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BUDGET CONSTRAINED FRONTIER MEASURES OF FISCAL EQUALITY AND EFFICIENCY IN SCHOOLING

by

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ABSTRACT

In this paper we employ a cost indirect output distance function to model school district technology and measure performance. This function has the advantage of readily modeling multiple output production where that production is subject to a budget constraint. It also provides a natural measure of performance which is closely related to Farrell type measures of efficiency. We use this distance function to analyze the performance of a sample of Texas school districts, and to simulate various equalization schemes by varying the budget constraint faced by individual school districts. We find that the school districts in our sample could make considerable gains in value added by allocating their resources more efficiently. The simulated equalization schemes can also provide gains in value-added, but they typically require additional funds. Our technique could be employed more generally to analysis of public sector performance. In the education context, it could also be employed to develop state aid formulas which are linked to performance and account for variations in cost of personnel and special needs of the student body.

Budget Constrained Frontier Measures of Fiscal Equality and Efficiency in Schooling

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1 Introduction

A prominent view among educators, policy makers and the population at large is that public schools need to provide more and better education than they have in the recent past. Many proposals for reform have been offered. On the one hand proposals calling for greater administrative accountability and greater parental, student, and teacher choice, have reflected the public's perception that the public schools are failing because of what has been called monopolistic bureaucratic control (Chubb and Moe (1990)). Put differently, this view suggests that the current organizational structure results in inefficient use of resources. On the other hand are proposals that call for greater public investment in education in order to attract and retain teachers vital to the educational process (Reich (1988)). In this view, there are not enough resources devoted to education. In either case, budgets, or control of budgets, is an important aspect of the proposed reforms.

This 'new' reform movement focuses essentially on efficiency. For years, much of the discussion concerning educational reform focused on school finance and issues of equalization or equity. The legacy of that era is legislation aimed at equalizing educational opportunity by equalizing access to resources across school districts. Simply mandating employment of equal resources across school districts may have undesirable efficiency consequences, particularly if differences in input prices across school districts are ignored. That is, schools should be allowed to choose the efficient mix of inputs given the input prices they face. Successful reform should account for differences in relative input prices (and the resulting production possibility sets) faced by school districts.

The purpose of this paper is to construct a multi-output production technology that allows us to determine how much education can be provided if a school district is allowed to

*The authors are listed in alphabetical order. Please direct correspondence to K. Hayes. The authors would like to thank Susan Porter-Hudak for helpful comments concerning econometric issues.

optimally choose inputs given the relative input prices they face and the total budget they have at their disposal. This model identifies those school districts which get the most from what they have, accounting for the prices they face and the total resources they have. Individual school districts are judged relative to this 'best practice frontier', and are compared to school districts with output mixes and input prices and budgets comparable to their own.

Using this model, we are able to simulate various equalization schemes by changing the budget faced by individual school districts. For example, we can analyze the effect on educational output of a policy to equalize per pupil budgets. The resulting change in output can be decomposed into a measure of efficiency and a measure of fiscal equality. Our efficiency measure allows us to compare the observed level of output for a school district with the level of output the school district could be expected to produce if they were using their current budget efficiently. Our fiscal equality measure compares the level of output the school district could produce if it operated efficiently given its current budget to the level of output that it could produce if it operated efficiently and faced an equalized expenditure level. We also simulate a policy intended to equalize the unit cost of education by letting all school districts face a common input price vector.[1] Notice that this technique allows us to address both reform issues. The early requests for equality of expenditures can be analyzed within this framework as well as the more recent proposals calling for improved resource usage.

In section 2 we review recent work on the empirical application of production functions and efficiency measurement in education. Section 3 reviews the distance function methodology for modeling multi-output production technologies and provides an empirical model for measuring efficiency and equity in schooling. In section 4 we employ data from Texas school districts to empirically implement the model developed in section 3. The final section of the paper offers policy implications and directions for future work in modeling school production processes.

2 Models of School Production and Efficiency

Much of the research examining school production has taken one of two paths. For many years researchers focused on estimating a single output, average production function for schooling. The single output was usually taken to be a measure of student achievement and was assumed to be produced using inputs related to school personnel, per pupil expenditures, and family background. The estimated production function gave estimates (based on average performance) of the marginal products of the inputs and allowed the researchers to infer which inputs would have the greatest marginal impact on achievement. Cohn and Geske (1990) have provided a thorough review of the output and input measures employed in these types of studies. See Levin (1974) and Hanushek (1979) for critical reviews of the

[1]The methodology can also be used to calculate 'lost' potential output due to resource misuse and 'lost' potential output due to input price differences or differences in total budgets across school districts.

production function approach.

Recent research on public school performance has taken a second path. Rather than assume that schools are efficiently producing some aggregate summary measure of student achievement, researchers have refined their modeling techniques to examine questions related to scale, technical, and allocative efficiency. One of the major contributions of this literature is the generalization to multiple outputs. Bessent and Bessent (1980) and Bessent et al. (1982, 1983, 1984) have employed data envelopment analysis (DEA)[2] to examine the performance of schools in Texas. Färe, Grosskopf, and Weber (1989) also used linear programming techniques to measure technical output efficiency for Missouri school districts. Using stochastic estimation techniques, Callan and Santerre (1990) found evidence that school districts in Connecticut produce primary and secondary education using inefficiently large quantities of capital and transportation services. In an earlier study, Jimenez (1986) found evidence that schools in Bolivia and Paraguay also used excessive amounts of capital. In addition, Jimenez found evidence that almost half of the schools in Bolivia that teach both primary and secondary students exhibited diseconomies of scale. Grosskopf, Hayes, Taylor and Weber (1991) calculate shadow prices of school district inputs in Texas and compare them to observed relative input prices using a distance function approach. They find that most school districts in the sample are not allocatively efficient.

