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**Working Paper**

## Optimal Contracting with Verifiable Ex Post Signals

Bonn Econ Discussion Papers, No. 19/2000

**Provided in Cooperation with:**

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*Suggested Citation:* Kessler, Anke; Lülkesmann, Christoph; Schmitz, Patrick (2000) : Optimal Contracting with Verifiable Ex Post Signals, Bonn Econ Discussion Papers, No. 19/2000, University of Bonn, Bonn Graduate School of Economics (BGSE), Bonn

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# BONN ECON DISCUSSION PAPERS

Discussion Paper 19/2000

## Optimal Contracting with Verifiable Ex Post Signals

by

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October 2000



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# Optimal Contracting with Verifiable Ex Post Signals

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and Patrick W. Schmitz\*

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October 2000

**Summary.** We study an adverse selection problem in which information that is imperfectly correlated with the agent's type becomes public ex post. Unbounded penalties are ruled out by assuming that the agent is wealth constrained. The following conclusions emerge. If the agent's utility is increasing in the contractual action (e.g. the quantity traded), the downward distortion in bad states may be strengthened. Hence, ex post information can reduce efficiency. In contrast, if the agent's utility is decreasing in the action level, there may be an upward distortion. Moreover, his rent may increase due to the ex post signal about his type. The qualitative results thus differ substantially depending on the specific situation under consideration, e.g., whether the agent is in a 'buyer' or 'seller' position. In both cases, however, additional information need not improve the efficiency of the relationship.

*JEL Classification:* D82

*Keywords:* Adverse Selection; Ex Post Information; Wealth Constraints; Upward Distortion

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# 1 Introduction

The workhorse of the literature on agency problems with adverse selection is a model in which a principal designs a contract for a privately informed agent.<sup>1</sup> The properties of this basic framework are by now well-understood: first, due to his superior information on his ‘type’, the agent can command a rent and second, because of this fact the principal optimally distorts the contracted decisions for all agents other than the ‘best type’ downwards, away from their efficient levels.

The present paper studies a natural extension of the standard principal-agent model. We consider a situation where some information about the agent’s type becomes available after the contractually specified action has been chosen, which seems to be a plausible assumption in many applications. In employment or procurement relationships, for example, ex post audits may reveal information on the agent’s productivity or production costs. Additional information may also be available without the principal taking action. Consider for instance an upstream firm that contracts as a principal with a downstream firm as an agent over the supply of an intermediate good. Clearly, the agent’s willingness to pay for the input will be related to the price which is later charged on the downstream market.<sup>2</sup> Finally, ex post information is prevalent in situations where the decisions of other parties are based on information correlated with the agent’s type as will often be the case, e.g., in an auction or product market competition context.

Obviously, if the agent’s private information is perfectly revealed ex post, he could simply be sufficiently punished whenever he did not report his type truthfully [see, e.g., Nalebuff and Scharfstein (1987)]. Indeed, even a weakly correlated ex post signal is sufficient to eliminate the agent’s rent and ensure the first-best outcome, provided the agent is risk neutral and has unlimited wealth.<sup>3</sup> For this reason, we will rule out unbounded penalties by assuming that the agent is wealth constrained. At first glance, one might guess that the presence of additional ex post information

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<sup>1</sup>See, among others, Baron and Myerson (1982), Maskin and Riley (1984), and Laffont and Tirole (1993). For models where the agent’s information is endogenously acquired, see, e.g., Cremer and Khalil (1992) and Kessler (1998).

<sup>2</sup>Similarly, Riley (1988) argues that the seller of an oilfield will want to condition the buyer’s payment on the quantity of oil extracted, which is readily observable and a noisy signal of oilfield profitability.

<sup>3</sup>General conditions on the distribution of signals for which this result holds are derived in Riordan and Sappington (1988) and Cremer and McLean (1988).

simply reduces the agent's rent and thus mitigates the downward distortion of the contracted action in some states of the world. While this intuitive reasoning is sometimes correct, there are also circumstances under which it does not apply and more surprising results hold. First, we show that if the agent's utility is increasing in the contracted action (e.g., the quantity of a good he buys from the principal), then the downward distortion may become more severe due to the presence of an ex post signal, i.e. allocative efficiency is further *reduced*. Second, if the agent's utility is decreasing in the action level (e.g., the amount of output he is to produce), then his action in bad states of the world may exceed the efficient level, i.e., it is distorted *upwards* instead of downwards. Moreover, the agent's rent may *increase* relative to a situation where the ex post signal is not available.

One general insight to be drawn from our analysis is thus that the specific situation under consideration matters. In other words, the qualitative conclusions can differ substantially depending on whether the agent is, e.g., in a 'buyer' or in a 'seller' position. Note that this is not true in the standard model where both scenarios lead to a downward distortion of the quantity traded in bad states of the world.<sup>4</sup> Intuitively, the dichotomy between 'buyers' and 'sellers' in our model emerges for the following reason. Suppose the ex post available signal indicates that the agent was not truthful, in which case the principal optimally imposes a penalty. Because the agent is wealth constrained, the extent to which he can be punished over and above the contractually specified payment depends on how large this payment is, which, in turn, depends on the contracted decision. Consider first an agent in a 'buyer' position. The lower the quantity of the good the agent purchases from the principal, the less he has to pay and the more money is left in his pocket that can serve as a penalty. Therefore, the principal optimally decreases the quantity traded beyond the level that would have been optimal in a situation where no ex post information is available and no penalties can be invoked. The converse is true if the agent is in a 'seller' position: the higher the amount of output he is to produce, the more he receives in compensation and the more severely he can be punished if the signal indicates non-compliance. Therefore, the principal optimally

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<sup>4</sup>What is important is that the single crossing property holds, i.e. absolute and marginal utility move in the same direction as we replace better types (high valuations or low costs) by worse types. This implies that better types suffer more (or benefit less) from a reduction of the quantity than worse types, so that a downward distortion makes it less attractive to mimic lower types. This logic does not depend on whether the agent is a buyer or a seller.

increases production and, as we will see, this effect may be sufficiently strong to yield production above the first-best level. Interestingly, the agent may then also enjoy a higher rent as additional information becomes available ex post. Finally, note that whether or not the principal actually wants to increase potential penalties depends on the informativeness of the ex post signal. If the signal is very precise, small penalties are sufficient and therefore the (upward or downward) distortion will be weakened in both cases.

