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An Empirical Two-Good, Two-Country Representative-Agent Model with Endogenous Growth

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Abstract

While a considerable amount of empirical research has been conducted on the subject of endogenous growth in national economies, much less has been carried out on the relationship between endogenous growth and international trade in real goods and financial assets. How an economy's rate of growth is influenced by conditions determined by the decisions of domestic and foreign agents, how this rate of growth affects the volume and direction of trade, and how changes in trade patterns in turn affect conditions influencing this rate of growth are questions that have received much attention in the theoretical literature but which remain largely unexamined in empirical settings. The principal objective of the research here presented is to begin empirical explorations of these questions in the context of a two-good, two-country representative-agent model. In this paper we introduce and motivate the model, discuss the details of its estimation with Italian and German data, present estimation results, evaluate the model's performance, and employ it in simulations to examine the effects of alternative tax policies and exogenous developments in adjustment costs, technology, and international markets.

JEL Classifications: C32, C51, D91, D92, F42, F43

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1. Introduction[1]

While a considerable amount of empirical research has been conducted on the subject of endogenous growth in national economies (e.g., Barro and Sala-i-Martin, 1995), much less has been carried out on the relationship between endogenous growth and international trade in real goods and financial assets. How an economy's rate of productivity growth is influenced by conditions determined by the decisions of domestic and foreign agents, how this rate of growth affects the volume and direction of trade, and how changes in trade patterns in turn affect conditions influencing this rate of growth are questions that have received much attention in the theoretical literature. (See e.g., Grossman and Helpman, 1991; Obstfeld and Rogoff, 1996; Jensen and Wong, 1997; Turnovsky, 1997; and Aghion and Howitt, 1998.) They have also been explored to a limited extent in numerical simulations (e.g., Backus et al., 1994; Baxter and Crucini, 1995; and Stockman and Tesar, 1995). But they remain largely unexamined in empirical settings. The principal objective of the research reported on in this paper is to begin empirical explorations of these questions.

The framework we adopt for our analysis is a two-good, two-country representative agent model. It is the sparest framework that will permit the analysis of the issues on which we focus and should thus permit the greatest amount of transparency. In spite of their small size, two-country models have proven to be capable of capturing international real-business-cycle dynamics (Backus et al., 1994) and, when extended to include international financial markets, they can accommodate risk-management and consumption-smoothing behaviors of agents in response to shocks of domestic or foreign origin (Stockman and Tesar, 1995). Moreover, they provide a plausible characterization of how effects from developments in one economy can be transmitted to another (Baxter and Crucini, 1995).

Increasingly macrodynamic theory in general and endogenous growth theory in particular is being developed in terms of the intertemporal optimization decisions of representative agents.[2] And, we should add, a preponderance of this work is formulated in continuous-time. While such models arguably contribute microfoundations and logical consistency that have hitherto been lacking, they pose difficult challenges to empirical analysts who might be interested in obtaining estimates of the model's parameters and testing theoretical assumptions. Possible responses to these difficulties include taking the representative agent assumption as an article of faith for theoretical research and abandoning empirical testing, conducting numerical simulations with models calibrated with plausible magnitudes for parameters, and examining the international transmission of shocks of various sorts via structural VAR models that give a less prominent role to intertemporal optimization and microfoundations. These are not the *only* possible responses, however. A second aim of this paper's research is to demonstrate how one may estimate, evaluate, and employ in analyses continuous-time models of intertemporally optimizing agents in a

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[1]We are most grateful to Professor Giancarlo Gandolfo for many contributions to this research undertaking of an analytical, interpretive, and constructively critical nature. The opinions expressed herein, however, are not necessarily completely shared by him.

[2]This development is not without its critics. See, e.g., Kirman (1992) and Hahn and Solow (1995).

statistically rigorous and logically consistent fashion.[3]

In the next section we introduce and motivate our model, which extends Turnovsky's (1997) endogenous-growth model of an open economy to a two-good, two-country world, and furthers the empirical analysis begun in Donaghy et al. (1999a). We discuss in the third section the details of the model's estimation with Italian and German data, the empirical results obtained, and the model's stability properties. In the fourth section we examine the model's responsiveness to changes in capital adjustment costs, tax policies, technology, world interest rates, and imports availability and conclude the paper with some general observations about the undertaking.

2. An Empirical Two-Good, Two-Country Representative-Agent Model with Endogenous Growth

In the two-good, two–country 'world' we consider, agents may hold domestic capital and traded bonds, through which they gain indirect ownership of foreign assets. Because they do not hold foreign assets directly, financial markets are 'incomplete' in the sense of Baxter and Crucini (1995). Each country specializes in the production of a single good, but agents in the two countries may consume both the domestically produced and imported goods. In each country the representative agent has access to the world bond market but is not free to borrow or lend as much as he or she might wish at the world interest rate, r^w. Rather, the effective interest rate faced by each agent is a function of his or her debt- or credit-equity position. The agent's borrowing and lending behavior must moreover satisfy an intertemporal solvency constraint. Adjustment to shocks occurs partly through the effective interest rates, partly through the relative price of foreign and domestic goods, and partly through adjustment costs associated with physical capital accumulation. While taxes on the representative agent of each country's economy play an important role in this model, tariffs do not, since they do not factor in the bi-lateral trading behavior of the countries whose data we use to examine the model empirically.

In the *domestic* economy, output, *Y*, is produced with the capital stock domiciled in the country according to the simplest of '*AK*' technologies, $Y=aK$, $a>0$. Domestic output is used partly for investment, *I*, partly for domestic consumption, *C*, and the rest is exported for consumption in the *foreign* economy, *MGS**.[4] In consuming this commodity and the foreign good, *MGS*, the domestic agent derives utility over an infinite time horizon, which is represented by an isoelastic intertemporal utility function,

$$\Omega \equiv \int_0^{\infty} \frac{1}{g}(C \bullet MGS^h)^g e^{-bt} dt, h > 0; -\infty < g < 1; h\ g < 1.; 1 > g(1+h). \qquad (2.1)$$

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[3]We agree with Sims (1996) that there is much to learn from each methodological response to the challenges posed by contemporary macroeconomic models but also with Wickens (1994) that applied macroeconometrics has the wherewithal to meet the challenges.

[4]In the sequel of this section we follow convention in referring to the economy of the first country as 'domestic' and that of the second as 'foreign.'

Edgeworth complementarity--i.e., $U_{C,MGS}, U_{MGS,C} > 0$--is not assumed, although is considered likely. In (2.1), the exponent ß denotes the agent's subjective discount rate and his or her intertemporal elasticity of substitution is given by $s=1/(1-?)$. We take the agent's supply of labor to be fixed. The constraints upon the parameters are implied by the assumption that the utility function is concave in its arguments.

