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# Bargaining behavior in insurance markets

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## Abstract

In this article, we analyze the role of consumer bargaining behavior on prices in competitive insurance markets. We set up a model in which firms, but not applicants themselves, have the option to screen an individual's risk of incurring a loss. If prices are not listed ex-ante, a sufficiently high share of applicants who consider only one single offer ensures credibility, fostering thereby competition on high risk bargainers. If firms compete on list prices, firms coordinate on non-persuasive equilibria. An ignorant firm earns strictly positive expected profits because her offers do not influence beliefs. This dampens competition also on the price listing stage and thereby affects all consumers. The markup on list prices increases in proportion to the share of bargainers in the market.

Keywords: Insurances; Asymmetric information; Informed principal; Bargaining

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# 1 Introduction

In most insurance markets, applications of consumers who are expected to have a high risk of incurring a loss are rejected. Typically, acceptance or rejection decisions are based on the outcome of a screening, which is conducted by an insurer. For instance, in the market for life or private occupational disability insurance, acceptance or rejection decisions are based on detailed information drawn from the applicant's medical record, which the consumer is required to provide upon application. Consumers are furthermore required to provide information on their application history.<sup>1</sup> This suggests that firms seek to protect themselves against consumers who try to bargain down their prices.<sup>2</sup>

In this study, we analyze how consumer bargaining, as opposed to price taking behavior, affects rejections and market prices.

It is not obvious why rejections happen at all. With informed consumers, higher risk types would not remain uninsured, since they would be willing to pay more for their insurance. On the other hand, firms dispose of advanced technological means to estimate the statistical risk of a loss. Although a consumer is aware of his medical condition, he usually does not have the necessary experience to evaluate his own risk. A relatively young stream of literature considers the possibility that insurers are better informed than consumers. In particular, ? studies a model in which applicants can infer their default probabilities only from the contract offers they receive. This signaling structure provides an explanation for the actual occurrence of rejections: since an applicant's willingness to pay for insurance directly depends on the beliefs he holds about his risk type, an informed firm may not be able to persuade consumers that they are high risks by setting high prices. In that case, the only way for a firm to credibly signal high risk is to reject the applicant.

In this paper we study a competitive insurance market in which firms are allowed to remain uninformed. We examine the role of consumer bargaining on the ability of firms to persuade high risk individuals of their type. In particular, price takers consider only one single offer while bargainers establish competition between firms. A firm that is approached in the first place makes an offer, and bargainers, as opposed to price takers, subsequently ask both firms for better offers. Hence, although firms cannot observe bargaining behavior, they are able to

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<sup>1</sup>In particular, insurers ask whether a consumer has applied for this kind of insurance before. Note that there are no incentives for misreporting, as far as the application history and the medical record are concerned. If misreporting is detected by the time a claim is submitted, or earlier, an insurer may rescind the contract without having to pay back any paid premiums.

<sup>2</sup>An alternative, or additional, and common way to obtain such information is by means of an information exchange system. In these systems only minimal information is provided, like accept/reject decisions, such is the case of the German Hinweis- and Informationssystem (H.I.S.) or others systems of medical information without assessment results, such as the U.S. Medical Information Bureau (MIB).

discriminate between both bargaining types. Equilibria that arise are such that a firm which is approached by all consumers, price takers and bargainers, acquires information on their risk types, while the other firm remains uninformed. Due to the monopoly position that the informed firm incurs on price takers, credible signaling is never possible in the first stage and high-risk-price-taking consumers necessarily end up uninsured. By contrast, credibility on high risk bargaining consumers may or may not be given.

Our first main finding is that if applicants randomly choose where to apply in the first place (for instance because prices are not listed ex-ante), a sufficiently high share of consumers that only considers one single offer allows the informed firm to credibly signal high risk in the bargaining stage. Since equilibria are fully revealing in the price taking stage, competition works efficiently on high risk bargainers (i.e. they are offered the actuarially fair rate).