As already indicated, our model applies to agency situations with auditing as a means to mitigate the prevailing incentive problem. For this reason, it is closely related to several contributions that investigate costly audits.<sup>5</sup> Kofman and Lawarrée (1993) consider a framework where the principal hires a (collusive) supervisor to collect information that is correlated with the agent's unknown productivity. It is shown that the principal uses the signal to extract informational rents, leaving the downward distorted action of a low-type agent largely unaffected.<sup>6</sup> The crucial difference is that the authors assume an exogenous upper bound on the maximum punishment. As a result, the penalty that can be inflicted on the agent in case of non-compliance does not vary with his transfer, and hence, with his action. Laffont and Tirole (1993) interpret limited liability in the same way we do, namely, as the inability of the principal to extract money from the agent.<sup>7</sup> In contrast to the present paper, the ex post signal (which is observed with a certain probability only) is perfectly revealing in their model. Monitoring then reduces the agent's rent and always mitigates the downward distortion. Khalil (1997) considers a setting where the principal cannot commit to costly auditing. He finds that whenever auditing occurs with positive probability, the agent receives no rent, and there is an upward distortion which increases the probability that the agent complies with the contract. The driving force behind this result is that the principal must be given sufficient incentives to audit. While it is thus different from ours, it also requires transfer-dependent penalties and relies on a similar intuition, as we discuss in more detail

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<sup>5</sup>Whether or not observing the signal involves a cost for the principal is largely irrelevant for our analysis. As the principal unambiguously benefits from additional information, all results apply under the condition that this cost is sufficiently low to be incurred in equilibrium.

<sup>6</sup>See also Baron and Besanko (1984). Only if the signal is very accurate and the informational rent is zero, the action is adjusted upward and eventually approaches its first-best level.

<sup>7</sup>The same assumption is made in Border and Sobel (1987) and Melumad and Mookherjee (1989).

below.<sup>8</sup>

Finally, upward distortions are also known from the literature on countervailing incentives, initiated by Lewis and Sappington (1989). Those models rely on the agent's reservation utility being type-dependent, which is ruled out by assumption in our framework.<sup>9</sup>

The remainder of the paper is organized as follows. Section 2 introduces the model. The optimal contract in the presence of a verifiable ex post signal is derived and discussed in Section 3. A final section concludes. All proofs are relegated to the Appendix.

## 2 The Model

Consider the following principal-agent model with adverse selection. The utility functions of the principal (P) and the agent (A) are, respectively,  $u_P = v(x, \theta) - t$  and  $u_A = u(x, \theta) + t$ , where  $x \in [\underline{x}, \bar{x}] \subset \mathbb{R}$  is a contractible action,  $\theta$  is a random parameter, and  $t \in \mathbb{R}$  is a (possibly negative) transfer payment from P to A. The parameter  $\theta$  is private information to the agent and can take only two values,  $\theta \in \{\theta_h, \theta_l\}$ . The ex ante probability that  $\theta = \theta_h$  is common knowledge and denoted by  $q \in (0, 1)$ .

To fix ideas, it will be helpful in what follows to keep in mind two applications of the above model that have been extensively studied in the literature. The first example is monopoly price discrimination where the principal is a monopolist who sells a quantity  $x$  of some good to the agent at cost  $c(x) \equiv -v(x, \theta) \geq 0$ . The buyer's utility from the good is  $u(x, \theta) \geq 0$  which is strictly increasing in the quantity bought, i.e.,  $u'(x, \theta) > 0$ . The second example is a procurement situation where the principal hires the agent to produce a quantity  $x$  of a good with  $v(x, \theta) \geq 0$  as her willingness to pay. The supplier's production costs are  $c(x, \theta) = -u(x, \theta) \geq 0$  and

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<sup>8</sup>See also Khalil and Lawarrée (2000), who analyze a principal-agent problem with unobservable effort, where the principal can commit to ex post verify either input (i.e., the agent's effort) or output (i.e., the principal's gross payoff) with a certain probability. Since one type cannot mimic another type's equilibrium input and output levels simultaneously, a dishonest agent could be punished with a certain probability, even though ultimately only one variable is observed. The out-of-equilibrium penalties are transfer-dependent and the agent may be induced to overproduce. Cf. also Footnote 19 below.

<sup>9</sup>Jullien (2000) provides a comprehensive analysis of this type of problem and further references.



strictly increasing in the quantity supplied, i.e.,  $c'(x, \theta) > 0 \Leftrightarrow u'(x, \theta) < 0$ .

Although the principal does not observe  $\theta$  directly, she has access to a verifiable signal  $s \in \{s_h, s_l\}$  which is realized after the action  $x$  has been taken and imperfectly correlated with  $\theta$ . In the monopolistic price discrimination example,  $s$  may be the final price that can be charged by the agent as an intermediate on the downstream market. In the procurement example,  $s$  could be information related to the agent's cost parameter that the principal can infer from the characteristics of the finished product or obtain through conducting audits.<sup>10</sup> Let  $\pi_{ij}$  be the probability that the signal  $s = s_i$  is realized, conditional upon a parameter value  $\theta = \theta_j$ ,  $i, j \in \{h, l\}$ . Following Kofman and Lawarrée (1993), we assume for simplicity  $\pi_u = \pi_{hh} \equiv \pi$ . Hence, the signal is 'correct' with probability  $\pi$  and 'incorrect' with probability  $1 - \pi$ . In what follows, we use  $\pi$  as a measure of the informativeness of the signal  $s$  and without loss of generality let  $\pi > \frac{1}{2}$ . If  $\pi_{hh} \neq \pi_u$ , the relevant measure would simply be  $\pi_{ii}/\pi_{jj}$ .

**Assumption 1.** The functions  $v(\cdot)$  and  $u(\cdot)$  are twice continuously differentiable, monotone, and concave in  $x$ . Total surplus  $S(x, \theta_i) \equiv v(x, \theta_i) + u(x, \theta_i)$  is strictly concave in  $x$ . Furthermore,

- a)  $u(x, \theta_h) > u(x, \theta_l) \forall x \in (\underline{x}, \bar{x}]$ ,
- b)  $u'(x, \theta_h) > u'(x, \theta_l) \forall x \in [\underline{x}, \bar{x}]$ ,
- c)  $x_i^{FB} = \arg \max S(x, \theta_i)$  satisfies  $x_l^{FB} \leq x_h^{FB}$  and  $x_i^{FB} \in (\underline{x}, \bar{x})$ ,  $i \in \{l, h\}$ ,
- d)  $\hat{x}_l(\pi) = \arg \max S(x, \theta_l) - \frac{q}{1-q} [u(x, \theta_h) - \frac{1-\pi}{\pi} u(x, \theta_l)]$  is unique and satisfies  $\hat{x}_l(\pi) \in (\underline{x}, \bar{x})$  for all values  $\pi \in [\frac{1}{2}, 1]$ .

Assumptions 1 a) and b) state that the agent's possible types  $\theta$  have a natural ordering both in absolute and marginal utilities (single crossing property). Parts c) and d) are made for convenience and allow us to focus on first-order conditions.<sup>11</sup>

<sup>10</sup>The assumption that  $s$  is observed at no cost is inconsequential for what follows. In the context of costly auditing, for instance, all our results remain valid provided it is optimal for the principal to incur the cost of an audit.

<sup>11</sup>Note that  $x_l^{FB} \leq x_h^{FB}$  is already implied by the single crossing property if the principal's utility  $v(\cdot)$  does not depend on  $\theta$ . Also observe that for any given  $q \in (0, 1)$  the objective function in Assumption 1 d) is concave if  $v(\cdot)$  is sufficiently concave.