Costs of adjustment to the physical capital stock brought about by investment expenditures, *I*, are represented by a quadratic function,

$$\Phi(I,K) = I + h\frac{I^2}{2K} = I\left(1 + \frac{hI}{2K}\right). \tag{2.2}$$

The agent accumulates net foreign assets in the form of bonds, *b*, which pay an effective interest rate, *r*, and pays taxes on income from physical capital, bond income, and consumption expenditures at rates t_k, t_b, and t_c. Tax revenues are rebated to the agent in the form of a lump-sum transfer payment, T. The agent's instantaneous budget constraint is thus given by

$$b = a\ (1-t_k)K + r(1-t_b)b - (1+t_c)(C + s \bullet MGS) - I\left(1 + \frac{hI}{2K}\right) + T, \tag{2.3}$$

in which we have substituted expression (2.2) for the cost function *F(I,K)*. Assuming non-depreciation of capital, the agent faces a physical constraint on capital accumulation,

$$K = I. \tag{2.4}$$

To maximize his or her intertemporal utility, the agent must choose his or her levels of consumption of the domestic and foreign goods, *C* and *MGS*, rate of investment, *I*, and rate of asset accumulation, b, subject to the equations constraining financial and physical capital accumulation and initial endowments of capital stocks

$$K(0) = K_0,\ b(0) = b_0. \tag{2.5}$$

The present-value Hamiltonian for this optimization problem is

$$\begin{aligned} H \equiv \frac{1}{g}(C \bullet MGS^h)^g e^{-bt} + l e^{-bt}[a\ (1-t_k)K + r(1-t_b)b \\ -(1+t_c)(C + s \bullet MGS) - I\left(1 + \frac{hI}{2K}\right) + T - b] + q' e^{-bt}(I - K), \end{aligned} \tag{2.6}$$

in which *s* is the real exchange rate, *?* denotes the shadow value--or marginal utility--of wealth, and *q'* is the shadow value of the agent's capital stock. The analysis can be simplified by using the shadow value of wealth as numeraire; in which case *q/q'/?* is the market value of capital.

The optimality conditions for *C*, *MGS*, and *I* are

$$(C \bullet MGS^h)^{g-1} MGS^h = l(1-t_c), \tag{2.7a}$$

$$h(C \bullet MGS^h)^{g-1} C \bullet MGS^{h-1} / s = l(1-t_c), \text{ and} \tag{2.7b}$$

$$1 + h\frac{I}{K} = q. \tag{2.7c}$$

These conditions imply that in macrodynamic equilibrium the marginal utility of consuming the domestic good equals the tax adjusted value of wealth, the marginal utility of consuming the

foreign good--evaluated in terms of the domestic good--also equals this tax adjusted value, and the marginal cost of purchasing and installing an additional unit of capital equals the market value of capital.

Rewriting (2.7.c) as

$$\frac{I}{K} = \frac{K}{K} = \frac{q-1}{h}, \tag{2.8}$$

gives the optimal rate of capital accumulation in terms of the market value of installed capital (Tobin's *q*), and, in view of the '*AK*' technology assumed, it also gives the rate of growth of the economy. Standard intertemporal macroeconomics optimality conditions with respect to *b* and *K* imply the arbitrage relationships

$$\beta - \frac{\lambda}{\lambda} = r(1-t_b), \text{ and} \tag{2.9a}$$

$$\frac{\alpha}{q}(1-t_k) + \frac{\dot{q}}{q} + \frac{(q-1)^2}{2hq} = r(1-t_b). \tag{2.9b}$$

The first (the Keynes-Ramsey consumption rule) equates the marginal return on consumption to the after-tax return on holding a foreign bond, whereas the second equates the after-tax rate of return on domestic capital to the after-tax bond yield. The second condition also emphasizes the importance of adjustment costs and the market price of capital in equilibrating the rate of return.

To ensure that the agent's intertemporal budget constraint is met, the following transversality conditions are imposed on the solution:

$$\lim_{t\to\infty} \lambda b e^{-\beta t} 0; \ \lim_{t\to\infty} q' K e^{-\beta t} = 0. \tag{2.10}$$

The foreign economy is analogous to the domestic one with appropriate variables and parameters being denoted by asterisks. The present-value Hamiltonian for the foreign agent's optimization problem is

$$H^* \equiv \frac{1}{\gamma^*}(C^* \bullet MGS^{*h^*})^{\gamma^*} e^{-\beta^* t} + \lambda^* e^{-\beta^* t}[\alpha^*(1-t_k^*)K^*$$

$$+ r^*(1-t_b^*)b^* - (1+t_c^*)(C^* + MGS^*/s) - I^*\left(1+\frac{h^* I^*}{2K^*}\right) \tag{2.6'}$$

$$+ T^* - b^*] + q^{*\prime} e^{-\beta^* t}(I^* - K^*).$$

The corresponding optimality and arbitrage conditions are

$$(C^* \bullet MGS^{*h^*})^{\gamma^*-1} MGS^{*h^*} = \lambda^*(1-t_c^*), \tag{2.7a'}$$

$$h^* s\,(C^* \bullet MGS^{*h^*})^{\gamma^*-1} C^* \bullet MGS^{*h^*-1} = \lambda^*(1-t_c^*), \tag{2.7b'}$$

$$\frac{I^*}{K^*} = \frac{q^*-1}{h^*}, \tag{2.8c'}$$

$$\dot{b}^* - \frac{\dot{l}^*}{l^*} = r^*(1 - t_b^*), \text{ and} \tag{2.9a'}$$

$$\frac{\dot{a}^*}{q^*}(1 - t_k^*) + \frac{\dot{q}^*}{q^*} + \frac{(q^* - 1)^2}{2h^* q^*} = r^*(1 - t_b^*). \tag{2.9b'}$$

Note that the marginal utility expressions are given in terms of the foreign good. The foreign agent's initial endowments are

$$K^*(0) = K^*_0, \; b^*(0) = b^*_0, \tag{2.5'}$$

and the corresponding transversality conditions are

$$\lim_{t \to \infty} l^* b^* e^{-b^* t} = 0, \; \lim_{t \to \infty} q^{*\prime} K^* e^{-b^* t} = 0. \tag{2.10'}$$

As stated above, we assume that the representative agents in the domestic and foreign economies face effective interest rates that are determined partly by the world interest rate and partly by their debt- or credit-equity positions, or, more precisely, by the ratios of their stocks of net foreign assets to their stocks of capital--i.e.,

$$r = r^w + v(b/K), \tag{2.11}$$

$$r^* = r^w + v^*(b^*/K^*). \tag{2.11'}$$

While for net-borrower nations one would expect the derivatives $v'(b/K)$ and $v^{*\prime}(b^*/K^*)$ to be negative, for net-lender nations they can be either positive or negative. The intuition underlying this ambivalence in the case of net-lender nations is that the functions (2.11) and (2.11') can represent, when the derivatives are positive, price-dependent supply functions and, when negative, yield curves for increasingly marginal investments in which r^w is the benchmark yield.

To close the model we require an additional relationship in which the real exchange rate is related to other endogenously determined variables. If we may help ourselves to an uncovered interest parity (UIP) assumption, the real exchange rate can be related to the effective foreign and domestic interest rates by

$$\frac{\dot{s}}{s} = r(b/K) - r^*(b^*/K^*). \tag{2.12}$$

In view of the mixed record of empirical testing of hypotheses about uncovered and covered interest parity (CIP) (discussed by Isard, 1995), this is probably not a bad initial assumption. Of course, a CIP relationship could be specified in terms of a foreign exchange risk premium (or spot and future exchange rates) *a la* McCallum (1994). A second type of structural relationship that could be availed is the domestic supply constraint of either economy. (See Turnovsky, 1995, Chapter 12.) Taking either of these alternative tacks would increase the number of variables in--and complexity of--the model.

