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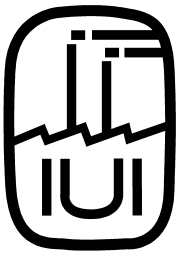

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# Campaigning and Ambiguity when Parties Cannot Make Credible Election Promises

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# Campaigning and Ambiguity when Parties cannot make Credible Election Promises*

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November 20, 2001

**Abstract**

This paper studies a model of how political parties use resources for campaigning to inform voters. Each party has a predetermined ideology drawn from some distribution. Parties choose a platform and campaign to inform voters about the platform. We find that, the farther away parties are from each other (on average), the less resources are spent on campaigning (on average). Thus, if parties are extreme, less information is supplied than if parties are moderate. We also show that if a public subsidy is introduced, we have policy convergence, given some mild technical restrictions on the public subsidy.

**Keywords** : Political Parties, Campaigning.

**JEL Classification** : C72, D72, D89

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# 1 Introduction

In political science, an important issue is the degree of information transmission between political representatives and the electorate. This issue has been studied in models with costless advertising by for example Alesina and Cuikerman (1990), Banks (1990), Harrington (1992,1993) and Martinelli and Matsui (1997). Banks finds that, if the realized policy of a party is far away from the median of the voter distribution, voters are able to infer the true policy from the announced platform of that party. If the platform is close to the median of the voter distribution, this is not the case. Martinelli (1997) has studied whether voters can learn from parties that have private information during the electoral process. Schultz (1996) studies a situation where parties posses more information about the true state of the world compared with voters. He finds that polarization leads to non-revealing sequential equilibria. However, in all of these papers advertising is costless. In reality, a feature of campaigning is that it is costly to send information to voters.

Another important aspect is how public subsidies affect campaign spending and the policy outcome. Ortuno Ortin and Schultz (2000) analyses this aspect. They find that a subsidy that depends on the vote share leads to policy convergence. Since the outcome in an election is a lottery between the two parties and voters are risk averse, policy convergence increases the utility of voters. A slight disadvantage in their model is that uninformed and informed voters have the same perception of the policy of the party. Even uninformed voters can infer the true policy of the parties. Thus, it does not really matter whether the subsidy is directed towards campaign spending or some other purpose.

The main aim of this paper is twofold. First, we analyze how the resources spent on campaigning depend on how close parties are to each other and how this in turn affects voters. Second, we show how public funding affects the policy implemented. Also, we provide theoretical support for the empirical fact that money affect voters most when races are close.

We study a symmetric model with two parties. Each party has a predetermined policy (or type) that is drawn from some distribution. One could think of the policy as the optimal

policy of the party, conditional of being elected. The parties care about the number of votes/the probability of winning as well as the consumption of some private good. The private good can be interpreted as a shortcut for spending in future elections. Each party has access to resources that can be used for campaigning. Parties choose a platform, not necessarily equal to the policy of the party, and a level of campaign spending. Since voters do not know the platform of the parties, the parties campaign to inform the voters about the platform. Given the platforms and the strategies of the parties, voters update their beliefs and then vote sincerely for the parties.

Since pooling equilibria can be ruled out by a restriction on beliefs, we focus on separating equilibria. We show that, if the policies of a given party are not too similar, there is a unique separating equilibrium.

We also find that, the farther away parties are from each other, in terms of the (prior) expected policy of the two parties, the less information is supplied (on average) in equilibrium. The motivation is the following. If a party informs a voter, the risk of voting for the party vanishes. Since voter preferences are flat when parties are close to each other, the effect of a risk reduction on voters is large when parties are close to each other. The effect makes parties gain more votes by informing when parties (on average) are moderate. Then spending increases and voters are (on average) more informed when parties are moderates. Thus, extremism leaves more voters uninformed.

Furthermore, we find that the introduction of a public subsidy on campaign spending leads to convergence in expected policy, given mild restrictions on how the public subsidy is constructed. Given these restrictions, the subsidy increases spending by types close to the mean (and median) of the voter distribution. These types are then more likely to win the election, leading to policy convergence.

The results in the paper are in line with some stylized facts of campaign spending. Empirical evidence in Erikson and Palfrey (2000) indicate that spending is largest and money matters most, i.e., affects the vote share most, in elections where parties have approximately the same probability of winning. We show that this result holds in the model. The reason

is the following. When parties are asymmetrically located they have different probabilities of winning. Then the voters that change their voting behavior when being informed are located far away from the median of the voter distribution. Since few voters are located asymmetrically, few voters change their mind when being informed. The effect of spending on the probability of winning is then small which leads to low spending.

One of the influences of this paper is Harrington and Hess (1996). In Harrington and Hess campaigning is explicitly modeled. Parties are assumed to have a fixed initial policy. Parties can use resources either to move their policy closer to the opponent (positive campaigning) or to move their opponents policy further away from the party's own policy (negative campaigning). However, there is no explicit model of why expenditures can affect voter's perceptions of the parties. Thus, the information processing by voters is modeled as a black box.

The paper by Chappell (1994) has a more sophisticated model of voter behavior. There are two parties that can choose either to spend an endowment on campaigning or not. In the model campaigning is assumed to be truthful. Also, only two possible levels of campaigning are allowed. Existence of equilibrium cannot be proven even in this simple setup. In contrast, in the model presented here, equilibria exist. Also, the paper does not address the issue studied in this paper.

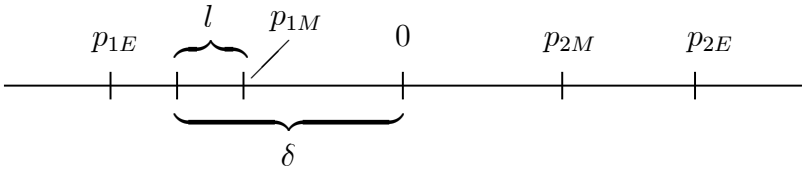
Another paper that analyzes campaigning is Potters, Sloof and van Winden (1997). In their paper, an incumbent faces challengers of two different types. In a separating equilibrium, only the "good" type spends resources on campaigning. Thus, the level of expenditures serves as a signal of the challengers' type. However, whether information transmission occur or not do not depend on the positions of the parties but only on the benefits of being in office and campaign costs. Thus, the effect of changes in extremism is not addressed.

In section 2 the model is described and in section 3 we characterize equilibria and study how spending depends on how extreme parties are and how this affects voters. Section 4 analyzes the effects of public subsidies, section 5 analyzes asymmetric equilibria and section 7 concludes.

# 2 The Model

There are two parties and a continuum of voters. The policy space is the real line. Each party has a predetermined policy (or type). The policy is interpreted as the optimal policy of the party, conditional of being elected. Let $P_k = \{p_{kE}, p_{kM}\}$ denote the set of possible policies for party $k$. Consider the location of the policies of party one. We assume that $p_{1M} = p_{1E} + 2l$ where $l > 0$ and $p_{1M} < 0$. Party 2 is located symmetrically on the opposite side of the origin, i.e., we have $p_{2M} = -p_{1M}$ and $p_{2E} = -p_{1E}$. Define $\delta = -\left(\frac{1}{2}p_{1M} + \frac{1}{2}p_{1E}\right)$ as the distance from the origin to the mean of the prior distribution for party 1. Thus, a party is either far away from the median voter or close to the median voter, relative to the expected policy. Note that $p_{1M} < 0$ implies $\delta > l$. The following figure illustrates how parties are located.

**Figure 1**. The locations of the policies of the parties.




Thus, the policy of party 1 is always smaller that the policy of party 2. A motivation for this assumption is that, for example, voters usually know that republicans are always to the right of democrats. [1]

For both parties, each policy is drawn with probability $\frac{1}{2}$. [2] Thus, the probability that a policy $p_{ki} \in P_k$ is drawn for party $k$ is independent of the policy drawn for party $j \neq k$. Let $P = P_1 \times P_2$ with generic element $p$.

[1] If this were not the case the following might occur in a separating equilibrium. As is shown below, voters are either informed about or uninformed about the platform of the parties. Suppose that $l < p_{2M} < 0 < p_{1M}$ and the strategy profiles are symmetric and reveals the type of each party. Then, any voter that is not informed by any of the parties and has ideal point below (above) zero prefers party 1 (2) while any voter informed by both parties that has ideal point below (above) zero prefers party 2 (1). Thus, the electorate "switches" completely.

[2] Most of the results hold without this assumption. However, the stability analysis is simplified.

A party is concerned about getting as many votes as possible/maximizing the probability of winning the election, as well as the consumption of some private good. The private good can be viewed as a shortcut for spending in future elections. Each party has access to some resource $\omega > 0$, which can be used either for informing voters or for consumption. Thus, the parties have access to the same amount of resources. Let $v_k$ denote the vote share/probability of winning for party $k$.

Parties do not present their true policy to voters. Instead, parties announce platforms that might be revealed to voters. We assume that it is costly for parties to announce a platform different from the policy. A motivation for this cost is that voters condition their voting behavior in an election on the difference between announced and implemented platforms in the past. If a party wins an election today with a given probability, the likeliness to win in the future decreases if the distance between the implemented policy and the platform increases. Thus, the cost can be thought as being caused by the effects on the performance of the party in future elections. In particular, given some policy $p_k \in P_k$ and some platform $q_k \in \mathbb{R}$, we assume the utility from the votes a party gets is given by $\theta(p_k, q_k)v_k$ where $\theta$ is concave and decreasing in the distance between $p_k$ and $q_k$. Given some spending level $c_k$, party $k$ of type $i = E, M$ has the following utility function

$$u_k(q_k, c_k, v_k, p_k) = \omega - c_k + \theta(p_k, q_k)v_k.$$

As we will see below, $v_k$ in general depend on spending and platforms of both parties.

An alternative interpretation of the model is that parties have to raise money for campaigning. Then $-c_k$ is the disutility from fund-raising.

Voters vote sincerely, i.e. vote for the party that gives them the highest expected utility, given their beliefs concerning the policies. Since there is a continuum of voters, strategic voting is not an issue. The (von Neumann-Morgenstern) preferences for voters are single peaked. In particular, we assume that, given some enacted policy $p_i$ of party $i$, the payoff when voting for party $i$ is

$$V(p_i - x) = -(p_i - x)^2,$$

where $x \in \mathbb{R}$ is the ideal point of the voter. The population of voters can be described by the distribution of the voters' ideal points. Let the density function be denoted by $f$ and the cumulative distribution function by $F$. We assume that $f$ is symmetric around zero and that $f(v) > 0$ for all $v \in \mathbb{R}$.

The timing in the model is the following. First, the policy of each party is revealed to the party. Then, the parties simultaneously choose platform and spending. Next, parties campaign and the election takes place. Finally, the policy of the winning party is implemented.

We assume that the policy of a party is not known to the other party when the platform and spending is chosen. Also, voters do not know the policies. A party can announce any platform in the policy space. We restrict attention to pure strategies. Let $q_k(p_k)$ denote the announcement of party $k$ with ideal policy $p_k \in P_k$ and let $q(p) = (q_1(p_1), q_2(p_2))$. Let $c_k(p_k) \in [0, \omega]$ denote the resources party $k$ of type $p_k \in P_k$ spends on campaigning. Also, let $c(p) = (c_1(p_1), c_2(p_2))$ and let $X_k = \mathbb{R} \times [0, \omega]$ denote the strategy space for party $k$.

## 2.1 Campaigning

Campaign expenditures and initially platforms are unobservable by voters. Parties use campaigning to inform voters about the platform of the party. That campaign expenditures are unobservable is open to discussion. During election campaigns, the levels are of campaign expenditures are sometimes debated. Thus, it might be reasonable that some voters have an idea of the amount of expenditures. On the other hand, it cannot be expected that all voters know the level of expenditures. For simplicity, we then focus on the case where expenditures are unobservable. Thus, voters can only extract information about the policy of a party when observing the platform of the party. [3]

If a party $k$ spends $c_k \in [0, \omega]$, voters are informed about the platform of the party with probability $\rho(c_k)$ where $\rho(0) = 0$, $\rho(c_k) < 1$ and $\rho'(c_k) \geq 0$. Also, $\rho$ is concave and twice continuously differentiable on $(0, \omega]$. Since the population is infinite, $\rho(c_k)$ is also the fraction

[3] If campaign expenditure were observable to voters, the voters could infer the policy of a party by observing the resources spent, if they depend on the policy of the party.

of the population informed the party.

The effects on voters of receiving information are twofold. First, informing a voter leads to a reduction in risk of voting for the party that informed. Since voters are risk averse, this effect makes the voter like the party that informed the voter more. Second, by revealing the platform (and hence the policy, if we study separating equilibria), a party also affects voter beliefs about the policy of the party. For example, if the expected policy is closer to the voters' ideal point than the actual policy, the utility of a voter decreases.

Messages cannot be directed to specific groups of voters. Also, the probability that a voter is reached by one party is independent of the probability that he is reached by the other.

## 2.2 Voter Beliefs

Voters form beliefs of the policy of a party, depending on whether they observe the platform of the party or not. Suppose that $q_1 \in \mathbb{R}$ and $q_2 \in \mathbb{R}$ are the platforms of parties 1 and 2. Given the spending choices of the parties, voters might observe both, one or none of the platforms. Let

$$o_k = \begin{cases} q_k & \text{if party } k \text{ has informed the voter} \\ \varnothing & \text{otherwise} \end{cases}$$

denote the observation of party $k$ and let $o = (o_1, o_2)$. Also, let $O$ denote the set of possible observations. Let $b_{ki}(o_k)$ denote the belief that party $k$ is of type $i$ when the voter observes $o_k$. Also, let $b(o) = (b_{1M}(o_1), b_{2M}(o_2))$.