The timing is as follows. Before contracting takes place, nature chooses  $\theta$  and the agent learns his type. Then, the principal proposes a contract to the agent which the latter can accept or reject. Focusing on the most interesting case, we assume throughout that the principal wants to contract with both types. If the agent accepts, the action  $x$  is taken in accordance with the contract. Next, the signal  $s$  is realized and the contractually specified transfer is paid. If the agent rejects, the parties obtain their reservation utilities. The sequence of events is summarized in Figure 1.

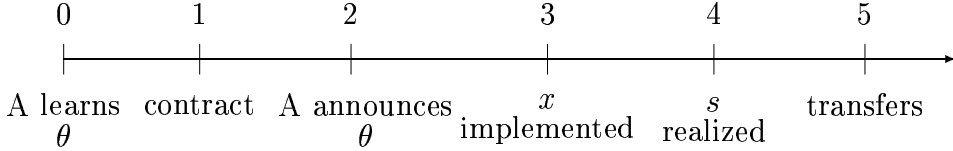


Figure 1

Assuming the principal can commit herself not to renegotiate the contract, we can invoke the Revelation Principle and confine attention to contracts  $\{x(\hat{\theta}), t(\hat{\theta}, s)\}$  that specify an action  $x$  and a transfer  $t$  as a function of the agent's report  $\hat{\theta}$  and the realization of the signal  $s$ .<sup>12</sup> Before solving for the principal's optimal contract offer, we briefly characterize as future points of reference the cases where information is symmetric and the signal is not available, respectively.

*First Best:*

If  $\theta$  is publicly observable, the principal maximizes her utility,  $u_P = v(x, \theta_i) - t$ , subject to the agent's participation constraint  $u(x, \theta_i) + t \geq \bar{u}$ , where  $\bar{u}$  denotes his reservation utility. For a type- $\theta_i$  agent, the first-best action is uniquely characterized by

$$S'(x_i^{FB}, \theta_i) = 0,$$

and the first-best transfer is  $t_i^{FB} = \bar{u} - u(x_i^{FB}, \theta_i)$ .

*Second Best:*

Next, suppose  $\theta$  is known only to the agent but the signal is not available or, equivalently, completely uninformative ( $\pi = \frac{1}{2}$ ). Let  $(x_i, t_i)$  denote the contract designed

<sup>12</sup>The reader may wonder whether it is restrictive that the signal  $s$  is realized only after  $x$  has been chosen. As will become clear below, the alternative assuming that  $s$  is observed prior the choice of  $x$  does not qualitatively affect our results.

for an agent who claims to be of type  $\theta_i$ . The individual rationality constraints are

$$u(x_i, \theta_i) + t_i \geq \bar{u} \quad i \in \{l, h\}. \quad (1)$$

In addition, truthful revelation requires

$$u(x_i, \theta_i) + t_i \geq u(x_j, \theta_i) + t_j \quad i, j \in \{l, h\}, i \neq j. \quad (2)$$

The principal maximizes her expected utility  $q[v(x_h, \theta_h) - t_h] + (1 - q)[v(x_l, \theta_l) - t_l]$ , subject to (1) and (2). It is easy to see that the incentive constraint for the low-type agent as well as the individual rationality constraint for the high-type agent are slack at the optimum and can be ignored. From the first-order conditions, the second best actions are determined by

$$\begin{aligned} S'(x_h^{SB}, \theta_h) &= 0 \Rightarrow x_h^{SB} = x_h^{FB}, \quad \text{and} \\ S'(x_l^{SB}, \theta_l) &= \frac{q}{1 - q} \phi'(x_l^{SB}) \Rightarrow x_l^{SB} < x_l^{FB}, \end{aligned}$$

where  $\phi(x) \equiv u(x, \theta_h) - u(x, \theta_l) > 0$  with  $\phi' > 0$ . The corresponding transfers are  $t_l^{SB} = \bar{u} - u(x_l^{SB}, \theta_l)$  and  $t_h^{SB} = \bar{u} - u(x_h^{FB}, \theta_h) + \phi(x_l^{SB})$ . Hence, under the no-signal second best contract, the action of a low-type agent is distorted downward and he obtains his reservation utility. In contrast, the action of the high-type agent is efficient and he earns an informational rent equal to  $\phi(x_l^{SB})$ .

### 3 Optimal Contracts with Verifiable Signal

We now turn to the case where the principal can condition the transfers specified in the contract on the verifiable signal  $s$ . Recall that due to  $\pi > \frac{1}{2}$ , the signal is informative and on average correct. Clearly, the principal will now want to lower the transfer whenever the realization of  $s$  contains evidence that contradicts the agent's claim  $\hat{\theta}$ . In order to rule out unboundedly low transfers (penalties) we assume that the agent is protected by limited liability:

**Assumption 2.** The agent is wealth constrained with initial wealth  $W \geq 0$ . Furthermore,

$$\bar{u} + W \geq \max \{u(x_h^{FB}, \theta_h), u(x_l^{SB}, \theta_l), u(x_l^{FB}, \theta_h)\}. \quad (3)$$

The first part of Assumption 2 requires transfers to satisfy the following wealth constraint:<sup>13</sup>

$$t_i(s_j) \geq -W \quad \forall i, j \in \{l, h\}. \quad (\text{WC}_{ij})$$

The second part of Assumption 2 ensures that this constraint is not binding at the first and second-best benchmark solutions determined above, so that those optimal contracts are unaffected by  $(\text{WC}_{ij})$ . This requirement also allows us to focus attention on the interesting case in which  $W$  is large enough so that the first-best is implementable for  $\pi = 1$  where the agent's type is perfectly revealed ex post.<sup>14</sup>

Given the distribution of the signal as described above, the agent's participation constraints can be written as

$$\pi t_i(s_i) + (1 - \pi)t_i(s_j) + u(x_i, \theta_i) \geq \bar{u} \quad i, j \in \{l, h\}, i \neq j. \quad (\text{IR}_i)$$

In addition, incentive compatibility now reads for  $i, j \in \{l, h\}, i \neq j$ ,

$$\pi t_i(s_i) + (1 - \pi)t_i(s_j) + u(x_i, \theta_i) \geq \pi t_j(s_i) + (1 - \pi)t_j(s_j) + u(x_j, \theta_j). \quad (\text{IC}_i)$$

The principal's problem is to choose  $\{x_i, t_i(s_j)\}$  so as to

$$\begin{aligned} \text{maximize} \quad & q \left[ v(x_h, \theta_h) - \pi t_h(s_h) - (1 - \pi)t_h(s_l) \right] \\ & + (1 - q) \left[ v(x_l, \theta_l) - \pi t_l(s_l) - (1 - \pi)t_l(s_h) \right] \\ \text{subject to} \quad & (\text{IR}_i), (\text{IC}_i) \text{ and } (\text{WC}_{ij}). \end{aligned} \quad (\text{P})$$

As can easily be seen, the principal's return from the relationship is strictly higher when the signal is available than under the optimal second-best contract. Also, if the signal is sufficiently precise, the possibility of contracting on  $s$  enables her to implement the first-best. The relevant cut-off value is given by<sup>15</sup>

$$\pi^{FB} \equiv \frac{\bar{u} + W - u(x_l^{FB}, \theta_l)}{2[\bar{u} + W - u(x_l^{FB}, \theta_l)] - \phi(x_l^{FB})} \in \left( \frac{1}{2}, 1 \right).$$

In the following, we focus our investigation on how ex post information affects the action  $x$  that is induced under the optimal contract. As this will crucially depend

<sup>13</sup>This implies in particular that penalties are transfer-dependent, which is an important property for our results as has already been indicated in the Introduction.

