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Working Paper

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CeDEx Discussion Paper Series, No. 2005-19

Provided in Cooperation with:
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Suggested Citation: Schmidt, Ulrich; Starmer, Chris; Sugden, Robert (2005) : Explaining preference reversal with third-generation prospect theory, CeDEx Discussion Paper Series, No. 2005-19, The University of Nottingham, Centre for Decision Research and Experimental Economics (CeDEx), Nottingham

This Version is available at:
https://hdl.handle.net/10419/28642

CeDEx

Centre for Decision Research and Experimental Economics

Discussion Paper Series

ISSN 1749-3293

CeDEx Discussion Paper No. 2005–19

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October 2005

The University of Nottingham

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Explaining preference reversal with third-generation prospect theory

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22 April 2005

Keywords: Prospect theory, preference reversal, reference dependence

JEL classifications: D81 (Criteria for decision-making under risk and uncertainty)

Abstract

We present a new theory of decision under risk called third-generation prospect theory. A novel feature of our version of prospect theory is that, by allowing reference points to be uncertain, it is able to accommodate the phenomenon of preference reversal. While several previous theories of preference reversal have been proposed, thus far it has resisted explanation via any empirically plausible model of preferences. We investigate whether our explanation is empirically plausible. We find that the standard patterns of preference reversal are predicted for typical parameterisations of prospect theory already established in the empirical literature. Consequently we suggest that our model constitutes a best buy theory: it offers the predictive power of previous variants of prospect theory and adds to that an explanation of preference reversal. The latter comes 'free of charge' since it involves no extra parameters and no re-parameterisation.

Acknowledgements

Robert Sugden and Chris Starmer are grateful to the Leverhulme Trust for Financial Support (award number: F/00 204/K).

1. Introduction

In this paper we present a new theory of decision under risk: *third-generation prospect theory* (PT^3 for short). The motivation for the theory is empirical: our model is presented as a descriptive theory intended to outperform the current 'best buys' in the literature. PT^3 has three key features: *reference dependence, decision weights* and *uncertain reference points* (i.e. reference points that can be lotteries). The first two features are the common characteristics of different versions of prospect theory including the original version (Kahneman and Tversky, 1979) and the later *second-generation* versions featuring cumulative decision weights (e.g. Starmer and Sugden, 1989; Tversky and Kahneman, 1992). Variants of second-generation prospect theory are increasingly widely applied in both theoretical and empirical work and some have argued that such theories may be serious contenders for replacing expected utility theory at least for specific purposes (see Camerer, 1989). No doubt this is partly because there is considerable empirical support for both reference-dependence and decision weights (see Starmer, 2000).

While second-generation prospect theory has been relatively successful in organising a range of experimental and field data, no variant to date has been able to explain an apparently robust and especially troubling failure of expected utility theory: the so-called *preference reversal phenomenon* (PR for short). By allowing reference points to be lotteries, our variant is also able to accommodate PR. This explanation of PR is highly parsimonious in the sense that, relative to other variants, it requires no extra parameters.

We should emphasise that our purpose is not merely to present another possible preference based explanation of PR; we seek to evaluate whether our explanation is empirically plausible. To investigate this, we explore the incidence and pattern of PR predicted by our theory given alternative parameterisations. We find that the standard patterns of PR are predicted for typical parameterisations of prospect theory already established in the empirical literature. Consequently we suggest that our model constitutes a best buy theory: it offers the predictive power of previous variants of prospect theory and adds to that an explanation of PR. The latter comes 'free of charge' since it involves no extra parameters and no re-parameterisation.

2. Existing Explanations of Preference Reversal

PR is one of the most notorious anomalies in individual decision making, but despite the large volume of literature it has generated, no satisfactory preference-based account of it has thus far been produced (see Cubitt, Munro and Starmer, 2004). The classic instances of PR involve decisions relating to pairs of gambles. In the simplest cases, gambles are binary lotteries with just one positive outcome (the *prize*); the other outcome is zero. One of the lotteries, usually called the 'P bet', gives the better chance of winning a prize while the other, the '$ bet' – has the larger prize. In a typical experiment investigating PR, agents' preference orderings over pairs of such bets are elicited in two ways: in a pairwise choice task, and by comparing willingness-to-accept (WTA) valuations of lotteries elicited separately for P and $ bets. PR is a widely observed tendency for agents to reveal a preference for the P bet in choice but the $ bet in valuation. We will call this pattern *standard PR*. Such inconsistencies between choice and valuation might arise through chance or error. But the opposite inconsistency, in which the $ bet is chosen but the P bet is given a higher value (*non-standard PR*), is much less frequently observed. It is this asymmetry between the two types of reversal which constitutes the puzzle of PR.

In the psychology literature it has been common to interpret PR as evidence that preferences do not satisfy procedural invariance but, instead, depend upon the method used to elicit them. On this view, if preferences are to be invoked at all in explaining PR, those preferences must be context-sensitive: that is, they must allow *different* preferences to govern decisions in choice and valuation tasks. We have no quarrel with the claim that in general behaviour is context sensitive and that specific forms of context sensitivity, such as the scale compatibility effect or the prominence effect, contribute to a full explanation of PR (Slovic, Griffin and Tversky, 1990). Our interest lies in exploring whether stable and context-independent features of agents' preferences also play an important explanatory role.

We will treat the use of WTA valuations as one of the defining characteristics of a PR experiment. In fact, there have been surprisingly few experiments in which willingness-to-pay (WTP) valuations of P and $ bets have been used. Such experiments have produced mixed results, but asymmetric PR is generally less pronounced than in WTA experiments, and sometimes is not present at all. It seems that WTP treatments tend to reduce the frequency of standard reversals and to increase the frequency of non-standard ones (Lichtenstein and Slovic, 1971; Knez and Smith, 1987; Casey, 1991). These findings are compatible with the hypothesis that PR is the product of several causal mechanisms, at least one of which is in some way linked to WTA valuations. We suggest that our model captures a mechanism of the

latter kind. In it, loss aversion imparts a tendency for PR that is specific to the case of WTA valuations.

Economists have suggested several models of context-free preferences as possible accounts of PR. All of them relax at least one of the axioms of expected utility theory. One subset of them retains transitivity and relaxes the independence and/or reduction axioms (Holt, 1986; Karni and Safra, 1987; Segal, 1988). Recent studies, however, continue to generate strong PR in experimental designs implementing controls for the explanations postulated in these theories (Tversky, Slovic and Kahneman, 1990; Cubitt, Munro and Starmer, 2004). Another possible explanation is that PR arises as a consequence of context-free, but non-transitive preferences. Persistent non-transitive cycles of choice analogous to PR have been observed in experimental studies (Loomes, Starmer and Sugden, 1989, 1991; Humphrey 2001), but the only preference theory that has been put forward to explain such behaviour is regret theory (Loomes and Sugden, 1983), which has failed other tests (Starmer and Sugden 1998).

A new preference-based explanation for PR is provided by Sugden's (2003) model of reference-dependent subjective expected utility (RDSEU). This model predicts PR when preferences are loss averse. The key novel feature of the theory is that, in contrast to previous reference-dependent theories, agents' reference points need not be constant but may be state-dependent, i.e. the reference point may be given by an act or lottery[1]. But although this model has the merit of explaining PR, it has a serious weakness as a potential 'best buy' for general use. Specifically, because it is linear in probabilities, it cannot accommodate other well-documented departures from expected utility theory, such as the Allais Paradox. Our model generalises Sugden's theory to allow non-linear probability weighting. The resulting model, PT^3, has the explanatory capacity of other variants of prospect theory, plus the added ability to explain PR.

3. Theory

In this section we introduce PT^3. In this theory, preferences are defined over (Savage) acts. Consider a finite state space S, consisting of the states s_i, i = 1, …, n, and a set of consequences X given by an interval of the real line. Each state s_i has a probability $\pi_i \geq 0$, with $\sum_i \pi_i = 1$. 'Probability' may be interpreted either subjectively or objectively; for convenience in this paper, we will use the objective interpretation. F is the set of all acts. A

particular act $f \in F$ is a function from S to X, i.e. an act f specifies for each state s_i the resulting consequence $f(s_i) \in X$.