Let $y(b(o))$ denote (the ideal point of) the indifferent voter when voter beliefs are given by $b(o)$. All voters observing $o$ with ideal points to the left of this point votes for party one and voters with ideal points to the right votes for party two. Thus, for a given observation, sincere voting implies that voter strategies can be described by finding the indifferent voter $y(b(o))$ for each $o \in O$.

Quadratic preferences implies that the indifferent voter, given beliefs $b(o)$, is

$$y(b(o)) = \frac{\bar{p}_1(b(o)) + \bar{p}_2(b(o))}{2} + \frac{\sigma_1^2(b(o)) - \sigma_2^2(b(o))}{2(\bar{p}_1(b(o)) + \bar{p}_2(b(o)))}. \quad (1)$$

where $\bar{p}_k(b(o))$ is the expected platform and $\sigma_k^2(b(o))$ the variance of party $k$, associated with beliefs $b(o)$.

## 2.3 Party payoffs

The payoff for party 1 of type $i$ is, given policies $p$, platforms $q = (q_1, q_2)$, party spending choices $c = (c_1, c_2)$ and voter beliefs $b(\cdot)$,

$$\tilde{u}_1(q, c, b(\cdot), p_{1i})$$

$$= \omega - c_1 + \theta(p_{1i}, q_1)\left\{\rho(c_1)\rho(c_2)\int_{-\infty}^{y(b(q_1,q_2))} f(v)dv + \rho(c_1)[1-\rho(c_2)]\int_{-\infty}^{y(b(q_1,\varnothing))} f(v)dv\right.$$

$$\left. +[1-\rho(c_1)]\rho(c_2)\int_{-\infty}^{y(b(\varnothing,q_2))} f(v)dv + [1-\rho(c_1)][1-\rho(c_2)]\int_{-\infty}^{y(b(\varnothing,\varnothing))} f(v)dv\right\}. \quad (2)$$

The vote share, i.e., the term within the curly brackets, deserves explanation. To see that the votes are as above, consider voters informed by both parties. The indifferent voter is $y(b(q_1, q_2))$ and the share of the electorate that observes both platforms is $\rho(c_1)\rho(c_2)$. Then the share of the total electorate that is informed by both parties and votes for party one is $\rho(c_1)\rho(c_2)\int_{-\infty}^{y(b(q_1,q_2))} f(v)dv$. A similar argument establishes that the share of the total electorate that is informed by party one and votes for party one is $\rho(c_1)[1-\rho(c_2)]\int_{-\infty}^{y(b(q_1,\varnothing))} f(v)dv$ and so on. The total vote share for party one is the sum over all possible observations for the voters. The expected payoff for party 2 is computed in a similar fashion.

# 3 Equilibrium

An equilibrium is a strategy profile for the parties and beliefs for voters such that, first, each party chooses a platform and a spending level, taking the strategy of the other party and voter beliefs as given and second, voters revise their beliefs using Bayes rule and vote sincerely given the strategy profile of the parties. Also, there is a consistency restriction on beliefs.

**Definition 1** *A **Voting Equilibrium** is a* $q^*(\cdot), c^*(\cdot)$ *and* $b^*(\cdot)$*such that*

*i) for* $k = 1, 2$, $l \neq k$ *and* $p \in P$, $(q_k^*(p_{ki}), c_k^*(p_{ki}))$
$= \arg \max_{(q_k, c_k) \in X_k} \sum_{j \in \{E,M\}} \frac{1}{2} \tilde{u}_k(q_k, q_l^*(p_{lj}), c_k, c_l^*(p_{lj}), b^*(\cdot), p_{ki})$

*ii) a) given* $q^*(\cdot)$ *and* $c^*(\cdot)$*, voters vote sincerely and revise beliefs using Bayes rule whenever possible*

*b) if* $q_k^*(p_{kM}) \neq q_k^*(p_{kE})$ *then* $b_{ki}^*(q_k^*(p_{ki})) = 1$ *for* $i = E, M$.

This definition almost conforms to a sequential equilibrium. We have added the requirement of sincere voting for the voters. The consistency condition in the general definition of sequential equilibrium requires that equilibrium strategies and beliefs is a limit of completely mixed strategies and beliefs where beliefs are computed from strategies using Bayes rule. Condition ii) b) is an implication of this requirement. As long as spending of one type is positive, Condition ii) b) follows from Bayes rule. Thus, the condition only has effect when both types of a party spend zero. In particular, the condition rules out the following awkward separating equilibrium, where both parties always spend zero and announce the ideal policy of the party, irrespective of the type of the party. Posterior voter beliefs when informed by a party are equal to prior beliefs. Since spending is zero, these beliefs do not violate Bayes rule, i.e., ii) a). It is easy to verify that the strategy profile and beliefs also satisfies i) in the definition above. However, the last part of ii) b) is violated.

We restrict attention to "symmetric" equilibria, i.e., parties with policy located with equal distance to the origin choose equal spending levels and platforms located with equal

distance to the origin.

## 3.1 Separating equilibrium

This section analyzes the spending and platform choices of the parties in a separating equilibrium. In general, there are a lot of separating equilibria. However, a mild restriction on beliefs is used to reduce the set of equilibria.

### 3.1.1 Extreme type spending and platform

Now consider the optimal spending choice when the policy of a party is extreme. Note that, since we study symmetric equilibria, we can deduce that the indifferent voter is at 0, when voters are informed by none of the parties.

**Lemma 1** *Suppose $q^*(\cdot), c^*(\cdot), b^*(\cdot)$ is a symmetric voting equilibrium. The extreme type of both parties spend zero.*

**Proof.** Consider party 1. By symmetry, a similar argument holds for party 2. Since we analyze separating equilibria, voters infer that party 1 is of type $E$ when observing $q_1^*(p_{1E})$. Note that, using (1), we have

$$y(b(o)) = \frac{(b_{2E}(o_2) - b_{1E}(o_1))\,\delta l}{\delta - (1 - b_{1E}(o_1) - b_{2E}(o_2))l}. \tag{3}$$

This expression is increasing in $b_{2E}(o_2)$ and decreasing in $b_{1E}(o_1)$.

**Step 1**: $y(b^*(q_1^*(p_{1E}), \varnothing)) \leq y(b^*(\varnothing, \varnothing))$ and $y(b^*(q_1^*(p_{1E}), q_2^*(p_2))) \leq y(b^*(\varnothing, q_2^*(p_2))$.

In a separating equilibrium we have $b_{1E}^*(q_1^*(p_{1E})) = 1$. Thus, $b_{1E}^*(q_1^*(p_{1E})) \geq b_{1E}^*(\varnothing)$. Consider $y(b^*(q_1^*(p_{1E}), \varnothing))$ and $y(b^*(\varnothing, \varnothing))$. Since $o_2 = \varnothing$ then $b_{2E}^*(o_2)$ is the same in both $y(b^*(q_1^*(p_{1E}), \varnothing))$ and $y(b^*(\varnothing, \varnothing))$. Since $b_{1E}^*(q_1^*(p_{1E})) \geq b_{1E}^*(\varnothing)$ and (3) is decreasing in $b_{1E}(o_1)$ we have

$$y(b^*(q_1^*(p_{1E}), \varnothing)) \leq y(b^*(\varnothing, \varnothing)).$$

A similar argument establishes that

$$y(b^*(q_1^*(p_{1E}), q_2^*(p_2))) \leq y(b^*(\varnothing, q_2^*(p_2))).$$

**Step 2**: $c_1^*(p_{1E}) = 0$.

Using (2), the effect of a change in spending on votes is

$$\rho'(c_1)\left\{\rho(c_2)\int_{y(b^*(\varnothing,q_2^*(p_2)))}^{y(b^*(q_1^*(p_{1E}),q_2^*(p_2)))} f(v)dv + [1-\rho(c_2)]\int_{y(b^*(\varnothing,\varnothing))}^{y(b^*(q_1^*(p_{1E}),\varnothing))} f(v)dv\right\}.$$

Since $y(b^*(q_1^*(p_{1E}),\varnothing)) \leq y(b^*(\varnothing,\varnothing))$ and $y(b^*(q_1^*(p_{1E}),q_2^*(p_2))) \leq y(b^*(\varnothing,q_2^*(p_2))$, by Step 1, this expression is negative and party 1 loses by informing. Then $c_1^*(p_{1E}) = 0$. A similar argument holds for party 2. ■

Thus, each party spends nothing if the realized platform is extreme. The reason is the following. First note that, since we analyze a separating equilibrium, if a voter is reached by a campaign message, the voter becomes aware of the true policy of the party. Second, to increase the vote share, a party must convince the voters in the middle of the voter distribution. If a voter in the middle of the voter distribution observes $q_1^*(p_{1E})$, the voter transfers probability from a good outcome, i.e., the party being of the moderate type and close to the ideal point of the voter, to a bad. This makes the voter like the party less and leads to zero spending.

Now consider the platform choice. First, since $f(v) > 0$ for any $v$ the vote share/probability of winning is positive. Also, since the extreme type do not spend any resources on campaigning, the payoff increases when the platform moves closer to $p_{kE}$. Thus, the extreme type always chooses $q_k(p_{kE}) = p_{kE}$.

### 3.1.2 Moderate type spending and platform

The spending and platform choice of the moderate type are not as easy to find as for the extreme type. To simplify notation, let $c_M = c_1(p_{1M}) = c_2(p_{2M})$ denote the spending choice and $q_M = q_1(p_{1M}) = -q_2(p_{2M})$ the platform choice of the moderate type.

To characterize equilibria, it is convenient to define

$$z(c_M, \delta) = \frac{l\delta}{\delta\,(2 - \rho(c_M)) - l\,(1 - \rho(c_M))}. \tag{4}$$

In a symmetric equilibrium when moderate type spending is $c_M^*$ and platform is $q_M^*$, using (1) and Bayes rule, some algebra shows that $z(c_M^*, \delta) = y(b^*(q_M^*, \varnothing)) = -y(b^*(\varnothing, -q_M^*))$. Thus, given spending $c_M$ and $\delta$, $z(c_M, \delta)$ is the ideal point of the indifferent voter; given that party one informs the voter. Note that $z(c_M, \delta)$ is increasing in $c_M$ and decreasing in $\delta$. Since $\delta > l$ and $\rho(c_M) < 1$, we have, for all $\delta \in \Delta$, $\frac{l}{2} < z(c_M, \delta) < l$. Also, in a symmetric equilibrium, we have $y(b^*(\varnothing, \varnothing)) = y(b^*(q_M, -q_M)) = 0$. Using symmetry and (2), the set of candidate equilibrium spending levels $c_M$ for a given $q_M$ and $\delta$ are given by

$$\theta(p_{1M}, q_M)\rho'(c_M) \int_0^{z(c_M,\delta)} f(v)dv - 1 = 0. \tag{5}$$

This is the first-order condition of the moderate type with respect to spending, assuming that $c_M \in (0, \omega)$ and that voters believe that parties spend $c_M$. In the remainder of the paper, we restrict attention to equilibria where the moderate type chooses an interior spending level, i.e., $c_M \in (0, \omega)$. We assume that $\theta(p_{1M}, p_{1M})\rho'(\omega) \int_0^l f(v)dv < 1$ and $\lim_{c\to 0} \theta(p_{1M}, p_{1M})\rho'(c) \int_0^{\frac{l}{2}} f(v)dv > 1$ which leads to interior choices by the moderate type. The first restriction guarantees that the party chooses to spend less than $\omega$ and the last that spending is positive.

Now consider the platform choice of the moderate type. Since the equilibrium is separating the platform must be chosen such that the extreme type do not want to mimic. Given that the moderate type chooses $q_M$ and $c_M$, let $c_E(q_M, c_M)$ denote the optimal spending level if the extreme type mimics the moderate type. Using (2) and symmetry, the (maximal) payoff for the extreme type when mimicking is

$$\omega - c_E(q_M, c_M) + \theta(p_{1E}, q_M) \left[\frac{1}{2} + \left[\rho(c_E(q_M, c_M)) - \frac{1}{2}\rho(c_M)\right] \int_0^{z(c_M,\delta)} f(v)dv\right]. \tag{6}$$

For some given spending level $c_M$, let $\breve{q}_M \geq p_{1E}$ be the value of $q_M$ such that

$$\omega - c_E(q_M, c_M) + \theta(p_{1E}, q_M) \left[ \frac{1}{2} + \left[ \rho(c_E(q_M, c_M)) - \frac{\rho(c_M)}{2} \right] \int_0^{z(c_M,\delta)} f(v)dv \right]$$

$$- \left\{ \omega + \theta(p_{1E}, p_{1E}) \left[ \frac{1}{2} - \frac{1}{2}\rho(c_M) \int_0^{z(c_M,\delta)} f(v)dv \right] \right\} = 0. \tag{7}$$

The term on the first row is the expected utility for the extreme type when choosing $q_M$ and the term on the second row is the expected utility when choosing $p_{1E}$. Since the first row is decreasing in $q_M$ when $q_M \geq p_{1E}$ the value of $\breve{q}_M$ is unique.

