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## An equilibrium model of turnover with belated information

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AN EQUILIBRIUM MODEL OF TURNOVER  
WITH BELATED INFORMATION

John McCall und Steven A. Lippman

Serie A — Nr. 167

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AN EQUILIBRIUM MODEL OF TURNOVER  
WITH BELATED INFORMATION

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März 1982

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This paper presents a unified treatment of matching and sequential job search. Both employer and employee follow sequential rules, and there is belated information on both sides of the market. The belated information gives rise to quits and firings. The main result is contained in Theorem 1 where equilibrium quit and firing probabilities are derived. With these probabilities equilibrium unemployment rates are then calculated.

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Serie A: Volkswirtschaftliche Beiträge

Serie B: Finanzwissenschaftliche Arbeitspapiere

Serie C: Betriebswirtschaftliche Beiträge

## 1. Introduction

The fundamental insight of search theory is that movements among jobs and between labor force participation and nonparticipation are sequential decisions based on incomplete information. A corollary to this is that labor market turnover can be viewed as a controlled stochastic process where the controls correspond to rational sequential decision-making by employers and employees when they are confronted with imperfect information. Of course, the intimate relation between turnover and unemployment accounts for the sustained interest in this problem by economists. Our view is that all forms of employment and unemployment contain a significant element of rational decision-making. This position differs with earlier theories of unemployment that concentrated on aggregates and ignored both individual decision-making and imperfect information. By emphasizing rational decision-making and imperfect information we can deduce optimal quitting policies for employees and optimal firing policies for employers. Given these optimal policies this paper then calculates the equilibrium unemployment and the equilibrium turnover associated with each permanent job. To achieve this the standard search model requires several modifications.

A major deficiency of the elementary sequential<sup>1</sup> search model is that employment constitutes an absorbing state: once an employee accepts a wage offer he remains in that job at that wage until retirement. Similarly, employers are assumed to retain workers until their retirement. Thus the elementary search model cannot comprehend quits or firings and certainly should not be asked to make predictions about either. Two models of quitting have been designed to redress these inadequacies. In the first, the individual has perfect information about the job before he accepts employment. However, he

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<sup>1</sup> The elementary model is described in Lippman and McCall (1976).

continues his search on-the-job knowing that better alternatives exist elsewhere. When a superior opportunity is revealed, he quits the current job; if the offer is sufficiently high, he ceases his on-the-job search and becomes permanently attached.<sup>2</sup> The second class of models recognizes the importance of belated information. The employer is not completely informed about the employee's productivity when he is hired. This information is conveyed only after employment commences. When this revelation occurs, the worker is either fired or retained. Just as imperfect information about the employee gives rise to fires, imperfect information about the employer generates quits. When the worker accepts a job, he is not fully informed about its attributes. When this information arrives, it may cause the worker to quit.<sup>3</sup> In this way both employer and employee engage in tentative decision-making until perfect information arrives. We are interested in studying the equilibrium amount of unemployment in this regime of joint decision-making.

Previous work on belated information analyzed the quit behavior of workers. These quit models studied only one side of the market; thus, their predictions about equilibrium phenomena like turnover and unemployment were flawed. It is the intention of this paper to repair this deficiency. This is accomplished first by constructing a belated information model of dismissals. The quit and firing models are then joined to give an equilibrium model of labor market turnover.

The first objective of this paper is to demonstrate that for given distributions of wages and productivity, an equilibrium turnover can be sustained in a two-sided model of sequential search. This turnover equilibrium is to be

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<sup>2</sup> For further discussion of on-the-job search see Burdett and Lippman and McCall (1976).

<sup>3</sup> This view of quits has been propagated by Wilde (1979), Hey and McKenna, Lippman and McCall (1980, 1981), and Viscusi.

distinguished from the equilibrium wage (price) distribution obtained by Wilde and Schwartz. The equilibrium distribution generated there required that sequential search be replaced by non-sequential search. Here the turnover equilibrium is sustained by sequential search given the wage and productivity distributions.<sup>3a</sup>

While there have been several equilibrium models of turnover, no equilibrium model has been designed in which there is sequential search and belated information on both sides of the market. These earlier models of equilibrium turnover have concentrated on other aspects of matchmaking and have not analyzed joint sequential decision-making and belatedness. The second objective of this paper is to join the theory of matching with the theory of sequential search.

Mortensen (1978) has generalized the on-the-job search model to permit both members (employer and employee) of a match to search for better partners.<sup>4</sup> For a specified employment contract the choice of search strategies by employer and employee is modeled as a two-person game. The quit and firing probabilities depend on these search strategies. Each search strategy is characterized by a reservation wage (determining when alternative matches are acceptable) and a search intensity. Both parties search without recall from known distributions. Mortensen shows that when the employment agreement specifies only the wage rate, the non-cooperative Nash solution of the game is such that the quit and firing probabilities are decreasing functions of the employee's and employer's shares of the value of the match. This solution

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<sup>3a</sup> Using different methods Feigin and Landsberger derive a stationary unemployment distribution for a sequential search model with no belated information.

<sup>4</sup> For extensions of this model see Diamond and Maskin (1979) and Mortensen (1982).

does not maximize the sum of the wealth acquired by both parties. The reason for this is that quits and fires do not consider the capital losses imposed on the other party.<sup>5</sup>

Jovanovic (1979a and 1979b) has designed two distinct equilibrium models of labor turnover. The first is a pure search theory of job change and as such is an elaboration of the standard sequential search model. Each period the worker contacts a new employer. Since there is no belated information, both employer and employee are immediately informed about marginal productivity and wages, respectively. The wage offer depends on the employer's perception of the match's quality. All search in this model is done by the worker. Given that he is currently working, the employee divides his time between on-the-job search and specific human capital accumulation. Search is conducted without recall. For a particular assumption about the evolution of the worker's productivity on his current job, the separation probability can be calculated. As would be expected, the optimal policy is such that both employees with superior matches and employees with high levels of specific human capital search less.

