Kuzin, Vladimir; Bremus, Franziska

Working Paper

Unemployment and Portfolio Choice: Does Persistence Matter?

FINESS Working Paper, No. D.4.5

Provided in Cooperation with:
German Institute for Economic Research (DIW Berlin)

Suggested Citation: Kuzin, Vladimir; Bremus, Franziska (2010) : Unemployment and Portfolio Choice: Does Persistence Matter?, FINESS Working Paper, No. D.4.5, Deutsches Institut für Wirtschaftsforschung (DIW), Berlin

This Version is available at:
https://hdl.handle.net/10419/119499

Project funded under the Socio-economic Sciences and Humanities

European Commission

Working Paper D.4.5

Unemployment and Portfolio Choice: Does Persistence Matter?

Vladimir Kuzin and Franziska Bremus

February 4, 2010

Unemployment and Portfolio Choice: Does Persistence Matter?*

Vladimir Kuzin
DIW Berlin
vkuzin@diw.de

Franziska Bremus
DIW Berlin
fbremus@diw.de

this version: February 4, 2010

Abstract

We use a life-cycle model of consumption and portfolio choice to study the effects of social security on the investment decisions of households for the European case. Our model is mainly based on the one developed by Cocco, Gomes, and Maenhout (2005). We extend it by unemployment risk using Markov chains to model the transition between different employment states. In contrast to most models in the life-cycle literature, our model allows for three different states, namely employment, short-term as well as long-term unemployment. This allows us to examine the effects of persistence in the unemployment process on portfolio choice. Our main findings are, first, that in case of short-term unemployment only, social security systems as those established in the EU are able to offset the negative impact of unemployment risk on the portfolio-share invested in risky assets. Second, the simulation results reveal that when allowing for long-term unemployment the equity-share is suppressed, especially for young investors. We show that this negative effect of unemployment is mainly driven by its persistence.

JEL Classification Codes: D91, E21, H31

Keywords: precautionary saving, unemployment insurance, long-term unemployment, income uncertainty

*This paper represents the authors' personal opinions and does not necessarily reflect the views of DIW Berlin. Financial support from the European Commission (7^{th} Framework Programme, Grant Agreement No. 217266) is gratefully acknowledged. We are grateful to seminar participants at the Institute for Applied Economic Research (IAW) in Tübingen for helpful comments. The codes for this paper were written in Matlab.

1 Introduction

In the last decade finite horizon life cycle models have been intensively used to study the optimal portfolio choice of individuals under different conditions. One important field of the literature has been dedicated to the optimal allocation of savings between riskless assets and risky stocks over the life cycle. In a seminal study, Cocco et al. (2005) employ a realistically calibrated life cycle model of consumption and portfolio choice with non-tradable labor income and borrowing constraints where the optimal share invested in equities is falling over life, since labor income constitutes a substitute for riskless asset holdings. In early years of their life individuals invest fully in stocks while in midlife saving for retirement becomes an important behavioral factor - the optimal share of savings invested in stocks thus declines. However, the modeling of so-called disastrous labor income shocks, originally introduced by Carroll (1997) which feature zero income with a small probability changes the results: it lowers the optimal share of risky assets especially for young agents. In a recent article, Chai, Maurer, Mitchell, and Rogalla (2009) derive optimal life cycle portfolio asset allocations for individuals who can choose their hours of work and their retirement age endogenously. Moreover, investment opportunities do not only include risky stocks and riskless bonds, but also survival-contingent payout annuities. They find, inter alia, that introducing annuities leads to earlier retirement and higher participation rates in financial markets by elderly agents.

Another strand of the literature is concerned with the analysis of social security issues, e.g. unemployment insurance, in the life cycle framework. Imrohoroglu, Imrohoroglu, and Joines (1995) develop a general equilibrium model to examine the welfare benefits of unemployment insurance. They find that an unfunded social security system is able to enhance economic welfare. On the other hand, Engen and Gruber (2001) show a negative impact of unemployment insurance on asset accumulation in a life cycle model and empirically confirm this result in a panel study for the US. However, these studies do not consider the optimal portfolio allocation between risky assets and bonds.

In this paper, our goal is to bring the above mentioned studies together in order to examine the portfolio implications of unemloyment insurance over the life cycle in Europe. The setup is simular to Cocco et al. (2005) who consider the impact of disastrous labor income shocks on the portfolio allocation between risky assets and riskfree bonds. However, in the European context a disastrous labor income shock in the sense of some zero income state is an unrealistic assumption, given that highly developed social security systems are in place. At this point we have to consider the key difference in the unemployment dynamics between the US and the EU. The data

show that long-term unemployment plays a much more important role in the EU than in the US, both in terms of the share of long-term unemployment in relation to total unemployment (between 2000 and 2008 19 percent of unemployed were long-term unemployed in the EU versus only 6 percent in the US) and in terms of the average duration of unemployment (the average duration of unemployment in the EU was 15.5 months versus 3 months in the US between 2000 and 2008). These empirical facts call for an explicit modeling of unemployment persistence in the European context.

To this end, we use the life cycle framework presented in Cocco et al. (2005) and augment it by unemployment risk where we allow for two different unemployment states: first, a state for short-term unemployment associated with a relatively high replacement ratio, and second, a much more persistent state for long-term unemployment with a low replacement ratio. Using this model, we theoretically study the impact of unemployment insurance systems, i.e. different replacement ratios, and long-term unemployment on the portfolio choice of households in Europe. Our results with unemployment risk are qualitatively similar to those including a small probability of a disastrous labor-income shock in Cocco et al. (2005), i.e. young agents reduce the optimal share of risky assets in their portfolios. We show that the high expected mean duration of the long-term unemployment state is essential for this result. Assuming alternative unemployment dynamics where the distribution of different income states is independent over time but nevertheless imposing the same unconditional distribution does not have any significant impact on optimal portfolio choice.

The remainder of the paper proceeds as follows. Section 2 discusses the model and section 3 the corresponding optimization problem. The calibration and parametrization is presented in section 4. Section 5 will be devoted to the results: the first subsection provides the model solution obtained by numerical methods, i.e. the policy functions for three different setups, while the second subsection presents our simulation results based on these policy functions. Section 6 concludes and proposes directions for future research.

2 The Model

Our model is mainly based on the life cycle framework with optimal consumption and portfolio choice presented in Cocco et al. (2005). We extend it by introducing unemployment risk which is modeled similar to that in Imrohoroglu et al. (1995). The model describes a partial equilibrium where households are *ex ante* homoge-

neous, that is they have identical preferences and are subject to the same mortality and labor income risks. *Ex post*, households differ with respect to age, employment status and wealth. They choose consumption and the share invested in risky assets endogenously, while labor supply and retirement age is assumed to be exogenous in this setup.

2.1 Preferences

The economy is inhabited by a continuum of individuals who live for a maximum of T periods, facing mortality risk in each period of life t. Let $t = 1, ..., T$ denote adult age. Each individual works up to period K when she reaches retirement age. K is assumed to be exogenous and deterministic. Individual i maximizes expected discounted lifetime utility

$$E_t \sum_{t=1}^{T} \delta^{t-1} \left[\prod_{k=1}^{t} p_k \right] u(C_t) \tag{1}$$

where δ is the subjective discount factor and p_t reflects the conditional probability of survival from age t to $t+1$[1]. Preferences are modeled by the constant relative risk aversion utility function

$$u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma} \tag{2}$$

which positively depends on consumption at age t, C_t, and γ is the coefficient of relative risk aversion. The intertemporal elasticity of substitution is given by $1/\gamma$.