In particular, the ability to persuade high risk bargainers of their type depends on the informed firm's deviation profits on low risks in the price taking stage. A low share of bargaining applicants implies high opportunity costs from rejecting consumers. To see why this ensures credibility, consider the following belief structure: whenever accepted in the first stage, a consumer believes that he is a low risk. He accepts that he is a high risk only if he is rejected in that stage. If the share of price taking applicants is high, he may be persuaded of being a high risk even if he obtains an offer that he considers acceptable - and which is preferred to her competitor's offer - in the second stage. The reason is that a firm, faced with a low risk type consumer, does not find it profitable to reject all consumers in the first stage.

If, on the other hand, the share of price takers is low, the same beliefs do not form an equilibrium. As a result, the informed firm can then never make an offer to a high risk consumer that is actually accepted. One possible equilibrium is that in order to persuade the high risk type of his own risk condition, the firm must reject him twice. Since credibility strengthens competition, the presence of price takers in the market benefits bargaining consumers. In other words, bargainers impose a negative externality on other bargainers.

Our second main finding is that if firms compete for low risk price takers in previously published list prices, firms manage to coordinate on non-persuasive equilibria. As a result, the ignorant firm, whose offers do not influence consumer beliefs, earns strictly positive profits on high risk bargainers. This impacts list prices: on account of the incentives of not winning the price competition and thereby incurring the ignorant firm's position list price competition remains imperfect.

In order to give credibility, list prices would need to be relatively high. In particular, the lowest list price would have to exceed some critical threshold. By the standard Bertrand logic, competition works up to a certain extend, but once price offers become such that

credibility is not given, being the ignorant firm becomes relatively more attractive.

Under list price competition, the 'natural' symmetric equilibrium in which both firms earn the same profits may not exist, either as a result of the non-credibility requirement or of the maximum willingness to pay for a low risk individual. As to the latter case, different list prices are offered in equilibrium: one firm (the future ignorant one) will offer a high price in the credibility interval. Thereby, the competitor, although earning less, has no incentives to switch roles. The observation that list prices differ between firms is in line with the real market situation.

If the lowest price is not determined by the maximum willingness to pay for a low risk, a higher share of bargainers in the market increases list prices. Since additional bargainers raise both the list price, which is necessary to ensure that credibility is not given, and the 'natural' equilibrium price. In that case, the presence of bargainers hurts price taking consumers.

Perhaps surprisingly, since credibility is then never given, competition in list prices may increase joint industry profits, provided that the share of price taking consumers takes on an intermediate value. The effect is present even when price takers usually benefit from first stage competition. Such an increase is only possible if credibility is indeed mitigated without list price competition, i.e. if the share of price takers is sufficiently high. On the other hand, since their presence fosters first stage competition, their market share must also not be too high.

*Related literature* A small stream of literature analyzes the effects of consumer bargaining behavior on prices and firm profits. In ?, firms first compete for price takers in list prices and only in a second step for bargainers. They find that a higher share of bargainers dampens competition both in the price listing stage and in the bargaining stage. Their result is in line with our findings, although the effect is somewhat different: there, list prices serve as an outside option for second stage prices and thereby reduce incentives to undercut the rival. In ?, the second stage effect is also present, while the unique market list price is determined through Cournot competition. In the present study, by contrast, list prices raise due to the inability of an informed firm to credibly signal high risk. ? analyze a model in which the bargaining strategy is an endogenous decision variable of firms. ? demonstrates that list prices can jump from the fully competitive outcome to the monopoly price if the share of bargainers exceeds some critical threshold. The logic is that in this juncture firms find it more worthwhile to negotiate prices with bargainers individually.

In our paper, meaningful signaling is possible since the beliefs restrict the maximum willingness to pay for insurance. Beliefs may be such that an insurer and a high risk applicant cannot agree on a price, forcing firms to reject these individuals. As a result, equilibria with

fixed and commonly known sender preferences need not necessarily be pooling but can be separating. Put differently, the only tool to signal high risk may be to reject an applicant. In this case, we say that an insurer is not credible, or persuasive. On the other hand, credibility is given if equilibria are separating and contract offers are acceptable by all types.<sup>3</sup>

To the best of our knowledge, we are the first to point out how customer bargaining behavior affects such credible signaling. We do not allow for firm tools to generate credibility, like reputation building (e.g. ?, ?), advertising (e.g. ?) or multiple senders (e.g. ?, ?). As to what the latter is concerned, with endogenous information acquisition firms will be asymmetrically informed in equilibrium.