Jovanovic's second equilibrium model is one in which the job match is a pure experience good. There is imperfect information on both sides of the market about the location of one's optimal assignment and there is no job search. The only way to gain information about the quality of a match is to work the job. Following an assignment, the quality of the match becomes better known and reassignment may be the preferred option. The job matching model generates turnover as the matches are broken and reassignments are effected. Dismissals are accomplished by lowering the worker's wage to such a

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<sup>5</sup> Joint wealth maximization can be achieved, for example, by requiring each party to compensate the other as a precondition for separation (Mortensen [1978, p. 578]).



point that a quit is induced. Again, all separations are initiated by the employees. Jovanovic demonstrates that the optimal wage contract requires that the worker be paid his expected marginal product at each moment of time. In this model turnover occurs because of the arrival of new information about the current job match; in the pure search model turnover is in response to the arrival of new information about alternative matches.

In the following two sections belated information theories of quits and firings are presented. The next section studies and establishes the existence of the equilibria that emanate from the joint belated decision-making by employer and employee. In the final section, we use the equilibrium to decompose the search unemployment into voluntary and involuntary components.

## 2. A Belated Information Theory of Quits<sup>6</sup>

We begin by assuming that certain aspects of the job are revealed to the worker only after the job has been accepted. If these characteristics prove to be unacceptable, the worker quits and looks for another job. A worker's remuneration consists of both pecuniary (wages) and nonpecuniary benefits. Whereas nearly all pecuniary benefits are stipulated prior to accepting the job, some nonpecuniary benefits like working conditions can be determined only after the job offer is accepted and work has commenced. Thus, the worker perceives the job as possessing both search and experience qualities: the wage rate corresponds to the search quality, for it is observable during the job search, while the nonpecuniary benefits are the experience qualities as they are observed only after the job has been experienced.

We designate the wage offer by the random variable  $W$  which has distribution function  $F$  and the nonpecuniary benefits associated with an offer of  $w$  by the random variable  $Y_w$ . As the worker can anticipate the mean of nonpecuniary benefits, we assume that  $E(Y_w) = 0$  for each  $w \geq 0$ . To simplify the analysis we assume that the distribution of  $Y_w$  is independent of  $w$ . Next, we measure uncertainty or the importance of nonpecuniary benefits by the parameter  $\alpha$ . We also assume that  $Y_w = \alpha Z$  where<sup>7</sup>

$$P(Z = 1) = P(Z = -1) = \frac{1}{2}.$$

<sup>6</sup> This section presents those aspects of the quit model, developed in Lippman and McCall (1981), that are pertinent to the equilibrium. The interested reader should consult this paper for a full discussion of the quit model.

<sup>7</sup> The essential features of the model are undisturbed when  $Z$  is viewed as an arbitrary random variable with distribution  $G$ . See Lippman and McCall (1981).

Then assuming that the utility of wages and nonpecuniary benefits are linear and additive,  $X_\alpha$ , the total per period benefit of a job offer, is simply  $W + \alpha Z$  and has distribution  $F_\alpha(w) = \frac{1}{2} \{F(w+\alpha) + F(w-\alpha)\}$ . Finally the model is specified by the per period search cost  $c$  and the discount factor  $\beta$ . (Search costs are incurred at the beginning of the period and offers are received at the end of the period.)

In order to convert the nonpecuniary benefits into an experience quality, we assume that they become known to the worker exactly one period after he has accepted the job.<sup>8</sup> At that time he may choose to quit or to remain on the job for the remainder of his working life.

Let  $V_\alpha(w)$  be the maximal expected discounted benefits attainable over an infinite horizon when  $F$  is the offer distribution,  $\alpha Z$  is the nonpecuniary benefit observable only after accepting a job, and  $w$  is the wage rate of the currently available job. Similarly, let  $C_\alpha$  be the optimal return when the currently available offer is rejected. Then

$$(2.1) \quad V_\alpha(w) = \max \{C_\alpha; w + \frac{1}{2} \beta [\max(\frac{w-\alpha}{1-\beta}; C_\alpha) + \max(\frac{w+\alpha}{1-\beta}; C_\alpha)]\}$$

$$\equiv \max \{C_\alpha; S_\alpha(w)\};$$

where

$$(2.2) \quad C_\alpha = -c + \beta \int_0^\infty V_\alpha(w) dF(w).$$

Clearly, the value of continued search is  $C_\alpha$  whereas  $S_\alpha(w)$  is the return to accepting the offer and then deciding (in an optimal manner), on the basis

<sup>8</sup> This assumption can be relaxed so that the time till revelation of the nonpecuniary benefits is a geometric random variable. See Lippman and McCall (1981). In addition, it is an easy exercise to generalize the model to include  $n$  nonpecuniary aspects (i.e., experience goods) so that the searcher pays  $c_i$  to observe the first  $i$  aspects prior to working the job (but after observing the wage). The essential character of the problem is unchanged.

of the newly revealed nonpecuniary benefits, whether to quit or to remain permanently on the job. The benefit function  $V_\alpha(w)$  can be rewritten as:<sup>9</sup>

$$\begin{aligned}
 & C_\alpha, & w \leq x_\alpha & \equiv (1-\beta)C_\alpha - \frac{\beta}{2-\beta} \alpha \\
 (2.3) \quad V_\alpha(x) = & w + \frac{1}{2}\beta\left[C_\alpha + \frac{w+\alpha}{1-\beta}\right], & x_\alpha < w < y_\alpha & \equiv (1-\beta)C_\alpha + \alpha \\
 & \frac{w}{1-\beta}, & y_\alpha & \leq w,
 \end{aligned}$$

where  $x_\alpha$  is the reservation wage rate.

Label the quit and retention probabilities  $q_\alpha$  and  $p_\alpha$ , respectively (so  $q_\alpha + p_\alpha = 1$ ). Study of the searcher's propensity to quit yields (see Lippman and McCall [1981; p. 142])

Proposition 1. The retention probability decreases in  $\alpha$ ; equivalently, the quit probability increases as the importance of the nonpecuniary benefits increases.

<sup>9</sup> See Lippman and McCall (1981).