2.2 Income

Individuals earn stochastic labor income during their working lifes. Since labor income risk is not completely insurable against shocks, the model exhibits a ceratin degree of market-incompleteness. As of retirement age K agents receive a constant fraction of their last labor income in terms of retirement benefits. Thus, labor income is stable during the retirement phase.

2.2.1 Worker's income

During professional life, individuals face a stochastic risk of getting unemployed. We extend the standard case of two employment states - unemployment and em-

[1] By definition $p_1 = 1$ and $p_t = 0$ for $t > T$.

ployment - by a third state, thus allowing for a differentiation between short- and long-term unemployment. Let $s \in S = \{e, u_s, u_l\}$ be the employment opportunities state which is assumed to follow a first-order Markov-chain. If $s = e$, the consumer is offered the opportunity to work. Whenever an individual is given the opportunity to work, he supplies labor inelastically. If $s = u_k, k = s, l$ the agent is short-term (u_s) or long-term (u_l) unemployed.

The transition matrix for the employment opportunities state is given by $\Pi(s', s) = \left[\pi_{ij}\right], i, j = e, u_s, u_l$ where each element $\pi_{ij} = Prob\{s_{t+1} = j | s_t = i\}$ reflects the probability that a particular state i is followed by state j so that

$$\Pi(s', s) = \begin{pmatrix} \pi_{ee} & \pi_{eu_s} & \pi_{eu_l} \\ \pi_{u_s e} & \pi_{u_s u_s} & \pi_{u_s u_l} \\ \pi_{u_l e} & \pi_{u_l u_s} & \pi_{u_l u_l} \end{pmatrix} . \tag{3}$$

Let $f(t, Z_{it}) = f_t$ be a deterministic function of age t and of a vector Z_{it} containing other individual characteristics which reflects the age-dependent labor income profile of agent i. Each individual's labor income can then be expressed as

$$Y_t = \begin{cases} f_t P_t \Theta_t & \text{for } t = 1, ..., K-1 \quad \text{if } s = e \\ \zeta_k f_{t-\tau} P_{t-\tau} & \text{for } t = 1, ..., K-1 \quad \text{if } s = u_k,\ k = s, l \end{cases} \tag{4}$$

where τ is the duration of the unemployment state, Θ_t is a transitory shock to labor income distributed as $ln(\Theta_t) \sim N(-\sigma_\theta/2, \sigma_\theta)$, and P_t is the permanent component of labor income. It evolves according to

$$P_{t+1} = \begin{cases} U_{t+1} P_t & \text{for } t = 1, ..., K-1 \quad \text{if } s = e \\ P_t & \text{for } t = 1, ..., K-1 \quad \text{if } s = u_k,\ k = s, l. \end{cases} \tag{5}$$

where U_{t+1} is a log-normally distributed shock to the permanent component of labor income with $ln(U_t) \sim N(-\sigma_u/2, \sigma_u)$. The growth rate of the age-specific deterministic component of labor income is given by $G_{t+1} = f_{t+1}/f_t$ if the agent is given the working opportunity. In case he is unemployed, he receives a constant fraction ζ_k of his permanent labor income of the last period he worked in and the deterministic growth rate equals one in this case. Overall, labor income is a serially correlated process subject to both temporary and permanent shocks as well as a positive probability of getting unemployed in every period.

2.2.2 Income during retirement

Once agents have reached retirement age K, they receive funding from the social security system. Similarly to unemployment benefits, retirement income is deterministic and modeled as a constant fraction λ of permanent income earned in the last period of working life

$$Y_t = \lambda f_{K-1} P_{K-1} \quad \text{for } t = K, ..., T \tag{6}$$

implying that $G_t = U_t = 1$ during retirement.

2.3 Asset market

On capital markets, the individual can either invest in bonds, B_t, or in risky assets, S_t. The riskless bond has a constant gross real return of R_f whereas stocks earn a gross real return of R_t. Excess returns are composed of the mean return on equity, μ, plus a disturbance term η:

$$R_t - R_f = \mu + \eta_t \ . \tag{7}$$

The expectation of the excess return is given by the mean equity-premium $E(R_t - R_f) = \mu$ and the return on equity is assumed to be independently and identically distributed as $ln(R_t) \sim N(ln(R_f + \mu) - \sigma_\eta/2, \sigma_\eta)$.

2.4 Budget constraint

Each period in his lifetime, the individual allocates his cash-on-hand, M_t, to bonds, risky assets, and consumption, C_t. Hence, cash-on-hand in period $t+1$ is defined as

$$M_{t+1} = \left[\alpha_t R_{t+1} + (1 - \alpha_t) R_f\right] A_t + Y_{t+1} \tag{8}$$

where $A_t = M_t - C_t$ reflects assets after all transactions have been taken in period t and can thus be interpreted as the agent's savings. The variable α_t represents the proportion of savings invested in stocks at time t.

3 Optimization problem

So far, we have two control variables, namely C_t and α_t together with the four state variables M_t, P_t, f_t and s_t. Given that our optimization problem is homogenous in P_t

and f_t, we normalize it by these two variables, such that the state space is reduced to two dimensions. For a detailed derivation see Appendix A. Defining $\frac{X_t}{P_t f_t} = x_t$, the normalized Bellman equation of the maximization problem can be written as

$$v_t(m_t, s_t) = \max_{c_t, \alpha_t} \left\{ u(c_t) + \delta p_t G_{t+1}^{1-\gamma} E_t \left[U_{t+1}^{1-\gamma} v_{t+1}(m_{t+1}, s_{t+1}) \right] \right\} \tag{9}$$

subject to the normalized budget constraint

$$m_{t+1} = \left[\alpha_t R_{t+1} + (1 - \alpha_t) R_f \right] \frac{(m_t - c_t)}{G_{t+1} U_{t+1}} + y_{t+1} \, . \tag{10}$$

Writing out the expectation over the employment state s explicitly, the individual's dynamic programming problem can be stated as

$$v_t(m_t, s_t) = \max_{c_t, \alpha_t} \left[u(c_t) + \delta p_t G_{t+1}^{1-\gamma} \sum_{s_{t+1}} \pi(s_{t+1}|s_t) \widetilde{E}_t U_{t+1}^{1-\gamma} v_{t+1}(m_{t+1}, s_{t+1}) \right] \tag{11}$$

where he maximizes the recursive value function v_t subject to the budget constraint (10) and the non-negativity constraint $a_t \geq 0$.

The levels of the value function, consumption and all other variables can be obtained from

$$V_t(M_t, P_t, f_t, s_t) = (P_t f_t)^{1-\gamma} v_t(m_t, s_t) \quad \text{and} \tag{12}$$

$$C_t(M_t, s_t) = P_t f_t c_t(m_t, s_t) \tag{13}$$

where we multiply the normalized functions with the appropriate income-factors as in Carroll (2009).