Further analysis is most conveniently conducted by defining new variables

$$c \equiv \frac{C}{K}; c^* \equiv \frac{C^*}{K^*}; mgs \equiv \frac{MGS}{K}; mgs^* \equiv \frac{MGS^*}{K^*}; z \equiv \frac{b}{K}; z^* \equiv \frac{b^*}{K^*}; \tag{2.13}$$

as the domestic and foreign rates of consumption of domestically produced and imported goods and the stocks of net foreign assets per unit of physical capital. By differentiating these identities

logarithmically with respect to time and making use of the optimality and arbitrage conditions, one obtains a set of equations that determine the evolution of *c, mgs, z, q, c*, mgs*, z*, q**, and *s*:

$$c = c\left(\frac{1}{h\ g - g - 1}[b - (1 - t_b)r(z) + h\ g\frac{s}{s}] - \frac{q-1}{h} \right), \tag{2.14a}$$

$$mgs = h\,c / s - h\,cs / s^2, \tag{2.14b}$$

$$z = -c - s \bullet mgs - \left(\frac{q^2 - 1}{2h} \right) + \left(\frac{q-1}{h} \right) z + a + r(z)z, \tag{2.14c}$$

$$q = (1 - t_b)r(z)q - (1 - t_k)a - \frac{(q-1)^2}{2h}, \tag{2.14d}$$

$$c^* = c^*\left(\frac{1}{h^* g^* - g^* - 1}[b^* - (1 - t_b)r^*(z^*) - h^* g^*\frac{s}{s}] - \frac{q^* - 1}{h^*} \right), \tag{2.14e}$$

$$mgs^* = h^* s\,c^* + h^* cs, \tag{2.14f}$$

$$z^* = -c^* - mgs^*/s - \left(\frac{q^{*2} - 1}{2h^*} \right) + \left(\frac{q^* - 1}{h^*} \right) z^* + a^* + r^*(z^*)z^*, \tag{2.14g}$$

$$q^* = (1 - t_b^*)r^*(z^*)q^* - (1 - t_k^*)a^* - \frac{(q^* - 1)^2}{2h^*}, \tag{2.14h}$$

$$s = s\left(r(z) - r^*(z^*)\right). \tag{2.14i}$$

The steady-state growth path for this model can be obtained by setting $c = c^* = mgs = mgs^* = z = z^* = q = q^* = s = 0$, eliminating *mgs*, *mgs**, and *s* by substitution, and solving the following equations for the steady-state values of *c, z, q, c*, z**, and *q**, which are denoted by tildes:[5]

[5]Note that $s = 0$ implies that $r(\tilde{z}) = r^*(\tilde{z}^*)$, and, from (2.7a), (2.7b), (2.7a') and (2.7b'), $m\tilde{g}s = h\,\tilde{c} / \tilde{s}$ and $m\tilde{g}s^* = h^* s^* \tilde{c}^*$. Because at the steady-state solution the system (2.14) reduces to six equations, we need only to solve for the levels of six endogenous variables to linearize the system for stability analysis. The first-order structural equations in mgs, mgs*, and s --(2.14b), (2.14f), and (2.14i)--characterize system dynamics away from the steady-state solution.

$$\frac{1}{h\,g+\,g-1}[b\,-(1-t_b)r(\tilde{z})]=\frac{\tilde{q}-1}{h},$$

$$(1+h)\tilde{c}+\left(\frac{\tilde{q}^2-1}{2h}\right)-\left(\frac{\tilde{q}-1}{h}\right)\tilde{z}-a\,-r(\tilde{z})\tilde{z}=0,$$

$$(1-t_k)\frac{a}{\tilde{q}}+\frac{(\tilde{q}-1)^2}{2h\tilde{q}}=(1-t_b)r(\tilde{z}),$$

$$\frac{1}{h\,*\,g\,*+g\,*-1}[b\,*-(1-t_b*)r\,*(\tilde{z}*)]=\frac{\tilde{q}\,*-1}{h*},\qquad(2.15)$$

$$(1+h\,*)\tilde{c}\,*+\left(\frac{\tilde{q}\,*^2-1}{2h*}\right)-\left(\frac{\tilde{q}\,*-1}{h*}\right)\tilde{z}\,*-a\,\,*-r\,*(\tilde{z}*)\tilde{z}*=0,$$

$$(1-t_k*)\frac{a\,*}{\tilde{q}\,*}+\frac{(\tilde{q}\,*-1)^2}{2h*\tilde{q}\,*}=(1-t_b*)r\,*(\tilde{z}*),$$

$$r(\tilde{z})=r\,*(\tilde{z}*).$$

The first , third, fourth and sixth equalities in (2.15) imply

$$(1-t_k)\frac{a}{\tilde{q}}+\frac{(\tilde{q}-1)^2}{2h\tilde{q}}=b\,+(1-\,g)\left(\frac{\tilde{q}-1}{h}\right),$$

$$(1-t_k*)\frac{a\,*}{\tilde{q}\,*}+\frac{(\tilde{q}\,*-1)^2}{2h*\tilde{q}\,*}=b\,*+(1-\,g*)\left(\frac{\tilde{q}\,*-1}{h*}\right),\qquad(2.16)$$

which suggests that at the steady-state equilibrium an increase in the tax on capital in either of the two economies will reduce the steady-state equilibrium growth rate in the economy, while an increase in the tax on bonds will leave this growth rate unaffected.

It is clear from (2.8) and (2.8') that changes in the costs of adjustment, h and h*, and developments that affect the market values of capital, q and q*, will affect the rates of growth in the two economies. Such developments might include changes in technology, a and a*, changes in the tax on bond revenue, t_b and t_b* (when the economies are not at the steady-state equilibrium), or changes in determinants of the effective interest rates, r and r*, some of which may originate from abroad. From the UIP assumption (2.12), developments in one country that affect the interest rate faced by its representative agent can be channeled to the corresponding agent in the other economy through changes in the real exchange rate.[6] The magnitudes of the

[6]Frankel, Razin, and Sadka (1991) provide the seminal analysis of the transmission of tax shocks in a world economy in a two-period model. Turnovsky and Bianconi (1992) adopt an intertemporally optimizing representative agent framework, which differs from the present model in distinguishing between the sources of income.

impacts will depend in large part on the values of the utility functions' parameters. From the budget constraints it is evident that exogenous changes in the availability of imported goods--possibly brought about by supply constraints or export/import quotas--can also affect an agent's debt- or credit-equity position, hence, effective interest rate, market value of capital, and rate of investment and growth in his or her economy. With (2.9a) and (2.9a'), the UIP assumption also implies that in the event discount rates and taxes on bond revenue in the two economies are the same, $ß = ß^*$ and $t_b = t_b^*$, the rates of change in the marginal utilities of wealth are related by

$$\frac{l}{l} = \frac{l^*}{l^*} + \frac{s}{s}. \tag{2.17}$$

Eliminating s, s, *mgs*, and *mgs** from the model by substitution and linearizing the resulting six-equation system around the steady-state solution , yields the following characterization of the local dynamics.

$$\begin{pmatrix} c \\ z \\ q \\ c^* \\ z^* \\ q^* \end{pmatrix} = \begin{pmatrix} 0 & A_{12} & A_{13} & 0 & A_{15} & 0 \\ A_{21} & A_{22} & A_{23} & 0 & 0 & 0 \\ 0 & A_{32} & A_{33} & 0 & 0 & 0 \\ 0 & A_{42} & 0 & 0 & A_{45} & A_{46} \\ 0 & 0 & 0 & A_{54} & A_{55} & A_{56} \\ 0 & 0 & 0 & 0 & A_{65} & A_{66} \end{pmatrix} \begin{pmatrix} c - \tilde{c} \\ z - \tilde{z} \\ q - \tilde{q} \\ c^* - \tilde{c}^* \\ z^* - \tilde{z}^* \\ q^* - \tilde{q}^* \end{pmatrix}, \tag{2.18}$$

where

$$A_{12} = \frac{-\tilde{c}}{h\ g + g - 1}[r'(\tilde{z})(1 - t_b) - h\ g r'(\tilde{z})]; A_{13} = -\frac{\tilde{c}}{h}; A_{15} = \frac{-h\ g}{h\ g + g - 1} r^{*\prime}(\tilde{z}^*);$$

$$A_{21} = -1; A_{22} = \frac{(\tilde{q} - 1)}{h} + [r(\tilde{z}) + r'(\tilde{z})\tilde{z}]; A_{23} = -\frac{(\tilde{q} - \tilde{z})}{h};$$

