.
- MACLENNAN, Andrew (2007). "Forced ranking: time to dismiss this underperformer?" In: *Strategy Execution* 3, pp. 28–30.

- MÜNSTER, Johannes (2007). “Selection tournaments, sabotage, and participation”. In: *Journal of Economics & Management Strategy* 16.4, pp. 943–970.
- ORRISON, Alannah; Andrew SCHOTTER; Keith WEIGELT (2004). “Multiperson tournaments: An experimental examination”. In: *Management Science* 50.2, pp. 268–279.
- ROSEN, Sherwin (1988). “Promotions, elections and other contests”. In: *Journal of Institutional and Theoretical Economics (JITE)/Zeitschrift für die gesamte Staatswissenschaft*, pp. 73–90.
- SCHLEICHER, Deidra J; Rebecca A BULL; Stephen G GREEN (2009). “Rater reactions to forced distribution rating systems”. In: *Journal of Management* 35.4, pp. 899–927.
- SCULLEN, Steven E; Paul K BERGEY; Lynda AIMAN-SMITH (2005). “Forced distribution rating systems and the improvement of workforce potential: A baseline simulation”. In: *Personnel Psychology* 58.1, pp. 1–32.
- VANDEGRIFT, Donald; Abdullah YAVAS (2010). “An Experimental Test of Sabotage in Tournaments”. In: *Journal of Institutional and Theoretical Economics JITE* 166.2, pp. 259–285.
- WELCH, Jack; Suzy WELCH (2005). *Winning: das ist Management*. Campus Verlag.

## A Appendix

### A.1 Effort level and sabotage activities for $q_i \leq z$

$$\begin{aligned}
 E(U_i) &= \left( \frac{z - \sum_{i=1}^n q_i}{z} \right) (W_3 - W_2) + W_2 - \frac{e_i^2}{c} - \frac{s_i^2}{c} \\
 &= \left( \frac{z - n(e^* - (n-1)s^*)}{z} \right) (W_3 - W_2) + W_2 - \frac{e_i^2}{c} - \frac{s_i^2}{c}.
 \end{aligned} \tag{14}$$

In order to determine optimal effort and sabotage levels, equation 14 is derived with respect to  $e_i$  and  $s_i$ :

$$\begin{aligned}
 \frac{\partial E(U_i)}{\partial e_i} = 0 &\Leftrightarrow -\frac{n}{z}(W_3 - W_2) - \frac{2e^*}{c} = 0 \\
 e^* &= -\frac{nc}{2z}[W_3 - W_2]
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial E(U_i)}{\partial s_i} = 0 &\Leftrightarrow \left( \frac{n(n-1)}{z} \right) (W_3 - W_2) - \frac{2s^*}{c} = 0 \\
 s^* &= \frac{nc(n-1)}{2z}(W_3 - W_2).
 \end{aligned}$$

The principal anticipates the agents' choice and determines the optimal prize levels  $W_1$ ,  $W_2$ , and  $W_3$ :

$$E(U_P) = [Vn(e^* - (n-1)s^*) - aW_1 - bW_2 - dW_3].$$

The principal utility per agent is

$$\begin{aligned} E\left(\frac{U_P}{n}\right) &= \left[ V(e^* - (n-1)s^*) - \frac{a}{n}W_1 - \frac{b}{n}W_2 - \frac{d}{n}W_3 \right] \\ &= [V(e^* - (n-1)s^*) - P_1W_1 - P_2W_2 - P_3W_3]. \end{aligned} \quad (15)$$

By  $E(U_i) \geq 0$ , the principal makes sure that the agents' participation constraint is fulfilled:

$$\begin{aligned} E(U_i) \geq 0 &\Rightarrow P_1W_1 + P_2W_2 + P_3W_3 - c(e_i) - c(s_i) \geq 0 \\ &\Rightarrow P_1W_1 + P_2W_2 + P_3W_3 \geq c(e_i) + c(s_i). \end{aligned}$$

By inserting the participation constraint in equation 15, we obtain

$$E\left(\frac{U_P}{n}\right) = V(e^* - (n-1)s^*) - c(e_i) - c(s_i). \quad (16)$$