Since there is no analytical solution to this finite-horizon maximization problem, numerical methods have to be used to obtain the optimal policy functions $c_t(m_t, s_t)$ and $\alpha_t(m_t, s_t)$. This is generally done by first specifying a terminal decision rule and then solving the problem by backward induction. Following Carroll (2006, 2009), we discretize the state space and compute the values of the policy functions at each grid-point of possible values of the state variables m_t and s_t. We then interpolate between these discrete points of the functions c_t and α_t in order to get an approximation to the optimal decision rules. Having computed the interpolated policy functions at time t, the corresponding value function can be determined. The solutions for earlier periods are constructed by recursion from $t = T$ to $t = 1$.

4 Calibration

We calibrate the model for the European context. If available, data for the EU27 is used. The model period corresponds to one year.

Table 4 summarizes the parameter values that are used in our benchmark simulation. Individuals enter worklife at age 20 and live up to a maximum age of 100 years so that our model accounts for $T = 81$ years. We set average retirement age to $K = 62$ according to Eurostat-data for 2008. The coeffcient of relative risk aversion, γ, is fixed at the value of 10 following Cocco et al. (2005), the subjective discount rate, δ takes on a value of 0.96 which corresponds to an annual interest rate of 4 percent. Following Cocco et al. (2005), we assume R_f, the real interest rate on the riskless asset, to be 2 percent while the mean return on stocks, μ, is set to 6 percent, hence implying an equity premium of 4 percent. The correlation between equity returns and shocks to labor income, ϕ, is set to zero as in Cocco et al. (2005).

According to OECD-data, the gross pension replacement rate of the median earner, i.e. pension benefits as a share of individual lifetime average earnings, was 65 percent in the EU in 2009 so that we set $\lambda = 0.65$. Concerning the gross replacement rate for unemployment benefits, we refer to the OECD Employment Outlook (2009) where the replacemet rate for those who are unemployed for a period up to one year is $\zeta_1 = 0.60$ whereas the replacemet rate drops to $\zeta_2 = 0.20$ for individuals who are long-term unemployed (five year unemployment spell).

Parameter	Description	Value
T	life span (20 to 100)	81
K	average retirement age	62
γ	coefficient of relative risk aversion	10
δ	subjective discount factor	0.96
μ	mean return on equity ($\mu - 1$)	0.06
σ_η^2	volatility of equity log-return	0.15^2
R_f	real riskless rate	1.02
σ_u^2	standard deviation of shock to permanent labor earnings	0.0106
σ_θ^2	standard deviation of transitory shock to labor income	0.07
ϕ	correlation between stock returns and earning shocks	0
ζ_1	benefit replacement rate (short term unemployment)	0.6
ζ_2	benefit replacement rate (long term unemployment)	0.2
λ	benefit replacement rate (retirement)	0.65

Table 1: Parameter values

The vector of conditional survival probabilities, p_t, is computed from the mortality tables provided by the Human Mortality Database (`http://www.mortality.`

org). Conditional survival probabilities for the EU are proxied by the average of the four largest EU-members' rates, that is Germany, France, Italy, and Spain.

The transition probabilities for the Markov process are chosen such that the unconditional probabilities of being short-term and long-term unemployed match European data. Taking into account that the average EU-unemployment rate between 2000 and 2008 was 8.5 percent with a share of long-term unemployment of roughly 20 percent of total unemployment, we calibrate the matrix Π such that the unconditional probaility of being short-term unemployed amounts to 6.8 percent while the corresponding probability for long-term unemployment is 1.7 percent. We define short-term unemployment as a period of being without a job of up to one year whereas long-term unemployment displays an average duration of six years in our model. Controlling for both unconditional probabilities as well as for the persistence of unemployment in the EU, the transition matrix we employ is given by

$$\Pi(s',s) = \begin{pmatrix} 0.931 & 0.069 & 0 \\ 0.865 & 0.1 & 0.035 \\ 0.165 & 0 & 0.835 \end{pmatrix} \tag{14}$$

where we set $\pi_{eu_l} = 0$, because an individual gets short-term unemployed first before being counted as long-term unemployed and hence the state $s = e$ cannot be followed directly by the state $s = u_l$. Moreover, once an individual is long-term unemployed in our model, it can either stay in this state or get back to work. However, in this setup it is not meaningful to switch from the state of long-term to short-term unemployment and consequently we set the corresponding probability $\pi_{u_l u_s}$ equal to zero, too.

For the scenario with two employment states only where $s \in S = \{e, u_s\}$, we adjust the transition matrix accordingly. Assuming a short-term unemployment rate of 6.8 percent and an average duration of one year, we get

$$\Pi(s',s) = \begin{pmatrix} 0.931 & 0.069 \\ 0.909 & 0.091 \end{pmatrix}. \tag{15}$$

Concerning the deterministic part of the labor income process, f_t, we use a simulated function $f(t, Z_t)$ which is hump-shaped as described in Cocco et al. (2005) and others. The calibration of this function for European household panel data is left for further research.

5 Results

In the following, we split up our analysis into three scenarios. First, we have a look at the policy functions and simulation results for the benchmark case where we abstract from any unemployment risk. This benchmark case reproduces the results presented in Cocco et al. (2005) for the European context. Secondly, we introduce unemployment risk, that is, the investor may find herself in two different states in each period of her working life. If $s = e$, she is given an employment opportunity. If $s = u_s$, she is short-term unemployed. In this second scenario we consider two subcases. First, only a minimum of insurance against unemployment is available ($\zeta = 0.1$). Second, we introduce unemployment insurance with an income replacement ratio of $\zeta = 0.6$ which is in line with European data.

Finally, we consider a third setup where the agent faces three possible employment states. Besides the two states $s = e, u_s$ the agent faces the additional risk of getting long-term unemployed, i.e. $s = u_l$. In this scenario, we again differentiate between two sub-cases: First, we do not consider the persistence of unemployment but rather calibrate the transition matrix Π such that the unconditional probabilities of being short-term or long-term unemployed match European data. In this case, we set the conditional probabilities equal to the unconditional one, such that the realizations of the possible states are independent over time. Put differently, the realizations are random draws from the same unconditional distribution. Second, we also take into account the average duration of being short-term or long-term unemployed and hence the persistence component of unemployment.

The key results from our analysis are the following. In case of short-term unemployment only (scenario 2), we show that unemployment insurance as established in the EU helps to offset the increased labor income risk. The share invested in stocks evolves thus very similarly to the benchmark case without unemployment risk. Hence, the replacement ratio seems to be important for portfolio choice. However, if long-term unemployment is taken into account (scenario 3), we observe that the equity-share is reduced even in the presence of unemployment insurance. This drop is particularly important for young investors. Thus, the model shows that the persistence of unemployment plays a key role in explaining low equity-shares in the portfolio of young investors.

5.1 Policy functions

In this section we discuss the policy function for the optimal share invested in stocks, $\alpha(t, M_t)$, for different scenarios. The function α mirrors the optimal decision rule for an investor of age t disposing of a certain amount of cash-on-hand M_t. We present the policy functions for the share invested in stocks as contour plots for each scenario studied. In each of the following graphs in this section, age t is plotted at the vertical axis while the level of cash-on-hand, M_t, can be read off the horizontal axis. The corresponding numerical values of the associated portfolio-share of stocks $\alpha(t, M)$ are indicated at the contour lines. The darker the area between the contour lines, the lower the associated values of α.

