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Working Paper

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CIBC Working Paper, No. 2012-5

Provided in Cooperation with:

Centre for Human Capital & Productivity (CHCP), Department of Economics, University of Western Ontario

Suggested Citation: Caucutt, Elizabeth M.; Lochner, Lance (2012) : Early and late human capital investments, borrowing constraints, and the family, CIBC Working Paper, No. 2012-5, The University of Western Ontario, CIBC Centre for Human Capital and Productivity, London (Ontario)

This Version is available at:

<https://hdl.handle.net/10419/70329>

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Investments, Borrowing Constraints, and
the Family**

by

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Working Paper # 2012-5

October 2012



CIBC Working Paper Series

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Early and Late Human Capital Investments, Borrowing Constraints, and the Family*

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October 18, 2012

Abstract

This paper investigates the importance of family borrowing constraints in determining human capital investments in children at early and late ages. We begin by providing new evidence from the Children of the NLSY (CNLSY) which suggests that borrowing constraints bind for at least some families with young children. Next, we develop an intergenerational model of lifecycle human capital accumulation to study the role of early versus late investments in children when credit markets are imperfect. We analytically establish the importance of dynamic complementarity in investment for the qualitative nature of investment responses to income and policy changes. We extend the framework to incorporate dynasties and use data from the CNLSY to calibrate the model. Our benchmark steady state suggests that roughly half of young parents and 12% of old parents are borrowing constrained, while older children are unconstrained. We also identify strong complementarity between early and late investments, suggesting that policies targeted to one stage of development tend to have similar effects on investment in both stages. We use this calibrated model to study the effects of education subsidies, loans and transfers offered at different ages on early and late human capital investments and subsequent earnings in the short-run and long-run. A key lesson is that the interaction between dynamic complementarity and early borrowing constraints means that early interventions tend to be more successful than later interventions at improving human capital outcomes.

*For helpful comments, we thank Flavio Cunha, Rick Hanushek, Jim Heckman, Alex Monge-Naranjo, and Aloysius Siow, as well as participants at the 2002 and 2012 Society of Economic Dynamics Annual Meetings, 2004 Federal Reserve Bank of Cleveland Workshop on Human Capital and Education, CIBC Human Capital Conference, 2011 CESifo Area Conference on Economics of Education, Workshop on Labour Markets and the Macroeconomy, 2011 Canadian Macro Study Group Meeting, Human Capital Conference at ASU, and 2012 AEA Annual Meeting. We also thank seminar participants at the Federal Reserve Banks of Chicago and St. Louis, Universitat of Autonomia de Barcelona, NYU, University of Virginia, Yale University, and the University of Saskatchewan.

1 Introduction

By limiting the incentives and capacity to invest in human capital, financing constraints and labor market uncertainty can play critical roles in determining aggregate productivity, national income distributions, social mobility, and economic growth and development (Becker 1975). The growing importance of parental income for child achievement and educational attainment (Belley and Lochner 2007, Duncan and Murnane 2011, Reardon 2011) raises serious questions about the ability (or willingness) of disadvantaged families to make efficient investments in their children. In this paper, we investigate the importance of family borrowing constraints in determining human capital investments in children at different ages. We also explore the extent to which policies targeted to different ages can address this market failure, potentially improving economic efficiency and equity.

Despite considerable evidence that adolescent skill levels are important in determining subsequent schooling and lifetime earnings (see, e.g., Cameron and Heckman 1998, Keane and Wolpin 1997, 2001, and Carneiro and Heckman 2002), only recently has the literature begun to consider whether borrowing constraints inhibit early investments in young children (Restuccia and Urrutia 2004, Cunha 2007, Del Boca, Flinn, and Wiswall 2010, Lee and Seshadri 2012). This is somewhat surprising, since studies of consumption behavior often find stronger evidence of binding liquidity constraints for younger households (e.g. Meghir and Weber 1996, Alessie, Devereux, and Weber 1997, Stephens 2008).¹ Indirect evidence suggests that constraints at early ages may play a more important role in determining human capital investment than constraints at later ages. For example, most empirical studies find high lifetime returns for early childhood programs, especially for disadvantaged children (e.g., see Karoly et al. 1998, Blau and Currie 2006, or Cunha, et al. 2006, Heckman, et al. 2010). Other studies find that family income received at early childhood ages has a greater impact on adolescent achievement when compared with income received at later ages (e.g. Duncan and Brooks-Gunn 1997, Duncan, et al. 1998, Levy and Duncan 1999, Caucutt and Lochner 2006).² More generally, a number of recent studies demonstrate that dif-

¹In related work, Attanasio, Goldberg and Kyriazidou (2008) conclude that many younger and middle-age American households are likely to be liquidity constrained based on differential car loan demand elasticities with respect to interest rates and loan maturity. In the presence of income uncertainty, Gourinchas and Parker (2002) empirically show that younger consumers behave as buffer-stock agents, saving small amounts (rather than borrowing) due to precautionary motives, while middle-age workers save primarily for lifecycle motives. The possibility of very low future income realizations causes young consumers to behave much like they are liquidity constrained.

²Carneiro and Heckman (2002) argue that early income should have a larger effect than later income due purely to discounting (e.g. \$1 at age 0 should have an effect that is $(1 + r)^{10}$ larger than income at age 10, where r is the annual interest rate). Accounting for this, they estimate similar effects of ‘early’ and ‘late’ family income on college enrolment in the Children of the National Longitudinal Survey of Youth (CNLSY); however, they also

ferences in child achievement by family income do not simply reflect unobserved differences in families and parenting styles that are correlated with family income. These studies show that exogenous increases in family income lead to real improvements in child development (e.g. Løken 2010, Duncan, Morris and Rodrigues 2011, Milligan and Stabile 2011, Dahl and Lochner 2012, Løken, Mogstad and Wiswall 2012).

Table 1 provides new evidence from the Children of the National Longitudinal Survey of Youth (CNLSY) that, in the U.S., early income is a more important determinant of educational attainment than is income earned at later ages. This table reports results from regressing educational attainment indicators on early and late family income.³ Estimates reported in Panel A control only for maternal education, while those in Panel B also control for other important child and mother characteristics. Specifications described in the first four columns use total reported parental earnings to measure family income, while the specifications reported in columns 5-8 use an adjusted ‘full’ earnings measure that adjusts for the possibility that some mothers work part-time to spend more time investing in their children.⁴ The estimated effects are quite similar across specifications and reveal that a \$10,000 increase in discounted annual income over ages zero to eleven significantly reduces high school dropout rates by about 2.5 percentage points, while it increases college attendance rates by as much as 4.6 percentage points. The same increase in discounted annual income when the child is 12-23 has smaller (and statistically insignificant) effects on these education margins; however, the effects of early and late income are more similar for college completion. For many specifications, differences in the effects of early and late family income on high school completion and/or college attendance are statistically significant at conventional levels.

Credit constraints are natural candidates to explain why some low-income families do not make

control for age 12 achievement levels which may absorb much of the effect of earlier income. Caucutt and Lochner (2006) estimate that (discounted) income received at earlier ages has a larger impact on age 5-14 math and reading achievement in the CNLSY than (discounted) income received at later ages.

³Income in Table 1 is measured in \$10,000 year 2008 dollars, using the CPI-U to account for inflation. Income is averaged over ages 0-11 (early income) and 12-23 (late income) after discounting income each year back to the year the child was born using a discount rate of 5%. These assumptions and age ranges are consistent with those used later in the calibration of our model.

⁴In Panel B, specifications include measures of child’s year of birth, race/ethnicity, and gender, while mother’s characteristics include educational attainment, whether she was a teenager when the child was born, living in an intact family at age 14, foreign-born, and Armed Forces Qualifying Test scores. The adjusted ‘full’ earnings measure inflates earnings for mothers working less than 1,500 hours per year to its 1,500 hour equivalent (i.e. multiplies their earnings by 1,500 and divides by reported hours). Since NLSY mothers were ages 14-22 in 1979, many of their children are still young. Thus, our sample sizes are smaller when looking at college attendance or completion at age 24 compared with measures of high school dropout as of age 21. We also lose some observations due to missing mother or child characteristics (Panel B) or missing measures of hours worked (columns 5-8). See Appendix A for additional details on the CNLSY data and our sample.

the same investments in their children as do higher income families and why early family income appears to be a more important determinant of educational outcomes than income received at later ages. While (generous) government student loan programs are available for college in the U.S. and other developed countries, neither governments nor private lenders typically offer loans to parents to help finance human capital investments in younger children. Even though elementary and secondary education is publicly provided, the quality of public schools available to poor families is often low, while high quality private schools and preschool programs are typically quite expensive. Kaushal, Magnuson, and Waldfogel (2011) find that families in the bottom family expenditure quintile spend 3% of their total expenditures on educational enrichment items, while families in the top quintile spend 9%. Parental spending on education-related items and activities rises with total family expenditures. Parental time is also an important input for a young child’s development that poor parents may be unable to afford.⁵ Finally, most parents of young children are young themselves, in the early stages of their labor market careers and without a solid credit history. Even young college-educated parents may be constrained by mortgages, their own schooling loans, and other liabilities. While we do not know exactly how any given parents might spend additional resources, recent studies clearly show that those resources lead to improvements in child outcomes (Løken 2010, Duncan, Morris and Rodrigues 2011, Milligan and Stabile 2011, Dahl and Lochner 2012, Løken, Mogstad and Wiswall 2012).

Investment in human capital is a multi-stage process that begins early in life.⁶ As a result, human capital investment is an intergenerational family problem.⁷ We develop a human capital-based theory of the family that incorporates the dynamic nature of investment in children, intergenerational transfers, and borrowing constraints faced by parents and college-age youth. Our theory accounts for the fact that later investments build on earlier investments, that early childhood investments are made by young parents at the beginning of their careers, and that desired borrowing may differ substantially over the lifecycle.

In our framework, young parents make early investments in their children and provide them with consumption. These parents also make their own consumption choices and borrow or save to intertemporally allocate resources. Constraints on their borrowing may limit consumption

⁵See Del Boca, Flinn, and Wiswall (2010). Using data from American Time Use Surveys, Guryan, Hurst and Kearney (2008) establish a robust positive education gradient in childcare. Higher-educated parents spend more time in childcare than less educated parents, whether or not one controls for employment status.

⁶See, e.g., Becker (1975), Todd and Wolpin (2003, 2007), Cunha, et al. (2006), Cunha (2007), Cunha and Heckman (2007), Cunha, Heckman and Schennach (2010), Del Boca, Flinn, and Wiswall (2010), and Lee and Seshadri (2012).

⁷See, e.g., Becker and Tomes (1979, 1986), Loury (1981), Glomm and Ravikumar (1992), Galor and Ziera (1993), Aiyagari, Greenwood and Seshadri (2002), Caucutt and Kumar (2003), and Restuccia and Urrutia (2004).

and investments in young children. Once children become young adults, they make additional investments in themselves (e.g. college), using their own earnings, transfers from their parents, and student loans to cover schooling costs and consumption. Again, choices may be constrained by imperfect credit markets. Older parents must decide how much to transfer to their college-age children and how much to borrow or save for their own current and future consumption. Once a child leaves the home to establish his own family, parents continue to work, save, and consume until retirement. This cycle repeats itself, as young adults grow into parenthood.

The dynamic nature of human capital investment has important implications for the role of borrowing constraints and economic policies. Consistent with the analysis of Cunha and Heckman (2007), we show that *dynamic complementarity* in investment – the complementarity between early and late investments in human capital – plays a central role in determining the impacts of family income, investment subsidies and borrowing constraints on investment over the lifecycle. When investments are sufficiently complementary, a policy that encourages investment at one stage of development will also tend to increase investment at other stages. By contrast, when investments are substitutable over time, a subsidy or loan increase at one stage of development tends to shift investment to that stage and away from others. Indirect evidence discussed in Cunha, et al. (2006) and estimates by Cunha, Heckman and Schennach (2010) suggest that investments are quite complementary. Calibration of our intergenerational model produces a similarly strong degree of dynamic complementarity.⁸

A large literature considers the impacts of college-age policies on schooling and labor market outcomes holding early investment and adolescent achievement levels fixed (e.g. Cameron and Heckman 1998, Heckman, Lochner and Taber 1998, Keane and Wolpin 2001, Caucutt and Kumar 2003, Hanushek, Leung and Yilmaz 2003, Johnson 2010, Gallipoli, Meghir and Violante 2011).⁹ The degree of dynamic complementarity we calibrate suggests that these policies not only affect college-going, but they also have significant impacts on earlier investments in children. Our quantitative analysis suggests that ignoring these earlier investment responses may lead researchers to under-estimate the total wage impact of college-age investment subsidies by as much as 60%.

The timing of borrowing constraints is also important and interacts with dynamic complementarity in investment. As discussed above, early income appears to be more important than later

⁸Unlike Cunha, Heckman and Schennach (2010), we do not distinguish between cognitive and non-cognitive skills. To the extent that these skills are combined to create a composite productivity (i.e. human capital) level used in the labor market, we effectively identify the technology mapping early and late investments (in cognitive or non-cognitive skills) into this productivity measure.

⁹Traditional difference-in-difference or reduced form studies of the impact of tuition or aid levels on college-going also implicitly hold early investment behaviors fixed.

income for educational attainment. Based on this feature of the CNLSY data, our calibration finds that borrowing constraints are more severe for young families with young children. Increasing borrowing opportunities or subsidizing investment for families with young children would lead to important (short-run) increases in human capital investments at early and college-going ages. By comparison, increasing borrowing limits for college students and older parents has very small effects. The latter is consistent with the findings of Keane and Wolpin (2001) and Johnson (2010). When compared with subsidies for early investments, subsidies for college also have weaker effects on human capital investment, since they come too late for many constrained families. Due to early borrowing constraints, many families cannot increase early investments to fully take advantage of later subsidies. Given strong complementarity of investments over time, additional later investments may not be worthwhile for these families. Thus, dynamic complementarity and early borrowing constraints interact in a way that limits the effectiveness of college-age policies for disadvantaged families.

While the impact of a policy depends on its timing, whether it is an in-kind transfer or subsidy is also important for understanding which groups benefit most. Our analysis suggests that increasing the amount of public spending on early investments disproportionately increases early investment for children whose parents are at the bottom of the education distribution. By contrast, increasing the subsidy to early investment disproportionately increases investment for those whose parents are at the top of the education distribution.

The intergenerational nature of human capital investment is also important. Keane and Wolpin (2001) and Johnson (2010) both emphasize the importance of differences in parental transfers by socioeconomic background in explaining differential schooling outcomes; however, parental transfers are exogenously determined and unaffected by policy and economic conditions in their models. By endogenizing parental transfers, we account for the fact that parents respond to different policies by adjusting transfers to their children. Furthermore, our dynastic approach to human capital investment enables us to study dynamic effects of lasting economic policies that are typically ignored by much of the literature.

First, we differentiate between the short-run effects of one-time, single-generation policies targeted to parents and similar policies that are put in place permanently. (Most empirical micro studies consider the former, while quantitative macro studies typically focus on the latter.) One-time policies indirectly affect children only through parental transfers, while permanent policy changes also affect children directly once those children grow up. Because these channels can

have opposing or reinforcing effects on investment depending on the type of intervention, the short-term effects of new policies depend heavily on how long they are expected to remain in place.

Second, our dynastic approach enables us to simulate the long-run effects of permanent policy changes in addition to the short-run effects typically measured in empirical studies. While short-run effects are based on the current distributions of wealth and human capital in the population, long-run effects take into account changes in these distributions over time. This is quite important when considering a policy to increase borrowing opportunities for young parents. In the short-run, such a policy has the expected effects of increasing borrowing and human capital investment. In the long-run, increases in borrowing cause parents to accumulate more debt and transfer less resources to their children. Over time, this leads to higher debt levels, pushing families nearly as close to borrowing limits as initial generations were before the policy change. While early generations accumulate more human capital, later generations do not. By contrast, investment subsidies appear to have greater long-run impacts (relative to short-run impacts), because they encourage human capital investment without building debt levels.

This paper proceeds as follows. In Section 2, we develop a lifecycle model of human capital investment with borrowing constraints. Allowing for two periods of investment, we analytically study the effects of changes in income in both periods. The results are useful in interpreting the empirical evidence in Table 1 on the relative importance of early vs. late income for educational attainment. We show that our findings are broadly consistent with strong dynamic complementarity in investments and binding borrowing constraints for many young families. We also analytically investigate the effect of relaxing borrowing constraints at different ages on investment behavior. This analysis establishes the importance of dynamic complementarity for the qualitative nature of investment responses. We demonstrate equivalence between the lifecycle model and an intergenerational problem where parents decide on their own consumption and transfer resources to their children to invest and consume. This highlights that endogenizing parental transfers does not alter the qualitative properties of the lifecycle problem.

In Section 3, we expand the model to a fully dynastic overlapping-generations framework in which altruistic parents value the utility of their children. We calibrate this model in Section 4 using data from the CNLSY on family income at different stages of child development, educational attainment by children and their parents, and the earnings outcomes of children. This analysis assumes a CES human capital production function without imposing any assumptions on the

complementarity of investments across different stages of development. It allows for unobserved heterogeneity in ability and post-school shocks to labor market earnings.

Based on our calibrated model, in Section 5 we simulate the impacts of various policy changes including increases in borrowing limits, investment subsidies, and income transfers. We discuss both short- and long-run effects of policies. We conclude in Section 6.