Recently McCarty and Yaisawarng (forthcoming) combined DEA and stochastic estimation techniques to measure efficiency for schools that employ discretionary inputs (such as teachers and administrators) and nondiscretionary inputs (such as socio-economic characteristics) in producing multiple outputs. They first used data envelopment analysis to construct efficiency measures for schools and then regressed the efficiency score on the non-discretionary inputs in the second stage. The advantage of this procedure is that it allows managerial inefficiency over the discretionary inputs to be separated from inefficiency that might occur as a result of differences in non-discretionary inputs.

Examining school performance using a production function or by defining a production possibility frontier for given inputs is only one way of examining the performance of public school outcomes. The cost function provides a dual way of specifying the production technology. The cost function gives the minimum cost of producing a given level of output for given input prices. Barrow (1991) estimated a cost function frontier for schools in England and found that actual costs were 4% to 16% above the minimum estimated cost for the schools in his sample. One problem with employing a cost function approach to measuring efficiency in the public schools is that public enterprises may not be cost minimizers. A second problem is that public schools often face a fixed budget, and are not free to adjust the level of expenditures. On the other hand, they do not take output as given (which is

[2]In this approach, the goal is to measure technical efficiency in a multiple output context. Technical efficiency in this case is equivalent to Farrell technical efficiency and is typically calculated using linear programming techniques.

essentially the way in which the cost function is defined, since output is considered to be exogenous when estimating a cost function). Rather, school districts act as though output is endogenous, i.e., they seek to provide maximum feasible educational services given the budget and input prices they face. The indirect output distance function provides a means of overcoming both of these potential problems and is discussed in greater detail in the next section.

3 The Indirect Output Distance Function

The indirect output distance function represents a convenient way of modeling the production technology of a firm that faces a budget constraint when hiring inputs, but does not necessarily take output as exogenous. While the cost function is capable of modeling a multi-output production technology, the indirect output distance function is more appropriate for firms that are cost constrained: in contrast to the cost function, the indirect output distance function takes cost as exogenous. In addition, the cost function implies cost minimizing behavior on the part of the economic agent while the indirect output distance function makes no equivalent a priori behavioral assumption.[3] The indirect output distance function should therefore be especially useful in modeling the technology of public enterprises that produce multiple outputs under conditions of budgetary constraint. Our purpose here will be to review the properties of the indirect output distance function as presented by Färe and Primont (1990) (see also Färe and Grosskopf (1991) and Färe, Grosskopf and Lovell (1986)) and to provide a functional form that can be employed to estimate it.

The following notation is employed throughout the paper

$$\begin{aligned} x &= (x_1, \ldots, x_n), \text{ a vector of variable input quantitites} \\ p &= (p_1, \ldots, p_n), \text{ a vector of variable input prices} \\ u &= (u_1, \ldots, u_m), \text{ a vector of output quantities} \\ z &= (z_1, \ldots, z_t), \text{ a vector of fixed input quantitites} \\ c &= (c), \text{ scalar cost or budget.} \end{aligned}$$

Define the set $G(p/c, z)$ as

$$G(p/c, z) = \{u : u \in P(x, z) \text{ and } p'x \leqq c\}, \tag{1}$$

where $P(x, z)$ is the production possibility set for a given (x, z) and $G(p/c, z)$ is the largest production possibility set allowing x to vary, but requiring that x satisfy the budget con-

[3]If the firm does minimize costs, however, and technology is homogeneous, the two functions are equivalent.

straint. It follows that $P(x,z)$ is a subset of $G(p/c,z)$ for all x which satisfy $p'x \leqq c$.

The (short run)[4] indirect output distance function can be defined as

$$ID_o(p/c,z,u) = \min_{\theta,x}\{\theta : u/\theta \in G(p/c,z)\}. \tag{2}$$

Figure 1 illustrates the construction of the indirect output distance function for a typical school district that produces two outputs. The set $G(p/c,z)$ gives all the possible combinations of two outputs that can be produced given the budget constraint faced by the school district. The school district is observed to produce outputs represented by point U in the diagram. The ratio OU/OA gives the value of the indirect output distance function and is the measure we will use to judge the efficiency of individual school districts. The reciprocal of the indirect output distance function (OA/OU) gives the factor by which all outputs could be expanded proportionately if the school district were operating efficiently. It follows that when the school district is producing efficiently (on the frontier of $G(p/c,z)$), the value of $ID_o(.)$ is 1.[5]

In order to ultimately estimate the indirect output distance function, we exploit several of its properties. From duality theory (see Färe and Primont (1990), p.883 or Färe and Grosskopf (1991)) we know that

$$ID_o(p/c,z,u) = \min_{x}\{D_o(x,z,u) : (p/c)'x \leqq 1\} \tag{3}$$

where $D_o(x,z,u)$ is the direct output distance function.[6] Since x in (3) is chosen to minimize $D_o(x,z,u)$ we can invoke the envelope theorem to yield

$$\partial ID_o(.)/\partial(p/c) = x(p/c,z,u), \tag{4}$$

where $x(\cdot)$ is the input demand function for normalized price vector p/c, fixed inputs z and multi-output level u.[7] This result will prove useful in identifying optimal input usage given school district budgets, and allows us to derive optimal subsidies. Furthermore, if we take the logarithm of the indirect output distance function and differentiate it with respect to log normalized prices we obtain the budget share equations for the inputs

$$\partial \ln ID_o(p/c,z,u)/\partial \ln(p/c) = w(p/c,z,u) = px/c, \tag{5}$$

[4]The indirect output distance function defined here may be thought of as short run in the sense that the choice of inputs is restricted to the subset of variable inputs, x.

[5]The reciprocal of the distance function can be thought of as a Farrell type output-increasing measure of 'indirect' technical efficiency. The measure is Farrell-like due to its definition as a proportional scaling. Farrell (1957) did not include budget constrained technology in his work. Färe and Grosskopf (forthcoming) and Färe, Grosskopf and Lovell (1985), (1988), and (forthcoming) generalize the Farrell efficiency measure to the indirect case.