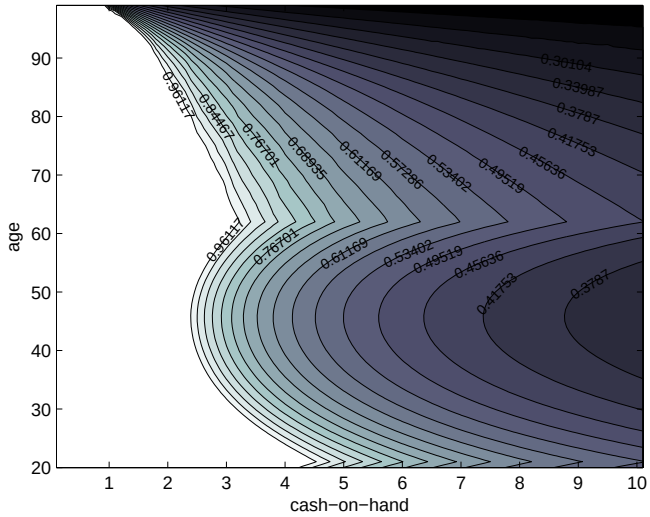


Figure 1: Contour lines for the equity-share, no unemployment risk

The contour plots can be read in the following way. Figure 1 illustrates the optimal decision rule for the benchmark scenario where we abstract from any unemployment risk. For a given level of cash-on-hand (imagine a vertical line at $M = 4$ for example), the contour lines show that the share invested in stocks falls from close to one down to 0.63 until age 48 approximately. Afterwards, α increases somewhat until retirement age $K = 62$ is reached. During the rest of her life, the investor continuously reduces her equity-share as she approaches end of life T.

Looking at the plot the other way around, let us fix age at 40 for instance and examine the evolution of α across different levels of cash-on-hand. The contour lines reveal that the equity-share is close to one up to $M = 2.5$. As M increases further, α_t

starts to descend, but at a diminishing rate as the contour lines lie farther away from each other for higher levels of cash-on-hand.

5.1.1 Benchmark scenario: no unemployment risk

We now turn to the interpretation of the baseline scenario without any unemployment risk. This scenario closely resembles the one analyzed in Cocco et al. (2005). However, we adjust the parameters to the European context.

Let us concentrate on the retirement period first where labor income is modeled under the simplifying assumption of being constant and certain. At any given age, the equity share decreases as cash-on-hand grows. The intuition for this observation can be explained as follows. Future retirement income (henceforth: RI) can be understood as a substitute for riskless asset holdings. In other words, the stream of future RI reflects implicit bond holdings in the individual's asset portfolio. Agents who dispose of little wealth buy more stocks, because their future retirement income and hence their implicit risk-free asset positions are larger relative to their wealth than for richer investors. Expressed in mathematical terms, the ratio of the present discounted value of future retirement income to wealth, $\frac{PDV(RI)}{M_t}$, is higher for poor households than for richer ones.

For any given level of wealth M_t, the amount of future RI diminishes as the agent ages. Hence, the investor demands a larger share of explicit risk-free assets in order to compensate for the decrease in risk-free RI. As a consequence, the portfolio share held in stocks gets smaller and smaller and finally approaches the complete markets solution which is determined by

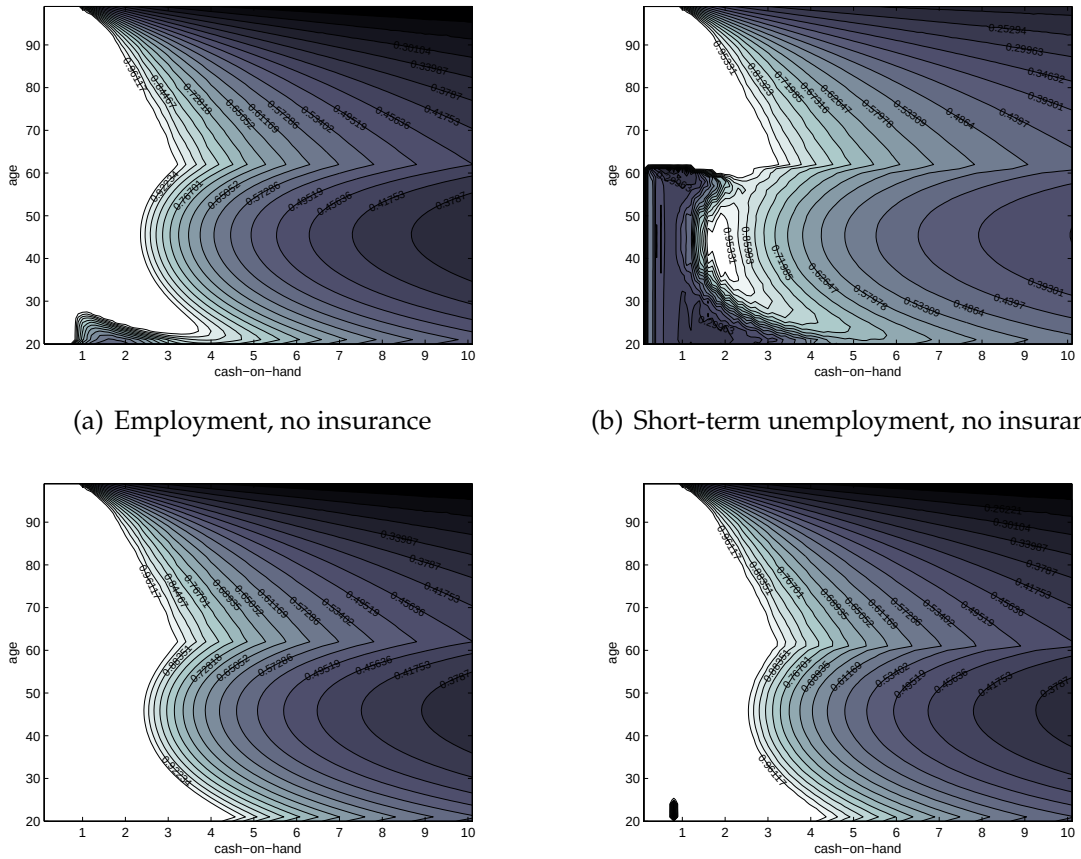
$$\alpha = \frac{\mu}{\gamma \sigma_\eta^2} \tag{16}$$

following Samuelson (1969) and Merton (1969). Plugging the above mentioned parameter values into equation (16), the equity-share amounts to roughly 20 percent for an investor close to the end of his life.

Having described the evolution of the equity share during retirement, we now turn to working life when labor income is stochastic. Holding age fixed, figure 1 reveals that the optimal decision rule for the equity share is still decreasing in cash-on-hand. Hence, stochastic labor income also seems to be a substitute for bonds rather than stocks and thus acts as implicit bond holdings. This is due to the fact that the shocks to the labor income stream are only weakly correlated with the disturbances to equity returns as in Cocco et al. (2005). On the other hand, for any given level of wealth M_t, the contour lines illustrate that during the first part of professional life

(up to age 48 approximately), α falls and this happens at a slower pace for higher levels of M_t. The reduction in the equity share can be explained by the fact that the present value of future labor income is high during the first years of adult life and then diminishes eventually. As of that point, investors start to substitute for implicit bond holdings by buying more bonds explicitly due to their precautionary savings motive: on the one hand, they built up buffers in order to insure against negative labor income shocks. On the other hand, investors accumulate wealth to prepare for retirement when income falls to the constant fraction λ of labor income, aiming at a smooth consumption path over their whole life. As of age 48, the equity share begins to rise again as investors know that they approach the retirement period where future RI will be certain. Moreover, they already have accumulated risk-free buffer stocks in order to protect against negative labor income shocks.