Condition (7) is a no-mimicking condition. If the moderate type chooses $\breve{q}_M$ and $c_M$ then the extreme type is just indifferent between mimicking or not. Since $p_{1M}$ is better for the moderate type than $\breve{q}_M$ when $\breve{q}_M < p_{1M}$, the moderate type never chooses $\breve{q}_M < p_{1M}$. Thus, let $\bar{q}_M = \max\{\breve{q}_M, p_{1M}\}$ denote the platform choice of the moderate type. If $q_M = p_{1M}$ then the extreme type weakly looses by mimicking.

To simplify notation we let, for $k = 1, 2$, $i = E, M$ and $l \neq k$,

$$\tilde{u}_k^* \left( q^* \left( \cdot \right), c^* \left( \cdot \right), p_{ki} \right) = \sum_{j \in \{E,M\}} \frac{1}{2} \tilde{u}_k(q_k^*(p_{ki}), q_l^* \left( p_{lj} \right), c_k^*(p_{ki}), c_l^* \left( p_{lj} \right), b^*(\cdot), p_{ki})$$

denote the expected equilibrium payoff for type $i$ of party $k$.

Without any further restrictions the set of separating equilibria is large. In particular, all $c_M$ and $q_M \geq \bar{q}_M$ such that the first-order condition (5) holds and the left hand side of (7) is nonpositive are separating equilibria. However, the following restriction on beliefs helps us reduce the set of equilibria.

Say that an equilibrium satisfies **Equilibrium Dominance** for party $k$ (and $l \neq k$ ) if whenever,

$$\tilde{u}_k^* \left( q^* \left( \cdot \right), c^* \left( \cdot \right), p_{ki} \right) > \sum_{j \in \{E,M\}} \frac{1}{2} \tilde{u}_k(\hat{q}_k, q_l^* \left( p_{lj} \right), \hat{c}, c_l^* \left( p_{lj} \right), b_{kM}(\cdot), b_{lM}^*(\cdot), p_{ki}),$$

for all $\hat{c} \in [0, \omega]$ and all $b_{kM}(\cdot)$ and

$$\tilde{u}_k^* \left( q^* \left( \cdot \right), c^* \left( \cdot \right), p_{kh} \right) \leq \sum_{j \in \{E,M\}} \frac{1}{2} \tilde{u}_k(\hat{q}_k, q_l^* \left( p_{lj} \right), \hat{c}, c_l^* \left( p_{lj} \right), b_{kM}(\cdot), b_{lM}^*(\cdot), p_{kh}),$$

for some $\hat{c} \in [0, \omega]$ and $b_{kM}(\cdot)$, then voters puts probability one on type $h$ when observing $\hat{q}_k \in \mathbb{R}$.

Thus, if type $i$ looses by choosing $\hat{q}_k$ for all possible beliefs and type $h$ weakly gains for some belief, then voters put probability one on type $h$ when observing $\hat{q}_k$.

Also, say that an equilibrium satisfies **Equilibrium Dominance** if it satisfies platform dominance for parties 1 and 2. The following result shows that the equilibria satisfying equilibrium dominance can be characterized by (5) and (7).

**Theorem 1** *$q^*(\cdot), c^*(\cdot), b^*(\cdot)$ is a separating voting equilibrium satisfying equilibrium dominance if and only if $q_k^*(p_{kE}) = p_{kE}$, $c_k^*(p_{kE}) = 0$ and expressions (5) and (7) (if $q_M^* > p_{1M}$) are satisfied for $q_M^* = q_1^*(p_{1M}) = -q_2^*(p_{2M})$ and $c_M^* = c_k^*(p_{kM})$ for $k = 1, 2$.*

**Proof.** Both i) and ii) follows from the following.

First, we show that when (5) (and when $q_M^* > p_{1M}$, (7) ) are satisfied for $q_M^*$ and $c_M^*$ as above, then $q^*(\cdot), c^*(\cdot), b^*(\cdot)$ is a voting equilibrium satisfying equilibrium dominance. Suppose that voters have the following beliefs. Voters put probability one on the extreme type if they see an announcement $r < q_M^*$ and probability one on the moderate type if they see an announcement $r \geq q_M^*$. Also, beliefs when being uninformed by some party are computed using $c_M^*$ and Bayes rule. By choice of $q_M^*$, the extreme type do not want to mimic the moderate type or choose an announcement $r > q_M^*$ . Then the extreme type choose to spend zero, by Lemma 1, and announces the ideal policy. The moderate type do not want to announce a platform smaller than $q_M^*$ since then voters believe that the party is extreme. Also, by construction of beliefs, it is optimal to announce $q_M^*$. The choice of spending then follows from (5). Thus, $q^*(\cdot), c^*(\cdot), b^*(\cdot)$ is an equilibrium.

Second, let us show that if $q^*(\cdot), c^*(\cdot), b^*(\cdot)$ is a voting equilibrium satisfying equilibrium dominance then (5) (and when $q_M^* > p_{1M}$, (7) ) are satisfied. We show this by contradiction. Suppose the conditions above are not satisfied and assume that $q^*(\cdot), c^*(\cdot), b^*(\cdot)$ is an equilibrium. If (5) is not satisfied, then spending cannot be optimally chosen, contradicting $q^*(\cdot), c^*(\cdot), b^*(\cdot)$ being an equilibrium. Consider (7). First, suppose $q_M^* < \bar{q}_M$ in the right

hand side of (7). If $q_M^* < p_{1M}$ it is profitable for the moderate type to deviate and choose $p_{1M}$ instead and if $q_M^* \geq p_{1M}$ it is profitable for the extreme type to mimic the other type, again contradicting $q^*(\cdot), c^*(\cdot), b^*(\cdot)$ being an equilibrium. The remaining possibility is that $q_M^* > \bar{q}_M$ in the right hand side of (7). Then, the payoff from mimicking for the extreme type is strictly smaller than the equilibrium payoff. To derive a contradiction, we show that the moderate type has a profitable deviation.

Before we proceed with the proof, consider a platform choice $q_1$ by party one and corresponding voter beliefs. First, consider voters that observe $q_1$ only. Since voters does not observe a deviation by party 2, $b_{2E}^*(\varnothing) = \frac{1-\rho(c_M^*)}{2-\rho(c_M^*)}$. If $b_{1E}^*(q_1) = 0$, i.e., $b_{1E}^*(q_1) = b_{1E}^*(q_M^*)$, then $y(b^*(q_1, \varnothing)) = z(c_M^*, \delta)$.

Second, consider voters that observe both platforms. Then $b_{2E}^*(-q_M^*) = 0$. If $b_{1E}^*(q_1) = 0$ then $y(b^*(q_1, \varnothing)) = z(c_M^*, \delta)$ and $y(b^*(q_1, -q_M^*)) = 0$. Furthermore,

$$\frac{\partial\left(y(b^*(q_1,\varnothing)) - y(b^*(q_1,-q_M^*))\right)}{\partial b_{1E}^*(q_1)}$$

$$= \frac{\left(y(b^*(q_1,\varnothing))\right)^2 b_{2E}^*(\varnothing)\left\{\left[\delta + \left(2b_{2E}^*(\varnothing) - 1\right)l\right]\delta - \left[\delta + \left(2b_{1E}^*(q_1) - 1\right)l\right]^2\right\}}{\left(\left(b_{2E}^*(\varnothing) - b_{1E}^*(q_1)\right)\delta\right)^2} \frac{\left(y(b^*(q_1,-q_M^*))\right)^2}{2\delta\left(2\delta l b_{1E}^*(q_1)\right)^2}$$

**Case 1.** $q_M^* > p_{1M}$.

Consider the following deviation by party one. Suppose party one chooses $q_1^\epsilon = q_M^* - \epsilon$ instead where $\epsilon$ is such that $q_1^\epsilon > \bar{q}_M$ and $q_1^\epsilon > p_{1M}$. Let $c_1^\epsilon(p_{1j})$ denote the optimal spending level of type $j$ when choosing $q_1^\epsilon$.

**Step 1.** Showing that the extreme type looses by choosing $q_1^\epsilon$ for $\epsilon$ small.

Using (2) and $\int_{-\infty}^{0} f(v)dv = \frac{1}{2}$, the payoff when deviating is, for the extreme type,

$$\omega - c_1^\epsilon(p_{1E}) + \frac{\theta(p_{1E}, q_1^\epsilon)}{2}\left\{1 + \rho(c_1^\epsilon(p_{1E}))\rho(c_M^*)\left[\int_0^{z(c_M^*,\delta)} f(v)dv - \int_{y(b^*(q_1^\epsilon,-q_M^*))}^{y(b^*(q_1^\epsilon,\varnothing))} f(v)dv\right]\right.$$

$$\left. + 2\rho(c_1^\epsilon(p_{1E}))\int_0^{y(b^*(q_1^\epsilon,\varnothing))} f(v)dv - \rho(c_M^*)\int_0^{z(c_M^*,\delta)} f(v)dv\right\}. \tag{10}$$

Note that, using (6) and since $c_1^\epsilon(p_{1E})$ need not be optimal when mimicking, the payoff when mimicking is at least as large as

$$\omega - c_1^\epsilon(p_{1E}) + \frac{\theta(p_{1E}, q_M^*)}{2}\left[1 + [2\rho(c_1^\epsilon(p_{1E})) - \rho(c_M^*)] \int_0^{z(c_M^*,\delta)} f(v)dv\right]. \tag{11}$$

**Subcase 1.** Suppose $b_{1E}^*(q_1^\epsilon) < \frac{1-\rho(c_M^*)}{2-\rho(c_M^*)} = b_{2E}^*(\varnothing)$. Note that $b_{2E}^*(\varnothing) > \frac{1}{2}$. Then we have $\delta > \delta + (2b_{2E}^*(\varnothing) - 1)\, l \geq \delta + (2b_{1E}^*(q_1^\epsilon) - 1)\, l > 0$ implying $\frac{\partial\left(y(b^*(q_1^\epsilon,\varnothing)) - y(b^*(q^\epsilon))\right)}{\partial b_{1E}\left(q_1^\epsilon\right)} > 0$. Since $b_{1E}^*(q_1^\epsilon) \geq b_{1E}^*(q_M^*) = 0$, we have $y(b^*(q_1^\epsilon, \varnothing)) \leq z(c_M^*, \delta)$. Also, since $\frac{\partial\left(y(b^*(q_1^\epsilon,\varnothing)) - y(b^*(q^\epsilon))\right)}{\partial b_{1E}\left(q_1^\epsilon\right)} > 0$ and since $y(b^*(q_1^\epsilon, -q_M^*)$ is decreasing in $b_{1E}^*(q_1^\epsilon)$ we have

$$\int_{y(b^*(q_1^\epsilon,-q_M^*))}^{y(b^*(q_1^\epsilon,\varnothing))} f(v)dv \geq \int_0^{z(c_M^*,\delta)} f(v)dv \geq \int_0^{y(b^*(q_1^\epsilon,\varnothing))} f(v)dv \tag{12}$$

with strict inequalities when $b_{1E}^*(q_1^\epsilon) > 0$ and equalities when $b_{1E}^*(q_1^\epsilon) = 0$.

If $b_{1E}^*(q_1^\epsilon) > 0$ then, using (12) and continuity of $\theta$, (11) is larger than (10) for $\epsilon$ small. Thus, it cannot be profitable to deviate for the extreme type, since the payoff when choosing $p_{1E}$ is strictly larger than (11), i.e., the payoff when mimicking.

If $b_{1E}^*(q_1^\epsilon) = 0$ then, using (12) and continuity of $\theta$, the payoff when choosing $q_1^\epsilon$ is approximately equal to the payoff when mimicking, for $\epsilon$ small. Also, since (7) is violated strictly, the equilibrium payoff is strictly larger than the payoff when mimicking. Then, for $\epsilon$ small, the payoff when choosing $q_1^\epsilon$ is smaller than the equilibrium payoff.

**Subcase 2.** Suppose $b_{1E}^*(q_1^\epsilon) \geq \frac{1-\rho(c_M^*)}{2-\rho(c_M^*)}$.

Note that $b_{1E}^*(q_1^\epsilon) \geq b_{2E}^*(\varnothing) = b_{1E}^*(\varnothing)$ implies that the voter puts a higher probability on party one being of the extreme type when observing $q_1^\epsilon$ than when not observing $q_1^\epsilon$. Then, by using a similar argument as in Lemma 1, the gain from informing must be nonpositive. Thus, we have $c_{1E}^\epsilon = 0$. Using (2), since the extreme type chooses zero spending both when announcing $p_{1E}$ and $q_1^\epsilon$ and since $\theta(p_{1E}, p_{1E}) > \theta(p_{1E}, q_1^\epsilon)$, the equilibrium strategy of announcing $p_{1E}$ gives the extreme type a higher payoff than choosing $q_1^\epsilon$.

Subcase 1 and 2 implies that there is some $\bar{\epsilon} > 0$ such that, for all $\epsilon \leq \bar{\epsilon}$, the extreme type looses by choosing $q_1^\epsilon$.

**Step 2.** Showing that there is some belief such that the moderate type gains by choosing $q_1^\epsilon$.

Suppose voters believe with probability one that party 1 is of the moderate type when observing $q_1^\epsilon$. Then we have $y(b^*(q_1^\epsilon, -q_M^*)) = 0$ and $y(b^*(q_1^\epsilon, \varnothing)) = z(c_M^*, \delta)$. Then the payoff for the moderate type is

$$\omega - c_1^\epsilon(p_{1M}) + \theta(p_{1M}, q_1^\epsilon)\left\{\frac{1}{2} + \rho(c_1^\epsilon(p_{1M}))\int_0^{z(c_M^*,\delta)} f(v)dv - \frac{\rho(c_M^*)}{2}\int_0^{z(c_M^*,\delta)} f(v)dv\right\}$$

$$\geq \omega - c_M^* + \theta(p_{1M}, q_1^\epsilon)\left\{\frac{1}{2} + \frac{\rho(c_M^*)}{2}\int_0^{z(c_M^*,\delta)} f(v)dv\right\} \tag{13}$$

The inequality follows since $c_M^*$ is not necessarily optimal when $q_1^\epsilon$ is chosen.