A large stream of literature analyzes quality signaling through prices (e.g. ?, ?, ?, ?). There, high quality sellers manage to signal their type by distorting prices upwards and thereby reducing sold quantities. ?) show that this might not be possible, i.e. high quality sellers drop out of the market if competition is strong, while ? demonstrate that equilibria exist even in a setting without additional product differentiation. This literature differs to the present study mainly in that we consider a market with inelastic demand. There, in order for separating equilibria to arise, additional conditions are required, such as buyers differing in their willingness to pay (?).

Our model adds to the literature of asymmetrically informed firms. As far as we know, a structure where one firm has private information over the customer and its competitor has only been considered in the credit lending literature, namely by ? and ?. Asymmetrically informed firms with informed customers have been studied by ? in an insurance setting and by ? and ? in the bank lending market. Asymmetrically informed firms are also considered in the literature on information sharing in competitive markets (see for instance ?, ?, ?, ? and ?). These works trade-off the ex-ante effects of decreasing uncertainty against those of a changing competitive environment. There, the symmetrically imperfectly informed firms agree ex-ante on a certain way of sharing information and are subsequently bound to report honestly.

Finally, this study provides an explanation for the advantages and disadvantages of list price competition and is thereby connected to a literature that investigates this topic (e.g. ?, ?). The thesis that intermediate shares of price takers make list price competition more attractive is a surprising and, as far as we are aware- unprecedented result.

We proceed as follows. Section 2 introduces the model. In Section 3, we analyze a market without a price listing stage. Instead, since firms are symmetric ex-ante, consumers approach one firm randomly. The competition in list prices takes place in Section 4, where we also

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<sup>3</sup>Formally, a rejection equals a contract offer that is not considered acceptable by anyone, whatever his beliefs.

shortly analyze the implications of list price setting. Section 5 concludes.

## 2 The setup

Consumers with wealth level  $W$  incur the risk of running a loss  $d$ , which is normalized to 1. Their utility is given by a VNM-function  $u(\cdot)$  where  $u'(\cdot) > 0, u''(\cdot) < 0$ . A loss occurs with some probability  $\theta \in \{L, H\}$  where  $0 < L < H < 1$ . We say  $\theta$  is a consumer's risk type. Let  $q_\theta$  with  $\sum_{\theta} q_\theta = 1$  be the commonly held priors on the distribution of risk types in the population. At the beginning of the game,  $\theta$  is unobservable for all players.

A strictly positive share  $\lambda$  of consumers does not bargain. In the following, we refer to these as price takers. Correspondingly, bargaining consumers are referred to as bargainers. Furthermore, we consider two competing risk-neutral insurance companies  $i \in \{A, B\}$ . The firms can perfectly observe an individual's risk type (although not his bargaining behavior) after having invested some arbitrarily small but strictly positive amount  $\gamma$ . We assume that this decision is not observable by other players. If a firm does not acquire information prior beliefs are preserved, unless the firm receives some meaningful signal from an informed firm. Consumers remain uninformed.

We will start by analyzing a game where prices are not listed ex-ante. The game proceeds as follows:

t=0 With probability 1/2, nature assigns consumers to firm  $i$  for  $i \in \{A, B\}$ .

t=1 Price taking stage: Firm  $i$  makes an investment choice  $\gamma_i \in \{0, \gamma\}$  and contingently observes  $\theta$ . The firm publicly announces a price offer  $p_{i,1}$ . Price takers accept the offer or choose no insurance.

t=2 Bargaining stage: Bargainers ask both firms to make better offers: firms choose whether to acquire information on consumers that have not been assigned to them. Firms simultaneously offer  $p_{A,2}^i, p_{B,2}^i$ . Bargainers choose among all offers, i.e.  $p_{i,1}, p_{i,2}^i, p_{j,2}^i$ ,<sup>4</sup> and no insurance.