2 The Economics of Early and Late Human Capital Investments and Borrowing Constraints

In this section, we study ‘early’ and ‘late’ investments in human capital and the roles of borrowing constraints and family resources at different stages of development. The analysis highlights the importance of dynamic complementarity in investments for determining the impacts of both borrowing constraints and family resources on investment decisions. Our analysis begins with a multi-period lifecycle model to focus on intertemporal tradeoffs central to the links between the timing of borrowing constraints, family resources, and human capital investments. We then show that this analysis extends to a multi-period intergenerational framework, in which parents decide on their own consumption and transfer resources to their children for investment and consumption. By mapping a two-generation problem directly into our initial lifecycle problem, we show that all of our ‘lifecycle’ results extend naturally to a two-generation framework.

2.1 Lifecycle Model with Early and Late Investments

We assume that people live through six periods in their lives. Human capital investment takes place in the first two periods (i.e. ‘childhood’), followed by three periods of work and a period of retirement. We are agnostic about the form of investments, instead focusing on the intertemporal nature of skill production and investment choices. Conceptually, investments may include various forms of goods inputs like computers and books, parental time in child development activities, formal schooling, and other time inputs by older children. Our main focus in this section is on the importance of multiple investment periods during childhood.

2.1.1 Technology for Human Capital Production

Denote an individual’s ability to learn by θ . (Below, this could also reflect ‘parenting ability’.) Investments in periods 1 and 2 are given by i_1 and i_2 , respectively. Together, these investments produce period 3 human capital:

$$h_3 = f(i_1, i_2, \theta). \tag{1}$$

The human capital production function $f(\cdot)$ is strictly increasing in all of its arguments and strictly concave in both investment arguments. We further assume that f_{13} and f_{23} are non-negative, so that ability and investments are complements. To guarantee appropriate second order conditions hold in the decision problems described below, we assume the following throughout our analysis:

Assumption 1. $f_{12} > \max \left\{ f_{22} \left(\frac{f_1}{f_2} \right), f_{11} \left(\frac{f_2}{f_1} \right) \right\}$ and $f_{12}^2 < f_{11} f_{22}$.

Most reasonable specifications for human capital production would have $f_{12} \geq 0$, satisfying the first part of this assumption. The second part limits the degree of complementarity in investments.

In our computational analysis below, we employ a CES human capital production function of the form

$$f(i_1, i_2, \theta) = \theta(ai_1^b + (1-a)i_2^b)^{d/b}, \quad (2)$$

where $a \in (0, 1)$, $b < 1$, and $d \in (0, 1)$; however, our theoretical analysis does not rely on any particular functional form. Assumption 1 holds for this production function. We impose decreasing returns to scale (i.e. $d < 1$); otherwise unconstrained individuals may want to invest an infinite amount.

Adult earnings depend on human capital acquired through childhood investments. Given our emphasis on childhood human capital investment (i.e. early childhood and schooling investments), we assume that human capital grows exogenously after childhood:

$$h_4 = \Gamma_4 h_3 \quad \text{and} \quad h_5 = \Gamma_5 h_4. \quad (3)$$

Adult earnings are given by

$$W(h_j) = wh_j, \text{ for } j \in \{3, 4, 5\}, \quad (4)$$

where $w > 0$ reflects the wage per unit of skill.

2.1.2 Lifecycle Decision Problem

We assume standard time separable preferences for consumption, $\sum_{j=1}^6 \beta^{j-1} u(c_j)$, where the time discount rate $\beta \in (0, 1)$ and utility function $u(c)$ is strictly increasing, strictly concave and satisfies standard Inada conditions. Individuals receive exogenous financial transfers y_j during childhood periods $j = 1, 2$, which could reflect transfers from parents or the government. They may also reflect earnings while in school for older children. Below, we explicitly endogenize transfers to children (from parents) in an intergenerational framework.

The gross rate of return on borrowing and saving is $R > 0$. Assets saved in period j are given by a_{j+1} , and total borrowing (negative a_{j+1}) may be limited by a restriction on debt carried over to the next period, $-L_j$. During retirement, individuals consume their savings and do not work.

For some of our analysis, it is convenient to separate the lifecycle decision problem into investment and post-investment periods. Consider individuals entering adulthood (period 3) with human capital h_3 and assets a_3 . They allocate consumption across their remaining life subject to any constraints:

$$V_3(a_3, h_3) = \max_{c_3, c_4, c_5, c_6} \sum_{j=3}^6 \beta^{j-3} u(c_j)$$

subject to budget constraints $a_{j+1} = Ra_j + W(h_j) - c_j$ for $j = 3, 4, 5$; borrowing constraints $a_4 \geq -L_3$ and $a_5 \geq -L_4$; and $c_6 = Ra_6$.

Children allocate their resources to consumption and investment, leaving some assets/debt for adulthood:

$$\max_{c_1, c_2, i_1, i_2, a_3} u(c_1) + \beta u(c_2) + \beta^2 V_3(a_3, f(i_1, i_2, \theta)) \quad (5)$$

subject to budget constraints:

$$a_{j+1} = Ra_j + y_j - i_j - c_j \quad \text{for } j = 1, 2;$$

and borrowing constraints:

$$a_2 \geq -L_1 \quad \text{and} \quad a_3 \geq -L_2.$$

2.1.3 Lifecycle Investment Behavior

Consumption allocations satisfy $u'(c_j) \geq \beta R u'(c_{j+1})$, $\forall j = 1, \dots, 5$, where the inequality is strict if and only if the borrowing constraint for that period (L_j) binds. First order conditions for investment imply:

$$u'(c_1) = \beta^2 \frac{\partial V_3(a_3, h_3)}{\partial h_3} f_1(i_1, i_2, \theta), \quad (6)$$

$$u'(c_2) = \beta \frac{\partial V_3(a_3, h_3)}{\partial h_3} f_2(i_1, i_2, \theta), \quad (7)$$

where $\frac{\partial V_3(a_3, h_3)}{\partial h_3} = w [u'(c_3) + \beta \Gamma_4 u'(c_4) + \beta^2 \Gamma_4 \Gamma_5 u'(c_5)] > 0$. Taking the ratio of these equations reveals that optimal investment equates the technical rate of substitution in the production of human capital with the marginal rate of substitution for consumption: $\frac{f_1(i_1, i_2, \theta)}{f_2(i_1, i_2, \theta)} = \frac{u'(c_1)}{\beta u'(c_2)} \geq R$.

Unconstrained optimal investments for an individual of ability θ , $i_1^u(\theta)$ and $i_2^u(\theta)$, satisfy $f_1(i_1^u(\theta), i_2^u(\theta), \theta) = \frac{R^2}{\chi}$ and $f_2(i_1^u(\theta), i_2^u(\theta), \theta) = \frac{R}{\chi}$, where $\chi = w(1 + R^{-1} \Gamma_4 + R^{-2} \Gamma_4 \Gamma_5)$ reflects the

discounted present value of an additional unit of human capital. Thus, unconstrained investments maximize the discounted present value of earnings net of discounted investment costs. They are independent of the marginal utility of consumption and income/transfers, since individuals can optimally smooth consumption across periods. This is not true when borrowing constraints bind.

Proposition 1. *Let i_1^* and i_2^* reflect optimal first and second period investment. Then, (i) $f_1(i_1^*, i_2^*, \theta) > f_1(i_1^u, i_2^u, \theta) = \frac{R^2}{\chi}$ if and only if any borrowing constraint binds; (ii) $f_2(i_1^*, i_2^*, \theta) > f_2(i_1^u, i_2^u, \theta) = \frac{R}{\chi}$ if and only if borrowing limits L_2 , L_3 or L_4 bind; (iii) there is under-investment in at least one period and possibly both (i.e. $i_1^* < i_1^u$ and/or $i_2^* < i_2^u$) if and only if any borrowing constraint binds.*

Proof: See Appendix B.

Facing a binding constraint at any point, even later in life, implies under-investment in human capital during at least one period. When the first period borrowing constraint binds, $f_1/f_2 > R$ and there is too little early investment relative to late investment. When only the second period (or later constraints) bind, both early and late investments tend to be too low even though $f_1/f_2 = R$. We next explore how investments in both periods depend on borrowing limits and financial transfers.

To focus on the role of borrowing constraints during investment periods, much of the remaining analysis is simplified considerably if we assume that credit constraints are non-binding in later periods after investments have been made. We state this assumption formally here and refer to it explicitly when used.

Assumption 2. *Unconstrained adulthood: $V_3(a, h) = v_3(Ra + \chi h)$ where*

$$v_\tau(z) = \max_{c_\tau, \dots, c_6} \sum_{j=\tau}^6 \beta^{j-\tau} u(c_j) \text{ subject to } \sum_{j=\tau}^6 R^{\tau-j} c_j = z. \quad (8)$$

Notice $v_\tau(z)$ reflects the maximized discounted lifetime utility for an unconstrained adult with total resources z as of period τ . It is strictly increasing and strictly concave in z given our assumptions on $u(\cdot)$. For an unconstrained adult in period 3, the total value of lifetime wealth is given by $Ra_3 + \chi h_3$, reflecting the value of current assets plus the discounted flow of labor income.

The complementarity of investments across periods plays a central role in determining individual responses to borrowing constraints. In particular, the following dynamic complementarity condition is important for a number of results:

Condition 1. $\frac{f_{12}}{f_1 f_2} > -\frac{v_3''(-RL_2 + \chi h_3)}{v_3'(-RL_2 + \chi h_3)} \chi$.

This condition simplifies nicely with the CES production function given in equation (2) and utility function $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. In this case, Condition 1 cannot hold if $d \leq b$, but this only rules out very strong substitution between early and late investments, since $d > 0$.¹⁰ For the more relevant case of $d > b$, Condition 1 simplifies to

$$\underbrace{\frac{1}{1-b}}_{\text{elast. of sub.}} < \underbrace{\frac{1}{\sigma}}_{\text{CIES}} \cdot \underbrace{\left(1 - \frac{RL_2}{\chi h_3}\right)}_{1 - \frac{\text{maximum debt}}{\text{lifetime income}}} \cdot \left(\frac{d-b}{d(1-b)}\right).$$

As the elasticity of substitution between early and late investments decreases (i.e. investments become more complementary) or the consumption intertemporal elasticity of substitution (CIES) increases (i.e. individuals become less concerned about maintaining smooth consumption profiles), this inequality is more likely to hold. For $b \leq 0$ (complementarity at least as strong as implied by a Cobb-Douglas production function) and $L_2 = 0$, this condition is satisfied whenever the elasticity of substitution between early and late investments is less than the CIES. More generally, when individual preferences for smooth consumption are strong, Condition 1 requires strong complementarity between early and late investments.

We now discuss how income transfers during early and late childhood affect investment decisions over both stages of development. In our two-generation model of Section 2.2, we show that these results also apply to parental income over the first two periods of a child's life. They are, therefore, helpful in interpreting evidence on the impact of family income on educational attainment (i.e. late investments i_2) found in Table 1. As noted above, changes in transfers y_1 and y_2 have no effect on investments for unconstrained individuals. The following proposition shows how constraints at different stages of development determine the responsiveness of investment to changes in financial transfers.

Proposition 2. *Assume no borrowing constraints during adulthood (Assumption 2).*

I. If borrowing constraints only bind in late childhood, then

$$\begin{aligned} (i) \quad & \frac{\partial i_1}{\partial y_1} = R \frac{\partial i_1}{\partial y_2} = \frac{\partial i_1}{\partial (R^{-1} y_2)} > 0; \\ (ii) \quad & \frac{\partial i_2}{\partial y_1} = R \frac{\partial i_2}{\partial y_2} = \frac{\partial i_2}{\partial (R^{-1} y_2)} > 0; \\ (iii) \quad & \frac{\partial h_3}{\partial y_1} = R \frac{\partial h_3}{\partial y_2} = \frac{\partial h_3}{\partial (R^{-1} y_2)} > 0. \end{aligned}$$

II. If borrowing constraints only bind in early childhood, then

$$(i) \quad \frac{\partial i_1}{\partial y_1} > 0; \text{ and } \frac{\partial i_1}{\partial y_2} < 0;$$

¹⁰When $d \leq b$, $f_{12} \leq 0$.

- (ii) $\frac{\partial i_2}{\partial y_1} > 0 \iff f_{12} > 0$; and $\frac{\partial i_2}{\partial y_2} < 0 \iff f_{12} > 0$;
 (iii) $\frac{\partial h_3}{\partial y_1} > 0$; and $\frac{\partial h_3}{\partial y_2} < 0$.

III. If borrowing constraints bind during both early and late childhood, then

- (i) $\frac{\partial i_1}{\partial y_1} > 0$; and $\frac{\partial i_1}{\partial y_2} > 0 \iff$ Condition 1 holds;
 (ii) $\frac{\partial i_2}{\partial y_1} > 0 \iff$ Condition 1 holds; and $\frac{\partial i_2}{\partial y_2} > 0$;
 (iii) $\frac{\partial h_3}{\partial y_1} > 0$ and $\frac{\partial h_3}{\partial y_2} > 0$.

Proof: See Appendix B.

We highlight two key implications of this proposition and relate them to the estimates in Table 1. First, note that if early borrowing constraints are non-binding, investments depend only on the discounted present value of financial transfers $y_1 + R^{-1}y_2$, not the timing of transfers (conditional on discounting y_2). When no constraints bind, investments are independent of all transfers, while investments depend positively on the discounted present value of transfers when only the late constraint binds. Second, when the early constraint binds, both the timing of income and the extent of dynamic complementarity are important factors determining the response of investments to changes in transfers. For example, early investments are always increasing in y_1 ; however, they are decreasing in y_2 if only the early constraint binds since a late transfer exacerbates this constraint. When only the early constraint binds, the impacts of transfers on late investment depend entirely on their effects on early investments and whether early investments raise or lower the marginal return to late investments. When constraints are binding throughout childhood, transfers in any period increase investments in both periods if and only if there is sufficient dynamic complementarity.

Table 1 shows that educational attainment – late investment in the context of our model – is significantly increasing in early income and unaffected by or marginally increasing in late income. Based on Proposition 2, the asymmetry in effects of early vs. (discounted) late income implies that early borrowing constraints are binding. The fact that late investments are increasing in early income suggests that early and late investments must be sufficiently complementary. Altogether, the results in Table 1 are most consistent with binding early and late constraints and substantial dynamic complementarity (case III of Proposition 2).¹¹

¹¹Introducing uncertainty in y_2 does not alter investment behavior in the absence of borrowing constraints. If late borrowing constraints may bind for some realizations of y_2 , i_2 will be increasing in y_2 for those realizations. With sufficient complementarity, this also tends to cause i_2 to increase with y_1 . If only early borrowing constraints bind, then i_2 will be increasing in y_1 if and only if $f_{12} > 0$. To the extent that y_2 is completely unpredictable, it will have no effects on i_1 or i_2 in this case. More generally, when only early borrowing constraints bind and y_2 is

We next analyze the effects of borrowing constraints on human capital investments. First, consider relaxing constraints on older children.

Proposition 3. *Assume that borrowing constraints bind during late childhood (i.e. $a_3 = -L_2$) but that there are no future borrowing constraints (Assumption 2).*

(i) *If the early borrowing constraint does not bind (i.e. $a_2 > -L_1$), then: $\frac{\partial i_1}{\partial L_2} > 0$, $\frac{\partial i_2}{\partial L_2} \in (0, 1)$, and $\frac{\partial h_3}{\partial L_2} > 0$.*

(ii) *If the early borrowing constraint also binds (i.e. $a_2 = -L_1$), then: $\frac{\partial i_1}{\partial L_2} > 0$ if Condition 1 holds; $\frac{\partial i_2}{\partial L_2} \in (0, 1)$; and $\frac{\partial h_3}{\partial L_2} > 0$.*

Proof: See Appendix B.

Relaxing borrowing constraints during late childhood unambiguously increases late investments. If early constraints are non-binding or if early and late investments are sufficiently complementary, then any increase in late investments encourages additional early investments as well. Even in the case of strong intertemporal substitutability when early investments may decline, individuals acquire more human capital h_3 when the late constraint is relaxed.

Next, consider relaxing the constraint on young children.

Proposition 4. *Assume that borrowing constraints bind during early childhood (i.e. $a_2 = -L_1$), but there are no borrowing constraints during adulthood (Assumption 2).*

(i) *If the late borrowing constraint does not bind, then $\frac{\partial i_1}{\partial L_1} \in (0, 1)$; $\frac{\partial i_2}{\partial L_1} > 0 \iff f_{12} > 0$; and $\frac{\partial h_3}{\partial L_1} > 0$.*

(ii) *If the late borrowing constraint also binds (i.e. $a_3 = -L_2$) and Condition 1 does not hold, then $\frac{\partial i_1}{\partial L_1} > 0$ and $\frac{\partial i_2}{\partial L_1} < 0$.*

Proof: See Appendix B.

When individuals are only constrained during early childhood, relaxing that constraint leads to an increase in early investment, which encourages late investment as long as the marginal productivity of i_2 is increasing in i_1 .