$$A_{32} = (1 - t_b) r'(\tilde{z})\tilde{q}; A_{33} = (1 - t_b) r(\tilde{z}) - \frac{(\tilde{q} - 1)}{h};$$

$$A_{42} = \frac{-h^*\ g^*}{h^*\ g^* + g^* - 1} r'(\tilde{z}); A_{45} = \frac{-\tilde{c}^*}{h^*\ g^* + g^* - 1}[r^{*\prime}(\tilde{z}^*)(1 - t_b^*) - h^*\ g^*\ r^{*\prime}(\tilde{z}^*)];$$

$$A_{46} = -\frac{\tilde{c}^*}{h^*}; A_{54} = -1; A_{55} = \frac{(\tilde{q}^* - 1)}{h^*} + [r^*(\tilde{z}^*) + r^{*\prime}(\tilde{z}^*)\tilde{z}^*]; A_{56} = -\frac{(\tilde{q}^* - \tilde{z})}{h^*};$$

$$A_{65} = (1 - t_b^*)r^{*\prime}(\tilde{z}^*)\tilde{q}^*; A_{66} = (1 - t_b^*)r^*(\tilde{z}^*) - \frac{(\tilde{q}^* - 1)}{h^*}.$$

We could speculate about the stability properties of the model in the neighborhood of the steady-state solution, but in view of the size of the system and the empirical orientation of this study it will be more expedient to evaluate the coefficients of the matrix from parameter estimates and compute the eigenvalues of the system directly.

3. Estimation of the Model

We proceed now to an examination of our two-good, two-country model in the case of Italy and Germany from the mid 1970s to the mid 1990s.[7] We note first that *q* and *q** (the market valuations of capital) have been defined as ratios of the two costate variables in each representative agent's optimization problem--*q*/*q'*/? and *q**/*q**'/?*. Eliminating q and q*, then, would effectively eliminate all four costate variables from the equations determining the evolution of the state variables. Considering the case of the domestic economy, recall that the time rate of change in *q* is determined in (2.14d) by

$$q = (1 - t_b)r(z)q - (1 - t_k)a - \frac{(q-1)^2}{2h}. \tag{3.1}$$

From (2.7) we note that q can easily be transformed into terms of *I/K* and *vice versa*:

$$q = 1 + h\frac{I}{K}, \frac{I}{K} = \frac{q-1}{h}. \tag{3.2}$$

Differentiating (3.2) w.r.t. time, we have

$$\dot{q} = h\frac{d}{dt}\frac{I}{K}. \tag{3.3}$$

Making use of (3.2) and rearranging enables us to rewrite the last term of (3.1) as

$$\frac{(q-1)^2}{2h} = \frac{h}{2}\frac{(q-1)^2}{h^2} = \frac{h}{2}\left(\frac{I}{K}\right)^2.$$

Hence, upon making substitutions and rearranging terms, (3.1) becomes

$$\frac{d}{dt}\frac{I}{K} = (1 - t_b)r(z)\left(\frac{1}{h} + \frac{I}{K}\right) - (1 - t_k)\frac{a}{h} - \frac{1}{2}\left(\frac{I}{K}\right)^2. \tag{3.4}$$

We next consider the second term on the r.h.s. of (2.14c) and observe that

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[7] This is not entirely in the 'two-country' spirit, as it does not exhaust the economies of the world with whom Italy and Germany enjoy trade relations. We had intended to examine the relationship between Italy and the rest of the industrialized nations but encountered difficulty in obtaining the necessary data in time to complete the estimation work and post-estimation evaluations and analyses.

$$\left(\frac{q^2-1}{2h}\right) = \frac{(q-1)}{h} + \frac{(q-1)^2}{2h} = \frac{I}{K} + \frac{h}{2}\left(\frac{I}{K}\right)^2 .$$

Similar substitutions and rearranging result in corresponding transformations of (2.14h) and the second term of (2.14g'). Defining the variables $k = I / K = K / K$, and $k^* = I^*/K^* = K^*/K^*$, the dynamic system (2.14) can be written equivalently as

$$c = c\left(\frac{1}{h\,g - g - 1}[b - (1 - t_b)r(z) + h\,g\frac{s}{s}] - k\right), \tag{3.5a}$$

$$mgs = h\,c/s - h\,cs/s^2, \tag{3.5b}$$

$$z = -c - s \bullet mgs - k - \frac{h}{2}k^2 + kz + a - r(z)z, \tag{3.5c}$$

$$k = (1 - t_b)r(z)\left(\frac{1}{h} + k\right) - (1 - t_k)\frac{a}{h} - \frac{1}{h}k^2, \tag{3.5d}$$

$$c^* = c^*\left(\frac{1}{h^*g^* - g^* - 1}[b^* - (1 - t_b)r^*(z^*) - h^*g^*\frac{s}{s}] - k^*\right), \tag{3.5e}$$

$$mgs^* = h^*s\,c^* + h^*cs, \tag{3.5f}$$

$$z^* = -c^* - mgs^*/s - k^* - \frac{h}{2}k^{*2} + k^*z^* + a^* - r^*(z^*)z^*, \tag{3.5g}$$

$$k^* = (1 - t_b^*)r^*(z^*)\left(\frac{1}{h^*} + k^*\right) - (1 - t_k^*)\frac{a^*}{h^*} - \frac{1}{h^*}k^{*2}, \tag{3.5h}$$

$$s = s\left(r(z) - r^*(z^*)\right). \tag{3.5i}$$

Choosing an appropriate homogeneous functional form for *r(z)* and *r*(z*)*, one could estimate the parameters of a linear approximation of equation system (3.5), in which the nonlinear terms have been linearized around the sample means or the steady-state solution values of *c*, *c**, *mgs*, *mgs**, *z*, *z**, *k*, *k**, and r^w, without imposing the transversality conditions of the underlying optimization problem that gave rise to the model's specification. This approach is often taken in estimating simple Euler-equation models. But it is not clear to what extent sample means provide meaningfully representative data points in a time-series model of economic growth, and the economy may be off the steady-state growth path for much of the sample period. Moreover, as Gandolfo et al. (1996) and Donaghy (1998) have shown, there can be substantial differences in maximum-likelihood estimates of the parameters of non-linear continuous-time models when the models are estimated in non-linear and linear forms. One could also employ the exact quasi-FIML non-linear continuous-time estimator of Wymer (1993), which is implemented according to the following two-step algorithm, again without imposing transversality conditions.

1. For a given set of parameter estimates (or initial values), the equation system is integrated forward over each observation interval by a numerical variable-order, variable-step Adams method, residuals are computed by comparing the one-period-forward solution values with the observed values, and the variance-covariance matrix is then formed.

2. The natural logarithm of the variance-covariance matrix is minimized by a quasi-Newton method to update parameter estimates. Convergence criteria are then checked and, if not met, another iteration is begun.

Across-equation restrictions on parameter values implied by theory are, of course, imposed in estimation.[8] But the one-period-forward solution of the model in this algorithm is inconsistent with the nature of the intertemporal optimization problem faced by the representative agents, which entails a 'dynamic' solution, and failure to account systematically for the transversality conditions may lead to omissions of important restrictions on values that parameter estimates can assume . These oversights may invalidate empirical tests of propositions emanating from representative agent models.

To estimate the model, then, we adopt a third approach introduced by Wymer (1997). This approach enables one to work directly with the complete set of theoretical conditions characterizing a macrodynamic equilibrium--i.e., the state and costate equations, initial endowments, and the transversality conditions that impose the representative agents' instantaneous budget constraints at every data point. Importantly, the intertemporal optimization assumed of the representative agents is *incorporated directly in the estimation algorithm*, which entails the following steps.[9]

1. A set of 'observations' on the unobserved variables and transversality conditions are generated. (For the first iteration this may be done via numerical simulation with plausible values of the parameters.)