[6]$D_o(x,z,u) = \min\{\theta : u/\theta \in P(x,z)\}$.

[7]This is true when technology is homogeneous.

which can be estimated simultaneously with the distance function to improve the efficiency of our estimated parameters.

Another property that we exploit in estimating an indirect output distance function is that the indirect output distance function is homogeneous of degree +1 in outputs. That is,

$$ID_o(p/c, z, \lambda u) = \lambda ID_o(p/c, z, u), \tag{6}$$

which follows from the definition of the indirect output distance function.

We estimate the indirect output distance function using the translog form,

$$\begin{aligned} \ln ID_o(p/c, z, u) \;=\; & \alpha_0 + \sum_{i=1}^{n} \alpha_i \ln(p_i/c) + 1/2 \sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_{ij} \ln(p_i/c)\ln(p_j/c) \\ & + \sum_{k=1}^{m} \beta_k \ln(u_k) + \sum_{i=1}^{n}\sum_{k=1}^{m} \beta_{ik} \ln(p_i/c)\ln(u_k) + \sum_{r=1}^{t} \gamma_r \ln z_r \\ & + \sum_{i=1}^{n}\sum_{r=1}^{t} \gamma_{ir} \ln(p_i/c) \ln z_r. \end{aligned} \tag{7}$$

As mentioned above, to improve efficiency in estimating the parameters of (7), we also estimate the budget share equations consistent with (7). Differentiating the above equation with respect to $\ln(p_i/c)$ yields the budget shares for the $i = 1, \ldots, n$ variable inputs

$$w_i = p_i x_i / c = \alpha_i + \sum_j \alpha_{ij} \ln(p_j/c) + \sum_k \beta_{ik} \ln(u_k) + \sum_r \gamma_{ir} \ln z_r, i = 1, \ldots, n. \tag{8}$$

If we set the distance function equal to its efficient (frontier) value, the left-hand side of equation (7) is zero for all observations. To avoid this problem recall from (6) that $ID_o(\cdot)$ is homogeneous of degree +1 in outputs. Therefore for each observation to be used in estimating (7) a value that is unique to that observation can be used to multiply all output values on the right-hand side and the value of $ID_o(\cdot)$ on the left-hand side. Let this value be $\lambda_s = c_s$. We choose to multiply all outputs by the total budget, c_s, that each school district has available to hire inputs. Since the left-hand side of (7) is now equal to $\ln c$, the indirect output distance function can be readily estimated. The parameter restrictions implied by homogeneity are

$$\begin{aligned} \sum_{k=1}^{m} \beta_k &= 1, \\ \text{and} & \\ \sum_{k=1}^{m} \beta_{ik} &= 0, i = 1, \ldots, n. \end{aligned} \tag{9}$$

The transformed indirect output distance function in logarithmic form and the budget share equations now take the following form

$$\begin{aligned}
\ln \lambda_s + \ln ID_o(p/c, z, u) &= \ln ID_o(p/c, z, \lambda_s u) \qquad (10)\\
\ln \lambda_s &= \alpha_0 + \sum_{i=1}^{n} \alpha_i \ln(p_i/c) + 1/2 \sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_{ij} \ln(p_i/c)\ln(p_j/c)\\
&\quad + \sum_{k=1}^{m} \beta_k \ln(\lambda_s u_k) + \sum_{i=1}^{n}\sum_{k=1}^{m} \beta_{ik} \ln(p_i/c)\ln(\lambda_s u_k) + \sum_{r=1}^{t} \gamma_r \ln z_r\\
&\quad + \sum_{i=1}^{n}\sum_{r=1}^{t} \gamma_{ir} \ln(p_i/x)\ln z_r,\\
w_i &= \alpha_i + \sum_{j=1}^{n} \alpha_{ij} \ln(p_j/c) + \sum_{k=1}^{m} \beta_{ik} \ln(\lambda_s u_k) + \sum_{r=1}^{t} \gamma_{ir} \ln z_r, i = 1, \ldots, n.
\end{aligned}$$

For estimation purposes we also divide each variable on the right-hand side of (10) by its mean value so that when each $(p_i/c), z_r$ and $\lambda_s u_k$ take on their mean value, the natural log of those values is zero. Because the budget share equations of the n inputs must sum to one we also impose the restrictions

$$\begin{aligned}
\sum_{i=1}^{n} \alpha_i = 1, \sum_{i=1}^{n} \alpha_{ij} = 0, j = 1, \ldots, n \qquad (11)\\
\sum_{i=1}^{n} \gamma_{ir} = 0, r = 1, \ldots, t,\\
\text{and}\\
\sum_{i=1}^{n} \beta_{ik} = 0, k = 1, \ldots, m.
\end{aligned}$$

Since the cost function is symmetric in normalized input prices, the indirect output distance function is also symmetric in normalized input prices leading to the final restriction, $a_{ij} = a_{ji}$, for $i \neq j$.

4 Empirical Results

To implement the model described in the previous section we employ data from 310 Texas school districts with enrollment between 1000 and 5000 students. Our variable inputs consist of various categories of employment, which represents more than 80 percent of current operating expenses. We include expenditures on maintenance and operations as a proxy for fixed capital inputs. We also construct a set of variables which represent fixed home produced inputs, which is explained in more detail below.