### 3. A Belated Information Theory of Firing

Prior to hiring an employee, firms are frequently unable to obtain perfect information about worker productivity. The situation is analogous to that of the job searcher. Tests can be performed to measure certain productivity characteristics before a worker is hired. These are search qualities and correspond to the wage rate. However, other characteristics such as punctuality and relations with co-workers are difficult to assess without direct on-the-job experience. We assume that these experience qualities are observed exactly one period after work commences. The search qualities are denoted by the random variable  $M$  which has distribution function  $G$  and density  $g$ ; the experience quality is given by the random variable  $N_m$  which is independent of  $m$  and has zero expected value. The magnitude of the experience quality is measured by the parameter  $\gamma$  so that  $N_m = \gamma V$ , where

$$P(V = +1) = P(V = -1) = \frac{1}{2}.$$

The search and experience qualities are linearly combined to yield a measure of worker productivity,  $L_\gamma$ . The random variable  $L_\gamma = M + \gamma V$  has distributions  $G_\gamma(m) = \frac{1}{2} \{G(m+\gamma) + G(m-\gamma)\}$ . The cost of generating an observation is  $k$  and the discount factor is  $\beta$ .

To keep the employer side of the market simple, assume that the firm receives a reward  $r > 0$  for finding a qualified worker. A worker is qualified if  $m + \gamma V$  exceeds  $L^*$ . The reward to the firm is independent of the amount by which  $L^*$  is exceeded. Similarly, the cost  $u$  for hiring an unqualified worker is independent of the amount by which  $L^*$  is undershot.<sup>10</sup>

<sup>10</sup> The firm's choice of  $L^*$  depends on  $G$  and the capital structure of the firm. The firm can adjust  $L^*$  by changing its capital structure, e.g.,  $L^*$  can be reduced by simplifying the worker's tasks. The choice of  $L^*$  is also contingent on the distribution of labor productivity. Of course, this distribution will also be responding to  $L^*$ . We do not attempt to solve this equilibrium problem.

Finally, assume  $u \geq k \geq 0$ .

Let  $R_Y(m)$  be the maximal expected discounted benefits attainable over an infinite horizon when  $G_Y$  is the distribution of total productivity and  $m$  is the total productivity level of the employee currently being interviewed. Then,

$$(3.1) \quad R_Y(m) = \begin{cases} r/[1-\beta], & \text{if } m > L^* \\ \phi_Y, & \text{if } m \leq L^* \end{cases}$$

where

$$(3.2) \quad \begin{aligned} \phi_Y &= -k + \beta \int_{-\infty}^{\infty} R_Y(m) dG_Y(m) \\ &= -k + \beta \{ \phi_Y G_Y(L^*) + \frac{r}{1-\beta} [1-G_Y(L^*)] \} \\ &= \{-k + \frac{\beta}{1-\beta} r [1-G_Y(L^*)]\} / \{1-\beta G_Y(L^*)\}. \end{aligned}$$

In addition,

$$(3.3) \quad \phi_Y' = \{g(L^*-\gamma) - g(L^*+\gamma)\} \beta(r+k) / 2 [1-\beta G_Y(L^*)]^2.$$

Now the time  $\tau_Y$  till a competent employee is found is a geometric random variable with parameter  $t_Y \equiv 1 - G_Y(L^*)$ . Observing that  $dt_Y/d\gamma = g(L^*-\gamma) - g(L^*+\gamma)$ , it follows from (3.3) that  $\tau_Y$  is stochastically decreasing and  $\phi_Y' > 0$  whence  $\phi_Y$  is increasing if  $g' \leq 0$ . If  $g' \geq 0$ , then  $\tau_Y$  is stochastically increasing and  $\phi_Y$  is decreasing in  $\gamma$ .

Turning to the firm receiving belated information, let  $W_Y(m)$  be the maximal expected discounted benefits attainable over an infinite horizon when  $G$  is the distribution of (immediately observable) productivity,  $\gamma V$  is the experience quality of productivity observable only after having hired the employee, and  $m$  is the observable productivity level of the employee currently being interviewed. Then

$$(3.4) \quad W_Y(m) = \begin{cases} B_Y, & \text{if } m \leq L^* - \gamma \\ \max [B_Y; T_Y], & \text{if } L^* - \gamma < m \leq L^* + \gamma \\ r/[1-\beta], & \text{if } m > L^* + \gamma \end{cases}$$

where

$$(3.5) \quad B_Y = -k + \beta \int_0^{\infty} W_Y(m) dG(m)$$

and

$$(3.6) \quad T_Y = r/2[1-\beta] + [-u + \beta B_Y]/2.$$

For this model, increases in  $\gamma$  cause the firm's expected profits  $B_Y$  to fall. Moreover, the reservation (observable) productivity rate  $z_Y$  is neither monotone nor continuous in  $\gamma$ . These results are summarized in

Proposition 2. The firm's expected profit  $B_Y$  decreases in  $\gamma$ , i.e.,

$$(3.7) \quad B'_Y < 0, \quad \gamma \geq 0.$$

Moreover, if  $B_0 - T_0 > 0$ , then there is a number  $b$ ,  $0 < b \leq \infty$  such that<sup>11</sup>

$$(3.8) \quad z_Y = \begin{cases} L^{*+\gamma}, & \text{if } \gamma < b \\ L^{*-\gamma}, & \text{if } \gamma > b \end{cases}$$

so  $z_Y$  increases on  $[0, b]$ , decreases on  $(b, \infty)$ , and jumps at  $b$ .