2. For a given set of parameter estimates (or initial values), a *shooting* algorithm is deployed in which the equilibrium conditions are solved by a variable-order, variable-step Adams method for each data point over a time horizon that (ideally) is sufficiently long for the

[8]This second approach was followed in the research reported in Donaghy et al (1999a), in which a one-good, one-country endogenous-growth model of an open economy was estimated with Italian data.

[9]The estimation algorithm bears some resemblance to the Fair-Parke algorithm for maximum-likelihood estimation of nonlinear rational-expectations models in discrete time (Fair and Taylor, 1983). For an early treatment of continuous-time estimation of linear rational-expectations models, see Hansen and Sargent (1991).

transversality conditions to be satisfied at an acceptable level of tolerance.[10] (The solutions must converge for each observation.) 'Observations' on the costate variables and transversality conditions are updated and residuals are computed for those variables on which there are empirical observations by comparing their dynamic solution values with their observed values and the variance-covariance matrix is then formed.

3. The natural logarithm of the variance-covariance matrix is minimized by a quasi-Newton method to update parameter estimates. Convergence criteria are then checked and, if not met, another iteration is begun.

Imposing the transversality conditions presents a problem since they embody a limit concept. In some cases one can quite reasonably assume that they are satisfied--e.g., if one is confident that the absolute values of the estimates of the agents' subjective discount rates, *ß* and *ß**, exceed the growth rates of the levels of consumption and the capital stock. But by taking the conditions, which in the limit must converge to zero, sufficiently far out in time, so that they satisfy a reasonably small tolerance level, one can gain information internal to the estimation process needed to impose a boundary point condition on the trajectories of the unobservable costate variables. (See e.g., Judd, 1998.) Because the solution algorithm entails solving the model *beyond* the period for which observations are available, a zero-order forcing function of time must be introduced for each exogenous variable other than time.

Although the first-order equations in *q* and *q** have been replaced by equations in *k* and *k**, the costate equations determining the levels of *?* and *?** must be included in the estimated model if the transverality conditions are to be imposed. From the arbitrage relationships (2.9a) and (2.9a'), these are

$$\dot{l} = l[b - r(z)(1 - t_b)], \text{ and} \tag{3.6a}$$

$$\dot{l}^* = l^*[b^* - r^*(z^*)(1 - t_b^*)]. \tag{3.6b}$$

The zero-order transversality conditions (2.10) and (2.10') can be rewritten in terms of the estimating model's variables as

$$lze^{-bt} = 0, l(1 + hk)e^{-bt} = 0; \text{ and} \tag{3.7a}$$

$$l^* z^* e^{-bt} = 0; l^*(1 + h^* k^*)e^{-b^* t} = 0. \tag{3.7b}$$

For the purposes of this exercise we defined *r(z)* and *r*(z*)* as

$$r(z) = r^w + qz, \text{ and} \tag{3.8a}$$

$$r^*(z^*) = r^w + qz^*. \tag{3.8b}$$

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[10]Minford (1992) reports that in many empirical discrete-time rational-expectations studies, in which a backward recursive solution approach is taken, the number of periods lagged is typically the number beyond which parameter estimates are unaffected. In solving the model forward we adopt a time horizon that extends as far out as still permits dynamic solutions to converge for each observation and beyond which parameter estimates are unaffected.

in which r^w is the US Treasury-bill yield.

To estimate the model we used quarterly data published for Italy and Germany (1977:1-1992:4) on private consumption, imports, capital formation and consumption, stocks of net foreign assets, 1990 GDP price deflators, exchange rates, and the US Treasury bill yield. (See the data appendix.) The time series were converted to real terms and deseasonalized around the trend.[11]

To obtain suitable initial values for the parameter estimates, we estimated the model first with the US Treasury bill yield exogenous, without the costate equations (3.6), and with the transversality conditions (3.7) not imposed using the one-period-forward solution algorithm discussed above. In so doing we imposed constraints on the values that several of the parameters could assume. We constrained the tax parameters to lie between 0.1 and 0.4 and the cost of adjustment parameters to not exceed 16.0. The bounds on the tax parameters are based on knowledge of their historical limits, whereas the choice of an upper bound on adjustment costs was informed by Barron and Sala-i-Martin's (1995) discussion of quadratic adjustment costs. We then estimated a forcing function for the US Treasury bill yield separately from the model to obtain the values of its coefficients, which, if r^w is indeed exogenous, should be determined independently of the model's parameters. The form of the function employed was

$$r^w = \gamma_1 + \gamma_2 t + \gamma_3 e^{(\gamma_4 t + \gamma_5)} + \gamma_6 e^{(\gamma_7 t + \gamma_8)} \cos(\gamma_9 + \gamma_{10}), \qquad (3.9)$$

for which the estimates of the coefficients and their asymptotic standard errors were as reported in Table 1.

Table 1. Estimates of Forcing Function Coefficients

Parameter	*Parameter Estimate*	*Estimate of ASE*	
γ_1	1.022	0.227	
γ_2	-0.006	0.001	
γ_3	-1.054	0.305	
γ_4	-0.007	0.003	
γ_5	-0.039	0.075	
γ_6	0.030		0.022
γ_7	0.003		0.015
γ_8	-1.384	0.482	
γ_9	0.145	0.018	
γ_{10}	-2.128	0.801	

The mean value of the quarterly yield was 0.02051, whereas the root-mean-square error of in-sample one-period forecasts of the forcing function was 0.00289. Employing the forcing function and the initial values of the parameter estimates obtained earlier, we re-estimated the model using

[11]Wymer's program TRANSF was used to prepare the data for estimation. All estimation work reported in the paper was carried out with his program ESCONA.

the dynamic boundary-point solution algorithm with the costate equations included, the transversality conditions imposed, and the same constraints on the parameters as before.

Table 2 presents the means and standard deviations of the endogenous variables and the root-mean-square errors (RMSEs) of in-sample one-period forecasts obtained with the model when estimated by the one-period-forward (OPF) and dynamic-boundary-point (DBP) solution algorithms, and the RMSEs of the out-of-sample dynamic forecasts produced by the model estimated by the two approaches.[12] While the fit of the model to the sample data is very good for parameter estimates obtained by both approaches, it is slightly superior for OPF estimates. In the case of the out-of-sample dynamic forecasts, the model estimated with the DBP algorithm is far superior. [13]

Table 2. Forecasting Errors of the Estimated Model

	Endogenous Variables		*In-Sample RMSEs*		*Out-of-Sample RMSEs*	
	Means	*Std. Dev'ns.*	*OPF†*	*DBP††*	*OPF*	*DBP*
c	0.060365	0.003323	0.000517	0.000718	0.001222	0.000544
mgs	0.003910	0.000418	0.000128	0.000190	0.000711	0.000159
z	0.012224	0.017259	0.008301	0.008359	0.140677	0.005113
k	0.009241	0.002286	0.000609	0.001123	0.007102	0.000680
*c**	0.038678	0.002270	0.000629	0.000876	0.003259	0.000500
*mgs**	0.001119	0.000109	0.000029	0.000050	0.000181	0.000061
*z**	0.027538	0.005284	0.003359	0.003471	0.020807	0.002992
*k**	0.005826	0.001345	0.000408	0.000557	0.005983	0.000428
s	0.872045	0.124082	0.015811	0.028283	0.096124	0.041179

†OPF denotes results obtained with model estimated by algorithm with one-period-forward solution.

††DBP denotes results obtained with model estimated by algorithm with dynamic boundary-point solution.