When children are constrained in both periods, relaxing the early constraint effectively shifts resources from late to early childhood. If early and late investments are very complementary,

partially unknown at the time early investments are made, late investments should be increasing in y_1 (assuming $f_{12} > 0$) and only weakly but positively related to y_2 . Unobserved heterogeneity in ability θ can also lead to a positive correlation between i_2 and both early and late income if ability is related to parental income; however, there is little reason to think that ability should be more correlated with early income than late income.

they will both tend to move in the same direction. In most cases, investments will increase; however, it is possible that investments could actually *decrease* in both periods. Intuitively, if late investments are very productive, then relaxing the early borrowing constraint can ‘starve’ those investments. By contrast, if investments are sufficiently substitutable over time, shifting resources from late to early childhood by relaxing the early constraint causes investment to shift from the late to the early period as well.

Finally, we show how policies targeted to adult ages can affect investments at earlier ages due to the forward-looking nature of investment decisions. These results are important in our intergenerational framework when we consider the consequences of permanent policy changes that affect the resources or constraints faced by young parents. These policies have two distinct effects on child investment behavior in equilibrium. First, they affect young children via their impact on parental transfers. Second, forward-looking children adjust their investment behavior in response to changes in future opportunities when they become adults.

Transfers and loans for constrained young adults affect the marginal returns to early and late investment. As seen in equations (6) and (7), these marginal returns are increasing in the marginal utility of consumption at adult ages and decreasing in consumption levels. Consider a financial transfer y_3 to adults in period 3 that are borrowing constrained in that period and all earlier periods. By increasing adult consumption, this income transfer reduces the marginal return on investments (both early and late) and unambiguously lowers investment. An equivalent increase in borrowing opportunities L_3 also raises period 3 consumption but lowers subsequent consumption as the loan is repaid. Thus, the effects on investment are weaker (or even positive). These results are summarized in the next proposition.

Proposition 5. *Assume that borrowing constraints bind throughout childhood (i.e. $a_2 = -L_1$ and $a_3 = -L_2$) and in period 3 (i.e. $a_4 = -L_3$), but there are no borrowing constraints after period 3. Let y_3 reflect an income transfer received in period 3. Then, $\frac{\partial i_j}{\partial y_3} < 0$ and $\frac{\partial i_j}{\partial L_3} > \frac{\partial i_j}{\partial y_3}$ for $j = 1, 2$.*

Proof: See Appendix B.

Suppose that an individual is constrained throughout childhood and investments are sufficiently complementary. Proposition 2 implies that individuals increase both early and late investments if they receive a transfer *while investing* in their human capital. We refer to this as the “current effect” of a transfer. If instead, the individual expects to receive a transfer *while earning the returns* on his investments, Proposition 5 implies that he will respond by reducing

investments. We refer to this effect as the “future effect” of a transfer. A similar dichotomy can be applied to other policies (e.g. increased borrowing opportunities for young adults). Considered together, the “current” and “future” effects of policy highlight the importance of quantitative work in an intergenerational context when children are affected both directly and indirectly by policies targeted at adults. Children are directly impacted, since they will become adults themselves one day. They are also indirectly impacted through changes in parental transfers. Since the “current” and “future” effects can push investment in opposite directions, it is unclear *ex ante*, whether income transfers or expansions in lending opportunities will encourage or discourage investment when the policies are operative over the entire lifecycle.

2.2 A Two-Generation Problem

We now extend the analysis to a two-generation family decision problem in which parents choose their own consumption and how much to transfer to their children for their investment and consumption. As we show, this problem can conveniently be mapped into the lifecycle problem described above.

Assume that both parents and children live for six periods, working in periods 3-5. Given our focus on investments in children, we concentrate on periods when children are alive, beginning with period 3 for parents. Young children are assumed to have no assets of their own and cannot borrow or save, while older children can borrow/save. Parents transfer resources during both investment periods of the child (i.e. periods 1 and 2 of the child, periods 3 and 4 of the parents). Borrowing constraints may limit intertemporal allocations as above. We abstract from complicated dynamic strategic interactions between parents and children by assuming no further interactions between parents and children following the investment periods.¹²

Parents are altruistic towards their children, valuing utility from their own consumption as well as the utility of their children. We use prime superscripts to denote variables for the child, while subscripts continue to refer to age/period. For parents with incomes (Y_3, Y_4, Y_5) and ‘altruism’ parameter $\rho > 0$, the family decision problem (beginning at period 1 of child, 3 of parent) can be

¹²Brown, Scholz and Seshadri (2012) show that with multiple periods of intergenerational transfers and the inability of parents to commit to future transfers, children have an incentive to under-invest early on in order to extract more resources from parents at later ages. These problems disappear with full commitment or the capacity for parents to effectively choose investment levels for their children (e.g. through tied transfers). Our problem is equivalent to both of these cases.

written as follows:¹³

$$\max_{c'_1, c'_2, c_3, c_4, c_5, c_6, a'_3, i'_1, i'_2} \left\{ \sum_{j=3}^6 \beta^{j-3} u(c_j) + \rho [u(c'_1) + \beta u(c'_2) + \beta^2 V_3(a'_3, f(i'_1, i'_2, \theta'))] \right\}$$

subject to

$$\begin{aligned} c'_1 + c_3 &= Y_3 - a_4 - i'_1 \\ c'_2 + c_4 &= Ra_4 + Y_4 - (a'_3 + a_5) - i'_2 \\ c_5 &= Ra_5 + Y_5 - a_6 \\ a_4 &\geq -L_3 \\ a_5 &\geq -L_4 \\ a'_3 &\geq -L_2 \end{aligned}$$

and $c_6 = Ra_6$. The child's continuation value $V_3(a'_3, h'_3)$ is defined earlier in Section 2.1.2, and parental transfers are given by $y_j = i'_j + c'_j$ for $j = 1, 2$.

2.2.1 A Two-Stage Solution to the Intergenerational Problem

If we assume that (i) children are unconstrained after their investment periods (Assumption 2) and (ii) borrowing when the child is old is only restricted by a single family borrowing constraint $A_3 \equiv a'_3 + a_5 \geq -(L_2 + L_4)$, then it is possible to write this family problem as a two-stage problem. Investment and total family resources over time are determined in the first stage (analogous to the lifecycle problem in Section 2.1), while individual consumption allocations are determined in the second stage. By considering a single borrowing constraint for families with old children (rather than separate constraints for the old child and his parent), we focus attention on cases when both old children and their parents are constrained or both are unconstrained.¹⁴

Defining the functions

$$W(C) = \max_z \{v_5(C - z) + \rho v_3(z)\} \quad (9)$$

and

$$U(C) = \max_x \{u(C - x) + \rho u(x)\}, \quad (10)$$

¹³Note that Y_3 may reflect parental income in period 3 and the value of any assets/debts they carry into that period.

¹⁴Whether or not the borrowing constraint for the parents of old children binds is irrelevant for investment. If only this constraint binds, then investments will be at the unconstrained optimal levels and will be independent of parental income. When the constraint on old children binds, the problem is very similar to the case described in this section where constraints bind for both old children and their parents. Results for i_1 and i_2 are qualitatively the same, except that changes in L_4 would have no effect since it is non-binding.

the family problem can be solved in two stages.¹⁵

Stage 1: Determine intertemporal total family resource allocations (C_1, C_2, C_3) and investments (i'_1, i'_2) :

$$\max_{C_1, C_2, C_3, i'_1, i'_2} \{U(C_1) + \beta U(C_2) + \beta^2 W(C_3)\}$$

subject to

$$\begin{aligned} C_1 &= Y_3 - a_4 - i'_1 \\ C_2 &= Ra_4 + Y_4 - A_3 - i'_2 \\ C_3 &= RA_3 + Y_5 + \chi f(i'_1, i'_2, \theta') \\ a_4 &\geq -L_3 \\ A_3 &\geq -(L_2 + L_4). \end{aligned}$$

Stage 2: (a) Determine the optimal allocation of total family consumption (C_1, C_2) to the child (c'_1, c'_2) and parent (c_3, c_4) for both investment periods based on equation (10). (b) Allocate post-investment period assets to children (a'_3) and parents (a_5) by solving equation (9). (c) Solve for child consumption allocations (c'_3, c'_4, c'_5, c'_6) given (i'_1, i'_2, a'_3) and parental consumption allocations (c'_5, c'_6) given a_5 by solving equation (8).

2.2.2 Investment Behavior and Endogenous Parental Transfers

We focus on the Stage 1 problem, since it determines investments. This problem is the same as the lifecycle problem of Section 2.1, replacing child consumption, transfers and assets with family consumption, income and assets, $u(\cdot)$ with $U(\cdot)$, and $v_3(\cdot)$ with $W(\cdot)$.¹⁶ As such, investments i'_1 and i'_2 have the same qualitative properties here as in the lifecycle problem (with a slight modification to Condition 1).¹⁷ Propositions 2-4 can be applied to an intergenerational context with comparative statics results for L_1 applying to changes in L_3 here, results for L_2 applying equally to changes in L_2 or L_4 here (as long as both old children and their parents are constrained), and results for y_1 and y_2 applying to changes in Y_3 and Y_4 , respectively.¹⁸

¹⁵The functions $v_5(\cdot)$ and $v_3(\cdot)$ are defined by equation (8).

¹⁶Since $u(\cdot)$ is strictly increasing and strictly concave, the functions $v_j(\cdot)$, $U(\cdot)$, and $W(\cdot)$ are also strictly increasing and strictly concave.

¹⁷Condition 1 must be slightly modified to Condition 1': $\frac{f_{12}}{f_1 f_2} > -\frac{W''(-R(L_2+L_4)+Y_5+\chi h'_3)}{W'(-R(L_2+L_4)+Y_5+\chi h'_3)} \chi$. In the special case with $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ and CES human capital production function given by equation (2), Condition 1' simplifies to $\frac{1}{1-b} < \frac{1}{\sigma} \left(1 - \frac{R(L_2+L_4)}{\chi h'_3} + \frac{Y_5}{\chi h'_3}\right) \left(\frac{d-b}{d(1-b)}\right)$ if $d > b$. For $Y_5 \geq 0$, if Condition 1 is satisfied, then Condition 1' is satisfied.

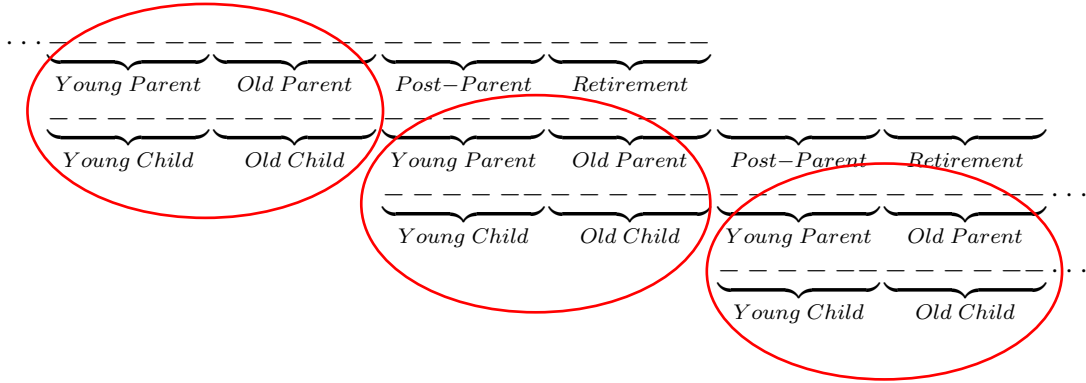
¹⁸Results analogous to Propositions 1 and 5 also apply; however, they require slightly more involved (and tedious) analyses.

Altogether, the results of Section 2.1 generalize naturally to an intergenerational context with respect to changes in borrowing limits or total family resources at different stages of the child's life. Proposition 2 shows that the timing of parental income and the extent of dynamic complementarity are important for investments in children when young parents are borrowing constrained. Furthermore, sufficient dynamic complementarity implies that both early and late investments tend to increase when constraints on parents or children are relaxed during either investment period (Propositions 3 and 4).

Propositions 2-4 are also useful for understanding how parental transfers respond to changes in borrowing limits or parental income. Without constraints on parental transfers, the consumption of parents and children positively co-move within any given period in response to policy or income changes. Increasing the income or borrowing limits for constrained old parents leads to contemporaneous increases in the parent's and child's consumption, as well as late investments and parental transfers. With sufficient dynamic complementarity in investments or if early family borrowing constraints are non-binding (i.e. $a_4 > -L_3$), parents will also increase early transfers and investments. By contrast, when early constraints are binding and investments are sufficiently substitutable over time, parents may respond to an (anticipated) increase in L_4 or Y_4 by transferring less to and investing less in their children when they are young. When increasing the borrowing limits or incomes of constrained young parents, transfers to young children increase as long as early investments increase (see Proposition 4); however, parents may reduce investments in and transfers to children at older ages if investments are quite substitutable. Altogether, we should generally expect transfers to co-move with investments in response to changes in parental resources. By contrast, a policy that relaxes borrowing constraints on older children, L_2 , would increase late investments i_2 but lead to a reduction in parental transfers as children and parents alike benefit from the additional resources. Parental transfers adjust to spread the gains (or losses) of a policy change throughout the family.

In the next section, we consider the fact that families may not always be able to freely spread resources across generations. In particular, parental transfers may be constrained to be non-negative if parents cannot easily extract resources from their children. This limits the ability of some families to spread the gains from policies directed at children (and future generations) to parents.

Diagram 1: Generations of a Dynasty



3 A Dynastic Framework

To understand the long-run effects of policy, we need to consider the evolution of assets and human capital over time. This requires an intergenerational dynastic framework. Assuming that children become parents themselves, valuing their own children’s utility, and that those children have children of their own, the problem becomes dynastic. Parents transfer resources to their children, who grow up, become parents of their own children, and transfer resources to them in an analogous fashion. In a dynastic framework, permanent policy changes not only affect human capital investments via their impact on opportunities today but also through their effect on opportunities in the future. As demonstrated in the previous section, current and future effects of new loans or transfers can move investment in opposite directions, requiring quantitative work to determine the sign of the total impact of a permanent policy.

As above, we assume that people live through six periods in their life. We refer to these periods as young and old childhood, young and old parenthood, post-parenthood, and retirement. The lifecycle of different generations in a dynasty is given by Diagram 1.

We assume that young children cannot borrow or save themselves (i.e. $a_2 = 0$), and that young parents make investment and consumption decisions for their young children. Although old children make investment decisions, we assume that it is their last period of financial interaction with their parents, so there is no scope for strategic behavior. It is, therefore possible to write the entire family problem from the point of view of parents. We assume that the ability of a child depends on that of the parent following a simple Markov process. Once a child is born, the parent’s ability becomes irrelevant. However, the child’s ability affects parental decisions, because it impacts the child’s ability to accumulate human capital, and it plays a role in determining the

future ability of the grandchild.

To make the problem more realistic and suitable for quantitative analysis, we extend the family problem in three important ways: We introduce earnings shocks, allow borrowing constraints to depend on human capital levels, and incorporate public spending on education and investment subsidies. We also constrain families so that parents must make non-negative transfers to their children. Due to the nature of assumed earnings shocks, it is useful to allow for human capital-specific growth rates, $\Gamma_4(h_3)$ and $\Gamma_5(h_4)$, as discussed further below.

To account for variation in earnings within education classes, we introduce period j -specific earnings shocks ϵ_j for young and old parents, so

$$W(h_j, \epsilon_j) = wh_j + \epsilon_j, \quad \text{for } j = 3, 4, \quad (11)$$

$$W(h_5) = wh_5. \quad (12)$$

These shocks are distributed such that earnings are always non-negative. To simplify computation, we abstract from shocks in period 5, when parents and children are no longer economically linked.

We allow borrowing constraints to depend on the future human capital of an individual to account for the possibility that higher education increases borrowing opportunities. This is both theoretically and empirically attractive for reasons discussed in Lochner and Monge-Naranjo (2011).¹⁹ We assume that borrowing limits depend on the lowest possible discounted value of future earnings, since that determines the amount a person can credibly commit to re-pay under any circumstances (Ayagari 1994). Letting $\underline{\epsilon}_j = \min\{\epsilon_j\}$ represent the lowest possible earnings shock in period j , we assume the following limits on borrowing:

$$\begin{aligned} L_2(h_3) &= \gamma[R^{-1}(wh_3 + \underline{\epsilon}_3) + R^{-2}(w\Gamma_4(h_3)h_3 + \underline{\epsilon}_4) + R^{-3}w\Gamma_4(h_3)\Gamma_5(h_3 + \Gamma_4(h_3))h_3], \\ L_3(h_3) &= \gamma[R^{-1}(\Gamma_4(h_3)h_3 + \underline{\epsilon}_4) + R^{-2}w\Gamma_4(h_3)\Gamma_5(h_3 + \Gamma_4(h_3))h_3], \\ L_4(h_3) &= \gamma R^{-1}w\Gamma_4(h_3)\Gamma_5(h_3 + \Gamma_4(h_3))h_3, \end{aligned}$$

where $\gamma \in [0, 1]$. Intuitively, the parameter γ reflects the efficiency of credit markets, since γ near zero implies that no borrowing is allowed while γ near one implies that individuals can borrow fully against guaranteed future earnings.²⁰

¹⁹Lochner and Monge-Naranjo (2011) argue that more skilled individuals can commit to re-pay higher debts, explaining why private lenders offer them more credit. Furthermore, the federal student loan system explicitly links loan amounts to post-secondary enrollment and the level of schooling attended.

²⁰Of course, γ could vary across stages of the lifecycle; however, we do not expect to be able to separately calibrate three different γ values given our data.