Our vector of outputs is based on batteries of test scores. Hanushek and Taylor (1990) have examined the potential problems that can arise in the use of test score data from a single year as a measure of output and found that value added test scores provide a more meaningful measure of output than test scores alone. We therefore estimate value added test scores for students in grades 3, 5, 9, and 11 as our output proxies. For each of the four grade levels we estimate the value added by the school district based on

$$\begin{aligned} TEAMS89_{sg} &= \delta_g + \sum_{i=1}^{3} \delta_{i,g} ETHNICITY_{i,s} + \delta_{4,g} SES_s + \delta_{5,s} XCOHORT_{s,g} \\ &\quad + \sum_{j=6}^{9} \delta_{j,g} TEAMS87_{sj,g-2} + \epsilon_{sg}, \; g = 3,5,9,11, \end{aligned} \tag{12}$$

where $TEAMS89_{sg}$ is the average total Texas Educational Assessment of Minimal Skills ($TEAMS$) scores for school district s for grade level g in 1989, $TEAMS87_{sj,g-2}$ is the average $TEAMS$ score in subject j (reading, writing, and mathematics) for the same cohort two years previously,[8] $ETHNICITY_{s,j}$ is the fraction of the student body of school district s that is Asian, black or Hispanic, respectively. SES_s is the fraction of the student body of school district s that is receiving free or reduced-price lunches (the best available proxy for socio-economic status) and $XCOHORT_{s,g}$ is the percentage change in the size of the grade g cohort between 1987 and 1989 (this controls for schools which try to improve scores by shedding students). The estimated residual, ϵ_{sg}, represents the average value added in school district s. Because the four value added equations share common regressors we estimate the system simultaneously using the SAS package for seemingly unrelated regressions (SUR). The parameter estimates of the equations in (12) are presented in Table 1.

Estimating school district outputs as equation residuals generates output measures that represent deviations from the state average. School districts that add less value than the state average have negative output measures. Since the distance function methodology cannot handle negative outputs, we transform the value-added residuals into tractable output measures by adding the estimated value of the intercept from each equation to the value-added residual for that equation.

Our proxies for the contribution of home production (treated as fixed inputs, z) are calculated for each school district and each grade as

$$\begin{aligned} STUINPUT_{s,g} &= \sum_{j=1}^{3} \hat{\delta}_{j,g} ETHNICITY_s + \hat{\delta}_{4,g} SES_s + \hat{\delta}_{5,g} XCOHORT_{s,g} \\ &\quad + \sum_{j=6}^{9} \hat{\delta}_{j,g} TEAMS87_{sj,g-2}. \end{aligned} \tag{13}$$

[8] Texas administers the $TEAMS$ test annually to odd numbered grades.

This is the predicted value (less the intercept) of student performance due to factors which are not subject to the control of the school district in the current period. This is the sense in which they serve to proxy fixed inputs.

We also have price data available for the four variable inputs of school administrators (AD), school teachers (TCH), school support staff (SUP), and teacher aides (AIDES). The budget each school district faces when hiring these four variable inputs is equal to the total cost per student of hiring the four inputs. Table 2 presents descriptive statistics for each of the four variable inputs, fixed inputs, four outputs, budget shares and costs. The value added in grades 3, 5, 9, and 11 are reported as XAG3, XAG5, XAG9, and XAG11.

Recall that our original specification has zero as the left-hand side variable in the first equation. To obtain a specification which can be estimated we exploit the homogeneity property of the indirect output distance function by multiplying both sides of (10) by c_s. As a result, c appears on both the left and right-hand sides of equation (10) which may lead to undesirable endogeneity on the right-hand side. To test whether the errors were correlated with the regressors,[9] we calculated Hausman's m-statistic.[10] The null hypothesis is that the parameter vector of the model specified in (10) and estimated using seemingly unrelated regression is consistent and efficient, while the alternative hypothesis is that the parameters of (10) are unbiased and efficient estimators only when estimated by three-stage least squares. We first estimate the equation system in (10) simultaneously using the seemingly unrelated regression algorithm in SAS. [11] We then reestimate the system of equations using three-stage least squares[12] and calculate Hausman's m statistic. Hausman has shown that the m statistic is distributed as χ^2_K, where K is the number of parameters estimated. In the indirect distance function given by (10) there are 66 parameters with 24 restrictions for a total of 42 free parameters to be estimated. The value of the test statistic is $m = 14.29$. The critical chi-square with 1% significance and 42 degrees of freedom is 66.206. We therefore cannot reject the null hypothesis and use the SUR regression estimates for further analysis. These are reported in Table 3.

An important research question in the economics of education and the school finance literature concerns how the outputs of school districts vary as their budget changes. Hanushek (1981) found that there does not seem to be any significant positive relationship between school district expenditures and student academic achievement. More recently Wahlberg and Fowler (1987) have found that per student expenditures are an insignificant determi-

[9]We would like to thank Susan Porter-Hudak for her advice on this issue.

[10]Hausman's m statistic is given as $m = N(\hat{\beta}_1 - \hat{\beta}_2)'[V(\hat{\beta}_1) - V(\hat{\beta}_2)]^{-1}(\hat{\beta}_1 - \hat{\beta}_2)$, where β_2 and β_1 are the parameter estimates under SUR and 3SLS, N is the number of observations, and $V(\beta_1)$ and $V(\beta_2)$ correspond to the inverse of the information matrix under each estimation method.

[11]To estimate the system of equations given by (10), the budget share equation for teacher aides is dropped to avoid exact linear dependence of the error terms.

[12]This requires that we specify instrumental variables to replace the output vector. We used second order and cross terms as instruments.

nant of student academic achievement in New Jersey school districts. Chubb and Moe (1990) in a comprehensive study of student academic achievement found that the organization of schools and resources within schools were a more important determinant of student achievement than the level of school spending. Even though the evidence seems to be mounting against 'throwing money at schools' as a way of promoting student achievement, the call for more money for schools is still strong. Furthermore, there are still numerous court cases that argue for greater equality of expenditures. We use our estimates of the indirect output distance function in order to simulate the effect of a change in the budget of schools on the output level of each school. Specifically, we wish to know: (1) the potential output gains that are possible if resources are used efficiently, and (2) the potential output gains that are possible if school districts have access to greater resources through a larger budget or equal input prices.