We are not aware of any statistical tests for cointegration of the variables or for autocorrelation in the residuals of *non-linear* simultaneous equation systems. Although they are perhaps inappropriate, we have nonetheless conducted single-equation augmented Engle-Granger (AEG) residual-based tests of cointegration because of the increasing sensitivity in the field to the

[12]Dynamic out-of-sample RMSEs for the model estimated with the OPF algorithm were computed using Wymer's program APREDIC. All others were produced in ESCONA.

[13]It would seem that these findings support Minford's (1992) speculation that models with feed-forward expectations (in this case, 'perfect foresight') are likely to forecast less well in-sample than models with myopic or adaptive expectations but forecast better than the latter out-of-sample.

charge that the institutional structures captured by structural-equation econometric models are 'fragile' artifacts of the data, do not represent 'long-run equilibrium relationships,' and hence cannot, when used in simulations, support robust theoretical inferences. The AEG tests we conduct are based on a five-period-lag auto-regression of the residuals of the 'constant, trend, and squared trend' type, i.e.,[14]

$$\Delta e_t = a + b_1 t + b_2 t^2 + c_0 e_{t-1} + c_1 \Delta e_{t-1} + \ldots + c_5 \Delta e_{t-5}. \quad (3.10)$$

This test is not invalidated by non-normality or heteroscedasticity in the residuals and accounts for the presence of serial correlation of order five or lower by virtue of its lag structure. It is known, however, to be biased *against* rejection of the null hypothesis of noncointegration (i.e., c_0=0) when data have been deseasonalized (Davidson and McKinnon, 1993). The results of the AEG tests, tabulated below, suggest that at the 95% level of confidence, noncointegration can be rejected for all equations except that determining *k*, the rate of growth of the capital stock in Italy. This finding gives us some encouragement that the model depicts non-spurious long-run equilibrium relationships and that inferences drawn from experiments conducted with the model should at least be plausible, if the model is acceptable on other terms.

Table 3. Augmented Engel-Granger Test of Noncointegration

Variable	*Estimate of c_0*	*Phillips' Non-parametric z*†*
c	-0.558	-31.2
mgs	-1.290	-72.1
z	-0.641	-35.9
k	-0.325	-18.2 (>-28.1)
*c**	-1.040	-58.1
*mgs**	-0.999	-55.9
*z**	-0.538	-30.1
*k**	-0.660	-37.0
s	-0.712	-34.9

†Davidson and McKinnon (1993).

Estimates of the parameters and their standard errors are given in Table 4. We include for purposes of comparison estimates obtained with both the OPF and DBP solution approaches. The latter estimates are clearly more efficient, with only the estimate of ?* not being discernible

[14]The rule of thumb for the number of lags discussed in Davidson and McKinnon (1993) is that there should be at *least* $n^{1/3}$ lags, which for a sample size of 64 would be 4.

from zero at a conventionally accepted level of statistical significance.[15] While the results are very similar for most of the parameters, there are noticeable differences, especially in estimates of the discount rates, *ß* and *ß**, which appear in costate equations (3.6a) and (3.6b), parameters appearing in the transversality conditions, *?*, *?**, and *h**, and the German imports substitution parameter, *?**.[16] This suggests that obtaining a dynamic boundary-point solution of the model including costate equations and imposing the transversality conditions in the estimation algorithm will be important for producing unbiased estimates of parameters for post-estimation model evaluation, hypothesis testing, forecasting, or simulation experiments.

Table 4. Quasi-FIML Estimates of Parameters and their Asymptotic Standard Errors

	OPF Solution Algorithm		*DBP Solution Algorithm*	
Parameter	*Parameter Estimate*	*Estimate of ASE*	*Parameter Estimate*	*Estimate of ASE*
a	0.07345	0.00120	0.07425	0.00118
ß	0.02138‘	0.02408	0.01500†	0.4E-6
?	-0.73826‘	2.17910	-1.66211	0.45212
?	0.04155‘	0.05549	0.04379	0.01239
?	0.61791	0.12860	0.62447	0.11789
h	16.0000††	0.2E-7	16.0000††	0.2E-7
t_b	0.10000†	0.6E-8	0.10000†	0.3E-8
t_k	0.40000††	0.1E-8	0.40000††	0.1E-8
*a**	0.04402	0.00057	0.04388	0.00053
*ß**	0.01794	0.00773	0.02260	0.00463
*?**	-0.65293‘	1.02887	-0.09286‘	0.58569
*?**	0.00579‘	0.00340	0.01120	0.00526
*?**	0.89346	0.22672	0.98350	0.14327
*h**	13.2785	5.01934	16.0000††	0.00182
t_{b^*}	0.40000††	0.5E-7	0.40000††	0.5E-6
t_{k^*}	0.30135	0.09662	0.25919	0.05839

‘ denotes estimate not discernible from zero at 5% level of statistical significance.
† denotes estimate at lower bound of interval constraint.
†† denotes estimate at upper bound of interval constraint.

[15]The insignificant estimate of *?** suggests that the parameter may be zero and hence the utility function for Germany may be logarithmic.

[16]Inspection of the RMSEs of the out-of-sample dynamic forecasts in Table 2 reveals why this is the case: the time path of *z* obtained with the model estimated by the algorithm with the OPF solution approach is *explosive*, whereas parameter estimates obtained with the BPD algorithm have been adjusted to ensure that the solution path of *z* is *convergent*.

The (DBP) estimates of *?* and *?** imply disparate intertemporal elasticities of substitution for Italy and Germany of 0.376 and 0.915. While, by virtue of the specification chosen for the utility function, the real-exchange-rate or relative-price-elasticities of demand are unity, the small values of the coefficients of the rate of change in the real exchange rate appearing in equations (3.5a) and (3.5a'),

$$\frac{h\ g}{h\ g - g - 1} = 0.027, \frac{h^{*}\ g^{*}}{h^{*}\ g^{*} - g^{*} - 1} = 0.001, \tag{3.10}$$

suggest that, in the case of Italy and Germany, determinants of changes in effective interest rates in one of these economies did not have a strong influence on the other over the sample period.

To examine the local stability of the model we solved the conditions for a steady-state equilibrium solution (2.15) numerically, used the steady-state solution values obtained for c, z, q, c*, z*, and q* (and the sample mean value of r^{w}) to reduce and linearize the equation system (2.14) as in (2.18), and computed the eigenvalues of the linearized system, which are those given in Table 5.[17]

Table 5. Eigenvalues of the Model

0.16624
0.09088
0.02368±0.76971i
-0.02126±0.53334i

Since four of the eigenvalues are complex, it is clear that the nature of economic growth over the sample period was cyclical, which confirms previous studies of the Italian economy. (See Gandolfo and Padoan, 1990.) And since the real parts of four of the eigenvalues are positive, the model will be saddle-path stable in the neighborhood of the steady-state solution just in case there are four jump variables (Gandolfo, 1997). Two logical candidates in the theoretical model are *q* and *q**, the market valuations of capital, or *k* and *k**, the growth rates, since, from (2.8) and the analogous expression for Germany, these are simple functions of *q* and *q**. Turnovsky (1997), in discussing a one-good, single-economy prototype of the present model, suggests that *c* and--by logical extension--*c** would be candidates for the other jump variables but that growth paths of *z* and *z** would be characterized by predictable transitional dynamics. Because in most industrial economies real consumption is smooth (see e.g., Quah, 1990), and because changes in *z* and *z** are--from (2.14) and (2.14')--heavily dependent upon the behavior of *q* and *q**, the finding of four eigenvalues with real parts would put in question *either* the stability *or* the realism of the model in the present case.[18]

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[17]Wymer's program CONTINEST was used for this purpose.

[18]An analysis of the model's global stability properties in terms of Lyapunov characteristic exponents remains to be performed.