We incorporate freely provided public investment in each period of childhood, denoting these public investments p_1 and p_2 . Thus, total investment in period j is given by $p_j + i_j$. We further assume that private spending on investment in each period is subsidized at rates s_1 and s_2 . Below, we consider the effects of policies that adjust these publicly provided investment levels and subsidy rates.

Letting prime superscripts denote the child's variables, the problem facing a young parent with a young child is given by:

$$V_3(h_3, \epsilon_3, a_3, \theta') = \max_{c_3, a_4, c'_1, i'_1} \{u(c_3) + \rho u(c'_1) + \beta E_{\epsilon_4} V_4(h_4, \epsilon_4, a_4, h'_2, \theta')\}$$

subject to

$$\begin{aligned} a_4 &= Ra_3 + W(h_3, \epsilon_3) - c_3 - i'_1(1 - s_1) - c'_1, \\ a_4 &\geq -L_3(h_3), \\ h'_2 &= p_1 + i'_1, \\ h_4 &= \Gamma_4(h_3)h_3, \end{aligned}$$

$c_3 \geq 0$, $c'_1 \geq 0$ and $i'_1 \geq 0$. Since young children are not allowed to borrow on their own, the only constraint on borrowing is that imposed on young parents. The expectation of V_4 is taken over the earnings shock the young parent will receive as an old parent.

The problem facing an old parent with an old child is given by:

$$V_4(h_4, \epsilon_4, a_4, h'_2, \theta') = \max_{c_4, a_5, c'_2, i'_2, a'_3} \left\{ u(c_4) + \beta V_5(h_5, a_5) + \rho \left[u(c'_2) + \beta E_{\theta'', \epsilon'_3} (V_3(h'_3, \epsilon'_3, a'_3, \theta'' | \theta')) \right] \right\}$$

subject to

$$\begin{aligned} a'_3 + a_5 &= Ra_4 + W(h_4, \epsilon_4) + W_2 - c_4 - c'_2 - i'_2(1 - s_2), \\ a'_3 &\geq W_2 - c'_2 - i'_2(1 - s_2), \\ a_5 &\geq -L_4(h_4), \\ a'_3 &\geq -L_2(h'_3), \\ h'_3 &= f(h'_2, p_2 + i'_2, \theta'), \\ h_5 &= \Gamma_5(h_4)h_4, \end{aligned}$$

$c_4 \geq 0$, $c'_2 \geq 0$ and $i'_2 \geq 0$. The second constraint ensures that parental transfers are non-negative. Both the old parent and the old child face constraints on their borrowing as shown in the third and fourth constraints. The expectation of V_3 is taken over the earnings shock the old child will

receive as a young parent and the ability level of the future grandchild, θ'' , conditional on the ability of the child, θ' .

The problem facing a post-parent with no child at home is a standard lifecycle consumption/savings problem:

$$V_5(h_5, a_5) = \max_{a_6} \{u(Ra_5 + W(h_5) - a_6) + \beta u(Ra_6)\}.$$

This is easily solved analytically (given our assumed utility function below) and incorporated into the old parent's problem.

4 An Empirically Based Quantitative Analysis

In our computational analysis, we assume a finite number of investment and ability levels but a continuum of asset levels. The finite investment and ability grids imply a finite number of human capital levels.

We assume a CES human capital production function, as in equation (2), and a CIES utility function, given by

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \quad \sigma \geq 0.$$

4.1 Calibration

We use data from the March Current Population Surveys (CPS), National Longitudinal Survey of Youth 1979 Cohort (NLSY79), and CNLSY to calibrate our model to the U.S. economy.²¹ The six model periods are mapped into ages 0-11, 12-23, 24-35, 36-47, 48-59, and 60-71. We consider four values of i_2 associated with different observed schooling levels: high school dropouts (less than 12 years of completed schooling), high school graduates (exactly 12 years of completed schooling), some college (13-15 years of completed schooling), and college graduates (16 or more years of completed schooling). An annual interest rate of $r = 0.05$ is assumed throughout, so $R = (1 + r)^{12} = 1.7959$. We assume $\beta = R^{-1}$. All earnings are in 2008 dollars (deflated by the CPI-U). We normalize $w = 1$, so human capital is measured in 2008 dollars per year. Finally, we choose the preference parameter $\sigma = 2$, which implies an intertemporal elasticity of substitution for consumption of 0.5. This is consistent with estimates in the literature (Browning, Hansen and Heckman 1999).

We assume that income shocks are *iid* log normally distributed.

²¹In this analysis, we use NLSY79 and Children of the NLSY79 collected through 2010 and CPS data from 2006.

Assumption 3. $\epsilon_j \sim \log N(m, s^2)$, for $j = 3, 4$.

We also assume a two-state Markov process for ability.

Assumption 4. $\theta \in \{\theta_1, \theta_2\}$ with $Pr(\theta_j = \theta'_j) = \pi_j$ for $j = 1, 2$.

Along with using data to guide our choice for the investment grids, the following parameters must be determined empirically: potential earnings in school (W_2), post-school income shock distributions (m, s), human capital growth rates (Γ_4, Γ_5), the human capital production function, (a, b, d), the Markov process for ability ($\theta_1, \theta_2, \pi_1, \pi_2$), parental altruism towards children (ρ), and the debt constraints (γ). We first discuss parameters that are chosen to match data outside of the model and then outline the calibration process for all remaining parameters.

4.1.1 Second Period Earnings and Investment Costs

We directly estimate potential earnings for ages 12-23, W_2 , using the CNLSY. We also estimate foregone earnings from these data, which are combined with direct educational expenditures by schooling level (from the Digest of Education Statistics 2008) to determine second period investment amounts, i_2 .

Using the random sample of the CNLSY, we estimate the discounted present value of average earnings for high school dropouts over ages 16-23.²² Dividing the average annual discounted income over this period by 12 yields an annualized potential income measure of $W_2 = 11,187$. This also reflects the total amount of foregone earnings for individuals in our highest schooling category: college completion. Foregone earnings for ‘high school graduates’ (those with ‘some college’) are given by the discounted present value of earnings for dropouts over ages 16-18 (16-20), dividing by 12 to annualize the amounts. We assume no foregone earnings for high school dropouts, since individuals cannot typically work before age 16.

We distinguish between total investment amounts and the amount privately paid by individuals themselves, since education is heavily subsidized in the U.S. Total investment amounts include foregone earnings and total public and private education expenditures. Consider first the investments made by old children ages 12-23. To calculate expenditures associated with grades 6-12, we use average expenditure per pupil for all public elementary and secondary schools. For the schooling category ‘some college’, we add two years of current-fund expenditures per student at all post-secondary institutions to the costs of high school. For ‘college graduates’, we add five years of current-fund expenditures per student at four-year post-secondary institutions to

²²A discount rate of $r = 0.05$ was used to discount earnings to age 18.

the costs of high school.²³ Combining foregone earnings with direct expenditures and dividing by 12 to annualize the amounts, we obtain total investment amounts ($p_2 + i_2$) of \$3,563, \$5,912, \$13,369, and \$29,805 for high school dropouts, high school graduates, some college, and college graduates, respectively.

Foregone earnings are borne by individuals, but we assume that primary and secondary schooling is otherwise publicly provided at no private cost. Since dropping out of high school entails no foregone earnings or other private costs, we set $p_2 = 3,563$. This amount is subtracted from total investment amounts to obtain private i_2 investments (inclusive of marginal subsidies) of \$0, \$2,260, \$9,374, and \$25,082 for high school dropouts, high school graduates, some college, and college graduates, respectively. High school graduates only pay foregone earnings (roughly two-fifths of their total investment), while college students pay both foregone earnings and a share of direct costs, which are heavily subsidized. Dividing revenue from tuition and fees by total revenue for all degree-granting post-secondary institutions in 1995-96 suggests that student tuition payments account for only 28% of college revenues. Altogether, individuals pay roughly 40% of their total marginal costs of finishing high school and about 55-60% of the total marginal costs of college. Striking a balance between these figures, we set the second period marginal investment subsidy rate to $s_2 = 0.5$.

Since there are no foregone earnings for young children, we take the annualized value of \$3,563 as the minimum period one investment.²⁴ Assuming this level of investment is completely subsidized for young children, we set $p_1 = 3,563$ and consider a grid for period one private investments i_1 ranging from zero to \$15,000, an amount very few parents wish to invest.²⁵ We set $s_1 = 0$, since private investments by parents in their young children are not typically subsidized in the U.S.

4.1.2 Earnings Growth Rates

We set $\Gamma_4(h_3)$ and $\Gamma_5(h_4)$ to match growth in average income levels, $E[W(h_4, \epsilon_4)]/E[W(h_3, \epsilon_3)] = 1.478$ and $E[W(h_5)]/E[W(h_4, \epsilon_4)] = 1.077$, using data from the NLSY79 and 2006 March CPS,

²³All schooling expenditure figures are taken from the *Digest of Education Statistics* (2008) and are adjusted to year 2008 dollars using the CPI-U. Primary and secondary expenditures (\$8,552 per year) are based on averages over the 1990-91 to 1994-95 period (Table 181). Post-secondary expenditures are based on all degree-granting institutions in 1995-96 (Table 360). Annual expenditures per student are \$25,902 at two-year institutions and \$32,712 at four-year institutions.

²⁴This corresponds to the sum of average annual expenditures per pupil of \$8,552 for grades 1-5 divided by 12 (to annualize the amount).

²⁵Our grid for early investments includes seven points from 0 to 15,000, with equally spaced increments of 2,500. Modest changes in the number of grid points or the upper limit produce very similar results.

respectively.²⁶ This approach assumes that individuals face the same expected growth in earnings regardless of their human capital level. Because earnings shocks ϵ_3 and ϵ_4 are non-negative, different human capital growth rates are needed for each level of human capital to produce the same growth rates in expected earnings.

4.1.3 Calibrating other Parameters Using Simulated Method of Moments

The remaining parameters are calibrated by simulating the model and comparing the resulting allocations with those observed in the data. In particular, we determine parameters of the earnings shock distribution (m, s) , the human capital production function (a, b, d) , parental altruism towards their children (ρ) , the ability distribution $(\theta_1, \theta_2, \pi_1, \pi_2)$, and the debt constraint parameter (γ) . We use a simulated method of moments procedure, which chooses parameters to best fit moments for educational and earnings dynamics using data from the CNLSY. This step requires fully solving the dynastic fixed point problem of Section 3 in steady state, simulating a number of conditional moment conditions, and comparing those moments with their empirical counterparts. In particular, we fit moments related to (i) the education distribution, (ii) the distribution of annual earnings for men ages 24-35 and 36-47 in the NLSY79, (iii) child schooling levels conditional on parental income and maternal schooling, and (iv) child wages at ages 24-35 conditional on their own educational attainment, maternal schooling, and parental income levels (when the child is ages 0-11). Appendix C provides greater detail on the calibration.

Table 2 shows the distribution of educational attainment for our NLSY calibration sample along with the calibrated steady state distribution produced by our model. While the model matches high school completion and college attendance rates quite well, it under-predicts college completion rates.

To help identify the earnings shock distribution and human capital levels, we match the mean and standard deviation of earnings from the random sample of men in the NLSY79 ages 24-35 and 36-47 (discounted to ages 30 and 42, respectively, using a 5% interest rate). Table 3 reports these statistics in the NLSY79 and the calibrated steady state for our model. In all cases, except the standard deviation of wages for old parents, the model matches the data quite well.

Child educational attainment (i'_2 in our model) should depend on early investments, child ability levels, second period parental income, and parental assets. Not all of these are observed in the NLSY data; however, we can simulate our model to fit $Pr(i'_2|Y_3, Y_4, i_2)$ where Y_3 and Y_4 reflect

²⁶In both cases, we use data for men deflated to year 2008 dollars. We discount within period earnings to ages 30, 42, and 54 using a 5% interest rate. We drop observations for respondents with annual earnings less than \$200 or greater than \$275,000 or those with less than 9 years of completed schooling.

total parental earnings in early and late parenthood. In practice, we condition on three categories of parental incomes Y_3 and Y_4 : bottom quartile, second quartile, and top half of the age-specific income distribution. These moments are most useful in identifying the credit constraint parameter γ , the complementarity of early and late investments, parental altruism, and the intergenerational correlation in ability. Proposition 2 shows that the extent to which early vs. late income affect educational attainment helps identify the importance of early borrowing constraints and the degree of dynamic complementarity. Later borrowing constraints and the extent of parental altruism also determine the relationship between parental income and child investments, but not the relative importance of early vs. late income. When any borrowing constraint or the non-negative transfer constraint binds, stronger altruism implies greater investment for any given level of parental income. Altogether, γ , b and ρ can be identified from the relationship between parental incomes and educational attainment as long as some families face binding constraints. The correlation between the educational attainment of children and parents for given income levels is useful for identifying the intergenerational correlation in abilities.

Table 4 reports the child's education distribution by parental education for our model and the NLSY79 data, while Table 5 shows the education distribution by parental income when the child is young and old.²⁷ Educational attainment is strongly increasing in parental education and income. The model slightly over-predicts the importance of parental education for high school completion, while it under-predicts its importance for college attendance and completion. The intergenerational correlation in i_2 investment amounts is 0.33 for our model and 0.32 in the NLSY data.

A few interesting patterns emerge from Table 5 when we simultaneously condition on both early and late parental income. Considering college completion, late income is relatively unimportant conditional on early income, while the reverse is not true. Early income is quite important for college even after conditioning on later income. These patterns are clearly evident in both the NLSY data and our model. When looking at high school completion, the model suggests that both early and late income are important (conditional on the other), while the NLSY suggests a more modest role for late parental income. Overall, the model replicates key features of the relationship between child educational attainment and parental education and income. As noted earlier, these relationships are central for identifying the human capital production technology and borrowing limits.

²⁷See Appendix C for the full set of moments (i.e. child education by parental education and income at young and old ages) used in calibration along with their steady state counterparts.

Wages during early adulthood depend on human capital levels, h'_3 . Conditional on schooling i'_2 , human capital and wages should be increasing in early childhood investments and the child's raw ability. Thus, we also fit $E(W'_3|Y_3, i_2, i'_2)$, using wage income for youth ages 24-35 as our measure of W_3 and three income categories for Y_3 as described above.²⁸ These moments are helpful in identifying parameters of the human capital production function, including the distribution of ability and its intergenerational correlation. Appendix C reports these moments in the data and our calibrated steady state.

Our calibrated parameter values are reported in Table 6. A value of $b = -1.1$ implies an elasticity of substitution between early investments and late investments of 0.48 – strong dynamic complementarity similar to that estimated by Cunha, Heckman and Schennach (2010). The model implies a similar weight on early and late investments, with a near one-half. Values for θ suggest that high ability individuals are roughly 2.5 times as productive as their low ability counterparts. Our calibration implies modest intergenerational persistence in ability: high ability parents have a high ability child 76% of the time, while low ability parents only have a high ability child 41% of the time. The calibrated value of $\rho = 0.67$ is far from ‘pure altruism’, but it still implies that considerable value is placed on children and grandchildren. Finally, our calibrated value for γ implies that individuals can only borrow up to 45% of their minimal discounted lifetime earnings at any age.²⁹ Thus, credit limits are far more stringent than the ‘present value’ limit of Aiyagari (1994).

4.2 Additional Features of the Baseline Steady State

Table 7 shows how average early (i_1) and late (i_2) private investment amounts vary with parental education in our benchmark steady state. On average, parents annually invest \$2,121 in their young children and \$7,227 in their older children. (Because of the 50% marginal subsidy on late investments, private investment expenditures for older children are only \$3,864.) Private investments in young (old) children are roughly five (four) times as great for the children of college graduates compared to high school dropouts. Kaushal, Magnuson, and Waldfogel (2011) find

²⁸Given enough data, we could fit $E(W'_3|Y_3, Y_4, i_2, i'_2)$. However, conditioning on Y_4 probably adds little additional identifying variation, since only i'_1 and θ' affect expected wages conditional on i'_2 and these are largely determined by Y_3 and i_2 . Note that we use weekly earnings for our measure of W_3 (due to data availability and the desire to best capture differences in human capital), while we use the distribution of annual earnings for men in helping identify earnings growth and the distribution of shocks (as described above). Since the units for these are quite different, we fit the ratio of $E(W'_3|Y_3, i_2, i'_2)$ for each category of (Y_3, i_2, i'_2) relative to the corresponding average for a baseline group of high school graduates with high school graduate mothers whose early parental income is in the lowest quartile. See Appendix C for additional details.

²⁹This implies average limits for L_2 , L_3 and L_4 of \$14,835, \$19,177, and \$17,528, respectively.

that high school dropout parents spend \$825 per child, annually on educational enrichment, while parents that graduated from college spend \$4,671.³⁰ Aggregating early and late private investment expenditures (i.e. $\frac{i_1+(1-s_2)i_2}{2}$), we obtain very comparable measures: \$1,115 and \$4,873, for high school dropout and college graduate parents, respectively.

Our calibrated steady state suggests that roughly half of all young parents and 12% of all old parents are borrowing at their limit, while no older youth are borrowing constrained. The share of young parents that are borrowing constrained is greater among those who attended (60%) or completed (68%) college relative to those who only finished high school (38%) or who dropped out (51%). This relationship is non-monotonic: high school graduates are less likely to be constrained than high school drop outs and college attendees and graduates. These patterns are consistent with a relatively high demand for credit among young high school dropouts that experience a bad income shock. For them, a low earnings shock is quite costly given already low expected income levels. More educated young parents tend to be constrained for other reasons. First, many already have debt from their own education. Second, more educated parents desire more credit to fund higher levels of investment in their children, since their children are more likely to be of high ability.