Proof. If  $B_Y \geq T_Y$ , then

$$B_Y = \{-k + \frac{\beta r}{1-\beta} [1-G(L^{*+\gamma})]\} / \{1-\beta G(L^{*+\gamma})\}$$

so that

$$B'_Y = -g(L^{*+\gamma}) [\beta k + \beta r] / [1-\beta G(L^{*+\gamma})]^2 < 0.$$

If  $B_Y < T_Y$ , then

<sup>11</sup> Suppose  $u = k$ . Then  $B_Y - T_Y$  is positive for all  $\gamma \geq 0$  if  $G(L^*) > \frac{1}{2}$  -- whence  $b = \infty$  -- and negative if  $G(L^*) < \frac{1}{2}$ . Suppose  $u \gg k$  and  $G(L^*) < \frac{1}{2}$  and  $\beta \cong 1$ . Then  $B_0 - T_0 > 0$ , but  $B_Y - T_Y < 0$  for  $\gamma$  large.

$$B_Y = \left\{ -2k + \frac{\beta}{1-\beta} r [2 - G(L^{*+\gamma}) - G(L^{*-}\gamma)] - \beta u [G(L^{*+\gamma}) - G(L^{*-}\gamma)] \right\} \\ / \{ 2 - 2\beta G(L^{*-}\gamma) - \beta^2 [G(L^{*+\gamma}) - G(L^{*-}\gamma)] \} \\ \equiv N_Y / D_Y,$$

so that

$$D_Y^2 B_Y' = [g(L^{*+\gamma}) + g(L^{*-}\gamma)] \left\{ -2\beta^2 k - \frac{2\beta r}{1-\beta} [1 - \beta^2 - \beta G(L^{*-}\gamma) (1-\beta)] \right. \\ \left. - \beta u [D_Y + G(L^{*+\gamma}) - G(L^{*-}\gamma)] \right\} - 2\beta N_Y g(L^{*-}\gamma) \\ < 0.$$

Consequently,  $B_Y' < 0$ .

Coupling  $B_Y' < 0$  and (3.8) yields  $B_Y' - T_Y' = (1 - \beta/2)B_Y' < 0$ , whence  $B_Y - T_Y$  is a decreasing (and continuous) function. By hypothesis  $B_0 - T_0 > 0$ , so the root  $b$  of  $B_Y - T_Y$  is strictly positive. Equation (3.8) easily follows from (3.4) and the definition of  $b$ . Q.E.D.

In contrast to the situation of the searcher with belated information who can and does profit from increases in the importance of the nonpecuniary benefits,<sup>12</sup> the increase in uncertainty ( $\gamma$ ) is totally detrimental to the firm's welfare; for it inhibits the firm's ability to locate qualified workers -- and there is no additional benefit to locating highly qualified vis-a-vis barely qualified workers. In fact if  $u = k = 0$ ,  $(1-\beta)B_Y/r$  is simply the minimal expected discounted time until the firm hires a qualified worker. Consequently, (3.7) reveals that the expected time until a qualified worker is hired increases in  $\gamma$ . Moreover, if  $\gamma > b$  the time  $\sigma_Y$  until the firm hires (and not merely interviews) a competent employee is not a geometric random variable, as is  $\tau_Y$ . Of course,  $\sigma_Y$  is a geometric random variable

<sup>12</sup>See Proposition 2 in Lippman and McCall (1981).



for  $\gamma < b$  and  $E\sigma_\gamma$  does increase with  $\gamma$  on  $(0,b)$  even when  $u \geq k \geq 0$ . And in any event the expected time that a vacancy waits to be filled is greater when  $\gamma V$  is an experience rather than a search quality. The difference in these expected waiting times measures the value of search information for the firm. The greater this difference the greater the incentive to acquire information about  $\gamma V$  before hiring, i.e., it may pay the firm to convert the experience quality into a search quality.

#### 4. An Equilibrium Model of Turnover With Belated Information

In this section we integrate the belated information theory of quits with the belated information theory of firing to obtain an equilibrium model of labor market turnover.

This section begins by describing the logic underlying our notion of equilibrium with a firing probability inserted in the employee model and a quit probability in the employer model. From this modified model of employee behavior the quit rate can be calculated as a function of the firing probability. Similarly, from the extended model of employer behavior, the firing probability can be calculated as a function of the quit probability. The fixed point(s) corresponding to the intersection(s) of these two functions gives the equilibrium value(s) of the quit and firing probabilities. From these the equilibrium levels of search and vacancy unemployment and the equilibrium probability of separation can be calculated. These equilibrium quantities can be studied to determine their response to variations in key parameters such as the cost of search, the distribution of marginal productivity, and the offer distribution.

##### Employee Search With Firing and Belated Information

First the employee search model with belated information of Section 2 is extended by adding the possibility of being fired. Let  $V_f(w)$  be the maximal discounted expected benefits attainable over an infinite horizon when  $F$  is the offer distribution,  $\alpha Z$  is the nonpecuniary benefit observable after the job is accepted,  $w$  is the wage rate of the currently available job, and  $f$  is the firing probability, i.e., the probability that the worker will be discharged after one period. A discharge occurs if the employer learns that

the worker's productivity (in this particular job) is below some minimally acceptable level. Similarly, let  $C_f$  be the optimal return when the current job offer is rejected.<sup>13</sup> Then

$$(4.1) \quad V_f(w) = \max\left\{w + \frac{(1-f)\beta}{2} [\max(\frac{w-\alpha}{1-\beta}, C_f) + \max(\frac{w+\alpha}{1-\beta}, C_f)] + f\beta C_f; C_f\right\},$$

where

$$C_f = -c + \beta \int_0^{\infty} V_f(w) dF(w).$$

The optimal solution to this problem induces a quit probability  $q(f)$  which we now study. We start the analysis by noting that the reservation wage rate  $x_f$  is now given by

$$(4.2a) \quad x_f = (1-\beta)C_f - \frac{\beta(1-f)}{2-\beta(1+f)} \alpha$$

where

$$(4.2b) \quad y_f = (1-\beta)C_f + \alpha.$$

The interval  $I_f$  of wages from which a quit can occur (with probability  $\frac{1}{2}$ ) is

$$(4.3) \quad I_f = (x_f, y_f).$$

In view of (4.3) it is clear that the quit probability  $q(f)$  is given by

$$(4.4) \quad q(f) = \frac{1}{2} [F(y_f) - F(x_f)] / [1 - F(x_f)],$$

The length  $J$  of the interval  $I_f$  is

$$(4.5) \quad J = \alpha \left\{ 1 + \frac{\beta(1-f)}{2 - \beta(1+f)} \right\},$$

and

$$(4.6) \quad \frac{dJ}{df} = - \frac{2\alpha(1-\beta)\beta}{[2 - \beta(1+f)]^2} < 0.$$

<sup>13</sup>

Because the parameter  $\alpha$  is not subject to variation and attention is now focused upon the firing probability  $f$ , we shall suppress the subscript  $\alpha$  and introduce a small abuse of notation by writing  $V_f$  and  $C_f$  as well as  $x_f$  and  $y_f$ .