4. Responsiveness of the Model to Changes in Adjustment Costs, Tax Policies, Technologies, and International Markets

It is evident--from (3.10) above--that in the bilateral trade relationship between the Italian and German economies during the estimating sample period changes in the real exchange rate, however induced, did *not* have a strong effect on the rate of change in consumption per unit of capital in either country. Nonetheless, it may be useful to examine how these economies--as characterized in the model--might have responded to developments in tax policies, technologies and international markets for insights regarding the *qualitative* nature of the responses. To do so, we re-solve the model over the three-year out-of-sample period of 1995:1-1997:4 (by the DBP solution approach) for twelve counterfactual scenarios featuring isolated changes. Taking the solution of the model as estimated to be Scenario 1 (the 'baseline' solution), we consider in Scenarios 2 and 3 25% reductions in capital adjustment costs, h and h^*. In Scenario 4 the estimate of the Italian bond revenue tax rate, t_b, is reduced from its upper-bound level, 0.4, to its lower-boundary level, 0.10, while in Scenario 5 the estimate of the German analog of this tax rate, t_b^*, is increased from its lower-bound level of 0.1 to 0.4. In Scenario 6 the value of the Italian capital-revenue-tax rate t_k is reduced to 0.25 (approximately the level of the estimate of t_k^*), whereas in Scenario 7 the estimated German tax rate t_k^* is increased to 0.4. We consider in Scenarios 8 and 9 one-point increases and decreases in the quarterly world interest (US Treasury bill) rate, r^w. To examine the effects of technological change we increase the output-capital ratios, a and a*, by 0.01 in Scenarios 10 and 11. Finally, we explore in Scenarios 12 and 13 the nature of impacts that import quotas (limits or purchase commitments) might have brought about by setting to zero the time rates of change in the import-capital ratios. Given model assumptions of perfect foresight and intertemporal optimization and the nature of the solution algorithm employed in estimating and simulating the model, this exercise is decidedly *not* vulnerable to the so-called 'Lucas Critique' of econometric policy evaluation (Lucas, 1981).

Table 6 presents the mean solution values of all endogenous variables in the model when the market values of capital, q and q', have been eliminated and growth rates, k and k^*, and marginal utilities of wealth, $?$ and $?^*$, included. In the left-most column of the table are the changes made to the estimated model in the scenarios. Figures in the body of the table in bold italics are means of variable levels that differ from those obtained in the baseline solution. Recall that because an '*AK*' technology has been assumed, the mean values of the levels of k and k^* give the average rates of quarterly endogenous growth in the two economies over the twelve-quarter period. From the solutions of the model in these scenarios and the preceding analysis one can make the following observations and inferences.

1. Not surprisingly, when the magnitudes of estimates of the utility-function parameters are small, the repercussions of developments in one economy are faint in the other. The real exchange rate is nonetheless effected by *all* the changes considered and the marginal utilities of wealth are perceptibly affected by many of them (although more so in Italy than in Germany). If imports from the trading partners factored more prominently in the utilities of the representative agents the spillover effects would surely have been larger as agents acted on these changes.

2. Changes in technology in either economy would have affected both economies through their influence--in equations (3.5c) and (3.5g)--on the credit- or debt-equity position of the country in which changes occurred and hence their influence on the real exchange rate. Interestingly, it appears that increased productivity at home would have led to increased consumption of imported goods abroad (i.e., exports) but increased consumption of only domestic goods at home, as the effects of increased consumption of the complementary domestic good and the relative price change cancel out in the imports equation (an artifact of the utility function). Technical change in Italy would seem to have induced *both* increased consumption *and* investment, hence growth, in Germany, but the converse would not have held.

3. In addition to changes in technology, developments that would have affected quantities of traded goods are changes in the German bond-revenue tax rate, the world interest rate, and the imposition of import quotas. From (2.9a) and (2.9a')--and the apparent complementarity of imported and domestic consumer goods--an increase in the world interest rate, would have increased the marginal utility of present consumption of both imported and domestic goods, hence consumption itself, while a reduction would have had the opposite effect.

4. The effects of an imports quota--in this case, imports stabilization--would appear to have been limited to the level of imports, the credit- or debt-equity position, and the marginal value of wealth in the economy in which the quota was imposed. It would also appear that these effects would have been completely internalized by that economy. This is the case because in the model's specification there is no channel--provided, say, by a domestic supply constraint--through which the change in foreign demand can be fed back to the domestic supplier.

5. A reduction in the cost of adjustment of Italy's physical capital stock would have induced increased investment (hence increased output) and consumption, an improvement in the debt-equity position, a slight increase in the real exchange rate, and a higher marginal value of wealth. A similar cost reduction in the case of the German economy would have elicited similar responses, except that the investment rate would have been puzzlingly *lower*.

6. It appears that the effects of changes in tax policies would have been internalized by the economy in which the changes occurred, perhaps because of the lump-sum nature of the tax rebate. Contrary to what we learn from studying the behavior of the model at its steady-state solution, however, it appears that an increase (decrease) in the tax on revenue from capital within an economy would have increased (decreased) the rate of endogenous growth, k or k^*, in that economy and an increase in the tax on revenue from bonds *would indeed* have affected the growth rate. The first result follows from the fact that in (2.14d) and (2.14h) as t_k and t_k^* increase , q and q^* must increase. The higher market values of

capital, for given adjustment costs, h and h^*, enhance the rates of growth of capital via the optimality conditions (2.7c) and (2.7c'). The intertemporal trade-offs between consumption and investment result in reductions in consumption (per unit of capital) and--by virtue of the nature of the enforcement of the intertemporal budget constraint, which determines changes in holdings of net foreign assets by residual--holdings of these assets per unit of capital are also reduced. As for the effects of changes in the taxes on bond revenues on these economies' growth rates, inspection of (3.5c) and (3.5g) suggests that increases in t_b and t_b^* will lead directly to reductions in the rates at which k and k^* grow, and, depending on the magnitudes of these reductions, also reductions in the changes in c and c^*. Meeting the intertemporal budget constraints then entails that the changes in z and z^* increase.

Table 6: Mean Values of the Variables of the Model in Alternative Scenarios (1995:1-1997:4)

Scenario	*c*	*mgs*	*z*	*k*	*?*	*c**	*mgs**	*z**	*k**	*?**	*s*
1. *Baseline Solution*	0.05437	0.00293	-0.00295	0.00151	0.00173	0.04155	0.00124	0.02257	0.00576	0.00155	0.94680
2. h=12.0	***0.05440***	0.00293	***-0.00251***	***0.00700***	***0.00182***	0.04155	0.00124	0.02257	0.00576	0.00155	***0.94690***
3. h^*=12.0	0.05437	0.00293	-0.00295	0.00151	0.00173	***0.04157***	0.00124	***0.02275***	***0.00555***	***0.00161***	***0.94674***
4. t_b =0.4	***0.05433***	0.00293	***-0.00288***	***0.00139***	***0.00135***	0.04155	0.00124	0.02257	0.00576	0.00155	***0.94682***
5. t_b^*=0.1	0.05437	0.00293	-0.00295	0.00151	***0.00174***	***0.04265***	***0.00125***	***0.02154***	***0.00651***	***0.00156***	***0.94713***
6. t_k=0.25	***0.05440***	0.00293	***-0.00260***	***0.00082***	***0.00199***	0.04155	0.00124	0.02257	0.00576	0.00155	***0.94690***
7. t_k^*=0.4	0.05437	0.00293	-0.00295	0.00151	0.00173	***0.04153***	0.00124	***0.02236***	***0.00615***	***0.00141***	***0.94687***
8. $?r^w$=+.01	***0.05456***	***0.00294***	***-0.00347***	***0.00209***	***0.00175***	***0.04216***	***0.00125***	***0.02227***	***0.00618***	***0.00156***	***0.94673***
9. $?r^w$=-.01	***0.05418***	***0.00292***	***-0.00244***	***0.00093***	***0.00171***	***0.04095***	0.00124	***0.02285***	***0.00535***	***0.00154***	***0.94687***
10. a=.084	***0.05445***	0.00293	***0.00720***	***0.00209***	***0.00175***	***0.04216***	***0.00125***	***0.02227***	***0.00618***	***0.00156***	***0.94673***
11.a^*=.054	***0.05436***	***0.00294***	-0.00295	0.00151	***0.00173***	***0.04190***	0.00124	***0.03300***	***0.00550***	***0.00175***	***0.94201***
12. mgs =0	0.05437	***0.00287***	***-0.00292***	0.00151	***0.00174***	0.04155	0.00124	0.02257	0.00576	0.00155	***0.94681***
13. mgs^*=0	0.05437	0.00293	-0.00295	0.00151	0.00173	***0.04172***	***0.00126***	***0.02256***	0.00576	0.00155	***0.94681***