Equation 16 is derived with respect to  $W_3$ - $W_2$ :

$$\begin{aligned} \frac{\partial E\left(\frac{U_P}{n}\right)}{\partial (W_3 - W_2)} &= V\left(\frac{\partial e^*}{\partial (W_3 - W_2)} - (n-1)\frac{\partial s^*}{\partial (W_3 - W_2)}\right) \\ &\quad - c'(e_i)\frac{\partial e^*}{\partial (W_3 - W_2)} - c'(s_i)\frac{\partial s^*}{\partial (W_3 - W_2)}. \end{aligned}$$

Since  $\frac{\partial s^*}{\partial (W_3 - W_2)} = \frac{nc(n-1)}{2z}$ ,  $\frac{\partial e^*}{\partial (W_3 - W_2)} = -\frac{nc}{2z}$ ,  $c'(e_i) = \frac{2}{c}e^* = -\frac{n}{z}[W_3 - W_2]$ , and  $c'(s_i) = \frac{2}{c}s^* = \frac{n(n-1)}{z}(W_3 - W_2)$ , it follows that

$$\begin{aligned} \frac{\partial E\left(\frac{U_P}{n}\right)}{\partial (W_3 - W_2)} &= V\left(-\frac{nc}{2z} - (n-1)\frac{nc(n-1)}{2z}\right) - \frac{n}{z}[W_3 - W_2]\left(-\frac{nc}{2z}\right) \\ &\quad - \frac{n(n-1)}{z}(W_3 - W_2)\frac{nc(n-1)}{2z} \end{aligned}$$

$$\begin{aligned} \frac{\partial E\left(\frac{U_P}{n}\right)}{\partial (W_3 - W_2)} = 0 &\Rightarrow V\left(-\frac{nc}{2z} - (n-1)\frac{nc(n-1)}{2z}\right) - \frac{n}{z}[W_3 - W_2]\left(-\frac{nc}{2z}\right) \\ &\quad - \frac{n(n-1)}{z}(W_3 - W_2)\frac{nc(n-1)}{2z} = 0 \\ &\Rightarrow -\frac{Vnc}{2z}\left(1 + (n-1)^2\right) \\ &= \frac{n^2c(n-1)^2}{2z^2}(W_3 - W_2) + \frac{n^2c}{2z^2}(W_3 - W_2) \\ &\Rightarrow -\frac{Vnc}{2z}\left(1 + (n-1)^2\right) = (W_3 - W_2)\frac{n^2c\left(1 + (n-1)^2\right)}{2z^2} \\ &\Rightarrow (W_3 - W_2) = -\frac{Vz}{n}. \end{aligned}$$

Agents' effort and sabotage levels:

$$e^* = \frac{Vc}{2}$$

$$s^* = -\frac{Vc(n-1)}{2}.$$

## A.2 Principal utility and the number of agents in the model of HARBRING/IRLENBUSCH (2008)

HARBRING/IRLENBUSCH (2008) investigate theoretically and empirically the impact of a variation in the number of agents and the distribution of prizes for the winner and loser on the agents' effort ( $e^*$ ) and sabotage levels ( $s^*$ ). The authors do not explicitly discuss principal utility. Based on the model assumptions and the model results, principal utility can be derived easily.

HARBRING/IRLENBUSCH (2008) define principal utility as follows:

$$E(U_P) = V \left[ E \left( \sum y_i \right) + nk \right] - aW_1 - bW_2$$

$$= V [n(e^* - (n-1)s^*) + nk] - aW_1 - bW_2$$

$$\Leftrightarrow E \left( \frac{U_P}{n} \right) = V [e^* - (n-1)s^* + k] - \frac{a}{n}W_1 - \frac{b}{n}W_2.$$

$k$  is the minimum output, which is realized by a "work-to-rule-behavior" (see HARBRING/IRLENBUSCH 2008).