Since $\theta(p_{1M}, q_1^\epsilon) = \theta(p_{1M}, q_M^* - \epsilon) > \theta(p_{1M}, q_M^*)$, the right hand side of (13) is larger than the equilibrium payoff. Thus, when $\epsilon < \bar{\epsilon}$, there are beliefs such that the moderate type gains when announcing $q_1^\epsilon$, while the extreme type always looses.

By equilibrium dominance, voters put probability one on the moderate type when observing $q_1^\epsilon$. Hence, using (13), it is profitable for the moderate type to deviate and announce $q_1^\epsilon$.

**Case 2.** $q_M^* < p_{1M}$.

A similar argument as in Step 1 establishes that it is profitable for the moderate type to deviate and announce $q_M^* + \epsilon$. ■

The intuition is the following. Clearly, (5) must hold in equilibrium. Suppose (7) does not hold. Then the payoff when mimicking, i.e., choosing $q_M^*$, is either strictly smaller or strictly larger than the equilibrium payoff for the extreme type. If it is profitable to mimic for the extreme type, we cannot have an equilibrium. The only remaining case is when the extreme type looses strictly when mimicking. We claim that it is not profitable to deviate and choose a platform $q_M^* - \epsilon$ for the extreme type. To see this, note that, since the extreme type looses strictly when mimicking and the payoffs of the parties are continuous in platforms, it cannot

be profitable to deviate and choose a platform close to $q_M^*$ if beliefs are the same as when $q_M^*$ is observed, i.e., voters believe that the party surely is of the moderate type. Also, it cannot be profitable to choose $q_M^* - \epsilon$, if voters put a positive probability on the extreme type, since voters are then less inclined to vote for the party. Thus it cannot be profitable to choose $q_M^* - \epsilon$ for any belief. Now consider the moderate type. If voters believes with probability one that party 1 is of the moderate type when observing $q_M^* - \epsilon$ (the same as when observing $q_M^*$), the party gains, since the payoff of the party is decreasing in the distance between the platform and the policy. Thus, there are beliefs such that the moderate type gains strictly when choosing $q_M^* - \epsilon$. Hence, by equilibrium dominance, voters put probability one on this type when observing $q_M^* - \epsilon$. Then the moderate type has a profitable deviation by choosing $q_M^* - \epsilon$.

## 3.2 Pooling Equilibrium

Now let us turn to pooling equilibria, i.e., both types announce the same platform. As is shown below, such equilibria are also possible in the model. They are, however, ruled out by a restriction on beliefs.

Note that, in a pooling equilibrium, at least one type has to spend a positive amount. If both types spend zero it is easily seen from (2) that the votes are unaffected by the announced platform. Then any type that announces a platform different from the policy gains by announcing the policy instead. Also, in equilibrium, the moderate type must spend a larger amount than the extreme type. To see this, note first that, by symmetry, for voters observing both or none of the parties, the ideal point of the indifferent voter is at zero. If spending of some type is to be positive, for voters observing only the platform of party one (two), the indifferent voter must be positive (negative). Otherwise, it cannot be profitable to inform voters and spending is zero for both types. The ideal point of the indifferent voter is positive (negative) only if the voter puts a higher probability on the moderate type, compared with the extreme type. This is the case only when the moderate type spends more than the extreme type.

The pooling equilibria can be characterized as follows. Let $c_M$ denote the spending level for the moderate type and let $c_E$ denote the spending level for the extreme type. Let $q_{ME}$ denote the platform choice (of both types) of party one. Then, for voters only observing the platform of party one, let $z(c_M, c_E, \delta)$ denote the indifferent voter. Using (2) and symmetry, the equilibrium payoff for type $i$ of party 1 is given by

$$\omega - c_i^* + \frac{\theta(p_{1i}, q_{ME}^*)}{2} \sum_{j \in \{M,E\}} \left\{ \frac{1}{2} + \left[\rho(c_i^*) - \rho(c_j^*)\right] \int_0^{z(c_M^*, c_E^*, \delta)} f(v)dv \right\}. \tag{14}$$

Spending and platform choices for party 1 in an equilibrium where both parties spend a positive amount is given by the solution to

$$-1 + \rho'(c_i^*)\theta(p_{1i}, q_{ME}^*) \int_0^{z(c_M^*, c_E^*, \delta)} f(v)dv = 0$$

for $i \in \{E, M\}$. Party 2 chooses the same spending levels and announces $q_2 = -q_{ME}^*$. We claim that both types actually must spend a positive amount. The reason is the following. Since the moderate type spends more than the extreme type, $q_{ME}^*$ is closer to $p_{1M}$ than $p_{1E}$. If $c_E^* = 0$, then type $E$ would choose to announce $p_{1E} \neq q_{ME}^*$, implying separation between the types.

Pooling equilibria are ruled out by a refinement on beliefs. In the refinements introduced in Cho and Kreps (1987) and Banks and Sobel (1987), receivers put probability zero on a type if there is some other type such that the set of receiver strategies for which the second type strictly gains is strictly larger than the set of receiver strategies for which the first type weakly gains.

Here a slightly different construction is used. To define refinement, first, let $D(p_{ki}, q_k)$ denote the set of beliefs such that, (where $l \neq k$ ),

$$\tilde{u}_k^*(q^*(\cdot), c^*(\cdot), p_{ki}) < \max_{c_k \in [0,\omega]} \sum_{j \in \{E,M\}} \frac{1}{2} \tilde{u}_k(q_k, q_l^*(p_{lj}), c_k, c_l^*(p_{lj}), b_{kM}(\cdot), b_{lM}^*(\cdot), p_{ki}) .$$

Thus, $D(p_{ki}, q_k)$ are all beliefs such that party $k$ of type $i$ is strictly better off than in the equilibrium when choosing $q_k$. Let $D^0(p_{ki}, q_k)$ denote the beliefs such that the above expression holds with equality.

Say that an equilibrium satisfies the **D1 property** if, for any out-of-equilibrium announcement $\hat{q}_k \in \mathbb{R}$ whenever

$$D\left(p_{ki}, \hat{q}_k\right) \cup D^0\left(p_{ki}, \hat{q}_k\right) \subset D\left(p_{kh}, \hat{q}_k\right) \tag{15}$$

then voters put probability one on type $h$ of party $k$.

Thus, if the set of beliefs for which type $i$ gains weakly are contained in the beliefs for which type $h$ gains strictly when choosing $\hat{q}_k$, voters put probability one on type $h$ when observing $\hat{q}_k$. Since voter strategies depend on beliefs, the definition above, while stated in terms of beliefs instead of receiver strategies is in line with the refinements introduced in Cho and Kreps (1987) and Banks and Sobel (1987). In particular, if the set $D\left(p_{ki}, q_k\right)$ becomes larger, then the set of voter strategies such that type $i$ of party $k$ gains also becomes larger.

**Lemma 2** *None of the pooling voting equilibria satisfies the D1 property.*

**Proof.** Consider some equilibrium where parties choose $q^*_{ME}, c^*_E$ and $c^*_M$.

Consider voter beliefs when party 1 chooses the platform $q^\epsilon_1 = q^*_{ME} + \epsilon$. If voters observe none of the platforms or only the platform of party 2 then voter beliefs are equal to equilibrium beliefs, since no deviation is observed. Thus, we have $y(b^*(\varnothing, \varnothing)) = 0$ and $y(b^*(\varnothing, -q^*_{ME})) = z(c^*_M, c^*_E, \delta)$.

Let $c^\epsilon_1\left(p_{1i}\right)$ denote the optimal spending for type $i$ when choosing $q^\epsilon_1$. The payoff when deviating is

$$\varphi\left(q^*_{ME}, b^*_{1E}\left(q^\epsilon_1\right), p_{1i}\right) = \omega - c^\epsilon_1\left(p_{1i}\right) + \frac{\theta(p_{1i}, q^\epsilon_1)}{2} \sum_{j\in\{E,M\}} \left\{ \frac{1}{2} + \rho(c^\epsilon_1\left(p_{1i}\right))\rho(c_j) \int_0^{y(b^*(q^\epsilon_1, -q^*_{ME}))} f(v)dv \right.$$

$$\left. + \rho(c^\epsilon_1\left(p_{1i}\right))\left(1-\rho(c_j)\right) \int_{z(c^*_M, c^*_E, \delta)}^{y(b^*(q^\epsilon_1, \varnothing))} f(v)dv + \left[\rho(c^\epsilon_1\left(p_{1i}\right)) - \rho(c_j)\right] \int_0^{z(c^*_M, c^*_E, \delta)} f(v)dv \right\}. \tag{16}$$

Note that, if $q^\epsilon_1 = q^*_{ME}$ then $\varphi\left(q^*_{ME}, b^*_{1E}\left(q^\epsilon_1\right), p_{1i}\right)$ is equal to the equilibrium payoff for type $M$ and the mimicking payoff of type $E$. Also, since $y(b^*(q^\epsilon_1, -q^*_{ME}))$ and $y(b^*(q^\epsilon_1, \varnothing))$ are decreasing in $b^*_{1E}\left(q^\epsilon_1\right)$, $\varphi$ is decreasing in $b^*_{1E}\left(q^\epsilon_1\right)$.

**Step 1.** Showing that the D1 property holds, i.e., $D\left(p_{kE},q_1^\epsilon\right)\cup D^0\left(p_{kE},q_1^\epsilon\right)\subset D\left(p_{kM},q_1^\epsilon\right)$, for $\epsilon$ small.

If $b_{1E}^*\left(q_1^\epsilon\right)>\frac{\rho(c_E^*)}{\rho(c_E^*)+\rho(c_M^*)}$, i.e., $b_{1E}^*\left(q_1^\epsilon\right)$ is weakly larger than equilibrium beliefs (when observing $q_{ME}^*$), the extreme type looses, since $\theta(p_{1E},q_1^\epsilon)<\theta(p_{1E},q_{ME}^*)$ and both $y(b^*(q_1^\epsilon,\varnothing))$ and $y(b^*(q_1^\epsilon,-q_{ME}^*))$ are decreasing in $b_{1E}^*\left(q_1^\epsilon\right)$. Thus, the extreme type can only gain compared with the equilibrium payoff if $b_{1E}^*\left(q_1^\epsilon\right)\leq\frac{\rho(c_E^*)}{\rho(c_E^*)+\rho(c_M^*)}$.

From the expression above, the effect of a change in $b_{1E}^*\left(q_1^\epsilon\right)$ on the payoff of type $i$ is

$$\frac{\partial\varphi\left(q_{ME}^*,b_{1E}^\epsilon,p_{1i}\right)}{\partial b_{1E}^*\left(q_1^\epsilon\right)}=\frac{\theta(p_{1i},q_1^\epsilon)}{2}\sum_{j\in\{E,M\}}\left\{\rho(c_{1i}^\epsilon)\rho(c_j)f(y(b^*(q_1^\epsilon,-q_{ME}^*)))\frac{\partial y(b^*(q_1^\epsilon,-q_{ME}^*))}{\partial b_{1E}^*\left(q_1^\epsilon\right)}\right.$$

$$\left.+\rho(c_{1i}^\epsilon)\left(1-\rho(c_j)\right)f(y(b^*(q_1^\epsilon,\varnothing)))\frac{\partial y(b^*(q_1^\epsilon,\varnothing))}{\partial b_{1E}^*\left(q_1^\epsilon\right)}\right\}.$$

Since $\frac{\theta(p_{1M},q_1^\epsilon)}{2}>\frac{\theta(p_{1E},q_1^\epsilon)}{2}$ and $c_{1M}^\epsilon>c_{1E}^\epsilon$ we have $\frac{\partial\varphi\left(q_{ME}^*,b_{1E}^\epsilon,p_{1M}\right)}{\partial b_{1E}^*\left(q_1^\epsilon\right)}<\frac{\partial\varphi\left(q_{ME}^*,b_{1E}^\epsilon,p_{1E}\right)}{\partial b_{1E}^*\left(q_1^\epsilon\right)}<0$. Thus, the effect of a decrease in $b_{1E}^*\left(q_1^\epsilon\right)$ is larger for the moderate type than the extreme type.

**Case 1.** First, suppose $q_{ME}^*<p_{1M}$ and choose $\epsilon$ such that $q_1^\epsilon\leq p_{1M}$. Since (16) is decreasing in $b_{1E}^*\left(q_1^\epsilon\right)$ and increasing in $q_{ME}^*$, the payoff of the moderate type is larger than the equilibrium payoff when $b_{1E}^*\left(q_1^\epsilon\right)\leq\frac{\rho(c_E^*)}{\rho(c_E^*)+\rho(c_M^*)}$. Hence the moderate type gains strictly, compared with the equilibrium payoff. Also, if $b_{1E}^*\left(q_1^\epsilon\right)\geq\frac{\rho(c_E^*)}{\rho(c_E^*)+\rho(c_M^*)}$ the extreme type looses since $q_1^\epsilon$ is farther away from $p_{1E}$ than $q_{ME}^*$. Thus, the set of beliefs for which the extreme type weakly gains is smaller than the set of beliefs for which the moderate type strictly gains. Thus, the D1 property (15) holds for $\hat{q}_k=q_1^\epsilon\leq p_{1M}$.