A key feature of our model is that preferences over acts are reference dependent. We formalise this following the approach of RDSEU. For any two acts f and g, $f \succeq_h g$ means that f is weakly preferred to g viewed from act h, the *reference act*. For present purposes the reference act can be interpreted as the status quo position. While reference dependence is one defining characteristic of prospect theory, in first and second-generation variants the point of reference is always a sure outcome (or in the current context, h is restricted to be a constant act). We relax this restriction by adopting a key innovation of RDSEU.

Sugden's axiom system implies maximisation of the function:

$$(1) \qquad V(f, h) = \Sigma_i \, v(f(s_i), h(s_i))\pi_i$$

In this expression, $v(f(s_i), h(s_i))$ is a *relative value* function. It can be interpreted as the desirability of the consequence of act f in state s_i relative to the consequence of a reference act h in the same state. This function is increasing in its first argument; $v(f(s_i), h(s_i)) = 0$ when $f(s_i) = h(s_i)$. The function V(f, h) is the expectation of relative value. It assigns a real value to any act $f \in F$ viewed from any reference act $h \in F$ (i.e. $V: F \times F \rightarrow \mathbf{R}$). It is a preference representation in the sense that, for all f, h, g in F, $f \succeq_h g \Leftrightarrow V(f, h) \geq V(g, h)$.

Notice, however, that the preference representation in (1) is linear in probabilties. PT^3 relaxes this restriction of RDSEU by generalising (1) to:

$$(2) \qquad V(f, h) = \Sigma_i \, v(f(s_i), h(s_i))W(s_i ; f, h)$$

where $W(s_i; f, h)$ is the *decision weight* assigned to state s_i when f is being evaluated from h. In principle, decision weights could be determined by a simple transformation of state probabilities (i.e. $W(s_i; f, h) = w(\pi_i)$) as in Handa (1977). In the contemporary literature on prospect theory it has become conventional to construct decision weights cumulatively using a rank-dependent transformation (Quiggin, 1982; Tversky and Kahneman, 1992). One of the key theoretical rationales for the cumulative construction is that, unlike the first-generation approach, it results in monotonic preferences. In PT^3 we retain the rank-dependent approach, but reconfigure it to work with statewise reference dependence.[2]

In order to construct cumulative weights for a given f, h pair, states must be ordered according to the 'attractiveness' of f's consequences in each state. This is because, in a cumulative construction, the weight attached to a given state depends not only on the probability of that state but also on the position of its consequence in the ranking of all consequences associated with f. In PT[3] the attractiveness of the consequence in each state of a given act depends on the corresponding consequence of the reference act. Hence, in general, the ordering of consequences must be constructed separately for each f, h pair.

Consider any f, h pair. Relative to that pair, there is a *weak gain* in a state s_i if $v(f(s_i), h(s_i)) \geq 0$, and a *strict loss* if $v(f(s_i), h(s_i)) < 0$. Let m^+ be the number of states in which there are weak gains and let $m^- = n - m^+$, be the number of states in which there are strict losses. We re-assign subscripts so that, for all subscripts i, j, we have $i > j$ if and only if $v(f(s_i), h(s_i)) \geq v(f(s_j), h(s_j))$, and so that the states with weak gains are indexed $m^+, ..., 1$ and the states with strict losses are indexed $-1, ..., -m^-$.[3]

Cumulative decision weights are then defined as follows:

$$W(s_i; f, h) =$$

$$(3) \quad \begin{cases} w^+(\pi_i) & \text{if } i = m^+, \\ w^+(\sum_{(j \geq i)} \pi_j) - w^+(\sum_{(j > i)} \pi_j) & \text{if } 1 \leq i \leq m^+ - 1, \\ w^-(\sum_{(j \leq i)} \pi_j) - w^-(\sum_{(j < i)} \pi_j) & \text{if } -m^- + 1 \leq i \leq -1, \\ w^-(\pi_i) & \text{if } i = -m^-, \end{cases}$$

where w^+ and w^- are, respectively, probability weighting functions for the gain and loss domains (w^+, w^- are strictly increasing mappings from [0, 1] onto [0, 1]).

PT^3 straightforwardly captures several models as special cases. RDSEU is the special case in which decision weights are untransformed state probabilities (i.e. $w^+(\pi_i) = w^-(\pi_i) = \pi_i$ for all i). Cumulative (or second-generation) prospect theory is the special case in which the relative value function takes the form $v(f(s_i), h(s_i)) = u(f(s_i) - h(s_i))$, where u(.) is a 'value' function, and in which reference acts are constrained to be certainties (i.e. $h(s_i) = h(s_j)$ for all i, j). Expected utility theory is the special case in which decision weights are untransformed state probabilities, as in RDSEU, and relative value is independent of the reference outcome (i.e. $v(f(s_i), h(s_i)) = u(f(s_i))$ where u(.) is a von Neumann-Morgenstern utility function).

4. Preference Reversal in PT^3: The General Case

Consider two acts with the basic structure of P-and $-bets. Specifically, let f^P represent an act giving an increment of wealth x with probability p and a zero increment otherwise, and let $f^\$$ be an act giving an increment of wealth y with probability q and a zero increment otherwise, with $y > x > 0$ and $1 > p > q > 0$. Standard PR is observed when (i) f^P is revealed preferred to $f^\$$ in a straight choice between the two gambles and (ii) $f^\$$ has a higher WTA valuation than f^P. Given PT^3, the condition for (i) is straightforward. As a normalisation, we define consequences as increments or decrements of wealth relative to the agent's wealth (treated as a certainty) prior to the PR experiment. Taking the agent's reference act to be her pre-experiment wealth, we may write:[4]

(4) $\quad f^P \succ_h f^\$ \Leftrightarrow w^+(p)\, v(x, 0) - w^+(q)\, v(y, 0) \geq 0.$

Now consider (ii). Given PT^3, we can define willingness to accept (WTA) as follows. Consider an agent selling a P-bet. Her situation is depicted as follows:

	p	1-p
h^P	x	0
k^P	WTA^P	WTA^P

In this case, the agent's reference act, denoted h^P, is the P bet. Her WTA valuation of this bet, denoted WTA^P, is the increment of wealth such that she is indifferent between retaining h^P or giving up h^P in exchange for the certainty of that increment. Hence, we define WTA^P as the sure payoff of some constant act k^P defined such that $V(k^P, h^P) = 0$. With $WTA^\$$ defined in an analogous way, the values of WTA^P and $WTA^\$$ are then determined, respectively, by the solutions to equations (5) and (6):

(5) $\quad w^-(p)\, v(WTA^P, x) + w^+(1 - p)\, v(WTA^P, 0) = 0$

(6) $\quad w^-(q)\, v(WTA^\$, y) + w^+(1 - q)\, v(WTA^\$, 0) = 0.$

Standard preference reversal is implied by the model when $w^+(p)\, v(x, 0) > w^+(q)\, v(y, 0)$ and $WTA^\$ > WTA^P$. The fact that none of the terms in expression (4) features in either of expressions (5) or (6) provides a clue to the fact that, under certain conditions, PT^3 predicts standard PR. In fact, as we demonstrate below, both standard and non-standard PR can occur

as a consequence of either loss aversion or probability weighting or both. However, our objective is to do significantly more than show that our model can accommodate PR in principle. Our aim is to explore whether PT^3 provides an empirically convincing account of observed instances of PR. To this end, we undertake calibration exercises designed to assess the empirical plausibility of our model's explanation of PR.

5. A Parameterised Form of PT^3

For the purpose of the calibrations it is necessary to adopt specific functional forms for our general model. In selecting these we are guided by three criteria. First, we seek a model flexible enough to allow us to investigate how the predicted incidence of PR varies with three key aspects of the agent's preferences: *attitudes to consequences*, *attitudes to probability*, and *attitudes to gain and loss*. Second, subject to that constraint, we seek to use the simplest model possible – that is, a model with just one parameter for each of the three attitudes we consider. Third, for comparability with existing evidence, we use wherever possible the functional forms that are most common in previously published research. By constraining ourselves to simple and widely used functional forms, we make the calibrations tougher and more meaningful tests of our model's explanation of PR.