Since  $P_1 = \frac{a}{n}$  and  $P_2 = \frac{b}{n}$ :

$$E \left( \frac{U_P}{n} \right) = V [e^* - (n-1)s^* + k] - P_1W_1 - P_2W_2. \quad (17)$$

The participation constraint is

$$E(U_i) \geq 0 \Rightarrow P_1W_1 + P_2W_2 - c(e_i) - c(s_i) \geq 0$$

$$\Rightarrow P_1W_1 + P_2W_2 \geq c(e_i) + c(s_i)$$

$$E(U_i) \geq 0 \Rightarrow P_1W_1 + P_2W_2 - c(e_i) - c(s_i) \geq 0$$

$$\Rightarrow P_1W_1 + P_2W_2 \geq c(e_i) + c(s_i).$$

The participation constraint is substituted in equation 17

$$E \left( \frac{U_P}{n} \right) = V [e^* - (n-1)s^* + k] - c(e_i) - c(s_i). \quad (18)$$

HARBRING/IRLENBUSCH (2008) compute  $e^*$  and  $s^*$  as follows:

$$e^* = \frac{(W_1 - W_2)c_e}{4\bar{e}}$$

$$s^* = \frac{(W_1 - W_2) c_s}{4\bar{\varepsilon}}.$$

The authors assume the following effort and sabotage cost functions:

$$c(e_i) = \frac{e_i^2}{c_e} = \frac{(W_1 - W_2)^2 c_e}{16\bar{\varepsilon}^2}$$

$$c(s_i) = \frac{s_i^2}{c_s} = \frac{(W_1 - W_2)^2 c_s}{16\bar{\varepsilon}^2}.$$

By substituting  $e^*$ ,  $s^*$ ,  $c(e_i)$ , and  $c(s_i)$  into equation 18, it follows that

$$\begin{aligned} E\left(\frac{U_P}{n}\right) &= V \left[ \frac{(W_1 - W_2) c_e}{4\bar{\varepsilon}} - (n-1) \frac{(W_1 - W_2) c_s}{4\bar{\varepsilon}} + k \right] \\ &\quad - \frac{(W_1 - W_2)^2 c_e}{16\bar{\varepsilon}^2} - \frac{(W_1 - W_2)^2 c_s}{16\bar{\varepsilon}^2} \\ &= V \left[ k + \frac{(W_1 - W_2)}{4\bar{\varepsilon}} (c_e - (n-1) c_s) \right] \\ &\quad - \frac{(W_1 - W_2)^2 (c_e - c_s)}{16\bar{\varepsilon}^2}. \end{aligned} \tag{19}$$

Equation 19 is derived with respect to  $W_1 - W_2$ :

$$\begin{aligned} \frac{\partial E\left(\frac{U_P}{n}\right)}{\partial (W_1 - W_2)} &= 0 \Leftrightarrow V \left[ \frac{c_e}{4\bar{\varepsilon}} - (n-1) \frac{c_s}{4\bar{\varepsilon}} \right] \\ &\quad - \frac{(W_1 - W_2) c_e}{8\bar{\varepsilon}^2} - \frac{(W_1 - W_2) c_s}{8\bar{\varepsilon}^2} = 0 \\ &\Leftrightarrow \frac{V}{4\bar{\varepsilon}} [c_e - (n-1) c_s] = \frac{(W_1 - W_2) (c_e + c_s)}{8\bar{\varepsilon}^2} \\ &\Leftrightarrow (W_1 - W_2) = \frac{2V\bar{\varepsilon} [c_e - (n-1) c_s]}{(c_e + c_s)}. \end{aligned} \tag{20}$$

Equation 20 is substituted into equation 19:

$$\begin{aligned} E\left(\frac{U_P}{n}\right) &= V \left[ k + \frac{2V\bar{\varepsilon} [c_e - (n-1) c_s]}{(c_e + c_s) 4\bar{\varepsilon}} (c_e - (n-1) c_s) \right] \\ &\quad - \frac{\left( \frac{2V\bar{\varepsilon} [c_e - (n-1) c_s]}{(c_e + c_s)} \right)^2 (c_e - c_s)}{16\bar{\varepsilon}^2} \\ &= V \left[ k + \frac{V [c_e - (n-1) c_s]^2}{2(c_e + c_s)} \right] - \frac{V^2 [c_e - (n-1) c_s]^2}{4(c_e + c_s)} \\ &= Vk + \frac{V^2 [c_e - (n-1) c_s]^2}{2(c_e + c_s)} - \frac{V^2 [c_e - (n-1) c_s]^2}{4(c_e + c_s)} \\ &= Vk + \frac{V^2 [c_e - (n-1) c_s]^2}{4(c_e + c_s)}. \end{aligned}$$

It can be easily seen that principal utility is exponentially increasing with an increase in the number of agents.

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