To assess the potential gains from fiscal equalization, we examine two potential reform proposals that equalize the size of the budget directly, and two reform proposals that equalize budgets indirectly by changing input prices. First, we consider equalizing total per-pupil expenditures across all school districts at the sample mean. This reform would be analogous to giving each school district a budget set of $G(p/c_{mean}, z)$ where c_{mean} is the average per-pupil expenditure. Second, we consider increasing the per-pupil expenditures of all school districts that are below the mean without changing the budgets of school districts that are above the mean. For this reform we adjust the budget sets to $G(p/c_{mean}, z)$ for only those school districts with per-pupil expenditures that are below the sample average. The remaining reforms adjust input prices so that all school districts face the minimum observed price and the mean observed price for each input. These reforms imply changing the budget sets to $G((p/c)_{min}, z)$ and $G((p/c)_{mean}, z)$, respectively.

For illustrative purposes, consider the option of allowing each school district to face a normalized input price vector, (p/c), equal to the minimum (p_n/c) for each variable input $n = 1, \ldots, N$ for the 310 Texas school districts in our sample. This implies that the budget set $G(p/c, z)$ will expand for all school districts which previously faced higher than minimum input prices.[13] Returning to Figure 1, the set $G((p/c)_{min}, z)$ gives the maximum production possibility set that could be attainable if the school district faced the minimum normalized variable input price vector, $(p/c)_{min}$ and its own fixed inputs.[14] The ratio OA/OB is our measure of fiscal equalization (which we call FE for short) for the typical school district. This ratio is equal to the value of the indirect output distance function for the school district that is put on its own frontier divided by the value of the indirect output distance function

[13]Since p/c is a vector and we seek the smallest cost-deflated prices for each element of that vector, the combination of input prices in $(p/c)_{min}$ may not be observed for any individual school district.

[14]Our construction of the budget constraint assumes homogeneity of inputs within each personnel category (the price of teacher services, for example, is the average salary paid in that school district). While that is clearly not accurate (there is variation in experience and wages, etc.), there is empirical evidence that characteristics of teachers which are correlated with higher wages may not be correlated with higher test scores. See, for example Hanushek (1986).

when that school district is given access to the lowest possible input prices. The reciprocal of the fiscal equity measure gives the amount that the school district could expand all outputs proportionally if they were originally operating efficiently (on their own $G(p/c, z)$ frontier) and then were faced with a new (minimum) input price vector. The total overall difference (TO) in output for the typical school district starting with their own price vector and performance, and ultimately facing the minimum normalized input price vector is then $TO = EFF * FE$. In terms of Figure 1, $TO = (OU/OA) * (OA/OB)$. It is important to note that for our data set the largest $G(p/c, z)$ set is for a hypothetical school district since the minimum normalized input price varies for each input by school district. Notice that allowing all school districts to face the same input price vector is not the same as simply giving each school district the same total budget since relative input prices are changing in the former case and not in the latter.

We can now demonstrate specifically how EFF and FE are calculated for the empirical specification (10). In order to calculate observation-specific efficiency EFF_s, we first calculate the estimated value of the indirect output distance function for each of the 310 school districts in our sample as

$$\begin{aligned} \widehat{\ln \lambda_s} &= \widehat{\ln ID_o}(p/c_s, z_s, \lambda_s u) \qquad (14) \\ &= \hat{\alpha}_0 + \sum_{i=1}^{n} \hat{\alpha}_i \ln(p_i/c) + 1/2 \sum_{i=1}^{n}\sum_{j=1}^{n} \hat{\alpha}_{ij} \ln(p_i/c)\ln(p_j/c) + \sum_{k=1}^{m} \hat{\beta}_k \ln(\lambda_s u_k) \\ &\quad + \sum_{i=1}^{n}\sum_{k=1}^{m} \hat{\beta}_{ik} \ln(p_i/c)\ln(\lambda_s u_k) + \sum_{r=1}^{t} \hat{\gamma}_r \ln z_r + \sum_{i=1}^{n}\sum_{r=1}^{t} \hat{\gamma}_{ir} \ln(p_i/c) \ln z_r. \end{aligned}$$

In theory, the value of the indirect output distance function should never exceed one for firms that are operating on their frontier. In the estimation of equation (14) however, an error term with mean zero, but positive variance is assumed. For some school districts the forecasted value of the indirect output distance function will therefore exceed the theoretically plausible value. To account for this problem we calculate the residuals and find the most negative residual for equation (14), which we call R_{min}. We then add that negative residual to the intercept term so that the corrected estimates for each school district of the indirect output distance function, $\hat{ID}$, never exceed the theoretically plausible value. We use this corrected value as our measure of efficiency, where efficient performance is consistent with $\hat{ID} = 1$:[15]

$$EFF_s = (\exp(\widehat{\ln \lambda_s} + R_{min}))/\lambda_s \leqq 1. \qquad (15)$$

Next we forecast what would happen to the value of $ID_o(\cdot)$ if the sth school district had access to the minimal input price vector. In order to account for any previous inefficiency,

[15] Bauer (1990) provides an overview of some of the econometric issues involved in estimating frontier functions by this approach and by alternative approaches.

we first make the school district efficient relative to its own frontier. That is, we inflate each output vector by the efficiency score, u_s/EFF_s, and then forecast

$$\begin{aligned}\ln\widehat{ID}_o((p/c)_{min}, \lambda_s u/EFF_s) &= \hat{\alpha}_0 + \sum_i \hat{\alpha}_i \ln((p_i/c)_{min}) \\ &+1/2 \sum_i \sum_j \hat{\alpha}_{ij} \ln((p_i/c)_{min}) \ln((p_j/c)_{min}) \\ &+ \sum_k \hat{\beta}_k \ln(\lambda_s u_k/EFF_s) \\ &+ \sum_i \sum_k \hat{\beta}_{ik} \ln((p_i/c)_{min}) \ln(\lambda_s u_k/EFF_s) + \sum_{r=1}^{t} \hat{\gamma}_r \ln z_r \\ &+ \sum_{i=1}^{n} \sum_{r=1}^{t} \hat{\gamma}_{ir} \ln(p_i/c) \ln z_r. \end{aligned} \tag{16}$$