In order to operationalise the model, we need to specify the form of reference dependence. We impose the restriction that the relative value function takes the form $v(f(s_i), h(s_i)) = u(z)$, where $z = (f(s_i) - h(s_i))$. When h is a constant act, this special case of statewise reference dependence is then equivalent to that built into earlier generations of prospect theory; u(.) is the counterpart of the value function in those theories.

Next we specify the class of value functions to be used in the calibration exercise. We adopt the power function which has been widely used in recent empirical literature (see Starmer 2000). Specifically,

$$(7) \qquad u(z) = \begin{cases} z^{\alpha} \text{ if } z \geq 0 \\ -\lambda |z|^{\alpha} \text{ if } z < 0. \end{cases}$$

The parameters α and λ are required to be strictly positive. The first of these parameters controls the curvature of the value function. If $\alpha < 1$, this function is concave in the domain of gains and convex in the domain of losses (the property of *diminishing sensitivity*).

Diminishing sensitivity imparts a tendency for risk aversion with respect to gains and risk-loving with respect to losses. While the empirical literature has suggested some differences in the exponents of the value function between the domains of gains and losses, in the interests of parsimony we will we apply the same exponent in both domains. The parameter λ controls attitudes to gain and loss. With $\lambda = 1$ there is loss neutrality. For λ values above unity, there is loss aversion: losses are weighted more heavily than gains. For values below unity, the opposite is the case.

We model decision weights via a single-parameter probability weighting function. Again, for reasons of parsimony we impose the restriction of identical weighting functions for gains and losses (i.e. $w^+(\pi) = w^-(\pi)$). Hence for the purpose of the calibration exercise the probability weighting function is denoted simply by $w(\pi)$; it takes the form

$$(8) \qquad w(\pi) = \pi^\beta / (\pi^\beta + (1 - \pi)^\beta)^{1/\beta}$$

with $\beta > 0$. This type of weighting function has been discussed by Tversky and Kahneman (1992) and Prelec (1998); variants of it have been widely used in the empirical literature. With $\beta = 1$, decision weights are linear (i.e. $w(\pi) = \pi$) but with $0.4 \leq \beta < 1$ the function generates an inverse-S pattern of weights with over-weighting (under-weighting) of probabilities below (above) some critical probability π^*. Inverse-S weighting has been reported across a wide range of empirical studies (Starmer, 2000).

That completes the specification of the generic model to be used in the calibrations. We will refer to this specification as *parameterised PT*^{3}.

Notice that, when applied to cases in which reference acts are certainties, parameterised PT^3 can also be interpreted as a parameterisation of cumulative prospect theory. In fact, models of this kind have already been estimated using experimental data (e.g. Tversky and Kahaneman, 1992; Loomes, Moffatt and Sugden, 2002). But because cumulative prospect theory does not allow reference acts to be lotteries, these estimations have not used data from PR experiments. Thus, parameter values from these estimations are applicable to our model, while it remains a genuine test of that model to ask whether, given those parameter values, it predicts observed patterns of PR.

We take the following to be relatively well-established stylised facts concerning the median values of the three parameters for experimental subjects. First, many studies suggest the existence of loss aversion, while its opposite is almost unknown; values of the loss aversion parameter in the range $1 \leq \lambda \leq 2.5$ would capture a reasonably wide range of

evidence. Studies fitting variants of prospect theory with power utility almost invariably find diminishing sensitivity. Although some studies have found values of α as low as 0.22 (Loomes, Moffatt and Sugden, 2002, note 17), values in the range $0.5 \leq \alpha \leq 1$ are typical. Inverse-S probability weighting, while not universal, is a very common finding; it would be reasonable to expect values of β in the range $0.5 \leq \beta \leq 1$. These ranges of values will be the focus for evaluating the predictions of our model.

6. Parameterised PT^3 and Preference Reversal

For simplicity, we restrict attention to P and $ bets which give either a positive payoff or zero. This case has been widely studied in the empirical literature. A feature of the power utility function is that model predictions are unchanged if all outcomes are multiplied by any positive constant. Exploiting this property, we may normalise the expected value of the P-bet to unity by setting its payoff $x = 1/p$. Given this normalisation, we can characterise any pair of P and $ bets by a three-parameter vector (p, q, r), where p is the probability of winning the prize in the P bet, q is the corresponding probability for the $ bet, and r is the expected value of the $ bet as a ratio of the expected value of the P bet (implying that the positive payoff of the $ bet is $y = r/q$). Notice that the condition $y > x$ (i.e. the $ bet has the higher prize) implies $rp > q$.

Substituting the functional form (7) into (4), the agent's choice between the two bets is determined by:

$$(9) \qquad f^{P} \succeq_h f^{\$} \Leftrightarrow w(p)/ w(q) \geq (pr/ q)^{\alpha},$$

or, equivalently:

$$(10) \qquad f^{P} \succeq_h f^{\$} \Leftrightarrow \alpha \leq \log[w(p)/ w(q)] / \log(rp/ q).$$

(For the moment, it is more convenient not to substitute in the parameterisation of the probability weighting function.) The following property of the model is an immediate implication of (10):

> *Property 1:* The choice between P and $ is independent of the value of λ. For any given value of β, there is a critical value of α at which the two bets are indifferent. At lower values of α, P is chosen; at higher values, $ is chosen.

This property reflects the fact that, in the choice task, all consequences are positive or zero. Because the negative domain of the value function is not relevant for this task, diminishing sensitivity (i.e. $\alpha < 1$) plays essentially the same role in PT[3] as diminishing marginal utility does in expected utility theory: the lower the value of α, the greater the attractiveness of the safer P bet relative to the riskier \$ bet.

Substituting (7) and (8) into (5) and (6) and then rearranging, we arrive at the following formulae for the valuations of the two bets:

(11) $WTA^P = (1/ p) / [((1 – p)/ p)^{\beta/\alpha} (1/ \lambda)^{1/\alpha} + 1]$ and

(12) $WTA^\$ = (r/ q) / [((1 – q)/ q)^{\beta/\alpha} (1/ \lambda)^{1/\alpha} + 1]$.

Thus:

(13) $WTA^P/ WTA^\$ = q[((1 – q)/ q)^{\beta/\alpha} (1/ \lambda)^{1/\alpha} + 1] / rp[((1 – p)/ p)^{\beta/\alpha} (1/ \lambda)^{1/\alpha} + 1]$.

Examination of (13) yields:

> *Property 2:* As λ increases, the value of $WTA^P/ WTA^\$$ falls; in the limit, as $\lambda \to \infty$, this value tends to q/ rp, where $q/ rp < 1$.

In other words, increases in the loss aversion parameter λ increase $WTA^\$$ relative to WTA^P; at sufficiently high values of λ, we have $WTA^\$ > WTA^P$. Intuitively, this is because the act of selling a bet carries the risk of losing the prize of that bet in the state in which the bet wins; since the \$ bet has the higher prize, the potential for loss in selling it is greater. Thus, loss aversion induces reluctance to sell low-probability high-prize bets.

Properties 1 and 2 are enough to give a preliminary sense of some of the combinations of parameter values that will induce PR. In order for the P bet to be selected in the choice task, α must be lower than some critical value. Given any such value of α, the \$ bet will have the higher WTA valuation if the value of λ is sufficiently high. Thus, standard PR is induced by the combination of sufficiently low α and sufficiently high λ.

It is convenient to explore the implications of the model graphically in (α, λ) space. This space is divided into quadrants by the lines $\alpha = 1$ and $\lambda = 1$. The stylised facts presented in Section 4 suggest that we should focus on the north-west quadrant, in which the value function is either linear or exhibits diminishing sensitivity (i.e. $\alpha \leq 1$) and in which there is either loss neutrality or loss aversion (i.e. $\lambda \geq 1$). We call this the *empirically plausible quadrant*.