The share of old parents that are borrowing constrained is monotonically increasing in educational attainment. Adverse income shocks at older ages increase the demand for credit less than at younger ages (especially for the least educated), since expected income levels are higher due to lifecycle wage growth and retirement is closer. Even though none of the older youth are borrowing constrained, the possibility of binding future borrowing constraints (during parenthood) may affect their current human capital investment behavior.

In our calibrated steady state, roughly 5% of all older parents are ‘transfer’ constrained, transferring zero to their old children. Unlike borrowing constraints, the least educated parents are most likely to be transfer constrained (15% of high school dropouts make zero transfers to their older children while all college graduates make a positive transfer). This relationship is driven by the fact that less-educated parents are more likely to have low income relative to what their children can expect to earn. Most high school dropouts are of low ability; however, 40% of those with low ability will have a high ability child. Without the non-negative transfer constraint, many of these parents would effectively take resources from their children.

³⁰See Table 3 of the online appendix from Kaushal, Magnuson, and Waldfogel (2011). Amounts reported here exclude enrichment spending allocated to parents.

5 Policy Analysis

We next simulate a series of policies to emphasize important economic forces affecting investment in human capital. In particular, we focus on policies that shed light on the interaction between borrowing constraints and investments at different ages. Intergenerational linkages through endogenous parental transfers play a key role in our analysis. First, we consider different loan policies to determine the importance of borrowing constraints at different stages of child development. We differentiate between the short- and long-run effects of increased borrowing, where the latter accounts for changes in human capital and asset distributions through intergenerational linkages. Second, we study fiscally equivalent early and late investment subsidy policies. Comparing these policies demonstrates the strong interaction between dynamic complementarity and early borrowing constraints. We also discuss the quantitative importance of incorporating early investment responses to policies that target subsidies at later ages. Third, we consider the effects of increasing the level of early public investment. This exercise underscores how different policies can target different ends of the education distribution. Lastly, we compare the effects of income transfers with those of loans. Here, we distinguish between “current” and “future” effects of these policies on investment. As noted earlier, policies that change the budget/borrowing constraints for parents not only affect children through parental transfer decisions (“current” effects); they also affect investment decisions by changing the returns to investment when children become parents themselves (“future” effects). This highlights the importance of considering the full effects of lasting policy changes in a dynamic intergenerational environment even if one is only interested in short-term responses.

5.1 Increasing Borrowing Limits

Given the level of complementarity that we find between early and late investments and the fact that borrowing constraints bind for many young parents in our baseline steady state, our analytical results suggest that relaxing early borrowing constraints should lead to increases in investment during both early and late childhood. To investigate this quantitatively, we simulate the ‘short-run’ and ‘long-run’ responses to a permanent \$2,500 increase in the borrowing limit for all young parents (leaving all other borrowing limits unchanged). The effects this has on early and late investments in children and on their average post-school wages are reported in Table 8. By ‘short-run’, we refer to responses of the first generation to be fully affected by the policy change. By ‘long-run’, we refer to decisions in the new steady state many generations later. The former

shows how families respond to the policy, given the distribution of assets and human capital in the baseline steady state, while the latter takes into account the fact that parental asset and human capital distributions change over time in response to expanded borrowing opportunities.

Focusing first on short-run impacts, Table 8 reveals that relaxing borrowing constraints on young parents would lead to sizeable increases in both early and late investments in children. Increases in early investments would be greatest among children from more educated households. This partly reflects the fact that college educated parents are the most likely to be borrowing constrained. It is also due to the fact that more educated parents want additional credit to bolster investment, while constrained high school dropouts appear to desire additional credit primarily to help smooth consumption. Parents with a college degree would increase early investments in their children by 19% on average, while there would be no early investment response among parents that dropped out of high school. Despite the lack of an early investment response among less-educated parents, their children are more likely to complete high school (with negligible average impacts on their final human capital and wage outcomes). Older children are willing to take on more debt to invest in their education, because they know they will be able to borrow more when they become young parents themselves (when they are likely to be constrained). As highlighted in Proposition 1, even if a person is not currently borrowing up to his debt limit, his investment decisions are adversely affected by the possibility of future binding constraints. Among children whose parents attended or completed college, effects on high school completion are small (almost all already complete high school) while there are sizeable increases in the probability of finishing college. The combined effects of increased early and late investment on average wage levels upon labor market entry are as high as 3.6% for the children of college graduates. Average short-run increases in wages among all young workers are about 1.5%.

The right half of Table 8 reports the long-run changes in investment and wages in the new steady state (many generations later). These changes incorporate the fact that many older children borrow more and find themselves in greater debt when they become young parents. While constraints on young parents are less likely to bind, more older parents (especially those with lower education levels) become borrowing and transfer constrained (see Table 9). Asset distributions at all ages shift left. Despite the fact that constrained persons with any given level of assets and human capital are likely to invest more in their children (this is precisely what the short-run effects demonstrate), the long-run shifts in asset distributions lead to lower overall early investment levels. This is most pronounced for children from the least educated families; although, the

16% drop in average i_1 among these families is only about \$100 given their low initial investment levels. Due to dynamic complementarity, these drops in early investment are accompanied by reductions in college completion rates among children of less-educated parents; however, high school completion rates actually increase slightly. By contrast, long-run responses by children of college graduates are more positive for both early and late investments.

These results suggest that relaxing borrowing constraints on young parents can be a double-edged sword in terms of investment in human capital. In the short-run, there are obvious gains in human capital investment among constrained families. Although the increased borrowing opportunities do not directly benefit unconstrained parents, they benefit their children and future generations who may become constrained. Parents take some of the ‘family’ gains by transferring less to their children. While this is good in terms of ‘family’ or ‘dynastic’ welfare, it can saddle future generations with more debt. This debt gets passed on across generations through smaller financial transfers and, in some cases, less human capital investment. In the long-run, asset distributions shift left and investment declines slightly. In general, these forces appear to be most pronounced at the bottom of the education distribution. While one may not typically be concerned about outcomes many generations into the future, we observe long-run-like investment responses for second- and third-generations affected by the policy. These results, therefore, underscore the importance of considering long-run policy impacts along with more immediate effects on current generations. They also highlight the fact that some policies may have important indirect effects on asset accumulation if future generations are affected: a policy may cause current generations to respond even if they themselves are not directly affected by the policy.

Because old children are not borrowing constrained in our baseline steady state, relaxing their borrowing limits has no effect on investment behavior.³¹ Yet, this does not mean that investment decisions for old children are optimal (even conditional on early investment choices), since many of these children will face binding constraints as young and old parents. Still, allowing them to borrow more as old children does nothing to alleviate these future constraints.

Relaxing constraints on older parents has fairly small effects. While this enables parents to smooth their consumption and transfer more wealth to their children, the magnitude of new transfers is small and has little effect on children’s investment behavior. In the long-term, the increase in parental transfers prevents the type of leftward shift in the asset distribution observed with increased borrowing for young parents. As a result, the long-run effects of increasing borrowing

³¹This result is roughly consistent with findings of Keane and Wolpin (2001) and Johnson (2010).

opportunities for older parents are positive and larger than the short-run effects, although they are still quite small. In the long-run, increasing loan limits for older parents by \$2,500 increases average early investment by 1.5%, average late investment by 2.3%, and average wages of young parents by .3%.

5.2 Subsidizing Education

We next study the consequences of increasing subsidy rates for early and late human capital investments. This analysis highlights the implications of dynamic complementarity in investments and borrowing constraints when considering policies targeted to different stages of development.

In comparing the effects of subsidies to early and late investments, we increase s_1 and s_2 so that total expenditures on all education subsidies increase by roughly the same amount. Given the complementarity of early and late human capital investments, subsidizing investments at one age will tend to increase investments at all ages. Because $s_2 > 0$ in the baseline economy, the total cost of subsidizing early investment includes both the direct cost associated with raising s_1 and the indirect cost associated with any increase in subsidized late investments. Since early investments are not subsidized in the baseline economy, an increase in s_2 only entails direct costs for additional outlays on second period investment.³²

Table 10 shows the short- and long-run effects of subsidizing early and late investments on average investments, the percent who graduate from high school and college, and average entry wage outcomes. The first row reports the effects of subsidizing early human capital investment at a rate of 10%. The per capita total cost of this policy is about \$900, with roughly two-thirds of this coming from the increased costs associated with subsidies for late investments. Not surprisingly, there are large increases in early investments in both the short- and the long-run (28% and 34%, respectively). Because investments are so complementary, this policy also increases late investments by roughly 12% in the short-run and 17% in the long-run. Most of the changes in the education distribution come from increases at the upper end. The percent who graduate college increases 30% in the short-run and 40% in the long-run. Changes in high school completion rates are negligible. Average post-school wages increase 3.1% in the short-run and 3.9% in the long-run. Unlike increases in borrowing limits, increased subsidy rates do not produce leftward shifts in asset distributions, since they enable families to invest more in their children without spending much more out-of-pocket.

³²The total per capita cost of increasing s_1 from zero to s'_1 is given by $s'_1 \bar{i}_1(s'_1, s_2) + s_2 [\bar{i}_2(s'_1, s_2) - \bar{i}_2(0, s_2)]$, where $\bar{i}_j(s_1, s_2)$ reflects average investment in period j under subsidy policy (s_1, s_2) . The total cost of increasing subsidies to late investment is $s'_2 \bar{i}_2(0, s'_2) - s_2 \bar{i}_2(0, s_2)$.

We next consider the effects of increasing the subsidy to late investments from 50% to 53% at a cost of roughly \$900 in the new steady state. We begin by discussing the effects of this policy when parents are aware of the higher subsidy rate when their children are young (row two of Table 10). Thus, both early and late investments may respond. We then discuss the short-term impact on families who are unaware of the policy when making early investments in their children, so only late investments respond (row three of Table 10). This effectively measures the short-run effects for families with older children when the policy is first announced and introduced.

The second row of Table 10 shows the effects of increasing s_2 on families who are aware of the program when their children are young. Although this policy costs the same as a 10% subsidy to early investment, it has much weaker effects on human capital accumulation. Early investments increase by only 4% and 7% in the short- and long-run, respectively, compared with 30-40% for the early investment subsidy. Perhaps more surprisingly, increases in average late investments are quite similar to those for an increase in early investment subsidies. (Notably, simulated effects on college attendance rates are consistent with typical estimates of the impacts of tuition and financial aid on college attendance in the U.S.)³³ While late subsidies have weaker impacts on college completion compared to early subsidies, they appear to increase high school graduation rates more. Altogether, these investment responses imply a much smaller increase (1.4% in the short-run and 1.8% in the long-run) in average entry wage rates relative to a policy that subsidizes early investment.

These results underscore the important interaction between credit constraints and the dynamic complementarity of early and late investments in human capital. The fact that many young parents are credit constrained means that they cannot easily finance additional early investments in response to policies targeted to later ages. While unconstrained families increase both early and late investments in response to an increase in s_2 , constrained young parents are limited in how much they can increase investments in their young children. Complementarity implies that if children do not receive adequate early investments, it may not be worth it for parents to make later investments, even if they are heavily subsidized. By contrast, early investment subsidies enable families to increase investments in their young children without having to sacrifice current consumption or borrow more. Those early investments can then be matched with later

³³Our s_2 increase of 0.03 is roughly equivalent to a \$1,300 reduction in annual tuition for the first two years of college. Our simulations suggest that this increases college attendance (i.e. some college or more) by 5-6 percentage points (depending on whether early investments are allowed to adjust). Kane (2006) and Deming and Dynarski (2009) provide recent surveys of the related empirical literature, concluding that a \$1,000 reduction in tuition leads to a 3-5 percentage point increase in college attendance.

investments, when constraints are less stringent.

Row three of Table 10 reports the effects of an increase in s_2 that is announced after early investments have already been made. Looking at the short-term impact of this policy, we see more modest effects on late investment and human capital accumulation, because early investment is held fixed. Overall, average late investment increases about 8.7%, a little more than half the effect observed when early investment is also able to adjust. This, coupled with no change in early investment, produces a much smaller increase in wages (0.6% vs. 1.4% when early investment adjusts). Increases in high school completion rates are quite similar whether or not early investment is able to adjust; yet, effects on college completion are negligible when early investment cannot respond, compared to a 23% increase when it can. In order for college to be productive, substantial early investments are needed. This is less true for high school.

These results demonstrate the importance of accounting for the interaction between early and late investments when considering education policies. Assuming that early investments and skill levels are fixed when analyzing policies that affect high school or college attendance decisions is not innocuous. Due to dynamic complementarity in investment, failing to account for adjustments in early investment not only neglects those responses, but it also leads one to underestimate the policy's true impact on late investments. Together, these imply substantial underestimation of policy effects on human capital and wages (except, of course, for those families with older children at the time of the policy change). Our results suggest that failure to account for early investment responses would cause researchers to underestimate the full impact of post-secondary subsidies on wages by almost 60%.³⁴

5.3 Public Provision of Early Investment

We next discuss the effects of increasing the amount of publicly provided early investment. Conceptually, changes in p_1 and s_1 are quite different. While an increase in the subsidy lowers the price of and encourages investment for all families, an increase in public investments primarily increases investments among those children who initially receive little or no private investment. Among families making sizeable private investments, any increase in public investment largely crowds out private investment. In fact, an increase in public investment is equivalent to an income transfer for any families initially investing more than the increase in p_1 . By contrast, children who

³⁴It is worth noting that these concerns not only apply to structural models of schooling decisions, but they also apply to more standard regression or differences-in-differences estimates of the effect of tuition or financial aid changes on college attendance. These strategies may identify the very short-run effects on older cohorts of college-age children when the policy is implemented, but they are unlikely to identify the medium-term effects on younger or future cohorts.

initially receive very little or no early private investments cannot reduce their private investments in response. For these children, total early investments increase one-for-one with increases in public investments.

We consider a public early investment increase of \$322, equivalent in cost to the early and late subsidies studied earlier.³⁵ On average, this increase crowds out \$116 of early private investment, which is 36% of the added public investment. High school completion rates increase by 12%, and the fraction that attends some college (or more) increases by 25%. Because the policy mainly increases total early investment for those who invest very little to begin with, it has no effect on college completion rates. Average wages increase by 1%, roughly one-third of the response to an increase in early subsidy rates.

It is noteworthy that increasing early public investments (p_1) and early subsidies (s_1) affect educational outcomes at opposite ends of the distribution. A modest increase in p_1 does not raise early investments enough to make college completion worthwhile for those who were investing little to begin with. By contrast, an increase in s_1 encourages those who were already making investments to invest more, pushing many of them across the college completion threshold. Yet, modest early investment subsidies are ineffective at raising high school completion rates, since most dropouts appear to be at a ‘corner’ solution during early childhood, wishing to invest less than is already publicly provided for free. Of course, these are precisely the children whose early investments increase one-for-one with increases in p_1 .

5.4 Income Transfers

Lastly, we investigate the short-run effects of income transfer policies on human capital investment and wages. Table 11 reports the investment effects of a \$2500 income transfer to young parents and, for comparison, the effects of a \$2500 increase in their borrowing limits. The loan policy provides liquidity only, while income transfers generate both liquidity and wealth effects. We distinguish between the short-run effects of these policies if implemented for a single generation (i.e. a “one-time” policy) or if put in place permanently for current and all future generations. One-time parental loan or transfer policies indirectly affect children through parental transfers, consistent with the “current” effects discussed in Section 2.1.3. A permanent introduction of these policies would also directly affect today’s children in the future when they become adults/parents. Thus, permanent policy changes induce both “current” and “future” effects.

According to Table 11, the effects of a permanent transfer policy on both early and late

³⁵The total cost includes the increase in late investment subsidies that result from increases in i_2 .

investments are *smaller* than the effects of a one-time transfer policy. This is consistent with negative “future” effects as discussed in Proposition 5. The opposite is true for a loan policy, implying positive “future” effects of expanded borrowing opportunities. This, too, is consistent with Proposition 5, which shows that the “future” effects are more positive for increased borrowing opportunities than for income transfers when parents are borrowing constrained.

The relative impacts of loan and transfer policies depend on whether we consider permanent vs. one-time policy implementations. Offering \$2500 in financial transfers to a single generation of young parents increases the average human capital and wage levels of their children more than a policy that increases their borrowing limits by the same amount. The opposite is true if these policies were to be put in place permanently. While both permanent loan and income transfer policies targeted to young parents would increase investment and wage levels for today’s children, the loan policy has nearly twice the effects due to the strong negative “future” effects of an income transfer policy.

These results emphasize the importance of taking intergenerational effects into account when evaluating policy. The short-term effect on wages of the one-time transfer policy is 37% greater than if that policy were put in place permanently. By contrast, the short-term wage effects of the one-time loan policy are 67% smaller than its permanent counterpart.

6 Conclusion

We show that family income received at earlier ages of child development improves educational outcomes more than income received at later ages. Our estimates from the CNLSY suggest that a \$10,000 increase in discounted annual income from birth to age 11 would reduce the probability of high school dropout by about 2.5 percentage points and increase college attendance rates by as much as 4.6 percentage points. The same increase in income over ages 12-23 has much smaller and statistically insignificant effects on these educational outcomes. The timing of family income is important, consistent with early borrowing constraints.