5.1.2 Scenario 2: short-term unemployment with and without unemployment insurance



(a) Employment, no insurance

(b) Short-term unemployment, no insurance

(c) Employment, insurance

(d) Short-term unemployment, insurance

Figure 2: Contour lines for the equity-share for $s \in S\{e, u_s\}$

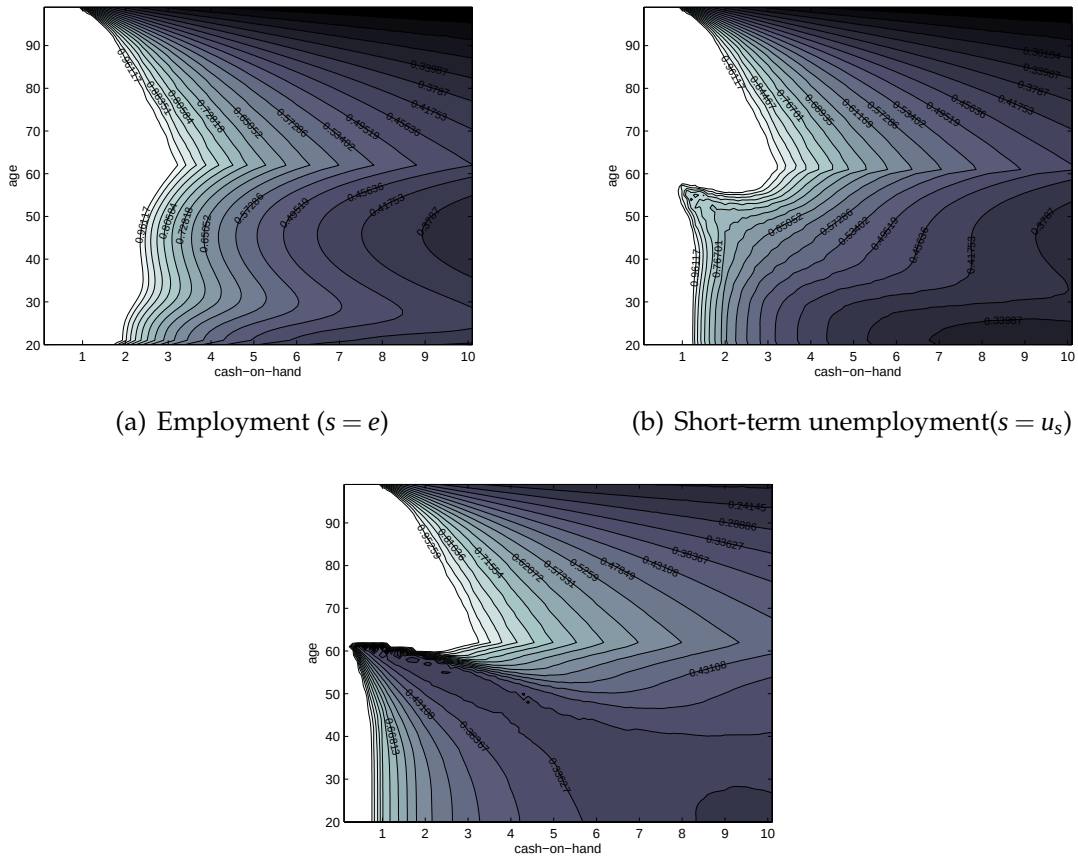
Figures 2(a) and 2(b) show the contour lines for the scenario with unemployment risk but only very basic insurance with a replacement ratio of 10 percent. In comparison to the baseline scenario without unemployment risk the following patterns appear. For high values of wealth and starting at age 30 approximately, the contour plots for the optimal share invested in stocks behave similarly to those in the benchmark scenario. Unemployment risk mainly affects young investors. In case of being employed (figure 2(a)), the equity share is lower for given M_t than without unemployment risk. This tendency is amplified in case of being unemployed (figure 2(b)) where the share invested in stocks is lower (i) during the whole working life for poor investors disposing of low levels of cash-on-hand only and (ii) for any given level of M_t compared to a situation where the agent is in employment. The small share invested in stocks by young investors, especially when being unemployed, results from the fact that young individuals start out with low levels of labor income. When being unemployed, they only get very basic benefits. Consequently, they invest a significant share of their (small amount of) savings in bonds in order to substitute for missing implicit risk-free asset holdings from labor income. During their last years in the labor force, agents quickly increase equity-shares as they approach constant and certain retirement income.

Holding age fixed, the optimal share invested in stocks starts out at a low level for young investors. As M_t increases over life, the equity-share increases and then decreases again. The rise in α kicks in at higher levels of cash-on-hand the younger the investor is, especially if being jobless. If a young person is unemployed, she only invests in risky assets if she is rich. Once the investor has reached midlife, she has already accumulated a certain amount of buffer stock savings, so that even at low levels of cash-on-hand she is able to invest more in stocks than a younger person.

Having discussed the effects of unemployment risk on the optimal decision rules $\alpha(t, M_t)$ in absence of unemployment insurance, let us now introduce unemployment insurance with a replacement ratio of 60 percent as in the EU. Figure 2(b) and 2(d) show the contour lines for $\alpha(t, M_t)$ with insurance for the employment and short-term unemployment state, respectively. When comparing with figure 1, it can be observed that the optimal policy rules are very similar to the benchmark case without any risk of getting unemployed. If the agent is jobless, figure 2(d) reveals that the optimal share invested in stocks is somewhat below the optimal share in the benchmark scenario and in the employment state. However, the negative effect of unemployment risk seems to be mainly absorbed if social security systems with relatively high replacement ratios are in place as is the case in the EU.

5.1.3 Scenario 3: short-term and long-term unemployment with and without persistence

We now extend the framework by one additional feature, namely the risk of getting not only short-term, but also long-term unemployed. To this end, we use the transition matrix Π which has been calibrated to European data as described in section 4. That is, we take the unconditional probabilities of getting short-term and long-term unemployed into account and also consider the persistence of the different employment states as reflected by average durations.