For any given pair of bets and any given value of the decision weight parameter β, (α, λ) space can be divided into four regions by identifying two boundaries. One boundary – *the choice boundary* – identifies the locus of (α, λ) pairs along which the P and $ bets are indifferent in choice (i.e. $f^P \sim_h f^\$$). We know from Property 1 that the choice boundary is a vertical line; P is chosen to the left of this line and $ is chosen to the right. A second boundary – *the valuation boundary* – is the locus of (α, λ) pairs along which the P and $ bets have equal WTA valuations. We know from Property 2 that P has the higher valuation below this boundary and $ has the higher valuation above it.

Figure 1 plots these two boundaries for a typical pair of bets, defined by (p, q, r) = (0.8, 0.2, 1), with $\beta = 1$. This particular combination of parameters will be called the *benchmark case*. Standard PR occurs in the region above the valuation boundary and to the left of the choice boundary; non-standard PR occurs in the region below the valuation boundary and to the right of the choice boundary. The two boundaries intersect at (1, 1). (This reflects the fact that, when $\alpha = 1$, $\lambda = 1$ and $\beta = 1$, our model reduces to the maximisation of expected value; since the two bets have equal expected value, they are equally preferred and have equal valuations.) In this benchmark case, the empirically plausible quadrant is made up of two sub-regions, separated by the valuation boundary. Above this boundary there is standard PR. Below it, the P bet is both preferred in the straight choice and valued more highly. Thus, our model predicts the classic asymmetry between standard and non-standard reversals: the former occur at parameter values within the empirically plausible quadrant, while the latter do not.

We now investigate the implications of moving away from the benchmark case. We begin by deriving some further general properties of parameterised PT[3] in relation to PR. First, we note that the benchmark case has the mathematically convenient property that $p = 1 - q$; pairs of bets with this property will be called *symmetrical*.

The following property of the valuation boundary can be derived by substituting $\lambda = 1$ and $\alpha = \beta$ into (11) and (12):

> *Property 3:* If $\alpha = \beta$ and $\lambda = 1$, then $WTA^P = 1$ and $WTA^\$ = r$, i.e. the valuation of each bet is equal to its expected value.

Notice that Property 3 implies that, if r = 1, the valuation boundary passes through the point (β, 1) in (α, λ) space. With rather more manipulation (see Appendix), it is also possible to prove:

Property 4: If the P and $ bets are symmetrical and if $\lambda = 1$ and $\beta = 1$, $WTA^P = WTA^\$$ when $\alpha = \log(p/q) / \log(rp/q)$.

Given that the probability weighting function is specified by (8), symmetry has the convenient implication that $w(p) / w(q) = (p/q)^\beta$. Substituting this equality into (10), we arrive at:

Property 5: If the P and $ bets are symmetrical, the choice boundary is the vertical line $\alpha = \beta \log(p/q) / \log(rp/q)$.

Notice that Property 5 implies that if $r = 1$, the choice boundary is $\alpha = \beta$. The following additional properties are derived in the Appendix:

Property 6: If $\beta < 1$ and $r = 1$, the choice boundary lies to the right of (respectively passes through, lies to the left of) the point $(\beta, 1)$ if $p + q$ is greater than (equal to, less than) 1.

Property 7: As $p, q \to 0$, $WTA^P / WTA^\$ \to (1/r)(q/p)^{1-\beta/\alpha}$. Thus, if $r = 1$, for any given α, β with $\alpha < \beta$ (respectively $\alpha = \beta$, $\alpha < \beta$), $WTA^P > WTA^\$$ (respectively $WTA^P = WTA^\$$, $WTA^P < WTA^\$$) for values of p and q sufficiently close to zero.

Property 8: If $q = 0.5$, $WTA^\$ > r \Leftrightarrow \lambda > 1$.

Property 7 tells us that, if $r = 1$, then as p and q approach zero, the valuation boundary approaches the vertical line $\alpha = \beta$. Since $WTA^P \to 1$ as $p \to 1$, it is an implication of Property 8 that, if $r = 1$ and $q = 0.5$, the valuation boundary converges to the horizontal line $\lambda = 1$ as p approaches unity.

Using Properties 1 to 8, we investigate configurations of choice and valuation boundaries as we move away from the benchmark case.

First, we maintain the benchmark assumptions $r = 1$ and $\beta = 1$, and consider the effects of variations in the values of p and q. As in the benchmark case, the choice and valuation boundaries intersect at (1, 1). We know from Property 7 that as p and q approach zero, the valuation boundary converges to the vertical line $\alpha = 1$, which is also the choice boundary; thus, in this limit, the region of standard preference reversal disappears. Now consider another limiting case, defined by $q = 0.5$ and $p \to 1$. We know from Property 8 that the valuation boundary converges to the horizontal line $\lambda = 1$; in this limit, the region of standard preference reversal takes up the whole of the empirically plausible quadrant. Figures 2a, 2b, 3a and 3b show the position of the valuation boundary for different values of p and q between these two limiting cases. Figures 2a and 2b show the effects of varying p, holding q constant

at 0.1 (Figure 2a) and 0.4 (Figure 2b). Figures 3a and 3b show the effects of varying q, holding p constant at 0.9 (Figure 3a) and 0.7 (Figure 3b). Together, these figures reveal a tendency for the region of standard PR to expand as a consequence of increases in either p or q. In all cases, however, standard PR occurs in some part of the empirically plausible quadrant while non-standard PR occurs only outside this quadrant.

Next, maintaining the benchmark assumptions of symmetry and $\beta = 1$, we consider the effect of variations in the value of r. It follows from Properties 4 and 5 that the choice and valuation boundaries intersect at $(\alpha^*, 1)$, where $\alpha^* = \log(p/q) / \log(rp/q)$. As r increases, α^* falls, expanding the regions in which the $ bet is favoured. The intuition for this is straightforward: an increase in r increases the $-bet prize relative to the P-bet prize, and so makes the $-bet relatively more attractive. Notice that if $r > 1$, the empirically plausible quadrant is made up of *three* sub-regions. Below the valuation boundary, the P bet is favoured in both choice and valuation. To the right of the choice boundary, the $ bet is favoured in both choice and valuation. Above the valuation boundary and to the left of the choice boundary, there is standard PR. Non-standard PR occurs only outside the empirically plausible quadrant. Figure 4 shows the configurations of choice and valuation boundaries for $r = 0.8$, $r = 1.2$ and $r = 1.4$ when the other parameters take their benchmark values (i.e. $p = 0.8$, $q = 0.2$, $\beta = 1$).

In the cases we have considered so far, our model has consistently predicted the classic asymmetry between standard and non-standard PR in the empirically plausible quadrant. However, it has failed to predict another stylised fact about PR experiments: that, even when the two bets have equal expected value, a significant proportion of subjects not only value the $ bet more highly but also choose it in preference to the P bet. Because of the role of diminishing sensitivity in choice, our model predicts that P will be chosen whenever $r = 1$, $\alpha < 1$, and $\beta = 1$. To show that this is not a problem for our approach, we note that the benchmark assumption $\beta = 1$ is an extreme case – the case in which the probability weighting function is linear. We now consider the implications of assuming lower values of β, that is, an inverse-S function.

So, maintaining the benchmark assumptions of symmetry and $r = 1$, we investigate the effect of variations in β. It follows from Properties 3 and 5 that the choice and valuation boundaries intersect at $(\beta, 1)$. Figure 5 plots these boundaries for the benchmark pair of bets (0.8, 0.2, 1) for three empirically plausible values of β, namely 0.9, 0.75 and 0.6. Essentially, the effect of reducing the value of β is to shift both boundaries to the left, expanding the

regions in which the $ bet is favoured. The intuition for this is that, as the value of β falls, small probabilities (such as 0.2, the probability that the $ bet wins) are increasingly overweighted while large probabilities (such as 0.8, the probability that the P bet wins) are increasingly underweighted. The resulting configurations of choice and valuation boundaries are similar to those generated by setting $r > 1$. Again, the empirically plausible quadrant of (α, λ) space is made up of three sub-regions. In one, the P bet is favoured in both choice and valuation; in another, the $ bet is favoured in both choice and valuation; in the third, there is standard PR.