*Comments on game structure.* Bertrand games may not have equilibria if utility functions exhibit discontinuities (see e.g. ?). This problem arises here because consumers update their beliefs based on the obtained contract offers which may render offers in some intervals unacceptable.<sup>5</sup> Equilibria in the bargaining subgame are affected whenever firms are asym-

<sup>4</sup>For technical convenience we assume that whenever a consumer is indifferent between offers, he chooses the offer from the firm to which he has been assigned in the first place.

<sup>5</sup>In particular, consider the following situation: let firm  $A$  be informed about  $\theta$ , while  $B$  only knows that  $\theta \in \{L, H\}$ . If the consumer, whose belief is  $L$ , is not willing to accept at least the fair price for  $H$ , i.e.  $H$ , there is an interval of prices that would never be accepted by the consumer if the offer is made by  $A$ .

metrically informed and the market situation is such that a consumer who believes to be an  $L$  type is not willing to pay at least  $H$  while, if priors are preserved, the willingness to pay is at least  $H$ . Since we are more concerned with the equilibria in which an informed firm reveals information in the price taking stage, we circumvent this problem by considering only situations where the willingness to pay of a consumer who holds prior beliefs is less than  $H$ .

*Offers.* Let  $\Omega$  be the set of possible price offers. Firm  $i$ 's offers  $p_{i,1}, p_{i,2} \in \Omega$ <sup>6</sup> and firm  $j$ 's offer,  $j \neq i$ ,  $p_{j,2} \in \Omega$  include acceptable and non-acceptable price offers. Formally, a rejection is identical to a price that is never acceptable to any consumer, whatever his beliefs. To make rejections more explicit, we denote by  $\Delta$  the set of all offers in  $\Omega$  that are not acceptable to a consumer who believes that he is an  $H$ -type. A consumer will accept an offer if his expected utility from it is at least as high as his expected utility from not buying insurance (his reservation utility). Beliefs in every period can be expressed as the expected risk which is a mapping of the set of vectors of offers in  $t$ ,  $P^t$  into the set of probability distributions over types. It is denoted as  $\hat{\theta}^t = \hat{\theta}(P^t) \in [L, H]$ . In  $t = 0$  it equals prior risk expectations, i.e.  $\hat{\theta}^0 = q_L L + q_H H$ . Then, his expected utility  $U(\cdot)$  from accepting an offer  $p$  is given by

$$U(p) = u(W - p),$$

while his expected utility from not buying any insurance is

$$U^0(\hat{\theta}^t) = (1 - \hat{\theta}^t)u(W) + \hat{\theta}^t u(W - 1).$$

A consumer judges an offer  $p$  acceptable if and only if  $U(p) \geq U^0(\hat{\theta}^t)$ .

In the following, the maximum willingness to pay for given beliefs will play an important role. In particular, three prices will be of relevance:  $\tilde{p}_L$  and  $\tilde{p}_H$  are the prices that correspond to the certainty equivalents for  $L$  and  $H$ -types, i.e.  $U(\tilde{p}_\theta) = U^0(\theta)$ . A consumer with the prior risk expectation  $\theta^0$  is indifferent between insurance at price  $\tilde{p}_{L,H}$  and no insurance. As mentioned above, to circumvent existence problems, we assume that  $\tilde{p}_{L,H} < H$ . From the definition of  $\Delta$  follows that all prices in  $\Delta$  are strictly greater than  $\tilde{p}_H$ . A firm's posterior profit is  $\pi_i(p_i) = p_i - \theta$  if her offer is selected by a consumer and zero otherwise. The lowest price a firm asks if she is informed about  $\theta$  is the fair price, i.e.  $\theta$ .

We restrict our analysis to pure strategy equilibria and full insurance contracts.<sup>7</sup>

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<sup>6</sup>For convenience, we consider the case that firm  $i$  has been approached in the first place and we drop the superscript.

<sup>7</sup>This may require further assumptions on the market situation. We discuss this topic in the last section.