This last fact suggests that the probability  $q(f)$  of a quit decreases as  $f$  increases. A straightforward induction argument (on  $n$  period versions of the problem in which a period corresponds to an opportunity of considering a job offer) reveals that

$$(4.7) \quad C_f' \leq 0.$$

Furthermore, if  $F$  has a continuous and strictly positive density, then it is not difficult to verify that  $C_f$  is a continuous function of  $f$  on  $[0,1]$ . It is then immediate from (4.2) that  $x_f$  and  $y_f$  are continuous so one can conclude from (4.4) that

$$(4.8) \quad q(f) \text{ is a continuous function of } f \text{ on } [0,1].$$

To ensure that  $q(f)$  is decreasing, we impose a mild restriction: the hazard rate  $h$  of the distribution function  $F$  is increasing. (Recall that the hazard rate  $h$  is defined by  $h \equiv F'/[1-F]$ .)

Proposition 3. If the hazard rate  $h$  is nondecreasing, then

$$(4.9) \quad q'(f) < 0.$$

and

$$(4.10) \quad x_f' < 0.$$

Proof: First rewrite equation (4.1) as follows:

$$\begin{aligned} V_f(w) &= f\beta C_f + (1-f\beta) \max\left\{\frac{w}{1-f\beta} + \frac{1}{2} [\max(\frac{w-\alpha}{1-\beta}, C_f) + \max(\frac{w+\alpha}{1-\beta}, C_f)]; C_f\right\} \\ &\equiv f\beta C_f + (1-f\beta) \max\left\{\frac{w}{1-f\beta} + Z_f; C_f\right\} \\ &\equiv f\beta C_f + (1-f\beta) \max\{S_f(w); C_f\}. \end{aligned}$$

Notice that

$$(4.11) \quad z_f^1 = \begin{cases} 0, & \text{if } C_f \leq \frac{w-\alpha}{1-\beta} \\ C_f^1/2, & \text{if } \frac{w-\alpha}{1-\beta} < C_f < \frac{w+\alpha}{1-\beta} \\ C_f^1, & \text{if } C_f \geq \frac{w+\alpha}{1-\beta}. \end{cases}$$

Coupling (4.7) and (4.11) yields

$$(4.12) \quad 0 \geq z_f^1 \geq C_f^1.$$

Observing that  $\frac{d}{df} \frac{w}{1-\beta f} > 0$ , it follows from (4.12) that  $S_f^1(w) > C_f^1$ . Consequently,

$$(4.13) \quad x_f^1 < 0.$$

Straightforward differentiation of (4.2) yields

$$(4.14) \quad x_f^1 = y_f^1 + \eta, \quad \text{with } \eta > 0.$$

Combining (4.14),  $y_f^1 < 0$ ,  $y_f > x_f$ , and  $h$  nondecreasing, we have

$$\begin{aligned} \frac{1-F(x_f)}{1-F(y_f)} q'(f) &= y_f^1 h(y_f) - x_f^1 h(x_f) \\ &= y_f^1 (h(y_f) - h(x_f)) - \eta h(x_f) < 0. \end{aligned}$$

Q. E. D.

### Employer Search With Quitting and Belated Information

Let us now investigate the employer side of the market. The firing model introduced in Section 3 is modified to include the probability that an employee may quit after one period. The quit is caused by receipt of unfavorable information regarding the nonpecuniary aspects of the job.

Let  $W_q(m)$  denote the maximum discounted expected benefits attainable over an infinite horizon when  $G$  is the distribution of immediately observable productivity,  $yV$  represents those aspects of productivity that are

observable only after the employee is hired,  $m$  is the observable productivity of the worker being interviewed, and  $q$  is the quit probability. Quits can occur when workers receive unfavorable belated information about the job. Similarly, let  $B_q$  be the optimal return when the currently available worker is not hired and  $T_q$  the optimal return from trying out this worker. The cost of search is  $k$ , the cost of hiring an unqualified worker is  $u$ , and the per period return from finding a qualified worker is  $r$ . Consequently,

$$(4.15) \quad W_q(m) = \begin{cases} B_q, & m \leq L^* - \gamma \\ \max[B_q, T_q], & L^* - \gamma < m \leq L^* + \gamma \\ r + \beta \left\{ (1-q) \frac{r}{1-\beta} + qB_q \right\}, & m > L^* + \gamma, \end{cases}$$

$$(4.16) \quad B_q = -k + \beta \int_0^{\infty} W_q(m) dF(m),$$

and

$$(4.17) \quad T_q = \frac{r-u}{2} + \frac{1}{2} \beta \left\{ (1-q) \frac{r}{1-\beta} + qB_q + B_q \right\}.$$

There will be no firing whenever  $B_q - T_q \geq 0$ . The next proposition asserts that this occurs if and only if the quit probability  $q$  exceeds a critical number; we label this critical number  $q^*$ .

Proposition 4. There is a number  $q^*$ ,  $0 < q^* < 1$ , such that

$$(4.18) \quad B_q - T_q \begin{cases} < 0, & \text{if } q < q^* \\ \geq 0, & \text{if } q \geq q^*. \end{cases}$$

Proof. Define  $\tau$  to be the number of workers interviewed until a worker whose observable productivity exceeds  $L^* + \gamma$  is found who also accepts the job offer. Clearly  $\tau$  is a geometric random variable with parameter  $p = [1 - G(L^* + \gamma)][1 - F(x_f)]$ . It is important to observe that  $p < 1$  so that  $\Phi < \beta$ , where  $\Phi \equiv E\beta^\tau$ .