(a) Employment ($s = e$)

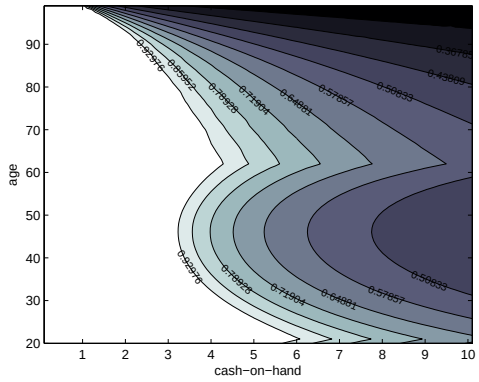
(b) Short-term unemployment($s = u_s$)

(c) Long-term unemployment($s = u_l$)

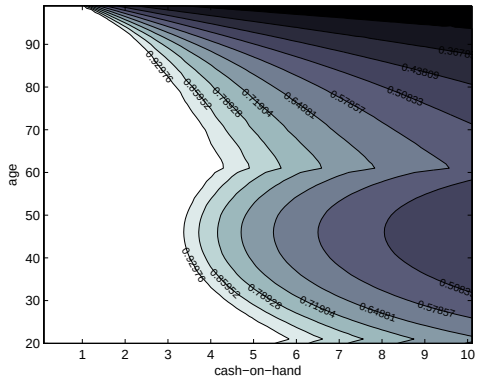
Figure 3: Contour lines for the equity-share, persistent long-term unemployment

Figure 3 illustrates the optimal policy functions $\alpha(t, M_t)$ for the three employment states $s = e, u_s, u_l$ allowing for persistence in the unemployment process. It can be observed that, in all three subfigures, the portfolio share invested in stocks is depressed when comparing the policy functions to the benchmark case. Apart from

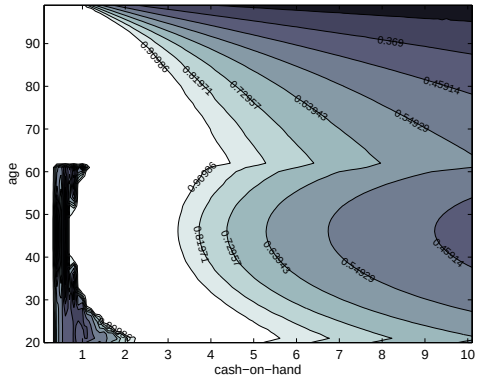
very low levels of cash-on-hand M_t, the equity-share lies below the one in the baseline scenario for a given level of wealth. This tendency is reinforced going from the employment over the short-term unemployment to the long-term unemployment state. Especially for those individuals who are close to retirement age and endowed with very little cash-on-hand the optimal equity-share is significantly reduced. Not surprisingly, the picture is especially pronounced in the long-term unemployment state (figure 3(c)) where the optimal equity share is heavily downsized. At age 40 and for a given level of wealth of $M_t = 4$, for instance, the optimal share invested in stocks drops to about 35 percent in case of long-term unemployment whereas if being short-term unemployed, the corresponding share amounts to roughly 55 percent. Hence, the risk of being jobless for an extended period of time is crucial for the investment decision of the household.



(a) Employment ($s = e$)



(b) Short-term unemployment($s = u_s$)



(c) Long-term unemployment($s = u_l$)

Figure 4: Contour lines for the equity-share, no persistence

In order to further analyze the effect that are responsible for the negative effect of unemployment risk on the equity-share chosen by households, we change the tran-

sition matrix Π such that the unconditional probabilities of being in one of the three employment states are still calibrated as before. However, we abstract from the persistence component of unemployment by equalizing conditional and unconditional probabilities. Consequently, the employment states do not mirror the actual autocorrelation displayed in the data. The resulting policy functions for the equity share α are presented in figure 4. It can be observed that without persistence the policy functions look qualitatively very similar to the benchmark scenario without unemployment risk apart from those for the long-term unemployment state where we have a non-monotone area at very low levels of cash-on-hand.

Summing up, the following key features can be deducted from figures 1 to 4. In all three scenarios, for a given level of cash-on-hand the equity share decreases during retirement as t approaches the final period T. The higher the value of M, the lower the speed of the fall in α_t, since the reduction in future retirement income is relatively less important for wealthy agents than for poorer ones.

During the working period, α_t decreases in M_t in the majority of cases, except for the unemployment state in scenario 2 and the long-run unemployment state without persistence in scenario 3 where we observe non-monotone behavior for low levels of wealth. Overall, the higher labor income risk - either presented by low unemployment benefits or by the risk of being long-term unemployed - the lower is the share invested in risky assets, particularly by young investors. Thus, we can state that labor income risk crowds out capital market risk for this age group. Finally, we find that the optimal portfolio share invested in stocks first decreases in the majority of cases until age 48 and then rises again towards retirement age. We will see in the next section that our simulation results also show this pattern when averaging the evolution of the equity-share over the life cycle for a large number of investors.

5.2 Simulation results

We simulate our model 10,000 times applying the Monte Carlo method and average over the 10,000 simulated investors to compute the representative evolution of the share invested in stocks over the life cycle. The following section starts with the baseline scenario without any unemployment risk. Afterwards, we will discuss the simulation results for the scenarios including short-term and long-term unemployment.

5.2.1 Benchmark scenario: no unemployment risk

Figure 5 shows the evolution of consumption, income, and cash-on-hand over the life cycle for our baseline scenario. The graph closely matches the results presented in Cocco et al. (2005). Income is slightly hump-shaped during working life, reaching its maximum at about age 48. A kink can be observed at average retirement age $K = 62$ when income drops to the fraction λ of the last labor income. Afterwards, during retirement, income is constant, as we impose the simplifying assumption that there are neither temporary nor permanent disturbances to retirement benefits.

Consumption follows a smooth path which closely matches income. A slight increase in consumption can be observed even after retirement age. When approaching the end of life, consumption slightly falls as wealth erodes quickly.

Cash-on-hand strongly increases during the first years of adult age due to the high growth rates of deterministic labor income. At about age 48, wealth is accumulated at a somewhat lower speed until the agent leaves the labor force. Once the retirement period starts, wealth is run down rapidly and at an increasing rate the closer the agent gets to the end of life. This is mainly due to mortality-enhanced impatience given that we abstract from bequest motives.

Consumption, income and wealth evolve very similarly for all scenarios studied here. That is why we only present the graphs once. The only difference which appears is that wealth peaks at a somewhat lower level in case of no unemployment insurance (scenario 2) and long-term persistent unemployment (scenario 3).

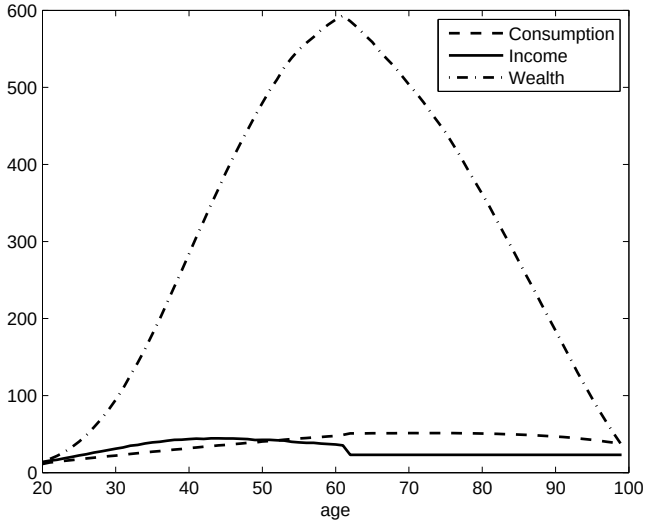


Figure 5: Simulation results for consumption, income and wealth, benchmark

Figure 6 plots the share invested in stocks for the benchmark scenario without unemployment together with the graphs for scenario 2 where short-term unemployment is introduced. The solid line represents the benchmark scenario. The graph shows that during the first years of professional life, all savings are invested in stocks. This results from the fact that the deterministic labor income profile is very steep during the first ten years of adult life while the level of wealth, M_t, is still low. Given that labor income acts as a substitute for riskless asset holdings, young investors' portfolio share held in stocks is elevated, because the ratio of the expected discounted future stream of labor income to wealth, $\frac{PDV(LI)}{M_t}$, is consequently very high.