**Case 2.** Suppose $q_{ME}^*\geq p_{1M}$.

Since $\frac{\partial\varphi\left(q_{ME}^*,b_{1E}^\epsilon,p_{1M}\right)}{\partial b_{1E}^*\left(q_1^\epsilon\right)}<\frac{\partial\varphi\left(q_{ME}^*,b_{1E}^\epsilon,p_{1E}\right)}{\partial b_{1E}^*\left(q_1^\epsilon\right)}<0$ and $\frac{\partial\varphi\left(q_{ME}^*,b_{1E}^\epsilon,p_{1E}\right)}{\partial q_{ME}^*}<\frac{\partial\varphi\left(q_{ME}^*,b_{1E}^\epsilon,p_{1M}\right)}{\partial q_{ME}^*}<0$ we have

$$\varphi\left(q_1^\epsilon,b_{1E}^*\left(q_1^\epsilon\right),p_{1M}\right)-\varphi\left(q_{ME}^*,b_{1E}^*\left(q_{ME}^*\right),p_{1M}\right)$$

$$>\varphi\left(q_1^\epsilon,b_{1E}^*\left(q_1^\epsilon\right),p_{1E}\right)-\varphi\left(q_{ME}^*,b_{1E}^*\left(q_{ME}^*\right),p_{1E}\right)$$

for any $b_{1E}^*\left(q_1^\epsilon\right)<b_{1E}^*\left(q_{ME}^*\right)$. Thus, if

$$\varphi\left(q_1^\epsilon,b_{1E}^*\left(q_1^\epsilon\right),p_{1E}\right)-\varphi\left(q_{ME}^*,b_{1E}^*\left(q_{ME}^*\right),p_{1E}\right)\geq 0, \tag{17}$$

i.e., the extreme type weakly gains, we have

$$\varphi\left(q_1^\epsilon, b_{1E}^*\left(q_1^\epsilon\right), p_{1M}\right) - \varphi\left(q_{ME}^*, b_{1E}^*\left(q_{ME}^*\right), p_{1M}\right) > 0, \tag{18}$$

i.e., the moderate type strictly gains.

Finally, let us show that the D1 property holds. Note that, for $\epsilon$ small, there is some belief $b_{1E}^*\left(q_1^\epsilon\right) < b_{1E}^*\left(q_M^*\right)$ such that the extreme type is indifferent (and hence, from above, the moderate type strictly gains) between deviating and choosing the equilibrium strategy. This follows, since $\varphi\left(q_1^\epsilon, b_{1E}^*\left(q_1^\epsilon\right), p_{1E}\right)$ converges to the equilibrium payoff when $\epsilon \to 0$ and $b_{1E}^*\left(q_1^\epsilon\right)$ converges to equilibrium beliefs $b_{1E}^*\left(q_{ME}^*\right)$.

Using 17, 18, continuity of $\varphi$ and that $\varphi$ is decreasing in $b_{1E}^*\left(q_1^\epsilon\right)$, the extreme type looses and the moderate type gains for beliefs slightly larger than $b_{1E}^*\left(q_1^\epsilon\right)$. Thus, (15) holds if $\epsilon$ is small.

**Step 2.** Showing the moderate type has a profitable deviation.

Case 1 and case 2 implies that the D1 property is satisfied, if $\epsilon$ is small. Then voters put probability one on the moderate type when observing $q_1^\epsilon$.

Since voters put probability one on the moderate type when observing $q_1^\epsilon$ instead of $\frac{\rho(c_M^*)}{\rho(c_E^*)+\rho(c_M^*)}$ when observing $q_{ME}^*$ we have

$$y(b^*(q_1^\epsilon, -q_{ME}^*)) = \frac{b_{2E}^*\left(-q_{ME}^*\right)\delta l^2}{\delta - \left(1 - b_{2E}^*\left(-q_{ME}^*\right)\right) l} > y(b^*(q_{ME}^*, -q_{ME}^*)) = 0$$

and

$$y(b^*(q_1^\epsilon, \varnothing)) = \frac{\left(b_{2E}^*\left(\varnothing\right)\right)\delta l^2}{\delta - \left(1 - b_{2E}^*\left(\varnothing\right)\right) l} > z(c_M^*, c_E^*, \delta) = \frac{\left(b_{2E}^*\left(\varnothing\right) - b_{1E}^*\left(q_{ME}^*\right)\right)\delta l^2}{\delta - \left(1 - b_{2E}^*\left(\varnothing\right) - b_{1E}^*\left(q_{ME}^*\right)\right) l},$$

where $b_{2E}^*\left(-q_{ME}^*\right) = \frac{\rho(c_E^*)}{\rho(c_E^*)+\rho(c_M^*)}$ and $b_{2E}^*\left(\varnothing\right) = \frac{1-\rho(c_E^*)}{1-\rho(c_E^*)+1-\rho(c_M^*)}$. Using (2), (16) and since $\theta(p_{1i}, q_1^\epsilon)$ is approximately equal to $\theta(p_{1i}, q_{ME}^*)$ for $\epsilon$ small, it is profitable for the moderate type to deviate and announce $q_1^\epsilon$ when $\epsilon$ is small. ■

The intuition is the following. We have two cases, depending on whether $q_{ME}^*$ is smaller or larger than $p_{1M}$. Suppose $q_{ME}^* < p_{1M}$. First, if the moderate type chooses $q_{ME}^* + \epsilon$ and

beliefs are equal to equilibrium beliefs, then the moderate type gains and the extreme type looses, because $q^*_{ME} + \epsilon < p_{1M}$ is closer to the policy of the moderate type and farther away from the policy of the extreme type. Second, for beliefs that put a higher probability on the moderate type the moderate type still gains and thirdly, for beliefs that put a higher probability on the extreme type, the extreme type always looses. Hence, the set of beliefs such that the moderate type strictly gains is larger than the set of beliefs such that the extreme type weakly gains. Thus, voters put probability one on the moderate type when observing $q^*_{ME} + \epsilon$. Since the probability that the party is moderate increases when voters observe $q^*_{ME} + \epsilon$ instead of $q^*_{ME}$, voters are more inclined to vote for the party. Hence, since $q^*_{ME} + \epsilon$ is closer to $p_{1M}$ than $q^*_{ME}$, the moderate type has a profitable deviation. In the case when $q^*_{ME} \geq p_{1M}$ a slightly more complicated argument is needed. Since $\theta$ is concave, the negative effect of an increase in the platform on the payoff of the moderate type is smaller than the effect on the extreme type. Taking this into account it is again possible to show that the moderate type has a profitable deviation.

Another motivation for ruling out pooling equilibria is empirical. In Budge and Hofferbert (1990) it is found that there is a correlation between party platforms and policies enacted, indicating separation.

## 3.3 Equilibrium Structure and Comparative Statics

### 3.3.1 Uniqueness

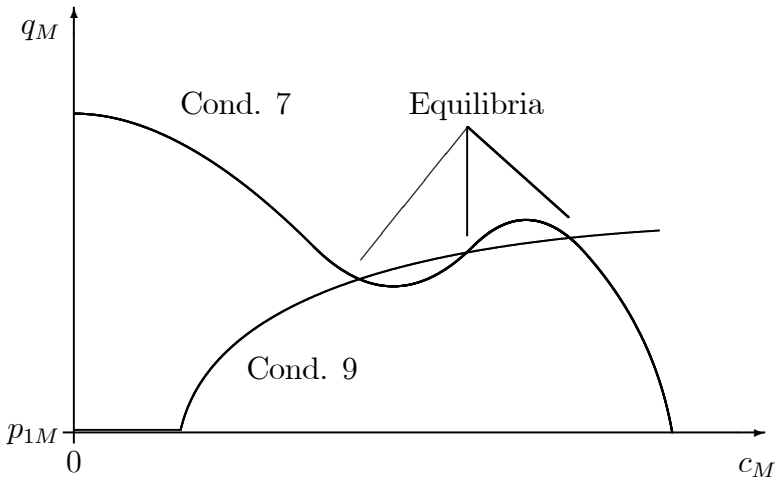
The set of equilibria are determined by conditions (5) and (7). Each of these conditions implicitly defines $q_M$ as a function of $c_M$. In general, these conditions are to complicated to ensure uniqueness. If $q_M > p_{1M}$, the slope of the implicit function defined by (7) is

$$\frac{dq_M}{dc_M} = -\frac{\theta(p_{1E}, q_M)\rho(c_E(q_M, c_M))f\left(z(c_M, \delta)\right)\frac{\partial z(c_M,\delta)}{\partial c_M}}{\frac{\partial\theta(p_{1E},q_M)}{\partial q_M}\left[\frac{1}{2} + \left[\rho(c_E(q_M, c_M)) - \frac{\rho(c_M)}{2}\right]\int_0^{z(c_M,\delta)} f(v)dv\right]}$$

$$-\frac{\left[\theta(p_{1E},p_{1E})-\theta(p_{1E},q_M)\right]\left[\frac{\rho'(c_M)}{2}\int_0^{z(c_M,\delta)}f(v)dv+\frac{\rho(c_M)}{2}f\left(z(c_M,\delta)\right)\frac{\partial z(c_M,\delta)}{\partial c_M}\right]}{\frac{\partial\theta(p_{1E},q_M)}{\partial q_M}\left[\frac{1}{2}+\left[\rho(c_E(q_M,c_M))-\frac{\rho(c_M)}{2}\right]\int_0^{z(c_M,\delta)}f(v)dv\right]}$$

Since $\theta(p_{1E},p_{1E})>\theta(p_{1E},q_M)$, $\frac{\partial z(c_M,\delta)}{\partial c_M}>0$ and $\frac{\partial\theta(p_{1E},q_M)}{\partial q_M}<0$, the slope is positive. If the no-mimicking condition does not bind, then the moderate type chooses $p_{1M}$. The slope of the implicit function defined by the first-order condition (5) cannot be determined in general. Conditions (5) and (7) are as illustrated in the figure below.

**Figure 2**. Illustrating conditions (5) and (7).




Note that, given our assumptions in section 3.1.2 guaranteeing that there is an interior equilibrium, if $q_M=p_{1M}$ then there is a value of $c_M$ that solves (5). If the implicit function defined by (5) is decreasing then there is (at most) one equilibrium. The following theorem shows that if the types are not to close to each other the slope is negative and hence there is a unique equilibrium, given some mild restrictions on the distribution $f$ and the technology $\rho$.

**Theorem 2** *If $l$ is sufficiently large, $\lim_{z\to\infty}\frac{f(z)z^2}{F(z)-F(0)}=0$, $\lim_{c\to\infty}\frac{\rho''(c)}{(\rho'(c))^2}<0$ and $\rho$ is strictly concave there is at most one equilibrium.*

**Proof.** As shown above, the function implicitly defined by (7) is non-decreasing in $c_M$.

Now consider (5). The slope of the function implicitly defined from (5) is

$$\frac{dq_M}{dc_M} = -\frac{\theta(p_{1M}, q_M)\left[\rho''(c_M)\int_0^{z(c_M,\delta)} f(v)dv + \rho'(c_M) f(z(c_M,\delta))\frac{\partial z(c_M,\delta)}{\partial c_M}\right]}{\frac{\partial\theta(p_{1M},q_M)}{\partial q_M}\rho'(c_M)\int_0^{z(c_M,\delta)} f(v)dv}$$

$$= -\frac{\theta(p_{1M}, q_M)\rho'(c_M)}{\frac{\partial\theta(p_{1M},q_M)}{\partial q_M}}\left[\frac{\rho''(c_M)}{(\rho'(c_M))^2} + \frac{f(z(c_M,\delta))\frac{(\delta-l)}{l\delta}(z(c_M,\delta))^2}{\int_0^{z(c_M,\delta)} f(v)dv}\right] \tag{19}$$

as long as $q_M > p_{1M}$. The last equality follows by using $\frac{\partial z(c_M,\delta)}{\partial c_M} = \frac{(\delta-l)\rho'(c_M)}{l\delta}(z(c_M,\delta))^2$. Since $\theta$ is strictly concave, we have $\frac{\theta(p_{1M},q_M)\rho'(c_M)}{\frac{\partial\theta(p_{1M},q_M)}{\partial q_M}} < 0$. Since $\rho$ is increasing, strictly concave and twice differentiable we have $\frac{\rho''(c)}{(\rho'(c))^2} < 0$ for all $c \in (0, \omega]$. Also, since $\frac{l}{2} \leq z(c_M,\delta) \leq l$ and $\lim_{z\to\infty}\frac{f(z)z^2}{F(z)-F(0)} = 0$, the term $\frac{\rho''(c_M)}{(\rho'(c_M))^2}$ dominates the last term, for large $l$. This implies that $\frac{dq_M}{dc_M} < 0$.

Since the function derived from (5) attains a value (weakly) larger than $p_{1M}$ at $c_M = 0$, the value $p_{1M}$ at some $c_M$ and is decreasing and continuous in $c_M$, there is a unique equilibrium. ■

The restriction on the distribution is satisfied, among others, by the Logistic and the standardized Normal distribution. Note also that, for the argument above to work, $\delta$ cannot be too small, since $\delta > l$.

### 3.3.2 Comparative statics

Now consider the effect of increasing $\delta$, i.e., increasing the (average) distance between the parties. As the following result shows, spending decreases when the distance between parties increases.