<sup>14</sup>To see this, suppose  $\pi = 1$  and consider the first-best contract with the transfer equal to  $-W$  in case the agent did not tell the truth. Under Assumption 2, this contract is incentive compatible because  $\bar{u} \geq u(x_l^{FB}, \theta_h) - W$  for the high-type and  $\bar{u} \geq u(x_h^{FB}, \theta_l) - W$  for the low-type agent.

<sup>15</sup>All derivations are relegated to the Appendix.

on whether the agent's utility is increasing or decreasing in  $x$ , we consider each of these two possibilities in turn.

Let us start with a situation where the agent's utility is increasing in the contractual action  $x$ , which corresponds for example to the application of monopoly price discrimination laid out above.

**Proposition 1.** *Suppose  $u(x, \theta_i)$  is a monotonically increasing function of  $x$ . Under the optimal contract with a verifiable ex post signal, the action taken by the low-type agent,  $x_l^S$ , is continuous in the informativeness  $\pi$  of the signal and efficient for  $\pi \geq \pi^{FB}$ . Otherwise, we have  $x_l^S \leq x_l^{FB}$  and there exists a value  $\underline{\pi} \in (\frac{1}{2}, \pi^{FB})$  such that*

$$\pi < \underline{\pi} \Leftrightarrow x_l^S < x_l^{SB}.$$

*The contracted action of the high-type agent is always efficient, i.e.,  $x_h^S = x_h^{FB}$  irrespective of  $\pi$ .*

Figure 2 illustrates the statement in the proposition. It depicts the action of the low-type agent as a function of the precision of the signal,  $\pi$ .

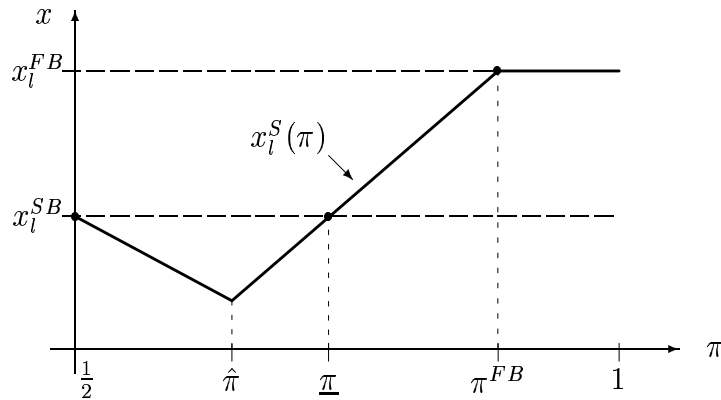


Figure 2

We see that allocative efficiency is improved relative to a situation where no signal is available *only if* the ex post signal is sufficiently precise. In particular, additional information leads the principal to reduce allocative efficiency whenever  $\pi < \underline{\pi}$ . This finding stands not only in contrast to the existing literature, but also seems to lack intuition at first glance. After all, we know that the optimal contract balances the principal's concern for efficiency with her desire to extract rents. Since additional

information mitigates the latter problem, one might expect this trade-off to change in favor of efficiency. This intuition is flawed, however, because the rent reduction achieved through the signal is in *absolute*, not *relative*, terms and it is the second effect that matters for the principal. To see this more clearly, recall that in the standard no-signal case with  $\pi = \frac{1}{2}$  the informational rent is strictly positive. For  $\pi$  not too large, this will continue to be the case and we can use (IR<sub>l</sub>) and (IC<sub>h</sub>) to write the informational rent of a high-type agent as

$$\begin{aligned} R(x_l, \pi) &= [u(x_l, \theta_h) + \pi t_l(s_h) + (1 - \pi)t_l(s_l)] - [u(x_l, \theta_l) + \pi t_l(s_l) + (1 - \pi)t_l(s_h)] \\ &= \phi(x_l) - (2\pi - 1)[t_l(s_l) - t_l(s_h)], \end{aligned} \quad (4)$$

where  $\phi(x_l) = R(x_l, \frac{1}{2})$  is the corresponding rent in the no-signal case. The last term on the right hand side of (4) reflects the expected rent reduction once ex post information is available. The principal detects a non-truthful announcement of the high-type agent with probability  $\pi$ , while for a truthful low-type agent,  $s = s_h$  only with probability  $1 - \pi$ . Since the rent reduction increases in the difference  $t_l(s_l) - t_l(s_h)$ , it is optimal for the principal to set the agent's compensation in case of confirmatory evidence as large as possible and as low as possible in case of contradicting evidence. Hence, we must have  $t_l(s_h) = -W$  (the wealth constraint is binding) and

$$t_l(s_l) = [\bar{u} - u(x_l, \theta_l) + (1 - \pi)W]/\pi$$

from (IR<sub>l</sub>). By inspection,  $t_l(s_l)$  decreases in  $x$  if  $u'(x, \theta) > 0$ . As a result, lowering  $x_l$  increases  $t_l(s_l)$  and, hence, diminishes the agent's rent relative to the no-signal second-best case as can be seen from (4). In terms of our monopoly price discrimination example where  $t_l(s_l)$  is negative, the higher the quantity sold to the buyer, the more he has to pay for the good. As the principal optimally wants to 'reward' a truthful low-type agent relative to an untruthful high-type agent, she lowers the payment of the former, which requires a further reduction in  $x_l$ . The availability of the ex post signal thus aggravates the inefficiency caused by the downward distortion of  $x_l$ , provided  $\pi$  is relatively small. Indeed, this effect becomes stronger the more precise the signal is for values  $\pi \leq \hat{\pi}$  [see Figure 2]. At  $\hat{\pi}$ , the agent's rent drops to zero and (IR<sub>h</sub>) becomes binding. Then, the above effect is no longer operative and the principal optimally increases  $x_l$  until the (IC<sub>h</sub>) constraint becomes slack at  $\pi = \pi^{FB}$ .

Next, we turn to the case where  $u'(x, \theta_i) < 0$ , which corresponds to our procurement example. A slightly different application that is often encountered and satisfies this property is the regulation of a firm with unknown cost. Let

$$\bar{\pi} \equiv \frac{u'(x_l^{FB}, \theta_l)}{u'(x_l^{FB}, \theta_l) + u'(x_l^{FB}, \theta_h)} \in \left(\frac{1}{2}, 1\right).$$

**Proposition 2.** *Suppose  $u(x, \theta_i)$  is a monotonically decreasing function of  $x$ . Under the optimal contract with a verifiable ex post signal, the low-type agent's action  $x_l^S$  is continuous in  $\pi$  and efficient for  $\pi \geq \pi^{FB}$ . Otherwise,  $x_l^S \geq x_l^{SB}$  and*

- a) if  $\bar{\pi} \geq \pi^{FB}$ , we have  $x_l^S < x_l^{FB}$  for all values  $\pi \geq \frac{1}{2}$ ;
- b) if  $\bar{\pi} < \pi^{FB}$ , we have  $x_l^S > x_l^{FB} \Leftrightarrow \pi > \bar{\pi}$ .