After the first ten years of working life, the asset share falls until age 55 approximately as investors demand more and more bonds during midlife in order to assemble savings for the retirement period. Put differently, the present discounted value of future labor income decreases as the investor ages - on the one hand because the future income stream shortens, on the other hand because the age-dependent component of labor income gets flatter and eventually falls - whereas the stock of cash-on-hand grows, leading to a decrease in the ratio $\frac{PDV(LI)}{M_t}$ of the two variables. Just before age 62, the portfolio share invested in stocks slightly increases again, since agents know that retirement benefits will be constant and deterministic. Approaching the end of life, the equity-share rises again. This can be attributed to the fact that wealth erodes at a faster rate than the present discounted value of future retirement income does just before the end of life. Thus, even though the share invested in stocks shifts in with age during this period, the net effect on α_t is positive.

5.2.2 Scenario 2: short-term unemployment with and without unemployment insurance

While there was no unemployment risk in the benchmark scenario, we now introduce two employment states, namely $s \in S = \{e, u_s\}$. First, let us look at a situation where there is only very rudimentary unemployment insurance available with a replacement ratio $\zeta = 0.1$. Hence, investors' labor income is now subject to a much higher risk. The dashed line in figure 6 reveals that under these circumstances, the evolution of the equity share significantly changes for young investors: it lies below 0.7 at the beginning of working life compared to a value of nearly one in the benchmark scenario. The share invested in risky assets sharply rises until age 30 before it starts falling again and comes back to normal at age 35. For the remaining life-time, the curve closely matches the one associated with the benchmark scenario, given that older investors already have accumulated precautionary savings and a certain

stock of wealth in earlier years so that they are less affected by unemployment risk than younger investors. Overall, the qualitative results of the benchmark scenario are preserved with decreasing equity-shares for older investors.

Once unemployment insurance is introduced with a replacement ratio of $\zeta = 0.6$ in line with European data, the dotted line in figure 6 reveals that we are basically back to the benchmark scenario with high stock-shares for young investors and lower ones for older individuals. Thus, the replacement ratio seems to be of vital importance for the investment decision of households facing a certain degree of unemployment risk. The results point out that the consequences of short-term unemployment for the portfolio-share held in risky assets can be compensated by a sufficient level of unemployment insurance in our model.

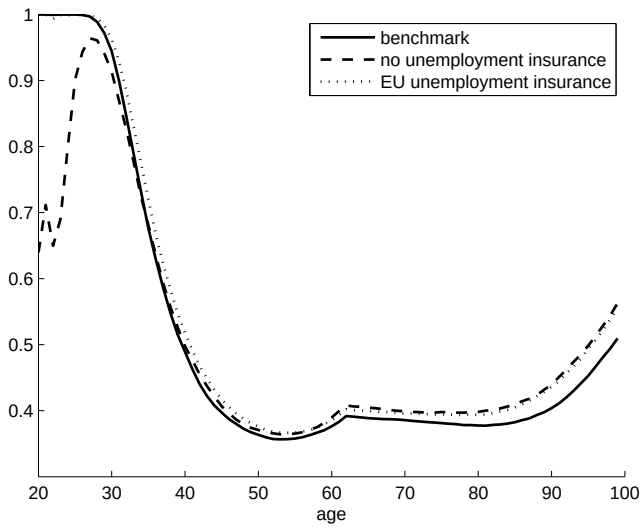


Figure 6: Simulation results for the equity-share, with and without unemployment risk

5.2.3 Scenario 3: short-term and long-term unemployment with and without persistence

In the third scenario, we allow for three different employment states adding the possibility of being long-term unemployed. When an individual is short-term unemployed meaning that he is out of work for at most one year he receives 60 percent of his last income. Once he is unemployed for more that one year, he is considered being long-term unemployed and the benefit replacement ratio reduces to 20 percent. In the following, we distinguish two setups. First, we take the transition

matrix Π which accounts for both the unconditional probabilities and the average duration of unemployment in line with the stilized facts for the EU. Second, we use Π such that only the unconditional probabilities are taken into account whereas we eliminate the persistence of the three possible employment states, thus abstracting from actual autocorrelation in the employment process.

From figure 7 it can be seen that if the Markov-chain for the employment state is calibrated realistically (dotted line), that is including both unconditional probabilities and persistence, the portfolio-share invested in risky assets is significantly below what we observe in the benchmark case (solid line). As before, the cohort of young investors is mainly affected by the risk of getting long-term unemployed. Until the age of 40, agents invest considerably less in stocks when confronted with the risk of getting short-term *and* long-term unemployed. Hence, even a quite generous social security system as the one established in Europe is unable to offset the risks associated with long-term unemployment and cannot avoid that young to middle-aged individuals significantly reduce their portfolio-shares held in risky assets.

The dashed line in figure 7 points to the key mechanism driving our results. Once we abstract from the persistence of unemployment, the evolution of the equity-share closely matches its path in the baseline scenario. We can thus conclude that the persistence component of unemployment is crucial for the investment decision of households; the autocorrelation of the unemployment states thus suppresses young workers' portfolio share invested in stocks.

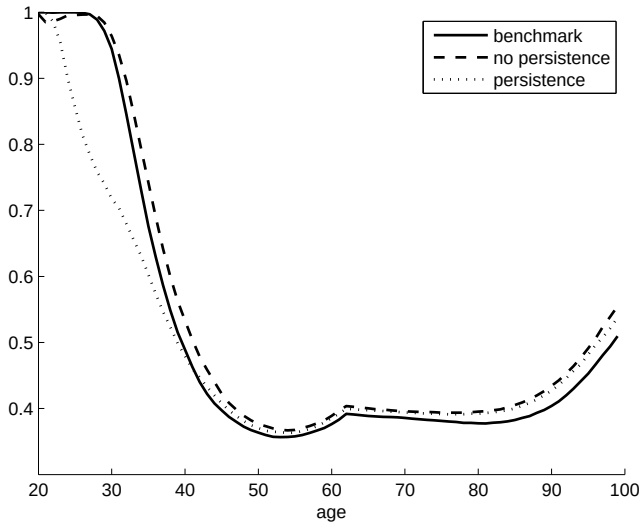


Figure 7: Simulation results for the equity-share, with and without persistence

6 Conclusion and Future Research

The goal of this paper has been to investigate the impact of social security systems on the investment decisions of households in the European context. To this end, we have used a calibrated life cycle model of consumption and portfolio choice which features unemployment risk. As opposed to most models in the life cycle literature, we allow for three employment states: besides the possibility of being employed or unemployed, we extend the state-space by explicitly differentiating between short-term and long-term unemployment. This extension is motivated by the fact that long-term unemployment plays an important role in describing the dynamics of European labor markets.