**Theorem 3** *Let $q^*(\cdot), c^*(\cdot), b^*(\cdot)$ be a separating voting equilibrium satisfying equilibrium dominance. If the conditions in Theorem 2 are satisfied and $l$ is not too small, we have $\frac{dc_M^*}{d\delta} < 0$.*

**Proof. Case 1.** $q_M^* > p_{1M}$.

Let $f_1(q_M, c_M, \delta)$ denote the left hand side of expression (5) and let $f_2(q_M, c_M, \delta)$ denote the left hand side of expression (7). In equilibrium, we have $f_1(q_M^*, c_M^*, \delta) = 0$ and $f_2(q_M^*, c_M^*, \delta) = 0$. Differentiating with respect to $c_M^*$, $q_M^*$ and $\delta$ gives

$$A \cdot \begin{pmatrix} \frac{dc_M^*}{d\delta} \\ \frac{dq_M^*}{d\delta} \end{pmatrix} = - \begin{pmatrix} \frac{\partial f_1(q_M^*, c_M^*, \delta)}{\partial \delta} \\ \frac{\partial f_1(q_M^*, c_M^*, \delta)}{\partial \delta} \end{pmatrix}.$$

From Cramer's rule we have

$$\frac{dc_M^*}{d\delta} = - \frac{\det \begin{pmatrix} \frac{\partial f_1(q_M^*, c_M^*, \delta)}{\partial \delta} & \frac{\partial f_1(q_M^*, c_M^*, \delta)}{\partial q_M} \\ \frac{\partial f_2(q_M^*, c_M^*, \delta)}{\partial \delta} & \frac{\partial f_2(q_M^*, c_M^*, \delta)}{\partial q_M} \end{pmatrix}}{\det A}.$$

The numerator is

$$\rho'(c_M^*) f(z(c_M^*, \delta)) \frac{\partial z(c_M^*, \delta)}{\partial \delta} \left\{ \theta(p_{1M}, q_M^*) \frac{\partial \theta(p_{1E}, q_M^*)}{\partial q_M} \left[ \frac{1}{2} - \frac{\rho(c_M^*)}{2} \int_0^{z(c_M^*, \delta)} f(v) dv \right] \right.$$

$$\left. + \left[ \theta(p_{1M}, q_M^*) \frac{\partial \theta(p_{1E}, q_M^*)}{\partial q_M} - \theta(p_{1E}, q_M^*) \frac{\partial \theta(p_{1M}, q_M^*)}{\partial q_M} \right] \rho(c_L^*(c_M^*, q_M^*)) \int_0^{z(c_M^*, \delta)} f(v) dv \right\}$$

Since $\frac{\partial z(c_M^*, \delta)}{\partial \delta} < 0$, $\frac{1}{2} - \frac{\rho(c_M^*)}{2} \int_0^{z(c_M^*, \delta)} f(v) dv > 0$, $\theta(p_{1M}, q_M^*) \frac{\partial \theta(p_{1E}, q_M^*)}{\partial q_M} < 0$ and

$$\theta(p_{1M}, q_M^*) \frac{\partial \theta(p_{1E}, q_M^*)}{\partial q_M} - \theta(p_{1E}, q_M^*) \frac{\partial \theta(p_{1M}, q_M^*)}{\partial q_M} < 0,$$

the numerator is positive.

Consider $\det A$. Since the slope of the function implicitly defined by (5) is negative and equal to $- \frac{\frac{\partial f_1(q_M^*, c_M^*, \delta)}{\partial c_M}}{\frac{\partial f_1(q_M^*, c_M^*, \delta)}{\partial q_M}}$ where $\frac{\partial f_1(q_M^*, c_M^*, \delta)}{\partial q_M} < 0$ we have $\frac{\partial f_1(q_M^*, c_M^*, \delta)}{\partial c_M} < 0$. Also, we have $\frac{\partial f_2(q_M^*, c_M^*, \delta)}{\partial c_M} > 0$ and $\frac{\partial f_2(q_M^*, c_M^*, \delta)}{\partial q_M} < 0$. Then we have $\det A > 0$. Thus, $\frac{dc_M^*}{d\delta} < 0$.

**Case 2.** $q_M^* = p_{1M}$. [4]

The equilibrium spending level is determined by $f_1(q_M^*, c_M^*, \delta) = 0$. Differentiating $f_1(q_M^*, c_M^*, \delta)$ with respect to $c_M^*$, $q_M^*$ and $\delta$ gives

$$\frac{dc_M^*}{d\delta} = -\frac{\theta(p_{1M}, p_{1M})\rho'(c_M^*)f(z(c_M^*, \delta))\frac{\partial z(c_M^*, \delta)}{\partial \delta}}{\frac{\partial f_1(q_M^*, c_M^*, \delta)}{\partial c_M}}.$$

As in Case 1, we have $\frac{\partial f_1(q_M^*, c_M^*, \delta)}{\partial c_M} < 0$. Since $\frac{\partial z(c_M^*, \delta)}{\partial \delta} < 0$ we have $\frac{dc_M^*}{d\delta} < 0$. ■

The reason behind the result is the following. Recall that $z(c_M^*, \delta)$, i.e., the ideal point of the indifferent voter, is the value of $x$ that solves

$$-(p_{1M} - x)^2 = -\left(\bar{p}_2\left(b^*\left(q_M^*, \varnothing\right)\right) - x\right)^2 - \sigma_2^2\left(b^*\left(q_M^*, \varnothing\right)\right). \tag{23}$$

The left hand side is the payoff when voting for party 1 and the right hand side the expected payoff when voting for party 2. Note that $z(c_M^*, \delta)$ must be closer to the expected policy of party 2, $\bar{p}_2\left(b^*\left(q_M^*, \varnothing\right)\right)$, than to $p_{1M}$, since otherwise the voter would strictly prefer to vote for party 1. Suppose parties move farther away from each other. First, as the distance $\delta$ between the parties increase, $z(c_M^*, \delta)$ decreases. To see this, suppose beliefs are fixed at some value $b^*\left(q_M^*, \varnothing\right)$ and suppose $x = z(c_M^*, \delta)$ solves (23). Then when $\delta$ increases for given beliefs, $p_{1M} - x$ and $\bar{p}_2\left(b^*\left(q_M^*, \varnothing\right)\right) - x$ increase by the same amount. Since preferences are quadratic, the utility when voting for party one decreases more than when voting for party 2. Then $x$ must decrease in order to ensure equality of (23). This makes parties of the moderate type gain fewer votes when informing, decreasing their incentives to spend. There are also secondary effects of an increase in $\delta$, since a change in spending affects $z(c_M^*, \delta)$ in the first order condition (5) and also the no-mimicking condition (7). However, these secondary effects are smaller than the direct effects. Thus, spending decreases. Informally, the risk reduction achieved by informing voters becomes relatively more important when parties are close to each other.

[4] We ignore the non-generic case when (7) holds for $\breve{q} = p_{1M}$.

Also, the marginal effect of spending on the vote share is larger when policy differences are negligible, i.e., $\delta$ is small, when the equilibrium is interior, i.e., $q_M^* > p_{1M}$. The reason is the following. From (2) and the first-order condition (5), the marginal effect of spending on the vote share is given by $\rho'(c_M^*) \int_0^{z(c_M^*,\delta)} f(v)dv$. Hence, there are two effects on the vote share as spending increases. First, since $c_M^*$ increases and $\delta$ decreases, the indifferent voter when informed by only one of the parties, $z(c_M^*, \delta)$, moves farther away from the median of the voter distribution. Thus, as $c_M^*$ increases, more voters switch their voting decision when receiving information from a party. Second, there is also a counteracting effect since as $c_M^*$ increases $\rho'(c_M^*)$ decreases. However, the second effect is dominated by the first. To see this, note that $q_M^*$ also increases. This follows since as $c_M^*$ increases it is more profitable for the extreme type to mimic the moderate type. Hence, the moderate type has to increase $q_M^*$ to prevent the extreme type from mimicking. Since $q_M^*$ increases $\theta\left(p_{1M}, q_M^*\right)$ decreases which, from the first-order condition (5), implies that $\rho'(c_M^*) \int_0^{z(c_M^*,\delta)} f(v)dv$ increases. At a boundary equilibrium $q_M^*$ do not change and hence the second effect exactly counteracts the first. Hence, the marginal effect on voters is unchanged.

### 3.3.3 Stability

As mentioned in the previous section, uniqueness of equilibrium cannot be proven in general. One way of selecting among equilibria is to assume that, given some starting values of $c_M$ and $q_M$, there is some tâtonnement-like process that changes $c_M$ and $q_M$. The change might depend on the value of the first-order condition (5) and no-mimicking condition (7) at $c_M$ and $q_M$. If, say, the first-order condition is positive, then $c_M$ increases. To define the adjustment process, recall that $f_1(q_M, c_M, \delta)$ is the left hand side of (5) and $f_2(q_M, c_M, \delta)$ is the left hand side of (7). For simplicity, we assume that the adjustment process is given by

$$\dot{c}_M = f_1(q_M, c_M, \delta) \tag{24}$$

and

$$\dot{q}_M = \begin{cases} 0 & \text{if } q_M = p_{1M} \text{ and } f_2(q_M, c_M, \delta) < 0 \\ f_2(q_M, c_M, \delta) & \text{otherwise.} \end{cases} \tag{25}$$

Let $\phi$ denote the solution to the system of differential equations. Existence of a (local) solution around some point $q_M$ and $c_M$ is guaranteed if $c_M > 0$. This follows, since both $f_1$ and $f_2$ are continuous for $c_M > 0$. (Beavis and Dobbs (1990)) We introduce the following definition, where $q_M^0$ and $c_M^0$ denotes the initial platform and spending values and $t$ denotes time. [5]

**Definition 2** *An equilibrium* $q^*(\cdot), c^*(\cdot), b^*(\cdot)$ *is* ***locally stable*** *if there exists a* $\eta > 0$ *such that*

$$\left\|(q_M^0, c_M^0) - (q_M^*, c_M^*)\right\| \leq \eta \Rightarrow \lim_{t\to\infty} \phi(t, q_M^0, c_M^0) = (q_M^*, c_M^*).$$

First, we give a partial existence result of a locally stable equilibrium. Let $\Delta \subseteq \mathbb{R}_{++}$ denote the (open) set of possible parameter values of $\delta$. To prove existence, we assume that, for each $p_k$, there is a $q_k \in \mathbb{R}$ such that $\theta(p_k, q_k) = 0$. A motivation for this assumption is that, if parties make announcements sufficiently far away from their implemented policy, this reduces the probability of winning future elections such that it is not worthwhile winning today. For example, if democrats announced a platform far to the right of republicans while implementing an extremely liberal policy after the election, the possibilities of winning the next election would be very small, since they loose credibility. We also need two technical conditions guaranteeing an interior choice of spending.

[5] Here, the choices of the extreme party is ignored. This is not a problem if $\breve{q}_M < p_{1M}$, i.e., if the extreme type strictly looses when mimicking. Then, for $q_M$ and $c_M$ close to $q_M^*, c_M^*$ the extreme type would also loose by mimicking. Hence, the extreme type chooses zero spending and the platform is the policy of the party. Thus, including an adjustment process for the extreme type would not pose a problem. However, when the moderate type is forced to announce a platform larger than the policy, difficulties arise. To see this, note that, given some equilibrium where the moderate type chooses $q_M^* = \breve{q}_M > p_{1M}$ and $c_M^*$, the extreme type is just indifferent between mimicking and following the equilibrium strategy. Then, for $q_M$ and $c_M$ close to $q_M^*, c_M^*$ such that $q_M < q_M^*$ and $c_M > c_M^*$ it must be the case that the extreme type gains by mimicking. This follows since the platform $q_M$ of the moderate type is closer to the policy than the equilibrium platform and that the party gains more votes by informing ($c_M > c_M^*$ implies $z(c_M, \delta) > z(c_M^*, \delta)$).