Our theoretical analysis of borrowing constraints and multi-period human capital investment establishes the central role played by dynamic complementarity. When investments are sufficiently complementary over the lifecycle, policies that encourage investment in one period tend to raise investment in other periods as well. When early borrowing constraints bind, increases in parental income when children are young can have a greater effect on investments at later ages than increases in income at those ages if early and late investments are sufficiently comple-

mentary. Thus, our finding (from the CNLSY) that early parental income has greater effects on educational attainment than later income implies strong dynamic complementarity. Based on this feature of the data, our calibration identifies a strong degree of dynamic complementarity, and our quantitative analysis suggests that early and late investments positively co-move in response to different policies. Our quantitative analysis yields a number of other important general insights.

We find that many young and old parents are borrowing constrained, especially those with higher education who took out loans to finance their own education and who tend to have high ability children. However, like Keane and Wolpin (2001) and Johnson (2010), our model suggests that there would be little impact on human capital investment (‘early’ or ‘late’) from relaxing borrowing constraints on college-age youth or older parents. At least in the short-run, relaxing constraints on young parents would substantially increase both ‘early’ investments in young children and ‘late’ investments in older children (e.g. high school completion and college). For example, we find that a modest increase in the borrowing limit faced by young parents would increase early investment by 11% and college graduation rates by 10%. The effects are greater for families with more educated parents, since these families are constrained and want more credit for investment in their children. Less-educated parents want more credit primarily for current consumption.

We also consider the long-run impacts of permanently relaxing borrowing constraints, allowing the distribution of assets and human capital to change in response. Here, the results are quite different. Since relaxing the borrowing constraint for young parents causes families to accumulate more debt over time, future generations find themselves constrained to nearly the same extent that initial generations were before the constraint was relaxed. On average, this shift in assets results in negligible long-run effects of relaxing the constraint on average human capital levels. Modest increases can be a double-edged sword, increasing human capital in the short-run but lowering family assets in the long-run.³⁶ These findings suggest that recent concerns about high student debt levels and their implications for future borrowing capacities may be well-founded.

We explore the impact of subsidies for ‘early’ vs. ‘late’ investment. Two interesting lessons emerge from this. First, subsidies for investment at either stage of development raise investments at both stages, calling into question traditional analyses of college-age policies that ignore the response of early investment. This omission would cause one to under-estimate the final impact on post-school wages by nearly 60%. Second, subsidies for early investment produce much greater

³⁶Of course, welfare of the dynasty is improved by relaxing the constraint; however, initial generations capture most of this gain.

short- and long-run gains in human capital than (fiscally equivalent) subsidies for late investment. Dynamic complementarity implies that families that are constrained when their children are young do not fully capitalize on subsidies at later ages, because it is too costly to adjust early investments. Those that receive inadequate early investments do not find it worthwhile to make additional later investments (especially college) even if it is heavily subsidized. By contrast, early investment subsidies enable families to increase investments in their young children without sacrificing current consumption or borrowing more. Those investments can then be matched with later investments when constraints are less severe.

In addition to subsidizing private investments, governments also provide a minimum level of free investment for all children in the form of public schooling. We show that efforts to increase public investments in young children can be effective in raising early investments and high school completion rates among children who would otherwise drop out of high school; however, new public investments largely crowd-out private investment for those who make more modest or sizeable investments in the first place

If the goal is to raise college completion rates, our results suggest that the most cost-effective policy is to subsidize early investment (i.e. raise s_1). Given strong dynamic complementarity in investments, college completion is only worthwhile if substantial early investments are also made. Modest increases in public investments do not raise early investment levels enough to affect college graduation rates, while investment subsidies at older ages come too late for many constrained families to make the earlier investments needed for success in college. Early investment subsidies produce sizeable increases in early investments for many children, making more individuals “college-ready”. If, instead, the goal is to increase high school completion rates, then increases in publicly provided early investments (and late investment subsidies) are more effective. Modest early investment subsidies are ineffective, since most dropouts appear to be at a ‘corner’ solution during early childhood, wishing to invest less than is already publicly provided for free. Of course, these are precisely the children whose early investments increase one-for-one with improvements in publicly provided investments. Altogether, these results suggest that the nature of investment policies can be as important as their timing for determining impacts on early investment and educational outcomes.

Lastly, we show that it is important to take into account intergenerational effects when evaluating policy. A one-shot policy that gives transfers to young parents increases human capital investments more than an equivalent loan to young parents. However, if the policy is permanently

put in place, the loan increases human capital investments more. Transfers today decrease the cost of investment, but transfers tomorrow decrease the benefit of investments. In our framework, this latter effect is quantitatively important.

Many simplifying assumptions have been made in order to make our intergenerational problem tractable. Future work should attempt to incorporate a richer structure for family size, marriage/divorce behavior, and labor supply decisions. Shrinking periods to one or two years would certainly enrich the nature of human capital production and other important lifecycle issues. General equilibrium concerns also deserve attention. While improvements along these lines should add credibility to any policy analysis, we have purposely focused on general lessons that should carry over to and guide future work in this area.

Appendix A Data from the Children of the National Longitudinal Survey of Youth

We use data from the CNLSY, which follows the children born to all women in the NLSY79. The mothers in our sample are original NLSY79 respondents from the random sample and were ages 14-22 in 1979 when the survey began.

The data contains measures of family income every year from 1979 to 1994 and biennially thereafter. Our analysis uses reported earnings for the father and mother as the main measure of family income. All income measures are deflated to 2008 values using the CPI-U.³⁷

Table 1 also uses a created measure of earned ‘full’ income. This measure uses reported hours worked by mothers to adjust their earnings to a 1500 hour (30 hours per week) annual equivalent. Specifically, for all mothers working less than 1500 hours, we multiply reported earnings by 1500 and divide by reported hours. We then add this to father’s earnings to get our measure of earned ‘full’ income.

We discount combined family earnings back to age zero of the child using a 5% annual interest rate. Our measure of ‘early’ income averages family earnings over child ages zero to eleven, while our measure of ‘late’ income averages earnings over ages 12-23. These assumptions and age groups are used in Table 1 and our calibration.

We categorize individuals (mothers and children) with less than 12 years of completed schooling as high school dropouts, 12 years of schooling as high school graduates, 13-15 years of schooling as some college, and 16 or more years of schooling as college graduates. In Table 1, we refer to those with 13 or more years of completed schooling as having attended college. For children, if educational attainment is unavailable at age 21 (24), we use reported education at ages 22-24 (25-27). For mothers, we use educational attainment as of age 28 (or ages 29 and 30 if missing at earlier ages).

The CNLSY contains measures of many child and mother characteristics that may affect educational attainment. Estimates in Panel B of Table 1 use reported year of child’s birth, indicators for whether the child is black or hispanic, gender, whether the mother was a teenager when the child was born, maternal education categories (high school dropout, high school graduate, some college, college graduate), whether the mother was living in an intact family at age 14, whether the mother is foreign-born, and the mother’s normed score on the Armed Forces Qualifying Test (AFQT) taken as part of the survey in 1980.

³⁷We impute missing earnings separately for mothers and fathers using individual-specific regressions of log earnings on an intercept, age and age-squared whenever at least 8 positive values are available and respondents are age 22 or older. Less than 10% of our final family earnings measures are imputed. Combined family earnings values of greater than \$500,000 and less than \$500 are set to missing.

Appendix B Proofs for Propositions 1-5

Proof of Proposition 1:

Combining FOCs for assets we have:

$$u'(c_1) \geq \beta R u'(c_2) \geq (\beta R)^2 u'(c_3) \geq (\beta R)^3 u'(c_4) \geq (\beta R)^4 u'(c_5), \quad (13)$$

where inequalities are strict when the relevant borrowing constraint binds.

(i) Using equations (6) and (13) we have:

$$u'(c_1) \leq \beta^2 w f_1 \left[\frac{u'(c_1)}{(\beta R)^2} + \beta \frac{u'(c_1)}{(\beta R)^3} \Gamma_4 + \beta^2 \frac{u'(c_1)}{(\beta R)^4} \Gamma_4 \Gamma_5 \right] = u'(c_1) w f_1 \frac{1}{R^2} \chi,$$

which implies $f_1 \geq \frac{R^2}{\chi}$, with strict inequality if any borrowing constraint binds.

(ii) A similar analysis shows that $f_2 \geq \frac{R}{\chi}$, with strict inequality if any of the borrowing constraints L_2 , L_3 , or L_4 binds.

(iii) We show that investment falls in at least one period when any borrowing constraint binds using proof by contradiction. Recall:

$$f_1(i_1^*, i_2^*, \theta) > f_1(i_1^u, i_2^u, \theta), \quad (14)$$

$$f_2(i_1^*, i_2^*, \theta) \geq f_2(i_1^u, i_2^u, \theta), \quad (15)$$

where the latter holds with strict inequality if borrowing limits L_2 , L_3 , or L_4 bind.

Case (1) Suppose $i_1^* = i_1^u$ and $i_2^* = i_2^u$. This contradicts (14).

Case (2) Suppose $i_1^* > i_1^u$ and $i_2^* = i_2^u$. Since $f_{11} < 0$, this implies that $f_1(i_1^*, i_2^*, \theta) < f_1(i_1^u, i_2^u, \theta)$, which contradicts (14).

Case (3) Suppose $i_1^* = i_1^u$ and $i_2^* > i_2^u$. Since $f_{22} < 0$, this implies that $f_2(i_1^*, i_2^*, \theta) < f_2(i_1^u, i_2^u, \theta)$, which contradicts (15).

Case (4) Suppose $i_1^* > i_1^u$ and $i_2^* > i_2^u$. If we take the total derivative of f_1 at the unconstrained optimum, we have $f_{11} di_1 + f_{12} di_2 > 0$, since $i_1^* > i_1^u$ and $i_2^* > i_2^u$ by assumption and $f_1(i_1^*, i_2^*, \theta) > f_1(i_1^u, i_2^u, \theta)$ by (14). Therefore, $\frac{di_1}{di_2} < \frac{-f_{12}}{f_{11}}$. Similarly, the total derivative of f_2 and (15) imply that $\frac{di_1}{di_2} \geq \frac{-f_{22}}{f_{21}}$. Together, these imply that $f_{11} f_{22} < f_{12}^2$, which contradicts Assumption 1.

The only cases that remain imply that either $i_1^* < i_1^u$ or $i_2^* < i_2^u$. Furthermore, a similar analysis shows that if $f_{12} > 0$, $i_1^* < i_1^u$ and $i_2^* < i_2^u$ when any borrowing constraint binds. \square

Proof of Proposition 2: (I) Re-write the problem given in equation (5) using Assumption 2 and directly substituting in the budget constraints for c_1 and c_2 . Because we are assuming this person is constrained as an old child, let $a_3 = -L_2$. The decision problem can be written as:

$$\max_{i_1, i_2, a_2} u(y_1 - i_1 - a_2) + \beta u(Ra_2 + y_2 - i_2 + L_2) + \beta^2 v_3(-RL_2 + \chi f(i_1, i_2, \theta)),$$

where the constant $\chi = w(1 + R^{-1}\Gamma_4 + R^{-2}\Gamma_4\Gamma_5) > 0$ and the value function $v_3(\cdot)$ is defined in the text by equation (8).

First order conditions for i_1 , i_2 and a_2 are:

$$-u'(c_1) + \beta^2 v'_3(-RL_2 + \chi f(i_1, i_2, \theta)) \chi f_1(i_1, i_2, \theta) = 0 \quad (16)$$

$$-\beta u'(c_2) + \beta^2 v'_3(-RL_2 + \chi f(i_1, i_2, \theta)) \chi f_2(i_1, i_2, \theta) = 0 \quad (17)$$

$$-u'(c_1) + \beta R u'(c_2) = 0. \quad (18)$$

Together, these first order conditions imply $f_1 = Rf_2$ at an optimum. Using this with Cramer's rule yields (dropping arguments of $f(\cdot)$ and $v_3(\cdot)$ for expositional purposes):

$$\begin{aligned} \frac{\partial i_1}{\partial y_1} &= \frac{\beta^3 u''(c_1) u''(c_2) v'_3 \chi (Rf_{22} - f_{12})}{\Delta_2} > 0, \\ \frac{\partial i_2}{\partial y_1} &= \frac{\beta^3 u''(c_1) u''(c_2) v'_3 \chi (f_{11} - Rf_{12})}{\Delta_2} > 0, \end{aligned}$$

$\frac{\partial i_1}{\partial y_2} = R^{-1} \frac{\partial i_1}{\partial y_1}$ and $\frac{\partial i_2}{\partial y_2} = R^{-1} \frac{\partial i_2}{\partial y_1}$, where

$$\begin{aligned} \Delta_2 &\equiv \beta^4 R^2 [u''(c_2)]^2 \chi (v'_3 f_{11} + v''_3 \chi f_1^2) + \beta^3 u''(c_1) u''(c_2) v'_3 \chi (f_{11} + R^2 f_{22} - 2Rf_{12}) \\ &\quad + \beta^4 v'_3 \chi^2 [u''(c_1) + \beta R^2 u''(c_2)] [v'_3 (f_{11} f_{22} - f_{12}^2) + v''_3 \chi (f_1^2 f_{22} + f_2^2 f_{11} - 2f_1 f_2 f_{12})] < 0. \end{aligned} \quad (19)$$

All of these expressions are signed using Assumption 1 and $f_1 = Rf_2$.

Finally, $\frac{\partial h_3}{\partial y_j} = f_1 \frac{\partial i_1}{\partial y_j} + f_2 \frac{\partial i_2}{\partial y_j} > 0$ for $j = 1, 2$, since all terms in this expression are strictly positive; $\frac{\partial h_3}{\partial y_1} = R \frac{\partial h_3}{\partial y_2}$ follows directly from the fact that $\frac{\partial i_j}{\partial y_1} = R \frac{\partial i_j}{\partial y_2}$ for $j = 1, 2$.

(II) Re-write the problem given in equation (5) assuming $a_2 = -L_1$ and Assumption 2:

$$\max_{i_1, i_2, a_3} u(y_1 - i_1 + L_1) + \beta u(-RL_1 + y_2 - i_2 - a_3) + \beta^2 v_3(Ra_3 + \chi f(i_1, i_2, \theta)), \quad (20)$$

where $\chi > 0$ is defined above. First order conditions for i_1 , i_2 and a_3 are:

$$-u'(c_1) + \beta^2 v'_3(Ra_3 + \chi f(i_1, i_2, \theta)) \chi f_1(i_1, i_2, \theta) = 0 \quad (21)$$

$$-\beta u'(c_2) + \beta^2 v'_3(Ra_3 + \chi f(i_1, i_2, \theta)) \chi f_2(i_1, i_2, \theta) = 0 \quad (22)$$

$$\beta u'(c_2) + \beta^2 R v'_3(Ra_3 + \chi f(i_1, i_2, \theta)) = 0. \quad (23)$$

Combining first order conditions, we have $f_2 = R/\chi$. However, $f_1 > Rf_2 = R^2/\chi$ since L_1 binds (see Proposition 1).

Cramer's rule yields (dropping arguments of $f(\cdot)$ and $v_3(\cdot)$):

$$\begin{aligned} \frac{\partial i_1}{\partial y_1} &= \frac{\beta^3 u''(c_1) v'_3 \chi f_{22} [u''(c_2) + \beta R^2 v''_3]}{\Delta_1} > 0 \\ \frac{\partial i_1}{\partial y_2} &= \frac{-\beta^5 R u''(c_2) v'_3 v''_3 \chi^2 f_1 f_{22}}{\Delta_1} < 0 \\ \frac{\partial i_2}{\partial y_1} &= \frac{-\beta^3 u''(c_1) v'_3 \chi f_{12} [u''(c_2) + \beta R^2 v''_3]}{\Delta_1} \\ \frac{\partial i_2}{\partial y_2} &= \frac{\beta^5 R u''(c_2) v'_3 v''_3 \chi^2 f_1 f_{12}}{\Delta_1} \end{aligned}$$

where

$$\begin{aligned}\Delta_1 &\equiv \beta^3 u''(c_1) v_3' [u''(c_2) + \beta R^2 v_3''] \chi f_{22} + \beta^5 u''(c_2) v_3' v_3'' \chi^3 f_1^2 f_{22} \\ &\quad + \beta^5 [u''(c_2) + \beta R^2 v_3''] (v_3')^2 \chi^2 [f_{11} f_{22} - f_{12}^2] < 0\end{aligned}\quad (24)$$

by Assumption 1. Clearly, $\frac{\partial i_2}{\partial y_1} > 0 \iff f_{12} > 0$, and $\frac{\partial i_2}{\partial y_2} < 0 \iff f_{12} > 0$. Since $\frac{\partial h_3}{\partial y_j} = f_1 \frac{\partial i_1}{\partial y_j} + f_2 \frac{\partial i_2}{\partial y_j}$ for $j = 1, 2$, Assumption 1 implies that $\frac{\partial h_3}{\partial y_1} > 0$ and $\frac{\partial h_3}{\partial y_2} < 0$.

(III) Now, re-write problem given in equation (5) assuming $a_3 = -L_2$, $a_2 = -L_1$, and Assumption 2:

$$\max_{i_1, i_2} u(y_1 - i_1 + L_1) + \beta u(-RL_1 + y_2 - i_2 + L_2) + \beta^2 v_3(-RL_2 + \chi f(i_1, i_2, \theta)) \quad (25)$$

where $\chi > 0$ is defined above. The first order conditions for i_1 and i_2 are given by equations (16) and (17), where $c_1 = y_1 - i_1 + L_1$ and $c_2 = -RL_1 + y_2 - i_2 + L_2$.