Our main findings can be summarized as follows. When considering employment and short-term unemployment only, we theoretically show that social security systems with income replacement ratios as those established in the EU are able to countervail the negative impact of unemployment risk on the portfolio share invested in risky assets. Consequently, under these circumstances investors choose their equity-shares as if there were no risk of loosing jobs. Yet, the picture changes when taking the long-term unemployment into account. In this case, even if social security systems help to insure against part of the increased labor income risk, the equity-share in the portfolio of young investors is significantly reduced due to enhanced precautionary savings. We show that this outcome is predominantly driven by the persistence of unemployment.

There are several tasks that are worth being treated in future research. As stated in section 4, we have so far taken a simulated hump-shaped function for the deterministic part of labor income $f(t, Z_{it})$ from Cocco et al. (2005). Since it is not granted that the age-dependent component of labor income evolves in the same way in Europe as it does in the US, we are going to estimate the function f_t as well as the error structure of the labor income process and its correlation with stock market returns for the European case using household panel data. The data could also be used to empirically study the effects of social security and the persistence of unemployment on the portfolio decisions of European households.

Another point that should be addressed in the future concerns labor supply which is exogenous in the current version of the model. Labor hours supplied could be made endogenous as in Chai et al. (2009). Being able to adjust their hours worked in response to income uncertainty would provide agents with an alternative means of tackling labor income risk. Consequently, individuals' behavior would be much closer to reality if they are able to flexibly adjust labor supply and hence their im-

plicit bond-holdings. Moreover, retirement income could be modeled more realistically by relaxing the assumption of constant pension income as well as taking average lifetime working income as a basis for pension benefits. Making retirement income uncertain would allow us to assess the change in the precautionary and retirement savings motive during professional life which should get more important as agents have to insure against negative shocks that hit when being pensionary. Beyond that, the dynamics of the equity-share which are so far very simplistic during the retirement phase would be actualized when introducing additional uncertainty.

Finally, it could be interesting to embed our model into a general equilibrium setup where the firm side is explicitly modeled. In such a framework the impact of firm's labor demand and the effects of the correlation between labor and capital income on the portfolio choice of households could be analyzed.

A Appendix

Abstracting from the state variable s_t for the moment, we normalize the optimization problem with P_t and f_t in the following way.

In a first step, consider equation (8) and divide by $P_{t+1} f_{t+1}$ such that

$$\frac{M_{t+1}}{P_{t+1} f_{t+1}} = \left[\alpha_t R_{t+1} + (1-\alpha_t) R_f\right] \left(\frac{M_t}{P_t f_t} - \frac{C_t}{P_t f_t}\right) \frac{P_t f_t}{P_{t+1} f_{t+1}} + \frac{Y_{t+1}}{f_{t+1} P_{t+1}} \quad . \tag{17}$$

Defining $\frac{X_t}{P_t f_t} = x_t$, (17) can be written as

$$m_{t+1} = \left[\alpha_t R_{t+1} + (1-\alpha_t) R_f\right] \frac{(m_t - c_t)}{G_{t+1} U_{t+1}} + y_{t+1} \tag{18}$$

where U_t is the stochastic growth rate of permanent labor income and G_t reflects the growth rate of the deterministic part of the labor income process, f_t. Normalized labor income y_t is given by

$$y_t = \begin{cases} \Theta_t & \text{for } t = 1, \ldots, K-1 \text{ if } s = e \\ \zeta_k & \text{for } t = 1, \ldots, K-1 \text{ if } s = u_k \, k = s, l \\ \lambda & \text{for } t = K, \ldots, T. \end{cases} \tag{19}$$

In a second step, we setup the Bellman equation for the consumer's optimization problem in the next-to-last period of life, abstracting for the moment from the employment state s_t. The consumer maximizes utility subject to equations (2)-(8) choosing C_{T-1} and α_{T-1}:

$$V_{T-1}(M_{T-1}, P_{T-1}, f_{T-1}) = \max_{C_{T-1}, \alpha_{T-1}} \quad \{u(C_{T-1}) + \delta p_{T-1} E_{T-1} V_T(M_T, P_T, f_T)\} \, . \tag{20}$$

Given that the consumer will die at the end of period T, she will consume all cash-on-hand implying that $M_T = C_T$ and hence

$$V_{T-1}(M_{T-1}, P_{T-1}, f_{T-1}) = \max_{C_{T-1}, \alpha_{T-1}} \quad \left\{ u(C_{T-1}) + \delta p_{T-1} E_{T-1} \left[\frac{M_T^{1-\gamma}}{1-\gamma} \right] \right\} . \tag{21}$$

Now, let us expand equation (21) by $P_t f_t$ in order to express it in lower case letters

$$V_{T-1}(\bullet) = \max_{c_{T-1},\alpha_{T-1}} \left\{ (P_{T-1} f_{T-1})^{1-\gamma} \frac{c_{T-1}^{1-\gamma}}{1-\gamma} + \delta p_{T-1} E_{T-1} \left[\frac{(P_T f_T)^{1-\gamma} m_T^{1-\gamma}}{1-\gamma} \right] \right\}$$

$$= (P_{T-1} f_{T-1})^{1-\gamma} \underbrace{\max_{c_{T-1},\alpha_{T-1}} \left\{ u(c_{T-1}) + \delta p_{T-1} (G_T)^{1-\gamma} E_{T-1} (U_T)^{1-\gamma} \left[\frac{m_T^{1-\gamma}}{1-\gamma} \right] \right\}}_{=v_{T-1}(m_{T-1})}$$

so that we finally have

$$V_{T-1}(M_{T-1}, P_{T-1}, f_{T-1}) = (P_{T-1} f_{T-1})^{1-\gamma} v_{T-1}(m_{T-1}) \quad . \tag{22}$$

The same logic can be applied for all earlier periods $t = 1, ..., T-2$.

References

CD Carroll. Buffer-stock saving and the life cycle/permanent income hypothesis. *Quarterly Journal of Economics*, 112(1):1–55, 1997.

CD Carroll. The method of endogenous gridpoints for solving dynamic stochastic optimization problems. *Economics Letters*, 91(3):312–320, 2006.

CD Carroll. Lecture notes on solution methods for microeconomic dynamic stochastic optimization problems. Technical report, John Hopkins University, Baltimore, March 2009.

J Chai, R Maurer, OS Mitchell, and R Rogalla. The effect of uncertain labor income and social security on life-cycle portfolios. NBER Working Paper 15079, June 2009.

JF Cocco, FJ Gomes, and PJ Maenhout. Consumption and portfolio choice over the life cycle. *Review of Financial Studies*, 18(2):491–533, 2005.

EM Engen and J Gruber. Unemployment insurance and precautionary saving. *Journal of Monetary Economics*, 47(3):545–579, 2001.

A Imrohoroglu, S Imrohoroglu, and Joines. A life cycle analysis of social security. *Economic Theory*, 6(1):83–114, 1995.

RC Merton. Lifetime portfolio selection under uncertainty: The continuous-time case. *The Review of Economics and Statistics*, 51(3):247–257, 1969.

OECD Employment Outlook. OECD Paris. 2009. URL `http://www.sourceoecd.org/employment/`.

PA Samuelson. Lifetime portfolio selection by dynamic stochastic programming. *The Review of Economics and Statistics*, 51(3):239–246, 1969.