**Theorem 4** *There is a $\bar{l}$ such that, for all $l \geq \bar{l}$, a locally stable voting equilibrium satisfying equilibrium dominance almost always exists.*

**Proof. Step 1.** Some properties of the candidate equilibrium.

Let $\underline{c}_M(\delta)$ be the smallest spending level $c_M$ such that the first-order condition (5) holds for $q_M = p_{1M}$. This is the equilibrium candidate. Since $\lim_{c_M \to 0} \theta(p_{1M}, p_{1M})\rho'(c_M) \int_0^{z(c_M,\delta)} f(v)dv > 1$, $f_1(p_{1M}, c_M, \delta)$ is positive for small $c_M$. Then, since $\underline{c}_M(\delta)$ is the smallest spending level, we have

$$\frac{\partial f_1(p_{1M}, \underline{c}_M(\delta), \delta)}{\partial c_M} = \theta(p_{1M}, p_{1M}) \left\{ \rho'(\underline{c}_M(\delta)) f(z(\underline{c}_M(\delta), \delta)) \frac{\partial z(\underline{c}_M(\delta), \delta)}{\partial c} \right.$$
$$\left. + \rho''(\underline{c}_M(\delta)) \int_0^{z(\underline{c}_M(\delta),\delta)} f(v)dv \right\} \leq 0. \tag{26}$$

Since $\frac{\partial z}{\partial \delta} \neq 0$, Proposition 8.3.1 in Mas-Colell (1989) together with $\theta(p_{1M}, p_{1M})\rho'(\omega) \int_0^{l} f(v)dv < 1$ implies that (26) holds strictly for almost all $\delta \in \Delta$. If we differentiate (5) with respect to $c_M$ and $\delta$ when (26) holds strictly, then $\frac{dc_M}{d\delta}$ is well defined and we have

$$\frac{dc_M}{d\delta} = -\frac{\rho'(\underline{c}_M(\delta)) f(z(\underline{c}_M(\delta), \delta)) \frac{\partial z(\underline{c}_M(\delta),\delta)}{\partial \delta}}{\rho'(\underline{c}_M(\delta)) f(z(\underline{c}_M(\delta), \delta)) \frac{\partial z(\underline{c}_M(\delta),\delta)}{\partial c} + \rho''(\underline{c}_M(\delta)) \int_0^{z(\underline{c}_M(\delta),\delta)} f(v)dv} < 0.$$

**Step 2.** Showing that only (5) hold in equilibrium for large $l$.

**Case 1.** First, let $l$ be arbitrary and choose $\delta = l + \epsilon$ where $\epsilon > 0$ and small. Then $p_{1E} = -2l - \epsilon$ and $p_{1M} = -\epsilon$. Let $\bar{l}$ be the smallest value of $l$ such that

$$\theta(-2l - \epsilon, -\epsilon) \left[ \frac{1}{2} + \left[ \rho(c_E(-\epsilon, \underline{c}_M(l + \epsilon))) - \frac{\rho(\underline{c}_M(l + \epsilon))}{2} \right] \int_0^{z(\underline{c}_M(l+\epsilon),l+\epsilon)} f(v)dv \right] + \omega$$

$$-c_E(-\epsilon, \underline{c}_M(l+\epsilon)) - \left\{ \omega + \theta(-2l - \epsilon, -2l - \epsilon) \left[ \frac{1}{2} - \frac{1}{2}\rho(\underline{c}_M(l + \epsilon)) \int_0^{z(\underline{c}_M(l+\epsilon),l+\epsilon)} f(v)dv \right] \right\} \leq 0 \tag{27}$$

for all $l \geq \bar{l}$. The expression above is equal to the no-mimicking condition (7) with $c_M = \underline{c}_M(l+\epsilon)$ and $\delta = l + \epsilon$. Too see that $\bar{l}$ exists, first note that the expression above is continuous in $l$ and that $\theta(p_{1E}, p_{1M}) = \theta(-2l - \epsilon, -\epsilon)$. Since there is some $q_k$ such that $\theta(p_k, q_k) = 0$ and since $\theta(p_k, q_k)$ is decreasing in the distance between $p_k$ and $q_k$, there is some $\tilde{l}$ such that $\theta(-2\tilde{l} - \epsilon, -\epsilon) = 0$ and $\theta(-2l - \epsilon, -\epsilon) < 0$ for $l > \tilde{l}$. Then, for $l \geq \tilde{l}$ we have $c_E(p_{1M}, \underline{c}_M(l+\epsilon)) = 0$. Since the extreme type spend the same amount as in equilibrium and announce a platform farther away from the policy than in equilibrium, the payoff when mimicking is smaller than the equilibrium payoff.

**Case 2.** Second suppose $\delta > l + \epsilon$ and $l > \bar{l}$. We claim that, if the moderate type chooses $q_M = p_{1M}$ and $\underline{c}_M(\delta)$ it cannot be profitable for the extreme type to mimic the moderate type. To see this, note that the effect of a change in $\delta$ on the left hand side of the no-mimicking condition (7), evaluated when the first-order condition (5) holds and when (26) holds strictly is

$$- \left[\theta(-2l-\epsilon, -\epsilon) - \theta(-2l-\epsilon, -2l-\epsilon\right] \left[ \frac{\rho(\underline{c}_M(\delta))}{2} f\left(z(\underline{c}_M(\delta), \delta)\right) \frac{\partial z(\underline{c}_M(\delta), \delta)}{\partial \delta} \right.$$

$$\left. + \left( \frac{\rho'(\underline{c}_M(\delta))}{2} \int_0^{z(\underline{c}_M(\delta),\delta)} f(v)dv + \frac{\rho(\underline{c}_M(\delta))}{2} f\left(z(\underline{c}_M(\delta), \delta)\right) \frac{\partial z(\underline{c}_M(\delta), \delta)}{\partial c} \right) \frac{dc_M}{d\delta} \right].$$

Using $\theta(-2l-\epsilon, -2l-\epsilon) > \theta(-2l-\epsilon, -\epsilon)$, $\frac{\partial z(\underline{c}_M(\delta),\delta)}{\partial c} > 0$, $\frac{\partial z(\underline{c}_M(\delta),\delta)}{\partial \delta} < 0$ and $\frac{dc_M}{d\delta} < 0$ the expression above is negative. Thus, the equilibrium payoff increases more/decreases less than the payoff when mimicking if $\delta$ increases. Thus, the extreme type do not want to mimic the moderate type, by our choice of $l$ for any $\delta > l + \epsilon$.

**Step 3.** Establishing stability.

We claim that the equilibrium strategy profile where the moderate type chooses $q_M = p_{1M}$ and $c_M = \underline{c}_M(\delta)$ is stable when (26) holds strictly. To see this, note that only $f_1(p_{1M}, c_M, \delta)$ is relevant for local stability, since $f_2(p_{1M}, c_M, \delta) < 0$ at $c_M = \underline{c}_M(\delta)$. From step 1, we have $\frac{\partial f_1(p_{1M}, c_M, \delta)}{\partial c_M} < 0$, establishing local stability. See Beavis & Dobbs (1990). ■

The intuition is the following. If the distance between the types is large enough, then it can be shown that the no mimicking condition (7) do not bind for any $q_M \geq p_{1M}$. Thus, the moderate type chooses $q_M = p_{1M}$ and spending is determined by the first-order condition (5). Since the first-order condition $f_1(p_{1M}, c_M, \delta)$ is positive when $c_M$ is small it must be non-increasing (and generically decreasing) in $c_M$ at the equilibrium with the smallest spending level. This implies that the equilibrium is stable.

Second, consider comparative statics. We have the following result.

**Theorem 5** *Let $q^*(\cdot), c^*(\cdot), b^*(\cdot)$ be a locally stable separating voting equilibrium satisfying equilibrium dominance. We have $\frac{dc_M^*}{d\delta} < 0$.*

**Proof. Case 1.** $q_M^* > p_{1M}$.

Using a linear approximation of (24) and (25) gives

$$\begin{pmatrix} \dot{c}_M \\ \dot{q}_M \end{pmatrix} = A \cdot \begin{pmatrix} c_M - c_M^* \\ q_M - q_M^* \end{pmatrix}$$

where $A$ is as in the proof of Theorem 3. The result then follows by noting that stability implies that $\det A > 0$ in Case 1 in the proof of Theorem 3.

**Case 2.** $q_M^* = p_{1M}$ and (7) holds for $\breve{q}_M < p_{1M}$.

Here, the linear approximation is given by

$$\dot{c}_M = \frac{\partial f_1(q_M^*, c_M^*, \delta)}{\partial c_M}(c_M - c_M^*). \tag{28}$$

Stability implies that $\frac{\partial f_1(q_M^*, c_M^*, \delta)}{\partial c_M} < 0$. Case 2 in the proof of Theorem 3 implies $\frac{dc_M^*}{d\delta} < 0$. ■

# 4 Public funding

Now consider the effects of introducing a public subsidy of campaign spending. If the subsidy is a lump sum transfer to the parties, there is no effect on the level of campaign spending, assuming that we focus on interior equilibria. Thus, let us focus on subsidies that depend on the vote share/probability of winning. Such subsidies are not uncommon in practice; Le Duc

et al (1996) finds that 17 out of 27 democracies have systems where public subsidies depend on the vote share. We let $\gamma$ denote the total amount paid out to both parties. The public subsidy $s(v_k, \gamma)$ is increasing in both the expected vote share/probability of winning and the total amount paid out to both parties. Also, we assume $\frac{\partial^2 s}{\partial v_k^2} < 0$. The utility function of the parties is

$$\omega - c_k + \theta(p_k, q_k)v_k + s(v_k, \gamma).$$

The effect of a subsidy on the parties is that the marginal benefit to spend changes. Note that this only affects the spending level of the moderate type. By an argument similar to Lemma 1, it can be shown that the extreme type spends nothing. The spending level of the moderate type is then determined by modified versions of (5) and (7). The following result shows that we need a technical restriction on the $s$ function to guarantee that spending increases when the public subsidy increases.

**Lemma 3** *Suppose the conditions in Theorem 2 are satisfied and that $l$ is large. Suppose $\frac{\partial^2 s(v_k,\gamma)}{\partial v_k \partial \gamma} v > \frac{\partial s(v,\gamma)}{\partial \gamma} - \frac{\partial s(1-v_k,\gamma)}{\partial \gamma}$ for all $v_k > \frac{1}{2}$ and all $v \in [1 - v_k, v_k]$. If $\gamma$ increases then party spending increases.*

**Proof.** Let $v_k^{mim}$ denote the expected vote share when mimicking and let $v_k^i$ denote the equilibrium expected vote share of type $i$ of party $k$.

**Case 1.** $q_M^* > p_{1M}$.

Spending and platform choices are determined by modified versions of (5) and (7);

$$f_1(q_M^*, c_M^*, \delta, \gamma) = \left[\theta(p_{1M}, q_M) + \frac{\partial s\left(v_k^M, \gamma\right)}{\partial v_k}\right] \frac{\partial v_k^M}{\partial c_M} - 1 \tag{29}$$

where $\frac{\partial v_k^M}{\partial c_M} = \rho'(c_M) \int_0^{z(c_M,\delta)} f(v)dv$ and

$$f_2(q_M^*, c_M^*, \delta, \gamma) = \omega - c_E(q_M, c_M)$$

$$+ \left[\theta(p_{1E}, q_M)v_k^{mim} + s\left(v_k^{mim}, \gamma\right)\right] - \left\{\omega + \theta(p_{1E}, p_{1E})v_k^E + s\left(v_k^E, \gamma\right)\right\}.$$

Using Cramer's rule as in Theorem 3 gives

$$\frac{dc_M^*}{d\gamma} = -\frac{\det\begin{pmatrix} \frac{\partial f_1(q_M^*,c_M^*,\delta,\gamma)}{\partial\gamma} & \frac{\partial f_1(q_M^*,c_M^*,\delta,\gamma)}{\partial q_M} \\ \frac{\partial f_2(q_M^*,c_M^*,\delta,\gamma)}{\partial\gamma} & \frac{\partial f_2(q_M^*,c_M^*,\delta,\gamma)}{\partial q_M} \end{pmatrix}}{\det A}.$$

The numerator is

$$\left[\frac{\partial^2 s\left(v_k^M,\gamma\right)}{\partial v_k \partial\gamma}\frac{\partial\theta(p_{1E},q_M)}{\partial q_M}v_k^{mim} - \left[\frac{\partial s\left(v_k^{mim},\gamma\right)}{\partial\gamma} - \frac{\partial s\left(v_k^E,\gamma\right)}{\partial\gamma}\right]\frac{\partial\theta(p_{1M},q_M)}{\partial q_M}\right]\frac{\partial v_k^M}{\partial c_M}$$

Note that, since $\theta\left(p_{1M},q_k\right)$ is concave and has a maximum at $q_k = p_{1M}$ we have $\frac{\partial\theta(p_{1E},q_M)}{\partial q_M} < \frac{\partial\theta(p_{1M},q_M)}{\partial q_M} < 0$. Also, using that $v_k^M = 1 - v_k^E$ and $v_k^{mim} \in \left(v_k^E, v_k^M\right)$ we have

$$\frac{\partial^2 s\left(v_k^M,\gamma\right)}{\partial v_k\partial\gamma}v_k^{mim} > \left[\frac{\partial s\left(v_k^{mim},\gamma\right)}{\partial\gamma} - \frac{\partial s\left(v_k^E,\gamma\right)}{\partial\gamma}\right]$$

implying that the numerator is negative.

A slight modification of the argument in section 3.3.1 shows that the implicit functions defined by the modified versions of (5) and (7) are decreasing in $c_M$ and non-decreasing in $c_M$, respectively. Using the same argument as in case 1 in Theorem 3 we have $\det A > 0$. Since $\det A > 0$ we have $\frac{dc_M^*}{d\gamma} > 0$.

**Case 2.** $q_M^* = p_{1M}$.

The spending choice is determined by (29). Then

$$\frac{dc_M^*}{d\gamma} = -\frac{\frac{\partial f_1(q_M^*,c_M^*,\delta,\gamma)}{\partial\gamma}}{\frac{\partial f_1(q_M^*,c_M^*,\delta,\gamma)}{\partial c_M}}.$$

Using the condition on $s$ for $v = 1 - v_k$ implies $\frac{\partial s\left(v_k^M,\gamma\right)}{\partial v_k\partial\gamma} > 0$. Then $\frac{\partial f_1(q_M^*,c_M^*,\delta,\gamma)}{\partial\gamma} = \frac{\partial s\left(v_k^M,\gamma\right)}{\partial v_k\partial\gamma}\frac{\partial v_k^M}{\partial c_M} > 0$. Using the same argument as in case 2 in Theorem 3 gives $\frac{\partial f_1(q_M^*,c_M^*,\delta,\gamma)}{\partial c_M} > 0$. Then we have $\frac{dc_M^*}{d\gamma} > 0$. ■

For the moderate type, as $\gamma$ increases the marginal benefit of spending increases. Then, $c_M$ increases.