Cramer's rule yields (dropping arguments of $f(\cdot)$ and $v_3(\cdot)$):

$$\begin{aligned}\frac{\partial i_1}{\partial y_1} &= \frac{\beta u''(c_1) [u''(c_2) + \beta v_3' \chi f_{22} + \beta v_3'' \chi^2 f_2^2]}{\Delta_{12}} > 0 \\ \frac{\partial i_1}{\partial y_2} &= \frac{-\beta^3 u''(c_2) \chi [v_3' f_{12} + v_3'' \chi f_1 f_2]}{\Delta_{12}} \\ \frac{\partial i_2}{\partial y_1} &= \frac{-\beta^2 u''(c_1) \chi [v_3' f_{12} + v_3'' \chi f_1 f_2]}{\Delta_{12}} \\ \frac{\partial i_2}{\partial y_2} &= \frac{\beta u''(c_2) \chi [u''(c_1) + \beta^2 v_3' \chi f_{11} + \beta^2 v_3'' \chi^2 f_1^2]}{\Delta_{12}} > 0\end{aligned}$$

where

$$\begin{aligned}\Delta_{12} &\equiv \beta u''(c_1) u''(c_2) + \beta^2 v_3' \chi [u''(c_1) f_{22} + \beta u''(c_2) f_{11}] + \beta^2 v_3'' \chi^2 [u''(c_1) f_2^2 + \beta u''(c_2) f_1^2] \\ &\quad + \beta^4 (v_3')^2 \chi^2 (f_{11} f_{22} - f_{12}^2) + \beta^4 v_3' v_3'' \chi^3 [f_2 (f_2 f_{11} - f_1 f_{12}) + f_1 (f_1 f_{22} - f_2 f_{12})] > 0.\end{aligned}\quad (26)$$

Assumption 1 ensures that $\frac{\partial i_1}{\partial y_1}$, $\frac{\partial i_2}{\partial y_2}$, and Δ_{12} are strictly positive. Both $\frac{\partial i_1}{\partial y_2}$ and $\frac{\partial i_2}{\partial y_1}$ are strictly positive if and only if Condition 1 holds. Using these results for investments, Assumption 1 implies that $\frac{\partial h_3}{\partial y_j} > 0$ for $j = 1, 2$. \square

Proof of Proposition 3: (i) Based on the problem discussed in the proof of Proposition 2 part (I), we can apply Cramer's rule obtaining:

$$\begin{aligned}\frac{\partial i_1}{\partial L_2} &= \frac{\beta^3 u''(c_2) v_3' \chi [\beta^2 R^2 v_3'' \chi f_2 + u''(c_1)] (R f_{22} - f_{12})}{\Delta_2} > 0 \\ \frac{\partial i_2}{\partial L_2} &= \frac{\beta^4 R v_3' v_3'' \chi^2 f_2 [u''(c_1) + \beta R^2 u''(c_2)] (f_{11} - R f_{12}) + \beta^3 u''(c_1) u''(c_2) v_3' \chi (f_{11} - R f_{12})}{\Delta_2} \in (0, 1),\end{aligned}$$

where $\Delta_2 < 0$ is defined previously by equation (19). All three of these expressions are signed using Assumption 1 and $f_1 = R f_2$. That $\frac{\partial i_2}{\partial L_2} < 1$ follows from Assumption 1, $f_1 = R f_2$, and

$f_2 < R/\chi$ when L_2 binds (see Proposition 1). Finally, $\frac{\partial h_3}{\partial L_2} = f_1 \frac{\partial i_1}{\partial L_2} + f_2 \frac{\partial i_2}{\partial L_2} > 0$, since all terms in this expression are positive.

(ii) Based on the problem used in the proof of Proposition 2 part (III), Cramer's rule yields:

$$\begin{aligned} \frac{\partial i_1}{\partial L_2} &= \frac{\beta^4 R v'_3 v''_3 \chi^2 (f_1 f_{22} - f_2 f_{12}) + \beta^3 R u''(c_2) v'_3 \chi f_1 - \beta^3 u''(c_2) \chi (v'_3 f_{12} + \chi v''_3 f_1 f_2)}{\Delta_{12}} \\ \frac{\partial i_2}{\partial L_2} &= \frac{\beta u''(c_2) [u''(c_1) + \beta^2 v'_3 \chi f_{11} + \beta^2 v''_3 \chi^2 f_1^2] + \beta^2 R u''(c_1) v'_3 \chi f_2 + \beta^4 R v'_3 v''_3 \chi^2 (f_2 f_{11} - f_1 f_{12})}{\Delta_{12}} \in (0, 1) \end{aligned}$$

where $\Delta_{12} > 0$ is defined previously by equation (26). Using Assumption 1, it is clear that $\frac{\partial i_1}{\partial L_2} > 0$ if Condition 1 holds and that $\frac{\partial i_2}{\partial L_2} > 0$. $\frac{\partial i_2}{\partial L_2} < 1$ follows from Assumption 1 and $f_2 < R/\chi$ when L_2 binds (see Proposition 1). Assumption 1 further implies that $\frac{\partial h_3}{\partial L_2} = f_1 \frac{\partial i_1}{\partial L_2} + f_2 \frac{\partial i_2}{\partial L_2} > 0$. \square

Proof of Proposition 4: (i) Based on the problem discussed in the proof of Proposition 2 part (II), we can apply Cramer's rule obtaining:

$$\begin{aligned} \frac{\partial i_1}{\partial L_1} &= \frac{f_{22} \{ \beta^3 u''(c_1) v'_3 \chi [u''(c_2) + \beta R^2 v''_3] + \beta^5 R^2 u''(c_2) v'_3 v''_3 \chi^2 f_1 \}}{\Delta_1} \in (0, 1) \\ \frac{\partial i_2}{\partial L_1} &= \frac{-f_{12} \{ \beta^3 u''(c_1) v'_3 \chi [u''(c_2) + \beta R^2 v''_3] + \beta^5 R^2 u''(c_2) v'_3 v''_3 \chi^2 f_1 \}}{\Delta_1} \end{aligned}$$

where $\Delta_1 < 0$ is defined previously by equation (24). $\frac{\partial i_1}{\partial L_1} < 1$ follows from $\chi f_1 > R^2$ and Assumption 1. Clearly, $\frac{\partial i_2}{\partial L_1} > 0 \iff f_{12} > 0$. Finally, $\frac{\partial h_3}{\partial L_2} = f_1 \frac{\partial i_1}{\partial L_2} + f_2 \frac{\partial i_2}{\partial L_2} > 0$ by Assumption 1.

(ii) Based on the problem used in the proof of Proposition 2 part (III), Cramer's rule yields:

$$\begin{aligned} \frac{\partial i_1}{\partial L_1} &= \frac{\beta u''(c_1) [u''(c_2) + \beta v'_3 \chi f_{22} + \beta v''_3 \chi^2 f_2^2] + \beta^3 R u''(c_2) \chi [v'_3 f_{12} + v''_3 \chi f_1 f_2]}{\Delta_{12}} \\ \frac{\partial i_2}{\partial L_1} &= \frac{-\beta R u''(c_2) [u''(c_1) + \beta^2 v'_3 \chi f_{11} + \beta^2 v''_3 \chi^2 f_1^2] - \beta^2 u''(c_1) \chi [v'_3 f_{12} + v''_3 \chi f_1 f_2]}{\Delta_{12}} \end{aligned}$$

where $\Delta_{12} > 0$ is defined previously by equation (26). Assumption 1 implies that $\frac{\partial i_1}{\partial L_1} < 1$. If Condition 1 does not hold, then $v'_3 f_{12} + v''_3 \chi f_1 f_2 < 0$, which implies that $\frac{\partial i_1}{\partial L_1} \in (0, 1)$ and $\frac{\partial i_2}{\partial L_1} < 0$. \square

Proof of Proposition 5: This proposition alters the problem discussed previously to include transfers y_3 in period 3. We consider two cases, each with binding borrowing constraints throughout childhood (i.e. $a_2 = -L_1$ and $a_3 = -L_2$). In the first case, the borrowing constraint binds during young adulthood (i.e. $a_4 = -L_3$), while the constraint is non-binding in the second (i.e. $a_4 > -L_3$).

Case 1: Borrowing constraints bind in periods 1-3. Substituting in all constraints, the decision problem can be written as:

$$\max_{i_1, i_2} u(y_1 - i_1 + L_1) + \beta u(-RL_1 + y_2 - i_2 + L_2) + \beta^2 u(-RL_2 + w f(i_1, i_2; \theta) + y_3 + L_3) + \beta^3 v_4(-RL_3 + \tilde{\chi} f(i_1, i_2; \theta)),$$

where $\tilde{\chi} = w[\Gamma_4 + R^{-1}\Gamma_4\Gamma_5]$.

First order conditions are:

$$\begin{aligned} -u'(c_1) + [\beta^2 w u'(c_3) + \beta^3 v'_4(-RL_3 + \tilde{\chi} f(i_1, i_2; \theta)) \tilde{\chi}] f_1(i_1, i_2; \theta) &= 0 \\ -\beta u'(c_2) + [\beta^2 w u'(c_3) + \beta^3 v'_4(-RL_3 + \tilde{\chi} f(i_1, i_2; \theta)) \tilde{\chi}] f_2(i_1, i_2; \theta) &= 0. \end{aligned}$$

Below, the arguments for $v_4(\cdot)$ and $f(\cdot)$ are dropped to streamline notation.

To simplify certain expressions, it is useful to define the following:

$$\begin{aligned} \Omega_1 &\equiv \beta^2 u'(c_3) w + \beta^3 v'_4(-RL_3 + f \tilde{\chi}) \tilde{\chi} > 0 \\ \Omega_2 &\equiv \beta^2 u''(c_3) w^2 + \beta^3 v''_4(-RL_3 + f \tilde{\chi}) \tilde{\chi}^2 < 0. \end{aligned}$$

Using Cramer's rule, it is straightforward to show that

$$\begin{aligned} \frac{\partial i_1}{\partial y_3} &= \frac{-\beta^2 w u''(c_3) [\beta u''(c_2) f_1 + \Omega_1 (f_1 f_{22} - f_2 f_{12})]}{\Delta_{123}} < 0 \\ \frac{\partial i_2}{\partial y_3} &= \frac{-\beta^2 w u''(c_3) [u''(c_1) f_2 + \Omega_1 (f_2 f_{11} - f_1 f_{12})]}{\Delta_{123}} < 0 \\ \frac{\partial i_1}{\partial L_3} &= \frac{[\beta^3 R v''_4 \tilde{\chi} - \beta^2 w u''(c_3)] [\beta u''(c_2) f_1 + \Omega_1 (f_1 f_{22} - f_2 f_{12})]}{\Delta_{123}} > \frac{\partial i_1}{\partial y_3} \\ \frac{\partial i_2}{\partial L_3} &= \frac{[\beta^3 R v''_4 \tilde{\chi} - \beta^2 w u''(c_3)] [u''(c_1) f_2 + \Omega_1 (f_2 f_{11} - f_1 f_{12})]}{\Delta_{123}} > \frac{\partial i_1}{\partial L_3}, \end{aligned}$$

where

$$\begin{aligned} \Delta_{123} &\equiv \beta u''(c_1) u''(c_2) + u''(c_1) (\Omega_1 f_{22} + \Omega_2 f_2^2) + \beta u''(c_1) (\Omega_1 f_{11} + \Omega_2 f_1^2) \\ &\quad + \Omega_1^2 (f_{11} f_{22} - f_{12}^2) + \Omega_1 \Omega_2 [f_2 (f_2 f_{11} - f_1 f_{12}) + f_1 (f_1 f_{22} - f_2 f_{12})] > 0. \end{aligned}$$

All of these expressions are signed using Assumption 1.

Case 2: Borrowing constraints only bind throughout childhood (periods 1 and 2). Substituting in all constraints, the decision problem can be written as:

$$\max_{i_1, i_2} u(y_1 - i_1 + L_1) + \beta u(-RL_1 + y_2 - i_2 + L_2) + \beta^2 v_3(-RL_2 + \chi f(i_1, i_2; \theta) + y_3)$$

where $\chi > 0$ is defined in the text. This problem is identical to that in equation (25) (see part III of the proof for Proposition 2) with y_3 included in the argument for $v_3(\cdot)$. As such, the first order conditions for this problem are the same as in that case (incorporating y_3).

Since $a_4 > -L_3$ by assumption, we have $\frac{\partial i_j}{\partial L_3} = 0$ for $j = 1, 2$. Using Cramer's rule,

$$\begin{aligned} \frac{\partial i_1}{\partial y_3} &= \frac{-\beta^3 v''_3 \chi [u''(c_2) f_1 + \beta v'_3 \chi (f_1 f_{22} - f_2 f_{12})]}{\Delta_{12}} < 0 \\ \frac{\partial i_2}{\partial y_3} &= \frac{-\beta^2 v''_3 \chi [u''(c_1) f_2 + \beta^2 v'_3 \chi (f_2 f_{11} - f_1 f_{12})]}{\Delta_{12}} < 0, \end{aligned}$$

where $\Delta_{12} > 0$ is given by equation (26). All expressions are signed using Assumption 1. \square

Appendix C Details on Calibration

We calibrate parameters of the earnings shock distribution (m, s) , the human capital production function (a, b, c) , parental altruism towards their children (ρ) , the ability distribution $(\theta_1, \theta_2, \pi_1, \pi_2)$, and the debt constraint parameter γ by simulating the model in steady state to best fit a number of moments in the NLSY79 and CNLSY data. In particular, we fit moments related to (i) the education distribution, (ii) the distribution of annual earnings for men ages 24-35 and 36-47 in the NLSY79, (iii) child schooling levels conditional on parental income and maternal schooling, and (iv) child wages at ages 24-35 conditional on their own educational attainment, maternal schooling, and parental income levels (when the child is ages 0-11).

When classifying individuals by education (either mother or child), we categorize them by highest grade completed (completing less than 12 years of school, 12 years of school, 13-15 years, or 16 or more years).

We minimize $ERR = \sum_{j=1}^4 \omega_j ERR_j$, where each ERR_j represents the error associated with one of the four sets of moments we fit and ω_j is the weight placed on that set of moments. We briefly describe each of these moments.

ERR_1 is the sum of squared differences between the model's steady state education probabilities and the corresponding sample proportions based on the random sample of all mothers in the NLSY79 (sample size of 2,478). See Table 2 in the paper for these moments in the data and our calibrated steady state.

ERR_2 reflects differences between moments associated with the model's steady state earnings distribution and their corresponding sample moments in the NLSY79 data. Let $E(W_j)$ and $SD(W_j)$ reflect the mean and standard deviation for steady state wages in period $j = 3, 4$ for the model. For corresponding sample moments in the data ($\hat{E}(W_j)$ and $\widehat{SD}(W_j)$ for $j = 3, 4$) we use annual earnings averaged over ages 24-35 and 36-47 (discounted at annual rate $r = 0.05$ to ages 30 and 42) for the random sample of men in the NLSY79. We then compute

$$ERR_2 = \frac{1}{N_3 + N_4} \sum_{j=3}^4 N_j \left[\frac{E(W_j) - \hat{E}(W_j)}{\hat{E}(W_j)} \right]^2 + \frac{1}{N_3 + N_4} \sum_{j=3}^4 N_j \left[\frac{SD(W_j) - \widehat{SD}(W_j)}{\widehat{SD}(W_j)} \right]^2.$$

Here, $N_3 = 2,696$ and $N_4 = 2,399$ reflect the number of observations used in each age-specific calculation.

ERR_3 is a weighted sum of squared differences between the model's steady state child education probabilities (conditional on parental income in periods 3 and 4 and parental schooling) and the corresponding sample proportions from the CNLSY. We separate our sample in the model and data depending on whether parental income (maternal plus paternal earnings) that period is in quartile 1, quartile 2, or above the median.³⁸ We use the maternal education categories discussed

³⁸In calculating empirical income cutoffs for the first quartile and median, we use the distribution of average

earlier. To determine child education probabilities, we use highest grade completed at age 21 to assign high school dropout and completion status, and age 24 to assign college attendance and completion status. We calculate

$$ERR_3 = \frac{1}{N} \sum_{j=1}^4 \sum_{k=1}^3 \sum_{l=1}^3 \sum_{m=1}^4 N_{jklm} [P(e' = j | Y_3 = k, Y_4 = l, e = m) - \hat{P}(e' = j | Y_3 = k, Y_4 = l, e = m)]^2,$$

where $P(e' = j | Y_3 = k, Y_4 = l, e = m)$ is the steady state probability a child chooses education category $e' = j$ conditional on family income categories $Y_3 = k$ and $Y_4 = l$, and maternal education in category $e = m$. $\hat{P}(e' = j | \cdot)$ reflects the corresponding conditional sample moment in the full sample of CNLSY. N_{jklm} is the number of observations used in calculating each conditional moment in the data and $N = \sum_{j,k,l,m} N_{jklm}$.³⁹ Table C1 reports estimated probabilities related to these moments from the CNLSY data and those obtained from our baseline calibration.