Assume  $B_q \geq T_q$ . Then

$$B_q = -k \left( \frac{1-\beta\phi}{1-\beta} \right) + \phi \left\{ r + (1-q)\beta \frac{r}{1-\beta} + \beta q B_q \right\}$$

so that

$$(4.19) \quad B_q = \frac{1}{1-\beta\phi q} \left\{ -k \left( \frac{1-\beta\phi}{1-\beta} \right) + \phi \left[ r + \frac{\beta r}{1-\beta} (1-q) \right] \right\}.$$

Thus,

$$(4.20) \quad B'_q \equiv \frac{dB_q}{dq} = \frac{\beta\phi}{1-\beta\phi q} \left\{ B_q - \frac{r}{1-\beta} \right\} < 0,$$

for (equality holds in (4.21) if  $u = k = 0$ )

$$(4.21) \quad B_q \leq \frac{\phi r}{1-\beta}.$$

Now,

$$T_q = \frac{r-u}{2} + \frac{1}{2} \beta \left\{ (1-q) \frac{r}{1-\beta} + qB_q + B_q \right\}$$

so that

$$(4.22) \quad T'_q = \frac{1}{2} \beta \{ B_q + qB'_q + B'_q \} - \frac{1}{2} \beta \frac{r}{1-\beta}.$$

Combining (4.20) and (4.22) yields

$$(4.23) \quad T'_q = \frac{1}{2} \beta B_q \left\{ 1 + \frac{\beta\phi(1+q)}{1-\beta\phi q} \right\} - \frac{1}{2} \beta \frac{r}{1-\beta} \left\{ 1 + \frac{\beta\phi(1+q)}{1-\beta\phi q} \right\}.$$

Now from (4.20) and (4.23) we obtain

$$\begin{aligned} T'_q - B'_q &= \frac{1}{2} \beta B_q \left\{ 1 + \frac{-2\phi + \beta\phi(1+q)}{1-\beta\phi q} \right\} - \frac{1}{2} \beta \frac{r}{1-\beta} \left\{ 1 + \frac{-2\phi + \beta\phi(1+q)}{1-\beta\phi q} \right\} \\ &= \frac{\frac{1}{2}\beta}{1-\beta\phi q} \left\{ \frac{r}{1-\beta} [2\phi - \beta\phi - 1] - B_q [2\phi - \beta\phi - 1] \right\}. \end{aligned}$$

Noting that  $\phi < \beta < 1$ , we see that

$$2\phi - \beta\phi - 1 = \phi(2-\beta) - 1 < \beta(2-\beta) - 1 = -(\beta-1)^2 < 0$$

so the coefficient of  $r$  is negative. We can now employ (4.21) to conclude

$$(4.24) \quad T'_q - B'_q = \frac{\frac{1}{2}\beta}{1-\beta\phi q} [2\phi - \beta\phi - 1] \left[ \frac{r}{1-\beta} - B_q \right] < 0 .$$

Summarizing,  $T'_q - B'_q < 0$  whenever  $B_q \geq T_q$ . Consequently, there is a number  $q^*$  that satisfies (4.18).

Q.E.D.

### Equilibrium Quit and Fire Probabilities

Using (4.18) it is immediate from (4.15) that  $f(q)$ , the probability of being fired, is given by

$$(4.25) \quad f(q) = \begin{cases} f^* & \text{if } q < q^* \\ 0 & \text{if } q \geq q^*, \end{cases}$$

where

$$(4.26) \quad f^* = \begin{cases} \frac{1}{2}[G(L^*+\gamma) - G(L^*-\gamma)]/[1 - G(L^*-\gamma)] & , \text{ if } \gamma > b \\ 0 & , \text{ if } \gamma < b. \end{cases}$$

Equation (4.25) has the following interpretation. The firm should try all job candidates whose quitting probabilities are below the critical level  $q^*$  and fire them if the belated information is unfavorable. On the other hand, if the job candidate has a high probability of quitting (in excess of  $q^*$ ), he should be hired only if the firm is certain he will not be fired when the belated information is revealed, i.e., observable productivity must exceed  $L^* + \gamma$ . In other words, if the quit probability is greater than  $q^*$ , the reservation productivity level is set equal to  $L^* + \gamma$  and there is no chance of firing the worker; on the other hand, if the quit probability is less than  $q^*$ , the reservation productivity level is set equal to  $L^* - \gamma$  and  $f^*$  is the firing probability.



We are interested in finding the firing and quit equilibrium probabilities. The pair  $(f_0, q_0)$  of firing and quitting probabilities is said to be an equilibrium if and only if  $f(q_0) = f_0$  and  $q(f_0) = q_0$ . Equation (4.25) exhibits the exceedingly simple structure of  $f(q)$ . This simple structure reveals that there are but two candidates  $(f, q)$  for an equilibrium:  $(0, q(0))$  and  $(f^*, q(f^*))$ .

Invoking (4.25), we see that the candidate  $(0, q(0))$  is an equilibrium if and only if  $q(0) \geq q^*$ . Similarly,  $(f^*, q(f^*))$  is an equilibrium if and only if  $q^*(f^*) < q^*$ .

As depicted in Figures 1 and 2, the continuity of the quit probability  $q(f)$  as a function of the firing probability  $f$  coupled with the analysis above and Proposition 3 yields the existence of sustainable equilibria.