The technical condition is for example satisfied by the "linear" function $s(v_k, \gamma) = \gamma v_k$. If $v_k$ is interpreted as the vote share, then a linear function seems to be in line with reality, since parties in many countries get a subsidy that depend linearly on the vote share. The restriction on $s$ guarantees that the extreme type does not gain too much when mimicking.

Without the restriction, it is difficult to draw any specific conclusions about the effect on the spending level. To see this, note first that a change in the $s$ function affects the first-order both directly through a change in $\frac{\partial s\left(v_k^M,\gamma\right)}{\partial v_k}$ and indirectly through a change in $q_M$ via the no-mimicking condition. In particular, suppose a public subsidy is introduced where the extreme type gains a lot when mimicking and the moderate type is marginally affected when following the equilibrium strategy. To prevent the extreme type from mimicking, the moderate type must then choose a platform farther away from the policy. This decreases the spending level chosen by the moderate type. If $\frac{\partial^2 s(v_k,\gamma)}{\partial v_k \partial \gamma}$ is too small, the effect of the change in platform on $\theta(p_{1M}, q_M)$ might dominate the effect of the change in public subsidy on $\frac{\partial s(v_k,\gamma)}{\partial v_k}$ and, using (29), lead to a decrease in spending.

Now consider the effect of a public subsidy on the expected policy. If $v_k$ is interpreted as the probability of winning, we have the following result.

**Theorem 6** *Suppose an increase in the public subsidy leads to an increase in campaign spending of the moderate type. Then, a public subsidy of campaign spending leads to convergence in expected policy.*

**Proof.** If both parties are of the same type, spending is affected similarly for both types, so each party still wins with probability $\frac{1}{2}$. If both parties are of different types, the probability that the moderate type wins is

$$\frac{1}{2} + \rho(c_M) \int_0^{z(c_M,\delta)} f(v)dv.$$

Since this expression is increasing in $c_M$, the probability that the moderate type wins increases. Since the public subsidy increases the probability that the moderate type wins when

parties are of different types, the expected policy is closer to 0, i.e., to the mean of the voter distribution. ■

A related result is found in Ortuno Ortin and Schultz (2000). They show that a public subsidy increases policy convergence, which if agents are risk averse, increases welfare. The reason is that, since public funds depend on the vote share, parties moderate their policy to increase the vote share and hence the public subsidy they receive. The result in this paper is closely related to their result. There is a small difference, though. To see this, note that instead of affecting the policy of the party, the subsidy increases the probability that "good" types win. Thus, while the actual policies do not converge as in Ortuno Ortin and Schultz (2000), expected policies do.

# 5 Asymmetries

Empirical evidence in Erikson and Palfrey (2000) seems to indicate that spending is largest when elections are close, i.e., each party wins the election with about the same probability. Also, the effect of spending on the vote share/probability of winning is largest when the election is close. In the analysis above, the ex ante vote share/probability of winning, i.e., before policies are realized, is always $\frac{1}{2}$ for each party, since we focus on a symmetric equilibrium. To analyze a setup where parties have different ex ante vote shares/probabilities of winning, the positions of the parties by are shifted by the distance $\pi > 0$. Thus, instead of being centered at 0 as in figure 1, the positions are centered at $\pi$. The voter distribution remains unchanged. We also assume that $\lim_{y\to\infty} f(y) = 0$. As before, we focus on separating equilibria.

Consider the indifferent voters. In general, some algebra shows that the indifferent voter is

$$\pi + \frac{\delta l \left[b^*_{2E}(o_2) - b^*_{1E}(o_1)\right]}{\delta - l\left[1 - b^*_{1E}(o_1) - b^*_{2E}(o_2)\right]}.$$

This expression is decreasing in $b^*_{1E}(o_1)$ and increasing in $b^*_{2E}(o_2)$. Since the equilibrium is

separating, we have $b_{1E}^*(q_1^*) = 0$ ($b_{2E}^*(q_2^*) = 0$), where $q_1^*$ ($q_2^*$) is the platform of the moderate type of party 1 (2). Consider voters informed by party one only. The indifferent voters ideal point is $\pi + \frac{\delta l\left[b_{2E}^*(\varnothing)\right]}{\delta - l\left[1-b_{2E}^*(\varnothing)\right]}$. Note that, since the moderate type spends at least as much as the extreme type, we have $b_{2E}^*(\varnothing) \geq \frac{1}{2}$. Using this and the fact that the expression is increasing in $b_{2E}^*(o_2)$, we have $\pi + \frac{\delta l}{2\delta - l} \leq \pi + \frac{\delta l\left[b_{2E}^*(\varnothing)\right]}{\delta - l\left[1-b_{2E}^*(\varnothing)\right]} \leq \pi + l$. Also, for voters that are not informed by any of the parties the indifferent voters ideal point is at least (when $b_{1E}^*(\varnothing) = 1$ and $b_{2E}^*(\varnothing) = \frac{1}{2}$) $\pi - \frac{\delta l}{2\delta + l}$ and at most $\pi + \frac{\delta l}{2\delta + l}$. Furthermore, if both parties have informed voters, the indifferent voters ideal point is at $\pi$, since voters then know that $p_{1M} = \pi - \delta + l$ and $p_{2M} = \pi + \delta - l$. Finally, note that the gain in vote share when informing voters is smaller than $\int_{\pi - l}^{\pi + l} f(v)dv$.

In an asymmetric equilibrium, the parties need not spend the same amount on campaigning. Let $c_{1E}(q_1, c)$ denote the optimal spending level for the extreme type of party 1 when mimicking, when the moderate type of party one chooses $q_1$ and $c = (c_1(p_{1M}), c_2(p_{2M}))$. The difference in payoff when mimicking and choosing $q_1$ and when following the equilibrium strategy for the extreme type is at most

$$\omega - c_{1E}(q_1, c) + \frac{\theta(p_{1E}, q_1)}{2}\left\{\rho(c_{1E}(q_1, c))\rho(c_2(p_{2M}))\int_{\pi - \frac{\delta l}{2\delta - l}}^{\pi} f(v)dv\right.$$

$$\left. + \rho(c_{1E}(q_1, c))\left[2 - \rho(c_2(p_{2M}))\right]\int_{\pi + \frac{\delta l}{2\delta + l}}^{\pi + l} f(v)dv + \int_{-\infty}^{\pi - \frac{\delta l}{2\delta - l}} f(v)dv + \int_{-\infty}^{\pi + \frac{\delta l}{2\delta + l}} f(v)\right\}$$

$$- \left\{\omega + \theta(p_{1E}, p_{1E})\left[\int_{-\infty}^{\pi - \frac{\delta l}{2\delta + l}} f(v)dv - \frac{\rho(c_2(p_{2M}))}{2}\int_{\pi - l}^{\pi - \frac{\delta l}{2\delta + l}} f(v)\right]\right\} \tag{32}$$

First, consider the platform choice of party one. Since $\lim_{\pi \to \infty} \int_{\pi - l}^{\pi + l} f(v)dv = 0$, the value of $q_1$ such that the expression above is zero is smaller than $p_{1M}$ for $\pi$ large. Thus, the extreme type does not want to mimic for any platform $q_1(p_{1M}) \geq p_{1M}$ when $\pi$ is large and hence, the moderate type chooses $p_{1M}$ in equilibrium.

Now consider the spending choice of the moderate type when $\pi$ is large. Using (2), the spending level is the value of $c_1$ that solves

$$-1+\theta(p_{1M},p_{1M})\frac{\partial v_1}{\partial c_1}(c)=0, \tag{33}$$

where $\frac{\partial v_1}{\partial c_1}(c)$ is the effect of party 1 spending on the vote share, when voters believe that parties spend $c$. Since $\frac{\partial v_1}{\partial c_1}\leq\rho'(c_1)\int_{\pi-l}^{\pi+l}f(v)dv$ and since $\lim_{\pi\to\infty}\int_{\pi-l}^{\pi+l}f(v)dv=0$, $c_1$ is close to zero for $\pi$ large. A similar argument shows that the moderate type of party 2 chooses $c_2$ close to zero.

Thus, given some symmetric equilibrium, there is a $\pi^*$ such that, for $\pi>\pi^*$, spending is smaller than $c_M^*$. Also, if we have $q_M^*>p_{1M}$ in the symmetric equilibrium, the effect of spending on $\frac{\partial v_1}{\partial c_1}(c)$ is smaller when $\pi$ is large. This follows, since $q_1$ converges to $p_{1M}$, since we have $\theta(p_{1M},q_1)\frac{\partial v_1}{\partial c_1}(c)=1$ in equilibrium and since $\theta(p_{1M},q_1)$ increases when $q_1$ decreases. Thus, money matters less than in the symmetric equilibrium for party one.

The reason for the result is that when parties are asymmetrically located and hence have different probabilities of winning, the effect of a marginal increase in spending has a small effect on the probability of winning since few voters are asymmetrically located, i.e., $f(y)$ is small. Then spending is small. Also, the extreme type do not want to mimic for any platform weakly larger than $p_{1M}$. Hence, the platform of the moderate type is equal to the policy. Assuming that we compare with a symmetric equilibrium where $q_M^*>p_{1M}$ (33) implies that the marginal effect of spending on the probability of winning decreases.

# 6 A note on the information structure

In the model presented above, it might seem somewhat artificial that the parties choose platforms and spending simultaneously. The following information structure might seem more reasonable. First, parties choose platforms simultaneously, without knowing the type of the other party. Then the platform choices are revealed to both parties and parties choose spending simultaneously. This unfortunately complicates matters. It can be shown that

the separating equilibria described above are also equilibria in the model with the modified timing. However, there might also be equilibria where (7) do not hold that cannot be ruled out by reasonable refinements. Results partly similar to the ones presented in this paper are shown to hold in a model with an information structure as described above where parties are restricted to use only truthful messages. See Westermark (1999).

# 7 Conclusions

The model described in this paper analyses political campaigning. We first characterize the equilibria. There are both separating and pooling equilibria. Furthermore, the set of separating equilibria is large. A mild restriction on beliefs restricts the separating equilibria significantly. Also, if the policies of a *given* party are not too close to each other, then there is a unique separating equilibrium. Pooling equilibria are ruled out by a restriction on beliefs.

When analyzing the separating equilibria, we show that more voters are informed when the difference (on average) in policy between the parties is small than when it is large. The motivation is the following. If a party informs a voter, the risk of voting for the party vanishes. Since preferences are flatter when parties are close to each other, the effect of the risk reduction is larger when parties are close to each other. Thus, the incentives to inform voters increase implying that parties spend more and that the share of informed voters increase.

Furthermore, we show that a public subsidy on campaign spending, given some mild technical restrictions on the public subsidy, results in an increase in the likeliness that "good" types win the election. Thus, introducing a public subsidy leads to convergence in expected policy.

We also show that spending is lower and has a smaller effect on the vote share/probability of winning when parties are asymmetrically located.

# References


Alesina, A. and Cuikerman A. (1990), "The Politics of Ambiguity", Quarterly Journal of Economics 105, 829-850.

Banks, J. (1990), "A Model of Electoral Competition with Incomplete Information", Journal of Economic Theory 50, 309-325.

Banks, J. and Sobel J. (1987), "Equilibrium Selection in Signalling Games", Econometrica 55, 647-662.

Budge, I. and Hofferbert, R. (1990), "Mandates and Policy Outputs: U.S. Party Platforms and Federal Expenditures" American Political Science Review 84, 111-131.

Chappell, H. (1994), "Campaign Advertising and Political Ambiguity", Public Choice 79, 281-303.

Cho, I.K. and Kreps, D.M. (1987), "Signalling Games and Stable Equilibria", Quarterly Journal of Economics 102, 179-221.

Erikson, R.S. and Palfrey, T.R. (2000), "Equilibia in Campaign Spending Games: Theory and Data", American Political Science Review 94, 595-609.

Harrington, J. and Hess, G. (1996), "A Spatial Theory of Positive and Negative Campaigning", Games and Economic Behavior 17, 209-229.

Harrington, J. (1993), "The Impact of Reelection Pressures on the Fulfillment of Campaign Promises", Games and Economic Behavior 5, 71-97.

Harrington, J. (1992), "The Revelation of Information Through the Electoral Process :An Exploratory Analysis", Economics and Politics 4, 255-275.

Le Duc, L., Niemi, R.G. and Norris, P. (1996), "Comparing Democracies. Elections and Voting in Global Perspective", Sage Publications, Thousand Oaks.

Martinelli, C. and Matsui, A. (1997), "Policy reversals : A Democratic Nixon and a Republican Clinton", mimeo.

Martinelli, C. (1997), "Information Revelation in Electoral Competition", mimeo.

Mas-Colell, A. (1985), "The Theory of General Economic Equilibrium: A Differentiable Approach", Cambridge University Press, New York.

Ortuno Ortin, I. and Schultz, C. (2000), "Public Funding of Political Parties", mimeo, University of Copenhagen.

Potters, J., Sloof, R. and van Winden, F. (1997), "Campaign Expenditures, Contributions and Direct Endorsements : The Strategic use of Information and Money to Influence Voting Behavior", European Journal of Political Economy 13, 1-31.

Schultz, C. (1996), "Polarization and Inefficient Policies", Review of Economic Studies 63, 331-343.

Westermark, A. (1999), "Extremism, Campaigning and Ambiguity", Working Paper 1999:9, Uppsala University.