The contracted action of the high-type agent is always efficient, i.e.,  $x_h^S = x_h^{FB}$  irrespective of  $\pi$ .

Again, the content of this proposition is best illustrated graphically. This is done in Figure 3, which displays  $x_l^S$  as a function of  $\pi$ . Case a) is sketched by the thin line, while the thick line corresponds to case b). Two possibilities have

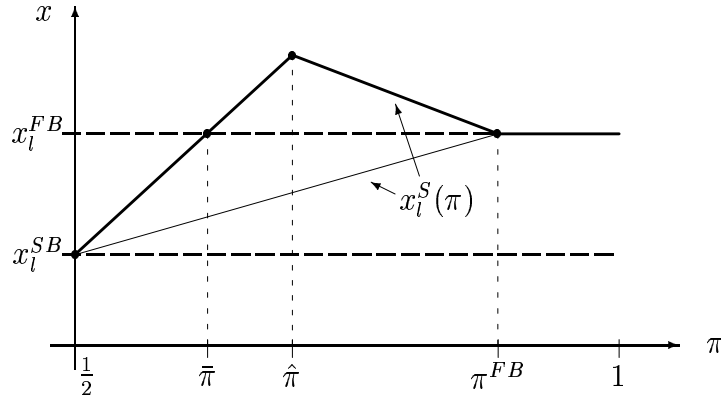


Figure 3

to be distinguished. First, if  $\bar{\pi} \geq \pi^{FB}$ , the downward distortion of  $x_l$  is simply alleviated and efficiency improves as additional ex post information is present. The more precise the signal, the smaller is the distortion, which is the obvious case that one might have expected. If  $\bar{\pi} < \pi^{FB}$ , however, the availability of noisy ex post information on the agent's type may lead the principal to distort  $x_l$  in the *opposite* direction. To understand this finding intuitively, note that the informational rent of

the high-type agent is again given by (4), provided  $R$  is strictly positive. As before, we also have  $t_l(s_h) = -W$  and  $t_l(s_l) = [\bar{u} - u(x_l, \theta_l) + (1 - \pi)W]/\pi$  under the optimal contract. Yet, contrary to the previous analysis, the reduction of the rent relative to the no-signal case,  $(2\pi - 1)[t_l(s_l) - t_l(s_h)]$ , is now increasing in  $x_l$  due to  $u'(x, \theta) < 0$ . Therefore,  $x_l$  is unambiguously larger than  $x_l^{SB}$ . While raising  $x_l$  raises  $\phi(x_l)$ , the rent reduction  $(2\pi - 1)(t_l(s_l) - t_l(s_h))$  now also increases. In terms of our procurement example where  $t_l(s_l)$  is positive, the higher the quantity the agent has to produce, the more he receives from the principal and the more he can be held liable if the signal indicates that he is a non-compliant high-type agent. This effect leads the principal to raise  $x_l$  and may be so strong that she sets  $x_l^S > x_l^{FB}$ . More formally, this situation emerges if the marginal effect on  $-u(x_l^{FB}, \theta_l)$  and, hence, on  $t_l(s_l)$  is sufficiently stronger than the marginal effect on  $\phi(x_l^{FB}) = u(x_l^{FB}, \theta_h) - u(x_l^{FB}, \theta_l)$ , i.e., if  $\bar{\pi}$  is sufficiently small. Again, the agent's rent  $R$  drops to zero and  $(IR_h)$  becomes binding at  $\hat{\pi}$  and this effect vanishes. For high values of  $\pi$ , the (downward or upward) distortion is thus alleviated and eventually, the first-best is implemented for  $\pi \geq \pi^{FB}$ .

The following corollary describes how the agent's rent is affected by ex post information.

**Corollary.** *If  $u(x, \theta_i)$  is an increasing function of  $x$ , the agent's rent is decreasing in the precision of the verifiable ex post signal,  $\pi$ . If  $u(x, \theta_i)$  is a decreasing function of  $x$ , the agent's rent decreases in  $\pi$  for  $\pi \geq \bar{\pi}$  but it may increase in  $\pi$  for some  $\pi < \bar{\pi}$ .*

Clearly, the principal's expected return never falls as additional ex post information becomes available because she could always offer the optimal second-best contract, which does not condition on the signal. Now consider Figure 2 where  $u'(x, \theta_i) > 0$ . For values of  $\pi < \hat{\pi}$ , the distortion of  $x_l$  is aggravated and expected total surplus decreases in  $\pi$ . Hence, the agent cannot be better off if additional ex post information becomes available. For  $\pi \geq \hat{\pi}$ , his rent is zero, so that a more precise ex post signal cannot help him either. However, a similar reasoning need not apply for  $u'(x, \theta_i) < 0$  as is indicated in the second part of the corollary: the agent may actually be *better* off when a verifiable ex post signal about his type becomes available. From Figure 3, it is straightforward to see that the agent's rent can only decrease for values  $\pi \geq \bar{\pi}$ . In case a), the agent's rent is zero. In case b),  $x_l$  increases above  $x_l^{FB}$ , so



that the expected total surplus again decreases (if  $\pi \leq \hat{\pi}$ ) or the agent's rent is zero (if  $\pi > \hat{\pi}$ ). However, if  $\pi < \bar{\pi}$  more precise ex post information reduces the distortion, so that total surplus increases. Then, not only the principal's profit, but also the agent's rent can increase. In particular, one can construct examples where the negative effect on  $R$  due to a higher  $\pi$  is overcompensated by the positive effect on  $R$  caused through the corresponding raise in  $x_l$ .<sup>16</sup>

To close this section, we briefly discuss how the above findings relate to Khalil (1997) who considers a situation where the principal cannot commit to conduct costly audits ex post. In his model, the agent carries out a productive task for the principal so that  $u'(x, \theta_i) < 0$  and penalties are transfer dependent.<sup>17</sup> In equilibrium, the agent lies with positive probability and the principal is indifferent between conducting an audit or not. Raising the transfer  $t_l$  increases her stakes in auditing and, as a consequence, the agent's compliance. Because the agent's rent is zero, the principal can raise  $t_l$  only by increasing  $x_l$ , which explains why there will always be overproduction in equilibrium. Although the author does not consider this case, our results and the preceding discussion suggest that there could also be further downward distortion in his model if  $u'(x, \theta_i) > 0$ . Still, the non-commitment setting is important for this intuition, as it implies that the agent's rent is always zero.<sup>18</sup> The key trade-off for the principal is therefore efficiency versus non-compliance rather than efficiency versus rent extraction as in our model.<sup>19</sup>

## 4 Concluding Remarks

In this paper, we have studied a straightforward variant of a simple principal-agent adverse selection problem in which information that is imperfectly correlated with the agent's type becomes public ex post and unbounded penalties are not feasible

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<sup>16</sup>See the Appendix for a straightforward example.