All the diagrams we have presented so far have the common feature that standard PR occurs only when $\lambda > 1$, and non-standard PR occurs only when $\lambda < 1$. The reader should not infer from this that loss aversion is essential if PT^3 is to predict PR. To the contrary, both standard and non-standard PR are compatible with $\lambda = 1$ for some pairs of bets. That this is the case follows from Properties 3 and 6. Let $\beta < 1$, and $r = 1$. Property 3 tells us that the valuation boundary passes through the point $(\beta, 1)$. Let α^* be the value of α at which the choice boundary crosses the line $\lambda = 1$. Property 6 tells us that if $p + q > 1$, then $\alpha^* > \alpha$. In other words, for values of α in the range $\beta < \alpha < \alpha^*$, standard PR occurs with $\lambda = 1$. Conversely, if $p + q < 1$, then $\alpha^* < \alpha$; for values of α in the range $\alpha^* < \alpha < \beta$, non-standard PR occurs with $\lambda = 1$. Figures 6a and 6b illustrate these possibilities for, respectively, the pairs of bets (0.8, 0.4, 1) and (0.6, 0.2, 1) with $\beta = 0.7$.

In principle, then, our model can predict PR in the absence of loss aversion. It can also predict PR in cases in which the model differs from expected utility theory *only* in respect of loss aversion. (Consider the case in which $p = 0.8$, $q = 0.2$, $r = 0.8$ and $\beta = 1$, shown in Figure 4. Notice that standard PR occurs at some points in (α, λ) space at which $\alpha = 1$ and $\lambda > 1$.) But these cases depend on special assumptions about the characteristics of the two bets. In contrast, the classic PR phenomenon occurs across a wide range of values of p, q and r. In particular, it occurs with $p + q < 1$, and it occurs with $r = 1$. It is a merit of our model that it explains PR across the range in which it has been observed.

The conclusion we wish to emphasise is this. Our model predicts the stylised facts of PR experiments on the assumption that subjects' values of the parameters α, λ and β are distributed over ranges that correspond with estimates derived from non-PR experiments – namely, values of α somewhat less than 1, values of λ somewhat greater than 1, and values of

β somewhat less than 1. Loss aversion, diminishing sensitivity and inverse-S probability weighting are all implicated in our explanation of PR.

7. Conclusions

In recent decades economists and psychologists have made considerable advances in understanding risky choice behaviour, prompted by anomalies relative to expected utility theory such as the Allais paradoxes. These developments have led economists to identify important new explanatory factors in decision making such as loss aversion and probability weighting. Moroever, theorists have found sophisticated ways of representing these factors in compact and tractable preference models. Explaining preference reversal (PR), however, has been a persistent stumbling block. While PR has long been recognised as an important departure from standard theory, and many theorists have attempted to provide preference-based explanations, we contend that no previous preference model has achieved this in an empirically satisfactory way.

We have presented a new model of risk preference: third generation prospect theory (PT^3). Our theory retains the empirically grounded features of prospect theory (loss aversion, diminishing sensitivity and non-linear probability weighting), but extends the model by allowing reference points to be lotteries. The resulting model retains all the predictive power of previous variants of prospect theory, but in addition provides a framework for determining the money valuation that an agent places on a lottery. Exploiting this feature of the model, we have shown that PR is consistent with PT^3. More significantly, when PT^3 is made operational by using simple functional forms with parameter values derived from existing experimental evidence, it predicts observed patterns of PR across a wide range of specifications of P and $ bets, consistent with the range in which PR has in fact been observed.

We do not claim that PT^3 provides a *complete* explanation of PR. We recognise that psychologists have proposed credible non-preference mechanisms of context-sensitive choice and valuation behaviour that are consistent with observations of PR. Predictions based on those mechanisms have been tested and confirmed in experimental tasks other than PR and, in some cases, outside the domain of theories of choice under uncertainty. This evidence clearly suggests that non-preference mechanisms contribute to PR. We assert only that PT^3 has a similar claim to be a model of mechanisms which contribute to that phenomenon. It too is based on psychologically credible hypotheses – loss aversion, diminishing sensitivity, the overweighting of small probabilities and the underweighting of large ones. It too is consistent

with observations of PR. It too has been tested and confirmed in experimental tasks other than PR – namely, pairwise choices between lotteries involving gains and losses. If one accepts prospect theory as an explanation of observed regularities in choice among lotteries, it seems reasonable to conclude that the mechanisms modelled by PT^3 play a significant role in the explanation of PR.

More generally, we offer PT^3 as a flexible and parsimonious model of choice under uncertainty which organises a large body of experimental evidence. We hope that it will find fruitful applications in future work.

Appendix: Derivations of Properties 4, 6, 7 and 8

Property 4: Substituting $\lambda = 1$ and $\beta = 1$ into (11) and (12) gives $WTA^P = (1/p) / [((1 - p)/p)^{1/\alpha} + 1] = (1/p) / [((1 - p)^{1/\alpha} + p^{1/\alpha})/p^{1/\alpha}]$ and $WTA^\$ = (r/q) / [((1 - q)/q)^{1/\alpha} + 1] = (r/q) / [((1 - q)^{1/\alpha} + q^{1/\alpha})/q^{1/\alpha}]$. But by symmetry, $(1 - p)^{1/\alpha} + p^{1/\alpha} = (1 - q)^{1/\alpha} + q^{1/\alpha}$. Thus $WTA^P/WTA^\$ = (q/rp)(p/q)^{1/\alpha}$ and so $WTA^P = WTA^\$ \Leftrightarrow rp/q = (p/q)^{1/\alpha}$, i.e. $WTA^P = WTA^\$ \Leftrightarrow \alpha = \log(p/q)/\log(rp/q)$.

Property 6: Substituting $\alpha = \beta$, $r = 1$ and (8) into (9) and rearranging, we arrive at $f^P \succ_h f^\$ \Leftrightarrow [q^\beta + (1 - q)^\beta]^{1/\beta} \geq [p^\beta + (1 - p)^\beta]^{1/\beta}$. Since $0 < p, q < 1$ and $\beta \leq 1$, this implies $f^P \succ_h f^\$ \Leftrightarrow q^\beta + (1 - q)^\beta \geq p^\beta + (1 - p)^\beta$. If $\beta < 1$, the function $\phi(\pi) = \pi^\beta + (1 - \pi)^\beta$ has an inverse U-shape, symmetrical around a maximum at $\pi = 0.5$. Thus, the P bet is chosen (respectively: the two bets are indifferent, the \$ bet is chosen) if q is closer than (respectively: equally close as, less close than) p to 0.5, i.e. if p + q is greater than (respectively: equal to, less than) 1.

Property 7: From (13), as $p, q \to 0$, $WTA^P / WTA^\$ \to (1/ r) [q^{1 - \beta/\alpha} (1/ \lambda)^{1/\alpha} + q] / [p^{1 - \beta/\alpha} (1/ \lambda)^{1/\alpha} + p]$. Since $0 < p, q < 1$ and $1 - \beta/\alpha < 1$, $(p^{1 - \beta/\alpha}/ p) \to \infty$ as $p \to 0$, and $(q^{1 - \beta/\alpha}/ q) \to \infty$ as $q \to 0$. Thus, $WTA^P / WTA^\$ \to (1/ r) [q^{1 - \beta/\alpha} (1/ \lambda)^{1/\alpha}]/ [q^{1 - \beta/\alpha} (1/ \lambda)^{1/\alpha}]$, i.e. $WTA^P / WTA^\$ \to (1/ r) (q/ p)^{1 - \beta/\alpha}$.

Property 8: Substituting $q = 0.5$ into (12) gives $WTA^\$ = 2r/ [(1/\lambda)^{1/\alpha} + 1]$. Thus $WTA^\$ > r \Leftrightarrow (1/\lambda)^{1/\alpha} < 1$. Since $\alpha > 0$, $(1/\lambda)^{1/\alpha} < 1 \Leftrightarrow \lambda > 1$.