We use (16) as the basis for our measure of fiscal equality. Specifically, we calculate the value of the indirect output distance function for each school district as if it had access to the lowest prices as follows

$$\begin{aligned} FE_s &= I\hat{D}_o((p/c)_{min}, z, u/EFF_s) \\ &= I\hat{D}_o((p/c)_{min}, z, \lambda_s u/EFF_s)/\lambda_s. \end{aligned} \tag{17}$$

We can now verify that the total effect of an increase in $G(p/c, z)$ is calculated for each school district according to the identity

$$TO_s = EFF_s \cdot FE_s. \tag{18}$$

For each of the four reforms under study, table 4 reports the mean values for our efficiency measure given by (15), our equity measure given by (17), and the mean of the overall difference in outputs (TO) given by (18). The mean value of EFF_s for the 310 Texas school districts in our sample is 0.708. This means that if each school district allocated their given budget efficiently, then outputs would increase by an average of about 29% (1-.71=.29). Note that we are judging performance relative to best practice (i.e., what is observed in our sample) rather than some theoretical standard. In a constant returns to scale world, that 29% increase in outputs is equivalent to current production with 29% lower cost. This result suggests that there are considerable gains to be made by improving the efficiency with which current resources are allocated.

Changing school district budget sets can also produce considerable gains in output. As Table 4 indicates, redistributing school district budgets so that all school districts can spend

the same amount per pupil would result in a 7.8% gain in output (1-.922=.078) if schools used their resources efficiently. If school districts all performed at the current average level of efficiency (.708), that gain would be reduced to approximately 6% (.078*.708=.055). The gains in output would be at least six times larger if we bring those school districts with below average expenditures per pupil up to the mean without reducing the budgets of those school districts that are above the mean. Of course, this second option would require additional funds.

How much would these equalization policies cost? For those reforms that equalize by changing the level of per-pupil expenditures, the calculations are rather straightforward. Changing expenditures so that all school districts spend the state average would be a purely distributional change, and would require no additional funds. Pulling those school districts that are below the mean up to the average would require an expenditure for each school district of the difference between observed expenditures and the mean level of expenditures. On average, such a plan would cost $295 per pupil for the 180 school districts below the mean.

Determining the cost of equalizing the opportunity sets for the school districts by changing input price vectors is somewhat more difficult. To do so we calculate the input demands, $x(\cdot)$, using our indirect Shephard's lemma from (4) for each school district. In this case we calculate $x(\cdot)$ as if they faced the minimum normalized input price vector and were producing their original level of output (made to be efficient) inflated by the reciprocal of our equity measure. That is we calculate $x = f((p/c)_{min}, (u/EFF)/FE)$. We also calculate $\delta = (\delta_1, \ldots, \delta_n)$ such that $(p/c)_{min} = \delta(p/c)_{act}$, where $(p/c)_{act}$ is the actual normalized input price vector faced by the school district. The vector δ then gives the amount each normalized input price must be deflated to reach the minimum normalized input price. If each school district were to receive an input price subsidy equal to s, where $\delta(p/c)_{act} = ((p-s)/c)_{act}$, then each school district would face the minimum normalized input price vector. The normalized input price subsidy would then be $s/c = (1-\delta)(p/c)$ with the total subsidy to the school district equal to $Subsidy = \sum_i (s_i/c)x_i c$. For the 310 school districts in our sample the additional cost per student of equalizing the production opportunity set ranges from $402 to $1146 with a mean value of $900. With per pupil spending currently averaging only $1921 for labor inputs it would appear that an equalization scheme of this magnitude would be prohibitively costly. We also calculate the subsidy necessary to equalize the mean normalized input price vector across school districts. In this case school districts would lose $43 per student on average, and would not require additional funds.

From the simulations presented in Table 4, we can draw two broad conclusions. First, when we compare those reforms that require no additional funds we find that the output gains from redistributing resources to equalize per pupil expenditures are greater than the output gains from equalizing input prices at the sample mean. Further, when we compare those reforms that require additional funding, we find that not only are the output gains from changing the total expenditures greater than the output gains from changing the input

price vector, but also that the cost of changing total expenditures is substantially less than the cost of changing the input prices. Therefore, for the reforms considered here, increasing the size of the budget is preferable to changing the input price vector: it generates greater output for no more money.

Second, perhaps surprisingly, our results suggest that throwing money at schools *can* help improve educational outcomes. Even if we assume that school districts will continue to perform as inefficiently as they do under their current budgets, our results suggest some gain in outputs. Nonetheless, it is obvious that such reforms would be much more effective if coupled with policies to improve efficiency. In fact, our results suggest that significant gains can be made by concentrating solely on reducing inefficiency. This has the clearcut advantage of requiring no increase in funding.[16]

5 Policy Implications

Our results indicate that school district access to resources does play an important role in determining the potential level of student achievement attainable in the school districts in our sample. We find that money can matter. Although the total budget available to the school district is important, we would argue that input price differences across school districts are also important in determining the ability of school districts to deliver educational outcomes. We show how subsidies to equalize real input prices could be derived.

This technique could be used to devise a state aid program incorporating a penalty for inefficiency and subsidies for districts which must pay above state average wages to their personnel. Consider two school districts with identical input prices (p/c), but one has EFF of .75 and the other EFF equal to 1.00 (perfect efficiency). The state aid formula could penalize the district with the below average efficiency score which would encourage school districts to maximize output given their input prices. Now consider two school districts with perfect efficiency scores but different input price vectors. The state aid formula could be designed to subsidize the school district which faces relatively high input prices based on the FE score.