<sup>17</sup>As the author himself notes, this property is crucial for the overproduction result to hold.

<sup>18</sup>One can easily verify that under Khalil's assumptions on the parameter values and functional form of  $u(\cdot)$ , only Proposition 2 a) applies and there is no overproduction if commitment is possible.

<sup>19</sup>Note that our analysis also suggests that the usual underproduction result could be worsened in a variant of Khalil and Lawarrée's (2000) model, where either input or output can be monitored ex post, if in contrast to the authors' assumption the agent's utility were increasing in the level of his hidden action. Thus, it may well be efficiency enhancing if in their framework the principal is legally required to ex ante specify with certainty which variable (input or output) will be monitored ex post.

because the agent is wealth constrained. It was demonstrated that the standard result of a downward distortion, which is possibly mitigated as additional information of the agent's type becomes available (e.g., through audits), is not robust with regard to this modification. First, the downward distortion may be strengthened. Second, there may be an upward distortion in the presence of ex post information when the agent is protected by limited liability. Moreover, the agent's rent may increase relative to the case where no additional information is available. Those predictions can be unambiguously tied to the nature of the underlying problem: while the first possibility emerges if the agent's utility is strictly increasing in the contractually specified action, the latter only arises in situations where the reverse holds.

Although we have used a very simple framework to highlight the economic forces at work, several plausible extensions of our model could be considered. For example, we have treated the informativeness of the ex post signal as exogenously given. One natural extension in this regard would be that the information is observed only if the principal conducts costly audits. Similarly, one could imagine that she can invest into an auditing technology that generates more informative signals. As her payoff is always higher if the signal is available and monotonically increasing in its precision, however, neither extension is likely to yield additional insights. Perhaps a more promising avenue could be to investigate a situation where the agent (rather than the principal) has some influence over the information that is observable ex post. Suppose for instance, the agent can manipulate ex post information at some personal cost. Our findings then suggest that efficiency may actually be enhanced if the agent's cost of manipulating ex post information decreases.<sup>20</sup> Moreover, the agent may be better off if he can release some public information ex post, which may be an interesting topic for future research.

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<sup>20</sup>See Maggi and Rodriguez-Clare (1995) for a related result in a model based on countervailing incentives.

# Appendix

The optimal contract  $\{(t_i^S(s_j), x_i^S)\}_{i,j=l,h}$  solves the program (P), the solution of which is characterized in the lemma below. Define  $\Phi(x, \pi) \equiv u(x, \theta_h) - \frac{1-\pi}{\pi}u(x, \theta_l) - \frac{2\pi-1}{\pi}(\bar{u} + W)$  and note that

$$\hat{x}_l(\pi) = \arg \max_{x \in [\underline{x}, \bar{x}]} S(x, \theta_l) - \frac{q}{1-q} \Phi(x, \pi). \quad (5)$$

**Lemma 1.** *The optimal contract is always characterized by  $x_h^S = x_h^{FB}$ . For values of  $\pi \geq \pi^{FB}$ ,  $x_l^S = x_l^{FB}$  and  $\Phi(x_l^S, \pi) \leq 0$ . For  $\pi < \pi^{FB}$ ,  $x_l^S$  is characterized by*

a)  $\Phi(x_l^S, \pi) = 0$  if  $\Phi(\hat{x}_l(\pi), \pi) \leq 0$ ,

b)  $x_l^S = \hat{x}_l(\pi)$  otherwise.

The transfers under this contract are  $t_l^S(s_h) = -W$ ,  $t_l^S(s_l) = [\bar{u} - u(x_l^S, \theta_l) + (1-\pi)W]/\pi$  and  $t_h^S(s_h) = t_h^S(s_l) = \bar{u} - u(x_h^{FB}, \theta_h) + \max\{\Phi(x_l^S, \pi), 0\}$ .

*Proof.* Note first that by invoking the Maximum Punishment Principle [Baron and Besanko (1984)], we can set  $t_l(s_h) = -W$  and let  $t_l(s_l) \equiv t_l$  for brevity of exposition. Second, provided the (IC<sub>l</sub>) constraint is slack, we can without loss of generality assume that  $t_h(s_h) = t_h(s_l) \equiv t_h$ . In what follows, we will ignore the (IC<sub>l</sub>) constraint and later verify that it is indeed not binding under the optimal contract. Furthermore, since  $t_h^S \geq \bar{u} - u(x_h^{FB}, \theta_h) \geq -W$  by (3), the wealth constraint for the  $\theta_h$ -type agent will not be binding and can be ignored. Similarly, for values  $\pi \geq \pi^{FB}$  we have  $x_l^S = x_l^{FB}$ , implying  $t_l^S \geq -W$  by (3). An argument that the wealth constraint  $t_l \geq -W$  is not binding for  $\pi < \pi^{FB}$  is deferred to the proofs of Propositions 1 and 2 below. Rewriting the principal's payoff and the remaining constraints, the Lagrangian of the principal's problem is

$$\begin{aligned} \mathcal{L} = & q[v(x_h, \theta_h) - t_h] + (1-q)[v(x_l, \theta_l) - \pi t_l + (1-\pi)W] \\ & + \lambda_l \{\pi t_l - (1-\pi)W + u(x_l, \theta_l) - \bar{u}\} + \lambda_h \{t_h + u(x_h, \theta_h) - \bar{u}\} \\ & + \mu \{t_h + u(x_h, \theta_h) - (1-\pi)t_l + \pi W - u(x_l, \theta_h)\}. \end{aligned}$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial x_h} = qv'(x_h, \theta_h) + (\lambda_h + \mu)u'(x_h, \theta_h) = 0 \quad (6)$$

$$\frac{\partial \mathcal{L}}{\partial t_h} = -q + \lambda_h + \mu = 0 \quad (7)$$

$$\frac{\partial \mathcal{L}}{\partial x_l} = (1-q)v'(x_l, \theta_l) + \lambda_l u'(x_l, \theta_l) - \mu u'(x_l, \theta_h) = 0 \quad (8)$$

$$\frac{\partial \mathcal{L}}{\partial t_l} = -(1-q)\pi + \lambda_l \pi - \mu(1-\pi) = 0, \quad (9)$$

Inserting (7) into (6), using  $S(x, \theta) = v(x, \theta) + u(x, \theta)$ , yields

$$S'(x_h, \theta_h) = 0 \quad \Rightarrow \quad x_h^S = x_h^{FB}.$$

Next, (9) implies  $\lambda_l = (1-q) + \mu \frac{1-\pi}{\pi} > 0$ . Thus, (IR<sub>l</sub>) is always binding. We can substitute  $\lambda_l$  in (8) to obtain

$$S'(x_l, \theta_l) = \frac{\mu}{1-q} \left[ u'(x_l, \theta_h) - \frac{1-\pi}{\pi} u'(x_l, \theta_l) \right]. \quad (10)$$