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Notes

[1] A related theory is proposed by Koszegi and Rabin (2004) who present a version of prospect theory in which reference points are expectations about outcomes. Koszegi and Rabin define preferences over prospects (i.e. probability distributions over outcomes) rather than acts. In terms of our framework, this approach can be thought of as assuming that the act being evaluated is stochastically independent of the reference act.

[2] Our analysis of PR does not depend on the cumulative transformation of probabilities. The acts that we analyse have no more than one strictly positive consequence and no more than one strictly negative one. For such acts, the cumulative transformation is observationally equivalent to Handa's simple transformation.

[3] If there are distinct states s_i, s_j such that $v(f(s_i), h(s_i)) = v(f(s_j), h(s_j))$, there may be more than one way of re-assigning subscripts consistently with these conditions. However, all permissible re-assignments generate the same value of V(f, h).

[4] In addition, $f^{\$} \sim_h f^{\$} \Leftrightarrow w^+(p)\, v(x, 0) - w^+(q)\, v(y, 0) = 0$. From now on, to avoid cluttering the exposition, we will not state conditions for indifference explicitly. In all cases, the condition for indifference can be constructed from the condition for weak preference by substituting an equality for a weak inequality.

Figure 1: The Choice and Valuation Boundaries
(p=0.8, q=0.2, r =1, beta=1)

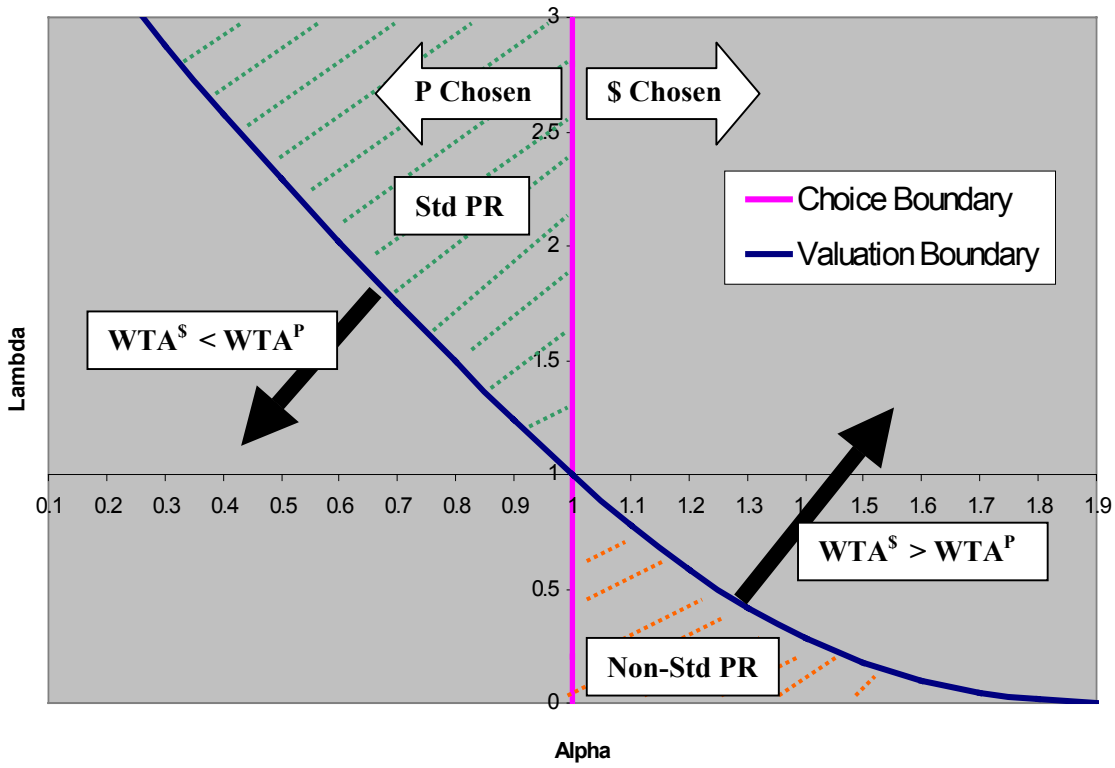


Figure 2a: Variation in P bet
(q=0.1, r=1, beta=1)

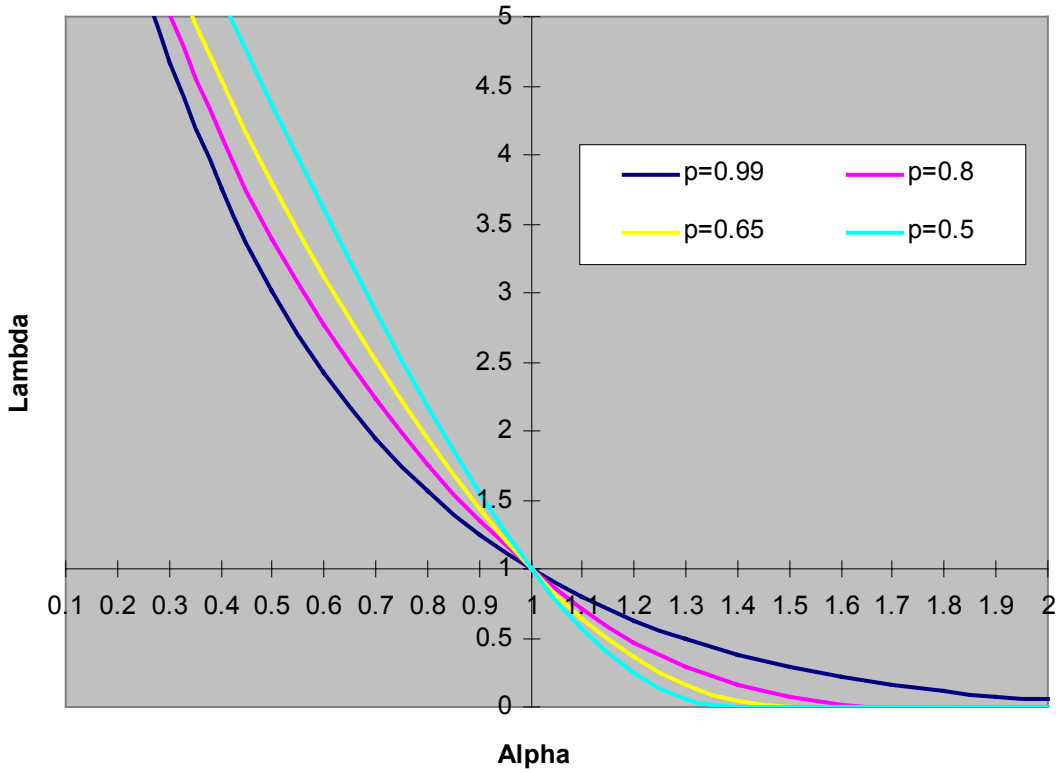


Figure 2b: Variation in P bet
(q=0.4, r=1, beta=1)