Since the FE score presumes efficiency, school districts that are not efficient should have their grants adjusted by their efficiency score. Less should be given to those districts which are inefficient, ceteris paribus. This is equivalent to using the TO score as a basis for state aid, since TO includes efficiency. This type of formula would be superior to many other state aid formulae because it incorporates a potential penalty for resource waste and it acknowledges that some schools face legitimately excessive or above average wages. Whereas power equalization formulas encourage school districts to tax at a higher rate to get more state aid, the formulation proposed here encourages school districts to use their current resources

[16]This presumes, of course, that efficiency improvements could be achieved costlessly.

more effectively without the added constraint of facing 'unfair' prices when compared with lower cost districts.

A formula of this type mitigates the need for a cost of education index, since major resource costs (personnel) are included. This formulation does not explicitly consider the additional cost of educating 'special needs' children, which is usually part of the design of state aid formulas. In fact, the formulation employed here eliminates the need for considering this additional cost since it can be included as a fixed cost in specifying the model as illustrated here (see (13). In this way, districts with relatively high proportions of special needs children are not penalized; in fact, if they are effective at creating value added they are rewarded. Including fixed inputs in the model puts school districts on 'equal footing' with respect to the diversity and needs of the student body.

Our results may appear to be at odds with previous researchers because in defining our output variables we attempted to purge the effects that home production of education and socioeconomic variables have on observed test scores. We have also accounted for any inefficiency that schools may be incurring so that our simulated changes in the budget are used efficiently. While it may be difficult for the state of Texas to foster the resources necessary to equalize indirect production possibility sets for its school districts, our results indicate that efficiency gains are possible without any new allocation of funds.

This paper has employed a new methodology for examining questions relating to efficiency and fiscal equalization in schooling. The method allowed a multiple output production technology to be specified for a public enterprise that is restricted by a budget constraint and has no *a priori* behavioral objective. This technique could be used to design state aid formulas which simultaneously address the issues of efficiency and equalization. Since our approach explicitly accounts for variation in input prices across school districts, there is no need to derive a cost of education index to adjust the state aid formula. Our formulation of outputs also accounts for differences in the needs or backgrounds of the students on performance, putting schools on equal footing.

Although researchers have recently concentrated on evaluating the performance of schools by examining improvement in test scores due to schooling, other outputs of importance are also produced by schools. For example, policy makers and parents are also concerned with the final outcome of the school process, such as school dropout rates, graduation rates, and the ability of schools to prepare their students for the job market or for further study in college. An examination of how well schools produce these alternative final outputs would add to our knowledge of the school production process and merits further study.

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Table 1

Estimates of School District Outputs

Variable	Grade 3	Grade 5	Grade 9	Grade 11
Intercept	676.37	616.90	431.21	417.63
	(27.97)	(25.70)	(31.25)	(20.55)
MATH PRETEST	0.03	0.03	0.08	0.24
	(0.06)	(0.04)	(0.03)	(0.03)
READING PRETEST	0.08	0.12	0.27	0.25
	(0.06)	(0.05)	(0.08)	(0.04)
WRITING PRETEST	0.15	0.17	0.17	0.02
	(0.05)	(0.04)	(0.04)	(0.02)
ASIAN	0.45	0.49	0.31	0.30
	(0.71)	(0.61)	(0.55)	(0.35)
BLACK	-0.01	-0.13	-0.23	-0.24)
	(0.11)	(0.10)	(0.09)	(0.06)
HISPANIC	-0.01	-0.003	-0.09	-0.15
	(0.10)	(0.09)	(0.06)	(0.05)
XCOHORT	-4.8	-0.38	-0.40	-0.35
	(0.10)	(0.09)	(0.06)	(0.05)
SES	-0.75	-0.57	-0.28	-0.17
	(0.11)	(0.10)	(0.09)	(0.06)
System weighted R-square is 0.4510				

Table 2
Descriptive Statistics
(Sample Size = 310)

VARIABLE	MEAN	ST. DEV.
Variable inputs		
AD	12.49729	5.838665
TCH	143.56516	66.118414
AIDES	23.18406	17.075055
SUP	13.47674	9.073291
Variable input prices		
ADPAY	38013.32258	3542.218458
TCHPAY	23046.04839	1562.410336
AIDPAY	9341.62903	1566.944875
SUPPAY	26855.51290	2503.316959
Budget shares		
W1	0.11043	0.021023
W2	0.76311	0.035799
W3	0.07779	0.024553
W4	0.04867	0.023688
Costs		
C	1867.62664	253.872446
LNC	7.52389	0.128812
ENROLL	2366.96129	1147.346218
Outputs		
XAG3	677.11503	24.349272
XAG5	616.20981	20.408941
XAG9	430.06668	20.543076
XAG11	417.19045	12.319643
Fixed Inputs		
$Z1(STUIN3)$	366.41439	17.106614
$Z2(STUIN5)$	358.80361	18.501925
$Z3(STUIN9)$	186.00187	20.364996
$Z4(STUIN11)$	139.22952	20.262567
Z5 (Capital)	364.22613	115.047542