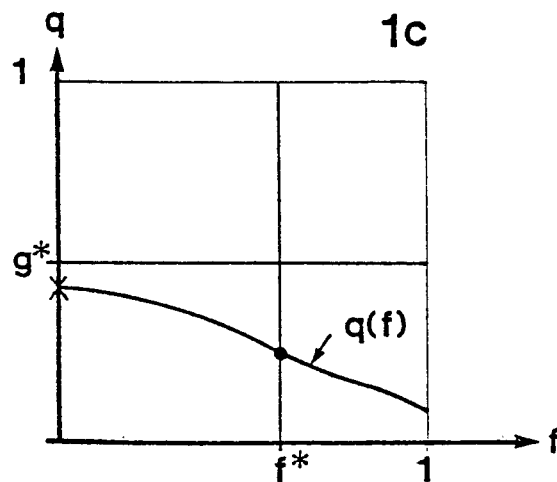
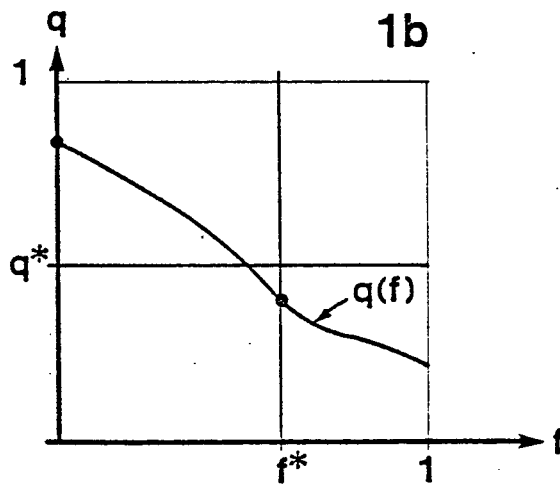
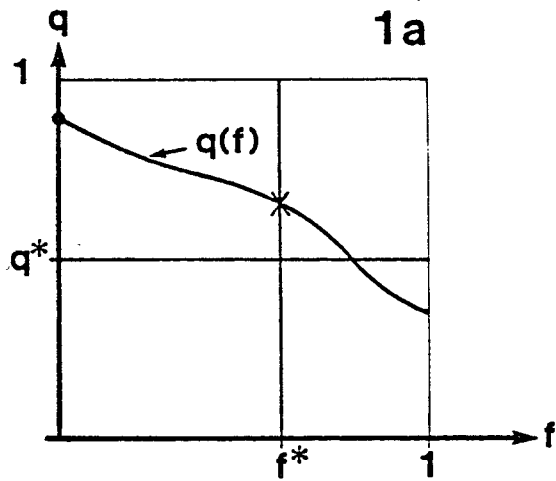
Theorem 1. If  $F$  has a nondecreasing hazard rate, there will be 1 or 2 equilibria. Otherwise, there can be 0, 1, or 2 equilibria.

Because both firing and quits are in fact observed, the equilibrium  $(0, q(0))$  is not interesting from an empirical viewpoint. Ignoring this potential equilibrium leaves us with the possibility of having either one equilibrium or no equilibrium, without regard to whether or not  $F$  has nondecreasing hazard rate.

Notice that this is the usual reactive type of strategic equilibrium. The firm makes an assumption about the average quitting behavior of the employee when it solves its optimization problem. Similarly, the employee solves his optimization problem assuming some average firing behavior for the firm. The optimization problems have an equilibrium when both of these expectational assumptions are confirmed. Hence the equilibrium indicates how agents respond to changes in reservation quantities; it does not show how they

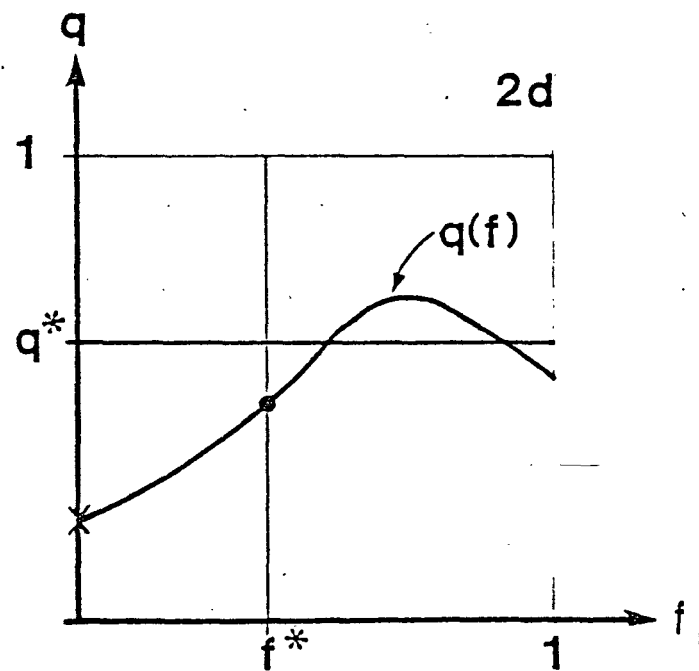
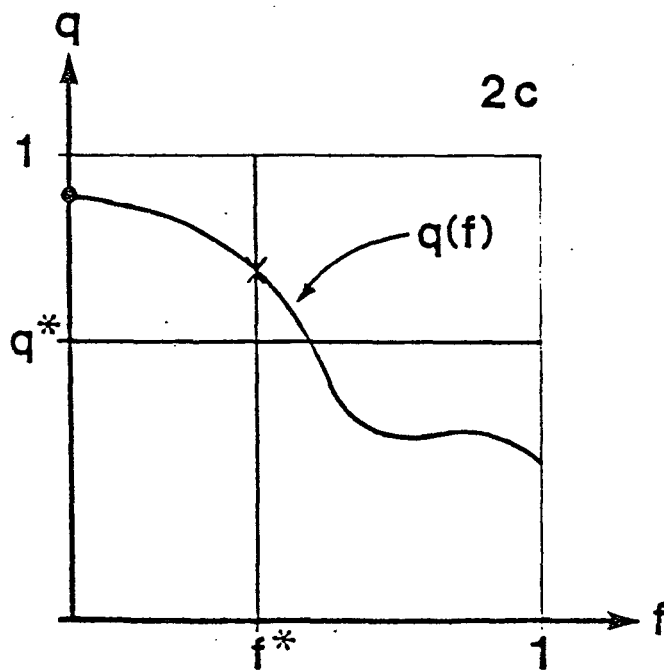
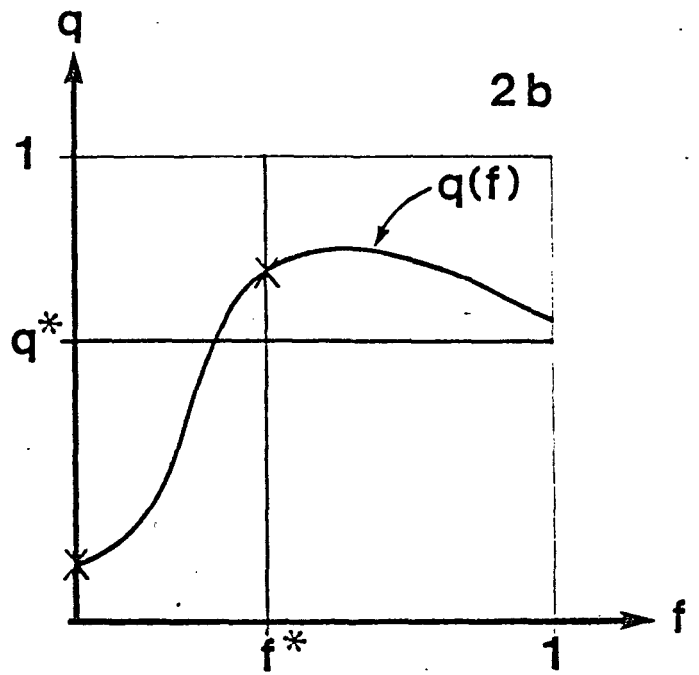
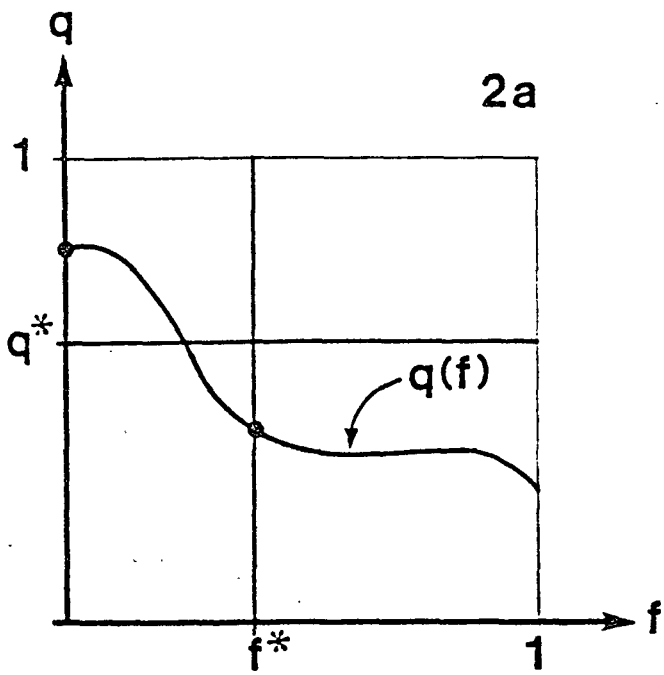
respond to changes in  $w$  and  $m$ . These latter responses can be studied only by making the wage and productivity distributions endogenous. This is an important topic for further research.