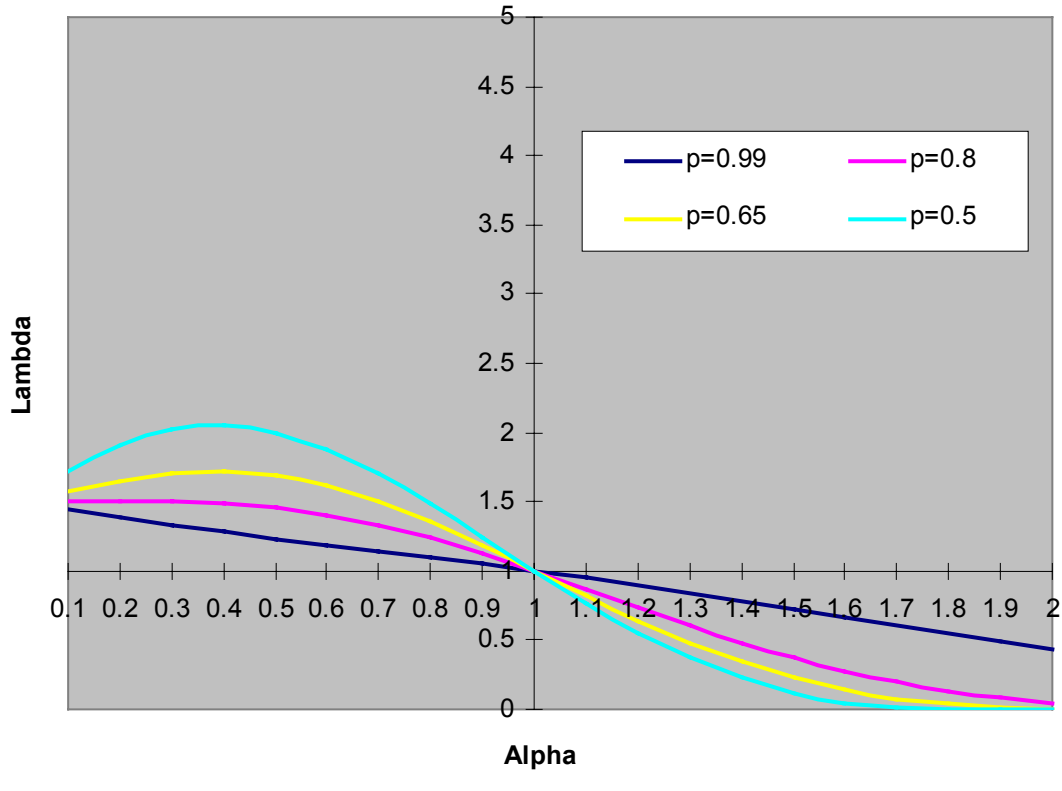


Figure 3a: Variation in $ bet
(p=0.9, r=1, beta=1)

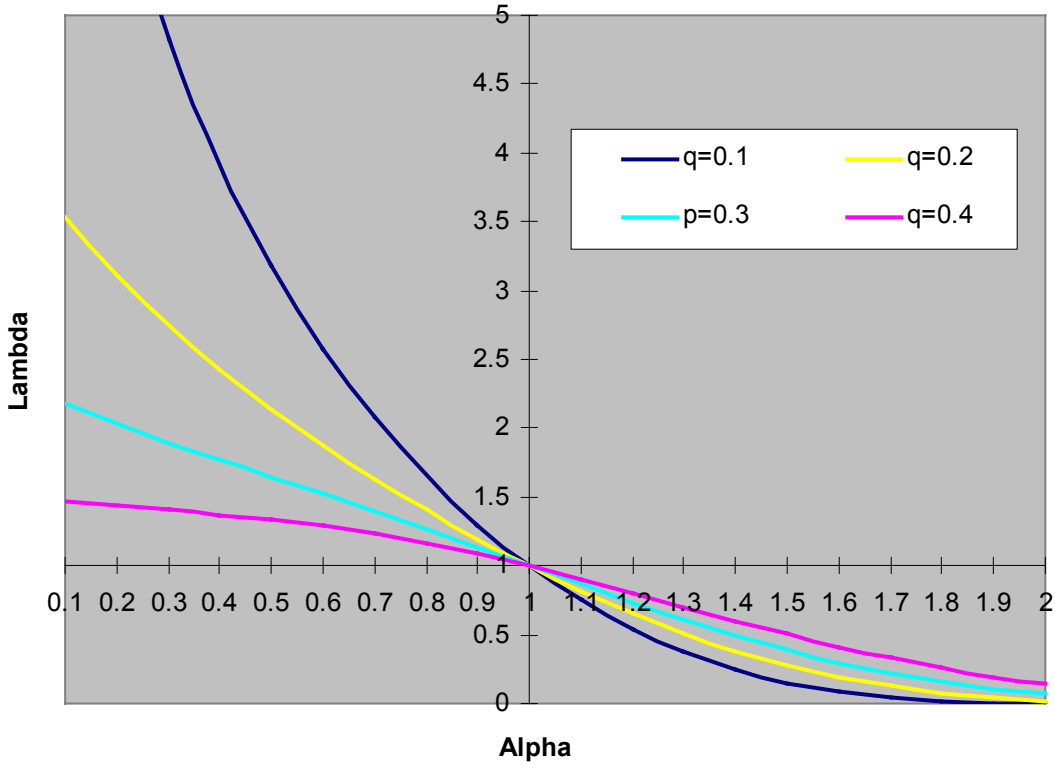
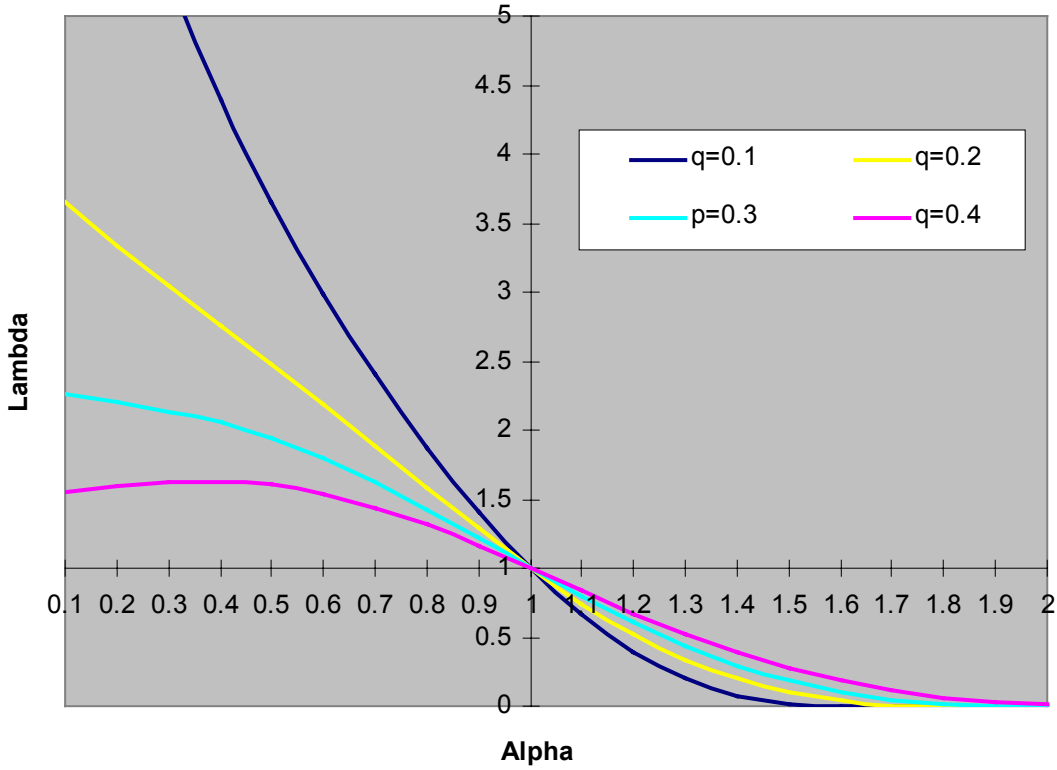


Figure 3b: Variation in $ bet
(p=0.7, r=1, beta=1)