Consider first  $\mu = 0$ . Then,  $x_l = x_l^{FB}$  by (10) and  $\lambda_h = q > 0$  from (7), so (IR<sub>h</sub>) is satisfied with equality. Since (IR<sub>l</sub>) is binding, in order for (IC<sub>h</sub>) to hold in this case, we must have

$$u(x_l^{FB}, \theta_h) - \frac{1-\pi}{\pi} [u(x_l^{FB}, \theta_l) - \bar{u}] - \frac{2\pi-1}{\pi} W \leq \bar{u},$$

which is equivalent to  $\Phi(x_l^{FB}, \pi) \leq 0$  or  $\pi \geq \pi^{FB}$  as defined in the text. Conversely, for  $\pi < \pi^{FB}$ ,  $\mu = 0$  yields a contradiction, so (IC<sub>h</sub>) is binding. There are two cases to distinguish:

a)  $\lambda_h > 0$  implies that both (IR<sub>h</sub>) and (IC<sub>h</sub>) are binding at the optimum.  $x_l^S$  is then implicitly characterized by

$$\Phi(x_l^S, \pi) = u(x_l^S, \theta_h) - \frac{1-\pi}{\pi} u(x_l^S, \theta_l) - \frac{2\pi-1}{\pi} (\bar{u} + W) = 0.$$

b)  $\lambda_h = 0$  yields  $\mu = q$  from (7) and  $x_l^S$  can be recovered from equation (10),

$$S'(x_l^S, \theta_l) = \frac{q}{1-q} \left[ u'(x_l^S, \theta_h) - \frac{1-\pi}{\pi} u'(x_l^S, \theta_l) \right], \quad (11)$$

implying  $x_l^S = \hat{x}_l(\pi)$ . Note that  $\lambda_h = 0$  requires  $\Phi(\hat{x}_l(\pi), \pi) \geq 0$ .

To complete the proof, it remains to show that (IC<sub>l</sub>) is not binding. Using (IR<sub>l</sub>) and (IC<sub>h</sub>), this constraint can be written as

$$u(x_h^{FB}, \theta_l) - u(x_h^{FB}, \theta_h) + \max \{ \Phi(x_l^S, \pi), 0 \} \leq 0. \quad (12)$$

Observe first that if  $\Phi(x_l^S, \pi) \leq 0$ , inequality (12) is implied by Assumption 1 a). Thus, we can confine attention to case b) where (IR<sub>h</sub>) is not binding and  $x_l^S = \hat{x}_l(\pi)$ . Next, note that for (12) to hold it suffices to prove that  $\Phi(x_l^S, \pi) \leq \Phi(x_l^{FB}, \frac{1}{2})$ , because  $\Phi(x_l^{FB}, \frac{1}{2}) = u(x_l^{FB}, \theta_h) - u(x_l^{FB}, \theta_l)$ , so (12) will again be satisfied by Assumptions 1 b) and c). Consider the following alternative contract:  $x_i = x_i^{FB}$ ,  $t_l(s_l) = t_l(s_h) = \bar{u} - u(x_l^{FB}, \theta_l)$  and  $t_h = \bar{u} - u(x_h^{FB}, \theta_h) + \Phi(x_l^{FB}, \frac{1}{2})$ . Since this contract satisfies all the constraints, the principal's expected utility under the optimal contract must weakly exceed her expected utility under the alternative contract. Hence,

$$S(x_l^S, \theta_l) - \frac{q}{1-q} \Phi(x_l^S, \pi) \geq S(x_l^{FB}, \theta_l) - \frac{q}{1-q} \Phi(x_l^{FB}, \frac{1}{2}),$$

by revealed preferences. The claim follows because  $S(x_l^{FB}, \theta_l) \geq S(x_l^S, \theta_l)$  by definition of  $x_l^{FB}$ .  $\square$

Proof of Proposition 1: As Lemma 1 already characterizes  $x_h^S$  as well as  $x_l^S$  for  $\pi \geq \pi^{FB}$ , we focus on how  $x_l^S$  varies with  $\pi$  for  $\pi < \pi^{FB}$ . Suppose  $u'(x, \theta_i) > 0 \forall x \in [\underline{x}, \bar{x}]$  and

recall from Section 2 that at  $\pi = \frac{1}{2}$ ,  $x_l^S = x_l^{SB} = \hat{x}_l(1/2)$  and  $\Phi(x_l^{SB}, 1/2) > 0$ . Since the objective function in (5) has decreasing marginal returns,  $\hat{x}_l(\pi)$  is a strictly decreasing function of  $\pi$  [see, e.g., Theorem 1 in Edlin and Shannon (1998)].<sup>21</sup> Also note that

$$\begin{aligned} \frac{d\Phi(\hat{x}_l(\pi), \pi)}{d\pi} &= \frac{\partial\Phi(\hat{x}_l(\pi), \pi)}{\partial\pi} + \frac{\partial\Phi(\hat{x}_l(\pi), \pi)}{\partial\hat{x}_l} \frac{\partial\hat{x}_l}{\partial\pi} \\ &= -\frac{1}{\pi^2} [\bar{u} + W - u(\hat{x}_l, \theta_l)] + \frac{1-q}{q} S'(\hat{x}_l, \theta_l) \frac{\partial\hat{x}_l}{\partial\pi}, \end{aligned} \quad (13)$$

where we have used the definition of  $\Phi(\cdot)$  and the fact that  $\hat{x}_l$  satisfies the first-order condition (11) to program (5). Due to  $\partial\hat{x}_l(\pi)/\partial\pi < 0$ , we must have  $\hat{x}_l < x_l^{SB}$  and, hence,  $S'(\cdot) > 0$  and  $\bar{u} + W - u(\hat{x}_l, \theta_l) > 0$  using (3) and  $u'(\cdot) > 0$ . Therefore,  $\frac{d\Phi}{d\pi} < 0$ , i.e., the agent's rent is decreasing in  $\pi$  as one would expect. Since  $\Phi(\hat{x}_l(\pi^{FB}), \pi^{FB}) < \Phi(x_l^{FB}, \pi^{FB}) = 0$ , there exists a unique  $\hat{\pi} \in (\frac{1}{2}, \pi^{FB})$  such that  $\Phi(\hat{x}_l(\hat{\pi}), \hat{\pi}) = 0$ . Now consider increasing  $\pi$  from  $1/2$  to  $\hat{\pi}$ , i.e., over the range where  $\Phi(x_l^S, \pi) > 0$  so that the  $(IR_h)$  constraint is not binding. Then, we are in case b) of Lemma 1 and  $x_l^S = \hat{x}_l(\pi)$ . Hence,  $x_l^S$  is decreasing in  $\pi$  for  $\pi \in [1/2, \hat{\pi}]$ . At  $\hat{\pi}$ , the  $(IR_h)$  constraint becomes binding. For values  $\hat{\pi} \leq \pi \leq \pi^{FB}$ , we are in case a) of Lemma 1, where  $x_l^S$  is implicitly determined by  $\Phi(x_l^S, \pi) = 0$ . By the implicit function theorem,