ERR_4 reflects the extent to which the model fits period 3 average wages of children conditional on their own education, parental education, and parental income when they were young. We classify parental income and education as we did for ERR_3 (in the model and data). We use average child weekly wages over ages 24-35 (all discounted to age 30 using $r = 0.05$) for children of the CNLSY.⁴⁰ Because we consider weekly wages for children (rather than annual income) to better reflect human capital levels at younger ages, we scale all average wage measures by those for children with a high school degree, whose mothers had a high school degree, and whose parental income was in the lowest quartile. We compute

$$ERR_4 = \sum_{k=1}^4 \sum_{l=1}^3 \sum_{m=1}^4 N_{klm} \left[\frac{E(W_3' | e' = k, Y_3 = l, e = m) - \hat{E}(W_3' | e' = k, Y_3 = l, e = m)}{\hat{E}(W_3 | e' = 2, Y_3 = 1, e = 2)} \right]^2,$$

where $E(W_3 | e' = k, Y_3 = l, e = m)$ is the average steady state period-three weekly wage W_3 for a child conditional on own education category $e' = k$, early parental income category $Y_3 = l$, and maternal education category $e = m$. Let $\hat{E}(W_3 | \cdot)$ reflect the corresponding conditional sample moment in the full sample of CNLSY. Let N_{klm} be the number of observations used in calculating each conditional moment in the data and $N = \sum_{k,l,m} N_{klm} = 3,049$. Table C2 reports relative average weekly wages (relative to average wages of high school dropouts from the lowest early income quartile whose mothers graduated high school) from the CNLSY data and those obtained from our baseline calibration.

family income over maternal ages 24-35 and 36-47 (discounted at annual rate $r = 0.05$ to ages 30 and 42) based on all mothers in the random sample of the NLSY79. We use family income averaged over child ages 0-11 and 12-23 for the CNLSY to categorize children by parental income in periods 3 and 4.

³⁹ N_{jklm} depends on the child education category, since we use different ages to determine high school dropout and graduate vs. some college and college completion. Altogether, our sample includes 3,928 individuals ages 21+ and 2,966 individuals ages 24+.

⁴⁰We drop observations with weekly wages less than \$40 or greater than \$2,500. To calculate more precise wage measures for high school dropouts and graduates, we also include weekly wage measures at ages 22-23 in computing average wages.

All of our ERR_j errors should be of similar magnitudes given the scaling of various moments. Reflecting sample sizes used to compute each error, we use weights $\omega_1 = 0.24$, $\omega_2 = 0.33$, $\omega_3 = 0.10$, and $\omega_4 = 0.33$. We generally fit all sets of moments well. Our calibration yields $ERR = 0.033$, with $ERR_1 = 0.003$, $ERR_2 = 0.07$, $ERR_3 = 0.02$, and $ERR_4 = 0.04$.

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Table 1: Random Sample: Effects of Early and Late Income (in \$10,000s PDV as of birth year) on Child Educational Attainment

	Earned Income			Earned "Full" Income				
	Sample Size (1)	Early Income (2)	Later Income (3)	Equal Effects (p-value) (4)	Sample Size (5)	Early Income (6)	Later Income (7)	Equal Effects (p-value) (8)
A. Controls Only for Maternal Education								
HS Dropout (Ages 21-24))	2,273	-0.026 (0.004)	-0.009 (0.006)	0.063	1,894	-0.023 (0.005)	-0.011 (0.006)	0.205
Attended Any College (Ages 24-27))	1,586	0.046 (0.007)	0.015 (0.009)	0.037	1,336	0.045 (0.008)	0.014 (0.010)	0.044
Graduated College (Ages 24-27))	1,586	0.028 (0.006)	0.024 (0.007)	0.765	1,336	0.033 (0.059)	0.025 (0.075)	0.500
B. Control for Maternal Education and Child/Family Background								
HS Dropout (Ages 21-24))	2,192	-0.023 (0.005)	-0.010 (0.006)	0.197	1,835	-0.022 (0.005)	-0.011 (0.006)	0.277
Attended Any College (Ages 24-27))	1,526	0.040 (0.008)	0.011 (0.009)	0.053	1,291	0.042 (0.008)	0.008 (0.010)	0.034
Graduated College (Ages 24-27))	1,526	0.025 (0.006)	0.021 (0.007)	0.760	1,291	0.032 (0.006)	0.021 (0.008)	0.359

Notes: Coefficient estimates in bold typeface are statistically significant at 0.05 level. Estimates reported in Panel A control only for maternal education, while those in Panel B also control for important child characteristics (year of birth, race/ethnicity, gender) and mother characteristics (educational attainment, whether she was a teenager when the child was born, living in an intact family at age 14, foreign-born, and Armed Forces Qualifying Test scores). Specifications in columns (1)-(4) use total reported parental earnings to measure family income, while those in columns (5)-(8) use an adjusted 'full' earnings measure that inflates earnings for mothers working less than 1,500 per year to its 1,500 hour equivalent. Early income reflects average discounted family income over child ages 0-11; late income reflects average discounted income over ages 12-23. A discount rate of 5% is used to discount income to age 0.

Table 2: Calibrated Education Distribution

Education	NLSY Data	Model
High school graduate or more	.82	.79
Some college or more	.42	.42
College graduate	.19	.13

Table 3: Calibrated Annual Earnings Distributions for Men Ages 24-35 and 36-47

Earnings Statistic	NLSY Data	Model
Mean (ages 24-35)	40,868	43,889
Standard deviation (ages 24-35)	23,108	26,362
Mean (ages 36-47)	60,392	64,826
Standard deviation (ages 36-47)	41,416	29,444

Table 4: Educational Attainment by Parental Education (Baseline)

Parental Education	Model			NLSY Data		
	High School Graduate or More	Some College or More	College Graduate	High School Graduate or More	Some College or More	College Graduate
High School Dropout	0.54	0.18	0.03	0.61	0.23	0.05
High School Graduate	0.77	0.39	0.08	0.78	0.42	0.14
Some College	0.90	0.55	0.20	0.84	0.53	0.21
College Graduate	0.99	0.66	0.30	0.93	0.76	0.46

Table 5: Educational Attainment by Parental Income (Baseline)

Parental Income Quartile:		Model		NLSY Data	
Early Ages	Late Ages	High School Graduate or More	College Graduate	High School Graduate or More	College Graduate
1	Any	0.58	0.01	0.65	0.07
2	Any	0.75	0.03	0.79	0.16
3 or 4	Any	0.91	0.24	0.89	0.29
Any	1	0.56	0.06	0.66	0.06
Any	2	0.72	0.11	0.77	0.14
Any	3 or 4	0.93	0.18	0.87	0.29
1	1	0.46	0.01	0.64	0.06
2	1	0.56	0.03	0.73	0.10
3 or 4	1	0.71	0.16	0.90	0.08
1	2	0.56	0.01	0.69	0.08
2	2	0.71	0.03	0.80	0.17
3 or 4	2	0.85	0.24	0.84	0.18
1	3 or 4	0.84	0.01	0.69	0.14
2	3 or 4	0.90	0.03	0.82	0.17
3 or 4	3 or 4	0.96	0.27	0.90	0.34

Table 6: Calibrated Parameter Values

Parameter	Value
a	0.45
b	-1.10
d	0.77
θ_1	7.82
θ_2	20.00
π_1	0.59
π_2	0.76
m	9.94
s	0.74
ρ	0.67
γ	0.45

Table 7: Average Baseline Investment Amounts by Parental Education

Parental Education	Average i_1	Average i_2
All Levels	2,121	7,227
High School Dropout	686	3,088
High School Graduate	1,711	5,989
Some College	2,970	9,473
College Graduate	3,643	12,205

Table 8: Effects of Increasing Young Parent's Borrowing Limit by \$2,500

Parental Education	Short-Run Effects (% Change)				Long-Run Effects (% Change)			
	Avg. i_1	HS+	College Grad.	Avg. W_3	Avg. i_1	HS+	College Grad.	Avg. W_3
All Levels	11.0	2.8	10.0	1.5	-0.4	1.7	-1.9	0.0
HS Dropout	0.0	5.2	0.0	0.1	-16.4	2.6	-14.2	-0.8
HS Graduate	9.7	3.9	12.7	1.4	-5.2	1.5	-3.1	-0.6
Some College	9.3	1.7	4.5	1.6	-2.1	0.9	-8.2	-0.4
College Graduate	18.8	0.4	17.6	3.6	10.5	0.3	8.1	2.0

Table 9: Effects of Increasing Young Parent's Borrowing Limit by \$2,500 on Fraction of Constrained Young and Old Parents, and Old Children

Parental Education	Fraction of Young Parents Constrained		Fraction of Old Parents Constrained		Fraction of Old Parents with $y_2 = 0$	
	Baseline SS	New SS	Baseline SS	New SS	Baseline SS	New SS
All Levels	0.51	0.46	0.12	0.18	0.05	0.12
High School Dropout	0.51	0.39	0.04	0.11	0.15	0.29
High School Graduate	0.38	0.35	0.04	0.10	0.04	0.14
Some College	0.60	0.56	0.14	0.22	0.01	0.03
College Graduate	0.68	0.66	0.41	0.46	0.00	0.00

Table 10: Effects of Early and Late Investment Subsidies

Policy	Short-Run Effects (% Change)					Long-Run Effects (% Change)				
	Avg. i_1	Avg. i_2	HS+	College Grad.	Avg. W_3	Avg. i_1	Avg. i_2	HS+	College Grad.	Avg. W_3
Announced early										
$s_1 = .10$	28.1	12.1	0.0	30.5	3.1	34.0	17.0	1.3	39.6	3.9
$s_2 = .53$	4.0	15.5	10.1	22.9	1.4	7.2	18.1	10.9	26.8	1.8
Announced late										
$s_2 = .53$	0.0	8.7	10.0	1.4	0.6	7.2	18.1	10.9	26.8	1.8

Table 11: Short-Run Effects of Permanent vs. One-Time Loan and Transfer Policies to Young Parents

Policy	Permanent Policy (% Change)			One-Time Policy (% Change)		
	Avg i_1	Avg i_2	Avg W_3	Avg i_1	Avg i_2	Avg W_3
\$2500 Transfer	5.7	4.0	0.8	7.8	5.0	1.1
\$2500 Loan	10.9	7.5	1.5	8.4	3.0	1.0

Table C1: Educational Attainment by Maternal Education and Early and Late Parental Income (CNLSY Data and Model)

Parental Income Quartile		High School Probabilities					College Probabilities				
		N	HS Dropout		HS Graduate		N	Some College		College Graduate	
Early	Late		Model	Data	Model	Data		Model	Data	Model	Data
A. Mother HS Dropout											
1	1	451	0.59	0.49	0.39	0.36	395	0.02	0.16	0.00	0.03
1	2	113	0.60	0.45	0.38	0.40	110	0.01	0.18	0.00	0.04
1	3 or 4	29	0.26	0.41	0.60	0.38	28	0.14	0.07	0.00	0.11
2	1	63	0.58	0.41	0.16	0.44	44	0.26	0.16	0.00	0.02
2	2	104	0.63	0.30	0.16	0.46	93	0.21	0.19	0.00	0.06
2	3 or 4	37	0.12	0.19	0.59	0.49	35	0.29	0.20	0.00	0.14
3 or 4	1	17	0.41	0.12	0.18	0.53	9	0.31	0.44	0.10	0.00
3 or 4	2	58	0.23	0.21	0.38	0.36	39	0.29	0.31	0.10	0.10
3 or 4	3 or 4	68	0.00	0.25	0.59	0.34	55	0.29	0.33	0.12	0.16
B. Mother HS Graduate											
1	1	418	0.50	0.34	0.25	0.39	363	0.24	0.26	0.01	0.06
1	2	187	0.34	0.27	0.52	0.40	162	0.14	0.28	0.01	0.07
1	3 or 4	35	0.11	0.40	0.60	0.23	28	0.29	0.21	0.01	0.18
2	1	110	0.35	0.27	0.32	0.45	83	0.28	0.27	0.05	0.12
2	2	214	0.24	0.21	0.49	0.38	165	0.25	0.25	0.02	0.19
2	3 or 4	153	0.08	0.20	0.53	0.39	118	0.37	0.29	0.03	0.15
3 or 4	1	45	0.20	0.16	0.27	0.33	26	0.34	0.50	0.19	0.08
3 or 4	2	149	0.13	0.18	0.23	0.40	95	0.44	0.38	0.19	0.17
3 or 4	3 or 4	378	0.02	0.11	0.30	0.31	250	0.48	0.36	0.20	0.28
C. Mother Some College											
1	1	203	0.06	0.24	0.48	0.37	176	0.17	0.31	0.29	0.11
1	2	89	0.19	0.28	0.31	0.37	74	0.40	0.19	0.09	0.14
1	3 or 4	26	0.08	0.04	0.42	0.46	22	0.42	0.45	0.07	0.09
2	1	50	0.16	0.16	0.05	0.46	31	0.61	0.29	0.18	0.16
2	2	116	0.21	0.11	0.41	0.28	87	0.30	0.34	0.07	0.24
2	3 or 4	80	0.11	0.18	0.53	0.34	67	0.32	0.28	0.04	0.24
3 or 4	1	27	0.05	0.00	0.19	0.37	14	0.17	0.43	0.59	0.14
3 or 4	2	76	0.14	0.12	0.22	0.26	44	0.32	0.41	0.32	0.18
3 or 4	3 or 4	263	0.06	0.10	0.30	0.21	167	0.37	0.34	0.27	0.32
D. Mother College Graduate											
1	1	20	-	0.15	-	0.35	14	-	0.43	-	0.29
1	2	19	0.00	0.21	0.50	0.42	15	0.00	0.27	0.50	0.13
1	3 or 4	8	0.00	0.38	0.29	0.00	4	0.00	0.25	0.71	0.25
2	1	5	-	0.20	-	0.00	5	-	0.40	-	0.20
2	2	23	0.00	0.09	0.27	0.39	12	0.00	0.42	0.73	0.33
2	3 or 4	21	0.01	0.10	0.29	0.24	15	0.01	0.60	0.70	0.27
3 or 4	1	1	-	0.00	-	1.00	1	-	0.00	-	0.00
3 or 4	2	27	0.00	0.11	0.24	0.26	6	0.00	0.17	0.76	0.67
3 or 4	3 or 4	245	0.01	0.03	0.33	0.12	114	0.37	0.30	0.30	0.58

Notes: High school dropouts (less than 12 years of schooling) and graduates (12 years of completed schooling) measured as of age 21. Some college (13-15 years of completed schooling) and college graduates (16 or more years of completed schooling) measured as of age 24. Data from CNLSY. In our baseline steady state, no old parents that graduated from college are in the bottom income quartile.

Table C2: Relative Average Child Wages by Own Education, Early Parental Income, and Maternal Education
(Wages relative to average wages for HS graduates from early income quartile 1 whose mother is a HS graduate)

Child's Education	Early Income Quartile	Mother's Education	N	Model	Data	
HS Dropout	1	HS Dropout	286	0.82	0.92	
	1	HS Graduate	208	0.82	0.89	
	1	Some College	74	0.82	0.92	
	1	College Graduate	7	-	1.23	
	2	HS Dropout	63	0.82	0.92	
	2	HS Graduate	96	0.82	0.99	
	2	Some College	33	0.82	0.88	
	2	College Graduate	4	0.82	0.83	
	3 or 4	HS Dropout	28	0.82	1.17	
	3 or 4	HS Graduate	56	0.82	1.21	
	3 or 4	Some College	25	0.82	1.42	
	3 or 4	College Graduate	5	0.82	0.90	
	HS Graduate	1	HS Dropout	217	1.02	0.95
		1	HS Graduate	246	1.00	1.00
		1	Some College	114	0.96	0.96
		1	College Graduate	13	0.84	0.80
2		HS Dropout	81	0.97	1.08	
2		HS Graduate	153	0.99	1.12	
2		Some College	61	1.01	1.12	
2		College Graduate	11	0.84	0.70	
3 or 4		HS Dropout	39	0.84	0.99	
3 or 4		HS Graduate	128	0.87	1.20	
3 or 4		Some College	60	0.94	1.07	
3 or 4		College Graduate	20	0.91	1.20	
Some College		1	HS Dropout	71	1.19	1.22
		1	HS Graduate	136	1.26	1.03
		1	Some College	57	1.29	1.09
		1	College Graduate	13	-	1.08
	2	HS Dropout	31	1.27	1.09	
	2	HS Graduate	77	1.27	1.15	
	2	Some College	51	1.25	1.15	
	2	College Graduate	12	1.28	1.65	
	3 or 4	HS Dropout	29	1.31	1.17	
	3 or 4	HS Graduate	114	1.32	1.38	
	3 or 4	Some College	56	1.29	1.15	
	3 or 4	College Graduate	19	1.22	1.27	
	College Graduate	1	HS Dropout	19	1.37	0.97
		1	HS Graduate	32	1.74	1.37
		1	Some College	29	1.76	1.46
		1	College Graduate	6	1.77	1.35
2		HS Dropout	9	1.37	1.24	
2		HS Graduate	53	1.76	1.23	
2		Some College	36	1.76	1.30	
2		College Graduate	4	1.77	1.38	
3 or 4		HS Dropout	12	1.77	1.77	
3 or 4		HS Graduate	63	1.77	1.44	
3 or 4		Some College	45	1.77	1.22	
3 or 4		College Graduate	47	1.77	1.15	

Notes: Ratios based on average weekly wages over ages 24-35, discounting all wages to age 30 using a 5% discount rate. Average wages for high school dropouts and graduates also use measures from ages 22 and 23. Data from CNLSY.