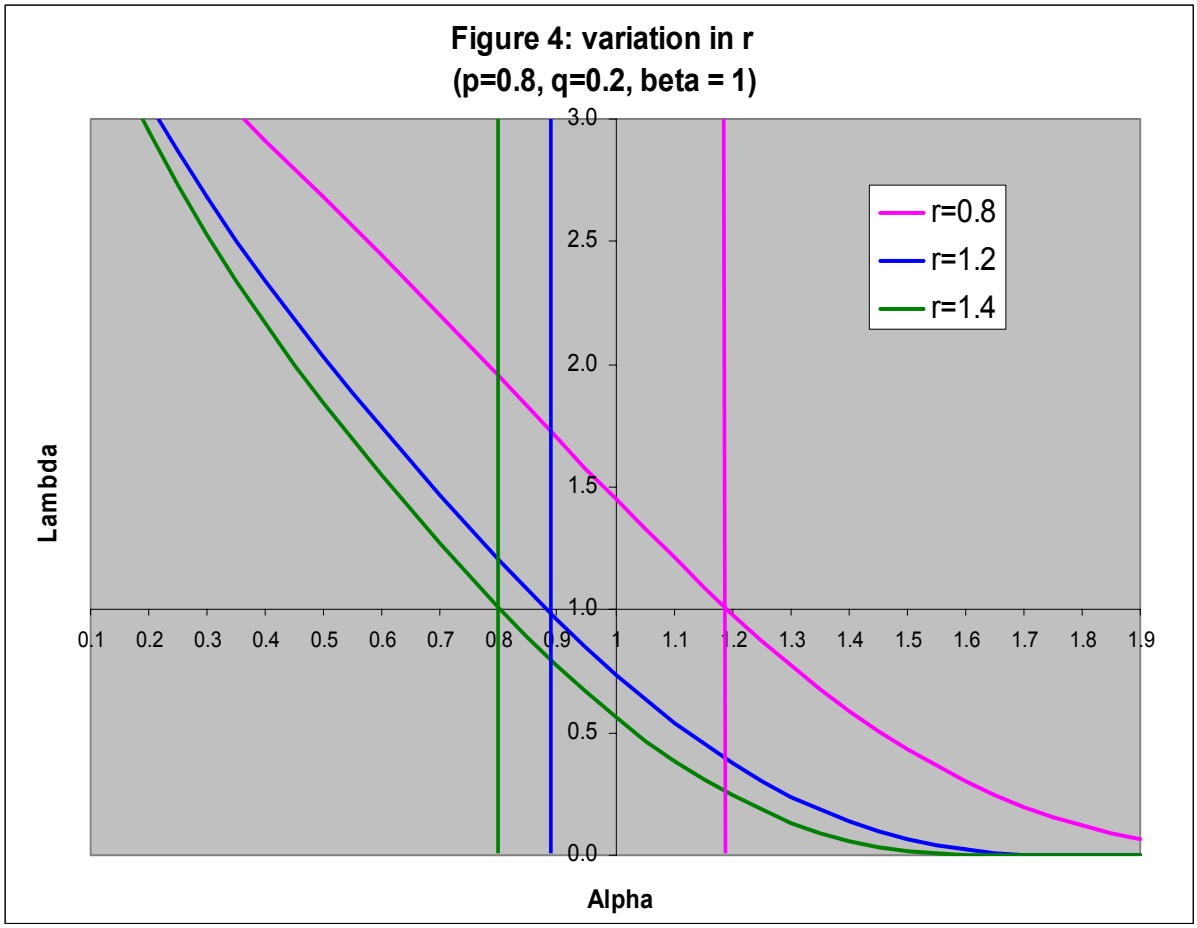


Figure 4: variation in r
(p=0.8, q=0.2, beta = 1)
Lambda
Alpha
r=0.8
r=1.2
r=1.4
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9

Figure 5: Variation in beta
(p=0.8, q=0.2, r=1)

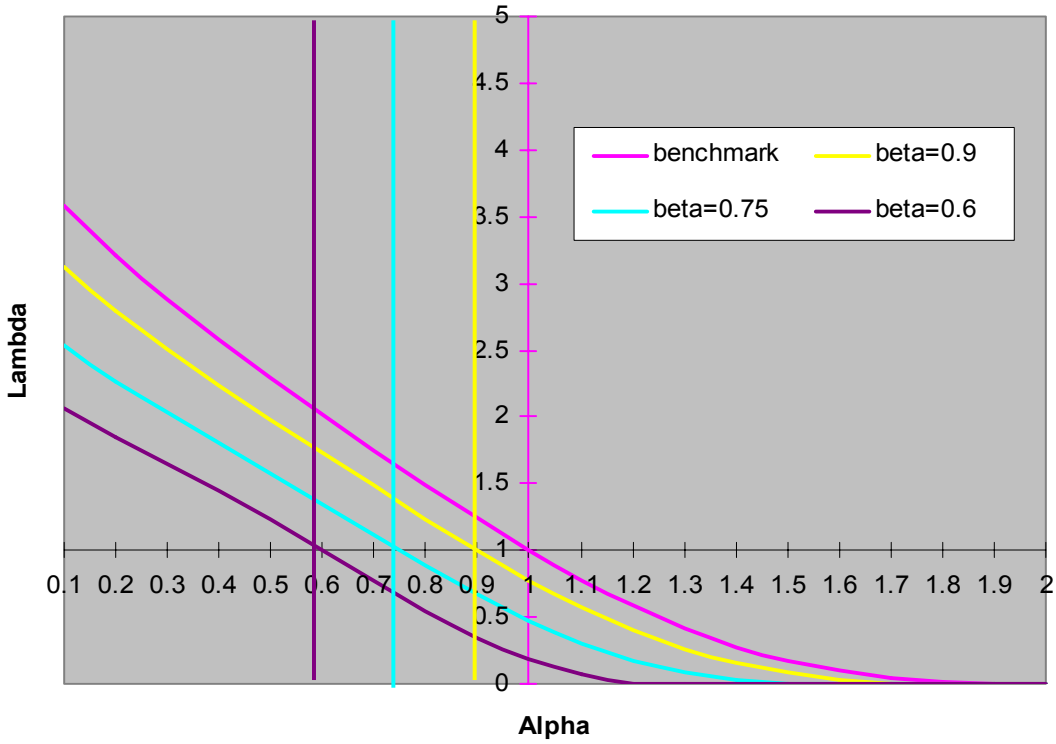


Figure 6a: bets (0.8, 0.4, 1) with beta = 0.7

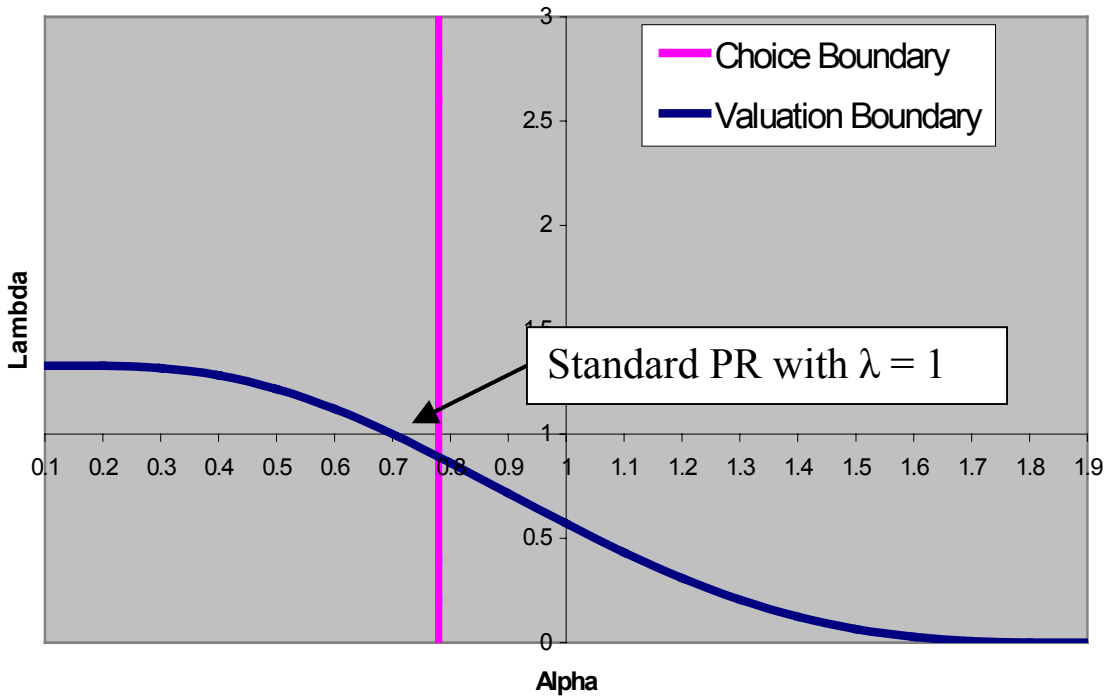


Figure 6b: bets (0.6, 0.2, 1) with beta = 0.7