It is easy to show that (i) the equilibrium firing rate is increasing in  $\gamma$ , (ii) the equilibrium quit rate is increasing in  $\alpha$ , and (iii) the equilibrium firing rate is increasing in  $L^*$  provided  $G' > 0$ .



● signifies an equilibrium  
 X signifies no equilibrium

FIGURE 1: The Equilibria When F Has a Nondecreasing Hazard Rate



- Signifies an Equilibrium
- X Signifies no Equilibrium

FIGURE 2: The Equilibria with No Restriction on  $F$

### 5. Voluntary and Involuntary Search Unemployment

Turnover results from quits and firing. We now show how the equilibrium can be utilized to decompose the total amount of search unemployment into voluntary and involuntary components. There are two cases to consider. In the first, the uncertainty facing the firm (as measured by  $\gamma$ ) is sufficiently large ( $\gamma > b$ ) to induce experimentation with workers that may prove to be unacceptable. In the second, uncertainty is so small ( $\gamma < b$ ) that experimentation by the firm is never worthwhile.

#### Case 1 ( $\gamma > b$ ):

Let  $L_i$  be the length of the  $i^{\text{th}}$  spell of unemployment, let  $N$  be the number of spells of unemployment (so  $N$  equals the number of temporary attachments plus the number ( $\equiv 1$ ) of permanent attachments), and notice that

$$(5.1) \quad S = \sum_{i=1}^N L_i$$

is the random amount of time, punctuated by quits and fires, that the worker remains unemployed until a permanent match is made. An attachment, be it temporary or permanent, occurs if both of the inequalities  $m \geq L^* - \gamma$  and  $w \geq x_\alpha$ , representing employer and worker willingness to form a match, respectively, are satisfied. This willingness is independent of their past histories. Consequently, the  $L_i$  are independent geometric random variables with

$$(5.2) \quad EL_i = 1/[(1-G(L^*-\gamma))(1-F(x_\alpha))],$$

for the denominator in (5.2) is the probability of both parties encountering a suitable employment opportunity. The geometric random variable  $N$  is independent of the  $L_i$  so that

$$(5.3) \quad EN = 1/[(1-f^*)(1-q^*)],$$

as  $(1-f^*)(1-q^*)$  is the probability that a random attachment is permanent. Applying Wald's equation to (5.1) yields

$$(5.4) \quad ES = E(S_1) E(N).$$

Each spell of unemployment can be decomposed into voluntary and involuntary components,  $L_i^V$  and  $L_i^I$ , respectively. Hence,

$$(5.5) \quad E(L_i) = \frac{1}{1-G(L^x-\gamma)} \cdot \frac{1}{1-F(x_\alpha)} \equiv E(L_i^I) E(L_i^V).$$

Similarly, the expected number of spells of unemployment can be written as

$$(5.6) \quad EN = \frac{1}{1-f^*} \cdot \frac{1}{1-q^*} \equiv E(N^I) E(N^V).$$

Substituting (5.5) and (5.6) into (5.4), we have

$$(5.7) \quad ES = E(S^I) E(S^V),$$

where  $S^I$  and  $S^V$  are the amount of involuntary and voluntary search unemployment.

This decomposition coincides with what most economists have in mind when they distinguish between voluntary and involuntary unemployment. Of course, voluntary unemployment on the part of the worker corresponds to an involuntary vacancy on the part of the firm and vice versa.

#### Case 2 ( $\gamma < b$ ):

In this case there is no firing so that

$$ES = ES^V = \frac{1}{(1-G(L^x+\gamma))(1-F(x_\alpha))} \cdot \frac{1}{1-q^*}$$

and all unemployment is voluntary.

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