Table 3
Distance Function Parameter Estimates

PARAMETER	VARIABLE	ESTIMATE	STD. ERROR	T-RATIO
A0	INTERCEPT	7.54341	0.00484	1559.41
A1	P1	0.11040	0.00121	90.95
A2	P2	0.076289	0.00183	416.27
A3	P3	0.07871	0.00134	58.65
B1	U1	0.37756	0.14054	2.69
B2	U2	0.36291	0.14185	2.56
B3	U3	0.01863	0.10806	0.17
A11	P1*P1	0.00648	0.01502	0.43
A12	P1*P2	-0.00446	0.01685	-0.26
A13	P1*P3	0.00137	0.01201	0.11
A22	P2*P2	0.12042	0.03011	4.00
A23	P2*P3	-0.09216	0.01970	-4.68
A33	P3*P3	0.08899	0.01859	4.79
B11	P1*U1	-0.02991	0.03521	-0.85
B21	P2*U1	0.13176	0.05311	-2.48
B22	P2*U2	-0.02754	0.05365	-0.51
B32	P3*U2	-0.04086	0.03932	-1.04
B33	P3*U3	-0.04316	0.03002	-1.44
C11	P1*Z1	0.09671	0.04802	2.01
C12	P1*Z2	0.00300	0.04947	0.06
C13	P1*Z3	-0.01774	0.02698	-0.66
C14	P1*Z4	-0.01736	0.01800	-0.96
C15	P1*Z5	0.00634	0.00447	1.42
C21	P2*Z1	-0.05845	0.07232	-0.81
C22	P2*Z2	0.08187	0.07465	1.10
C23	P2*Z3	0.06163	0.04064	1.52
C24	P2*Z4	0.04235	0.02706	1.56
C25	P2*Z5	-0.02104	0.00670	-3.14
C31	P3*Z1	0.07086	0.05304	1.34
C32	P3*Z2	-0.08225	0.05468	-1.50
C33	P3*Z3	0.02311	0.02978	0.78
C34	P3*Z4	-0.01652	0.01985	-0.83
C35	P3*Z5	0.01891	0.00492	3.84
D1	Z1	0.02949	0.19108	0.15
D2	Z2	0.045865	0.19710	2.33
D3	Z3	-0.43113	0.10710	-4.03
D4	Z4	-0.10683	0.07141	-1.50
D5	Z5	0.17948	0.01753	10.24

Hausman's m=14.2958

Table 4
Efficiency and Fiscal Equalization Measures

VARIABLE	OBS	MEAN	ST. DEV.	MIN	MAX
1. Equalize budgets to mean: (p/c_{mean})					
EFF	310	0.708	0.058	0.567	1.000
FE	310	0.922	0.125	0.641	1.601
TO	310	0.649	0.072	0.481	0.998
COST/PUPIL	310	$0	0	0	0
2. Level budgets below mean up to mean: $(p/c)_{mean^*}$					
Districts below mean c					
EFF	180	0.726	0.055	.595	1.000
FE	180	0.843	0.058	.641	.921
TO	180	0.611	0.043	.481	.727
COST/PUPIL	180	$295.00	90.00	$151.42	$548.55
Districts above mean c					
EFF	130	0.682	0.052	0.567	0.810
FE	130	1.000	0	1.000	1.000
TO	130	0.682	0.052	0.567	0.810
COST/PUPIL	130	0	0	0	0
3. Equalize input prices at min: $(p/c)_{min}$					
EFF	310	0.708	0.058	0.567	1.000
FE	310	0.674	0.068	0.472	0.895
TO	310	0.474	0.030	0.391	0.564
COST/PUPIL	310	$900	115.33	$402	$1146
4. Equalize input prices at mean: $(p/c)_{mean}$					
EFF	310	0.708	0.058	0.567	1.000
FE	310	1.011	0.103	0.705	1.342
TO	310	0.711	0.046	0.584	0.849
COST/PUPIL	310	-$43	215.21	-$969.50	$391.54

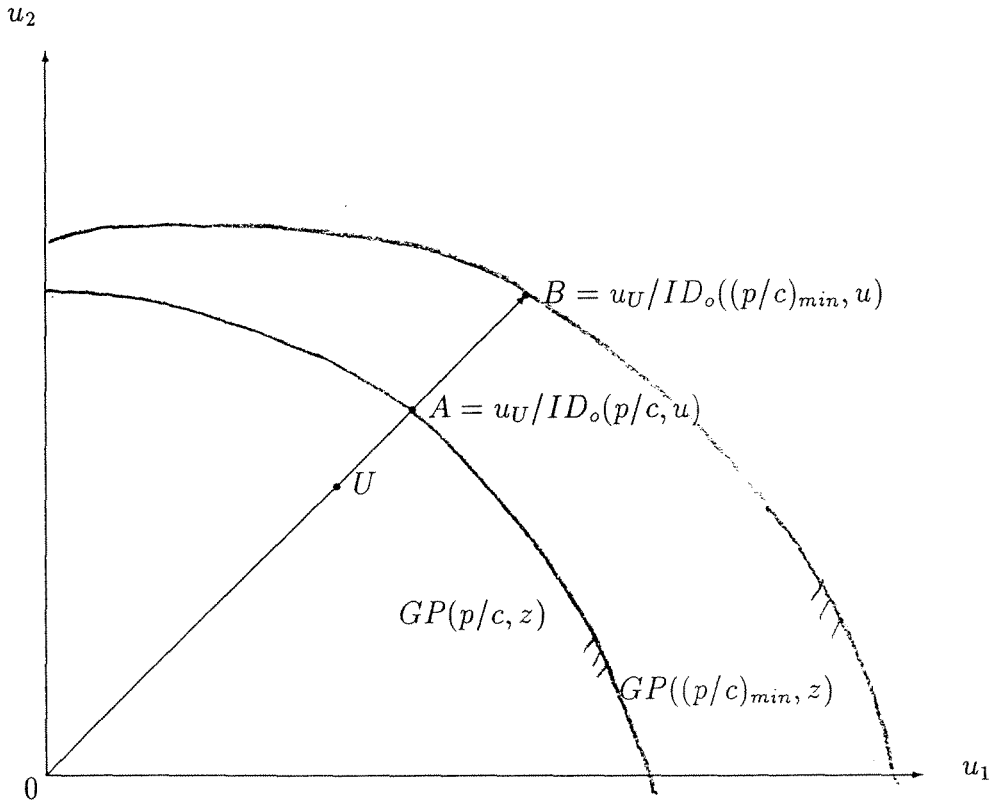

Figure 1: The Indirect Output Distance Function