5. Discussion and Conclusions

In this paper we have developed a representative-agent model for examining the relationship between endogenous growth and international trade and the transmission of externalities from one economy to a second. We have demonstrated how this model may be estimated in a manner that is consistent with its underlying behavioral assumptions and have evaluated its empirical performance in terms of tests commonly used to assess structural-equation models. We have moreover employed the model in counterfactual deterministic simulations to explore how the Italian and German economies might have responded to a number of developments emanating from within their economies and abroad. Among the substantive findings of this research are the following. Most developments considered within either of these economies would appear to have had some repercussion in the other, no matter how faint. Import quotas would not seem to have noticeably affected the rate of growth in the exporting economy and most of the effects of changes in domestic tax policies would appear to have been internalized in the economy in which they occurred. The latter results would appear to be due largely to two features of the model's specification--its demand-side orientation and the lump-sum nature of tax rebates--and so perhaps tell us more about the model than bilateral relations of trade and growth between Italy and Germany. In the 'real-business-cycle' type of framework we have investigated it would appear that technical change is the mechanism propagating changes in one economy that would have had the greatest impact on the other. Although there is much symmetry in the response patterns of the two economies, it is not perfect. From a formal perspective, this should not be surprising in view of the degree of nonlinearity of the model. From a researcher's point of view it is reassuring that institutional differences can emerge from empirical work with a highly stylized equilibrium framework. Among the methodological findings to come out of this work is clear evidence that the algorithm used to solve the model in the estimation *does* make a difference and implementation of an algorithm that takes into account the full set of constraints upon the model's dynamic solution will be important for purposes of testing assumptions about parameter values or institutional stability. We also find that the out-of-sample dynamic forecasts of the model estimated using the dynamic-boundary-point solution algorithm were significantly better than those made with the model estimated with the one-period-forward solution algorithm.

While we have shown how a continuous-time model of the representative-agent genre can be estimated, evaluated, and employed in analyses of possible developments, we have not discussed how such basic assumptions as intertemporal optimizing behavior, perfect foresight, and equilibrium growth that it embodies might be tested.[19] In applied econometrics it is difficult to test just one of a model's behavioral assumptions in the presence of its others. Hence nearly all hypothesis tests are compound. Models are, moreover, not meaningfully tested in isolation but against other models that derive from competing theoretical explanations of the same phenomena. (See Chow, 1983.) It is reasonable, then, to expect that the present (or any other) empirical representative-agent model of endogenous growth and international trade and the set of assumptions it embodies would be judged in comparison with competing empirical models that are equally ambitious in terms of what they purport to explain. While in- and out-of-sample forecasting performances will be important, they cannot be completely decisive in the choice between models if what we are after ultimately is grasping more fully the nature of the relationship

[19]But see Donaghy (1999) for some observations on these issues.

between endogenous growth and international trade. And since the preferred combination of model selection criteria is a field-dependent issue, we cannot hope to resolve it here by further speculation.

In the mutual accommodation of models and data that goes on in empirical research, we use models to interrogate the world and data to assess our models. Proceeding directionally from 'data'--broadly construed--to 'model,' we have found that there are developments the present model fails to capture. In opting for transparency and parsimony in this initial foray, we have perhaps delimited too severely the range of macroeconomic phenomena that can arise within the stylized world our model represents. This finding puts in question the extent to which this particular model is useful in understanding its intended subject and calls for specification changes but does not lead to wholesale rejection of the modeling approach. By demonstrating how a model of this type can be operationalized we hope to have contributed to the development of an environment in which representative-agent models in international economics can be constructively engaged by and fairly compared with models from other traditions within the field.

Our assessment of the model suggests consideration of changes in both its specification and derivation. With regards to its specification, the results of the simulations clearly indicate the need to relate explicitly changes in foreign consumption of domestic exports to domestic behavior. Moreover, the model could (and should) be extended to allow minimally for explicit treatment of government spending--and possibly household labor-supply--decisions made subject to appropriate intertemporal budget constraints. (See, e.g., Turnovsky, 1997.) The functional forms employed to characterize utility and technology, while popular for their transparency and ease of use, are perhaps not realistic and are certainly heroic in what they assume of households and firms. The present specification should be tested against others that are derived from more general (and flexible) functional forms. With regards to the manner in which the conditions characterizing a macrodynamic equilibrium are derived, we note that it may be possible to usher in greater realism and complexity without increasing the difficulty of implementation if one were to employ results from intertemporal duality theory, thereby obviating the need to work directly with the primal of the optimization problem (the present-value Hamiltonian). (On this see Cooper, 1999.) Issues of model testing, specification, and derivation will be followed up in subsequent studies.

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Data Appendix

Published Sources of Data:

BBI Bulletin of the Bank of Italy
DOT *Direction of Trade,* International Monetary Fund.
IFS *International Financial Statistics,* International Monetary Fund.

Variable Definitions:

s Real exchange rate: Lit/$ exchange rate divided by 1,000xDM/$ exchange rate, multiplied by the German 1990 GDP price deflator, and divided by the Italian 1990 GDP price deflator. The latter was computed as the ratio of GDP at current prices to GDP at constant 1990 prices. Sources: BBI and IFS.

C Real Italian consumption of domestic goods and services. Total domestic consumption of goods and services at current prices, less government domestic consumption at current prices, divided by the Italian 1990 GDP price deflator. Source: BBI.

MGS Real Italian imports of German Goods and services. Goods and services imported from Germany by Italy in current $US, multiplied by the Lit/$ exchange rate and divided by the 1990 GDP price deflator. Sources: BBI, DOT, and IFS.

b Real Italian net foreign assets. Net foreign assets in lire divided by the 1990 GDP

price deflator. Sources: BBI and IFS.

K Real Italian capital stock. Gross capital formation at current prices less capital consumption at current prices divided by the 1990 GDP price deflator and accumulated on a base stock of 3,554.99 trillion lire for 1990:4. Source: BBI.

*C** Real German consumption of domestic goods and services. Private consumption of domestic goods and services at current prices divided by the 1990 GDP price deflator. Source: IFS.

*MGS** Real goods and services imported from Italy by Germany in $US multiplied by the DM/$ exchange rate and divided by the 1990 GDP price deflator. Sources: DOT and IFS.

*b** Real German net foreign assets. Net foreign assets in deutchmarks divided by the 1990 GDP price deflator. Source: IFS.

*K** Real German capital stock. Gross capital formation at current prices less capital consumption at current prices divided by the 1990 GDP price deflator and accumulated on a base stock of 6,704 billion DM 1978:4.

r^w World interest rate. Quarterly US thirty-year Treasure-bill rate. Source: IFS.

All time series were deseasonalized around the trend, as recommended by Durbin (1963).