$$\frac{\partial x_l^S}{\partial \pi} = -\frac{\partial\Phi(\cdot)/\partial\pi}{\partial\Phi(\cdot)/\partial x_l^S} = -\frac{-\frac{1}{\pi^2} [\bar{u} + W - u(x_l^S, \theta_l)]}{\frac{1-q}{\mu} S'(x_l^S, \theta_l)}, \quad (14)$$

which is strictly positive as  $x_l^S \leq x_l^{FB}$  using a similar argument as above. Continuity follows from the Theorem of the Maximum. To summarize,  $x_l^S$  at first decreases in  $\pi$  below  $x_l^{SB}$ , obtains a minimum at  $\hat{\pi}$  and then increases again until  $x_l^S = x_l^{FB}$  at  $\pi = \pi^{FB}$ . The existence of the critical value  $\underline{\pi}$  as stated in Proposition 1 now follows from an intermediate value argument. Finally, note that (3) and  $x_l^S \leq x_l^{FB}$  together with  $u' > 0$  ensure that  $\bar{u} + W - u(x_l^S, \theta_l) \geq 0$  over the entire range. This implies  $t_l^S \geq -W$ , so that the contract characterized in Lemma 1 also satisfies the wealth constraint for the  $\theta_l$ -type agent.  $\square$

Proof of Proposition 2: Suppose  $u'(x, \theta_i) < 0 \forall x \in [\underline{x}, \bar{x}]$ . Again, we confine attention to the case where  $\pi < \pi^{FB}$  and investigate how  $x_l^S$  varies with  $\pi$ , starting from  $x_l^S = x_l^{SB} = \hat{x}_l(\frac{1}{2})$  at  $\pi = \frac{1}{2}$ . Due to  $u' < 0$ , the objective function in (5) has increasing marginal returns and  $\hat{x}_l(\pi)$  is therefore a strictly increasing function of  $\pi$ . Since  $\Phi(\hat{x}_l(\frac{1}{2}), \frac{1}{2}) > 0$ , case b) of Lemma 1 again applies for  $\pi$  sufficiently close to  $\frac{1}{2}$ , implying  $x_l^S = \hat{x}_l(\pi)$ . Moreover, note that  $\bar{\pi}$  as defined in the text is such that  $\hat{x}_l(\pi) \geq x_l^{FB} \Leftrightarrow \pi \geq \bar{\pi}$ . There are two possibilities to consider.

First, if  $\pi^{FB} < \bar{\pi}$ , then  $\hat{x}_l(\pi) < x_l^{FB}$  for all  $\pi \leq \pi^{FB}$ . Since  $\frac{\partial\Phi}{\partial\hat{x}_l} > 0$  for  $\hat{x}_l < x_l^{FB}$ , we have  $\Phi(\hat{x}_l(\pi^{FB}), \pi^{FB}) < \Phi(x_l^{FB}, \pi^{FB}) = 0$ , so that by continuity there must exist a  $\hat{\pi} \in (1/2, \pi^{FB})$  with  $\Phi(\hat{x}_l(\hat{\pi}), \hat{\pi}) = 0$  such that  $x_l^S$  is characterized as in Lemma 1 a) for  $\pi \geq \hat{\pi}$ . Over this range,  $\Phi(x_l^S, \pi) = 0$ , so that  $x_l^S$  continues to be increasing [see (14) and

<sup>21</sup>A function  $f(x, t)$  is said to display increasing (decreasing) marginal returns, if  $\frac{\partial f}{\partial x}$  is strictly increasing (decreasing) in  $t$ . This property is sometimes also referred to as (strict) supermodularity.

note that  $x_i^{SB} < x_i^S < x_i^{FB}$ . Hence,  $x_i^S$  monotonically increases in this case from  $x_i^{SB}$  to  $x_i^{FB}$  as  $\pi$  varies from  $\frac{1}{2}$  to  $\pi^{FB}$ .

Second, we may have  $\bar{\pi} < \pi^{FB}$ , so that  $\hat{x}_l(\pi) > x_i^{FB}$  for all  $\pi \in (\bar{\pi}, \pi^{FB}]$ . From  $\frac{\partial \Phi}{\partial \hat{x}_l} < 0$  for  $\hat{x}_l > x_i^{FB}$ , it follows again that  $\Phi(\hat{x}_l(\pi^{FB}), \pi^{FB}) < \Phi(x_i^{FB}, \pi^{FB}) = 0$ . Furthermore,  $\Phi(\hat{x}_l(\bar{\pi}), \bar{\pi}) = \Phi(x_i^{FB}, \bar{\pi}) > \Phi(x_i^{FB}, \pi^{FB}) = 0$ , because  $\frac{\partial \Phi}{\partial \pi} < 0$ . By continuity there thus exists a  $\hat{\pi} \in (\bar{\pi}, \pi^{FB})$  with  $\Phi(\hat{x}_l(\hat{\pi}), \hat{\pi}) = 0$  such that  $x_i^S$  is characterized as in Lemma 1 a) for  $\pi \geq \hat{\pi}$ . Over this range,  $x_i^S$  is decreasing in  $\pi$  as can be seen from (14) using  $x_i^{FB} < x_i^S$ . To summarize, for  $\bar{\pi} < \pi^{FB}$ ,  $x_i^S$  at first increases in  $\pi$  above  $x_i^{FB}$  for  $\pi > \bar{\pi}$ , obtains a maximum at  $\hat{\pi}$ , and then decreases again until  $x_i^S = x_i^{FB}$  at  $\pi = \pi^{FB}$ . Again, it is straightforward to show that  $\bar{u} + W - u(x_i^S, \theta_l) \geq 0$  over the entire range, which implies  $t_l^S \geq -W$ , so that the contract characterized in Lemma 1 is indeed optimal.  $\square$

Proof of the Corollary: The first part of the corollary follows immediately from the proof of Proposition 1. In order to prove the second part, suppose  $u'(x, \theta_i) < 0$ . The agent's rent is given by  $\max\{\Phi(\hat{x}_l(\pi), \pi), 0\}$ , where  $\Phi(\hat{x}_l(\pi), \pi)$  varies with  $\pi$  according to (13). The first term in (13) is negative due to  $\hat{x}_l \geq x_i^{SB}$  and (3). The second term is also negative if  $\hat{x}_l(\pi) \geq x_i^{FB}$ , which is equivalent to  $\pi \geq \bar{\pi}$ . For  $\hat{x}_l(\pi) < x_i^{FB}$ , however, the second term is positive. It is straightforward to construct examples where the second effect overcompensates the first effect for some  $\pi < \bar{\pi}$ . For instance, let  $u(x, \theta) = -\frac{1}{2}(x - \theta)^2$ ,  $v(x, \theta) = x$ ,  $q = .5$ ,  $\theta_l = .5$ ,  $\theta_h = 1$ , and  $W = \bar{u} = 0$ . It can easily be verified that  $\bar{\pi} = 2/3$  and  $\pi^{FB} = .8$ , and that the agent's rent is increasing for  $\pi \in [.5, .6]$ .  $\square$

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