*Equilibrium selection* At the beginning of the game, consumers hold beliefs  $\hat{\theta}^0$  about their risk type and the investment behavior of the two firms. Updating takes place after the price taking stage, i.e. after the first offer has been made, and after the bargaining stage. Since bargainers do not have to make a decision after the price taking stage, we are not concerned with their beliefs after the price taking stage. We assume that whenever consumers observe off-path behavior, they make inferences only about their risk type. Put differently, beliefs on investment behavior remain fixed and are never updated. We require that off-path beliefs satisfy the Intuitive Criterion by ?.

### 3 No list price competition

We start by analyzing the bargaining game in  $t = 2$ . For the moment we will assume that in  $t = 1$  firm  $i$  has acquired information. We are interested in whether the second firm  $j$  will choose to be informed.

*Symmetrically informed firms.* Consider an equilibrium in which firm  $j$  acquires information in  $t = 2$ . Let  $p_{i,t}^{\theta*}$  for  $\theta \in \{L, H\}$  be the subgame equilibrium offers made by an informed firm  $i$  in period  $t$ . Then the following properties hold.

**Lemma 1** (?) *Whenever both firms are informed, in any equilibrium it holds that  $(p_{A,2}^{L*}, p_{B,2}^{L*}) \neq (p_{A,2}^{H*}, p_{B,2}^{H*})$  and that  $p_{A,1}^{L*} = p_{B,2}^{L*} = L$ .*

The lemma states that any equilibrium in the bargaining game is separating and the competitive outcome arises on the  $L$ -type. Because profits are zero on the  $L$ -type, the second firm may as well not acquire information and always offers the equilibrium contract for the  $H$ -type in order to save costs. Furthermore, whenever no firm is informed in equilibrium, firms will always benefit from clandestinely acquiring information. The following proposition follows immediately.

**Proposition 1** *An equilibrium where firms are symmetrically (un)informed does not exist.*

*Asymmetrically informed firms.* We can therefore focus on equilibria in which firm  $j$  remains uninformed. Since  $\tilde{p}_{L,H} < H$  in any such equilibrium, the consistency condition requires first, that price taking consumers must rule out the possibility that they are a high risk type whenever accepted and offered the corresponding equilibrium price for  $L$ -type applicants and second, that high risk applicants are indeed rejected in the price taking stage. At the beginning of the bargaining stage, the uninformed firm shares the price taker's (consistent) belief. Consequently, the uninformed firm's information sets on the equilibrium path are

singletons:  $I^A = \{L\}$  is the information set when a person has been accepted and was offered the equilibrium price for an  $L$ -type in  $t = 1$ . The corresponding information set for a rejection is  $I^\Delta = \{H\}$ . We denote by  $p_{j,2}^{I^A*}$  and  $p_{j,2}^{I^\Delta*}$  equilibrium prices that are offered by the uninformed firm. The following proposition characterizes the set of possible equilibria.

**Proposition 2** *Any equilibrium where firm  $i$  has acquired information, verifies that*

- (1) *in the price taking stage,  $p_{i,1}^{L*} = \tilde{p}_L$ ,  $p_{i,1}^{H*} \in \Delta$ .*
- (2) *in the bargaining stage,  $p_{i,2}^{L*} = L$  and  $p_{j,2}^{I^A*} = L$ . Furthermore, there exists a  $\overset{\circ}{\lambda} \in (0, 1)$  such that if  $\lambda \geq \overset{\circ}{\lambda}$ , then  $p_{i,2}^{H*} = p_{j,2}^{I^\Delta*} = H$ . Otherwise,  $p_{i,2}^{H*} > H$  and  $p_{j,2}^{I^\Delta*} = \min\{p_{i,2}^{H*} - \varepsilon, \tilde{p}_H\}$  with  $\varepsilon$  being arbitrarily small. It is given by*

$$\overset{\circ}{\lambda} = \frac{H - L}{H - L + \tilde{p}_L - L}.$$

Proposition ?? states that all equilibria are separating in both the price taking stage and the bargaining stage. Low risk applicants are offered the monopoly price  $\tilde{p}_L$  in the price taking stage. It cannot be lower since any deviation from a lower price offer will be profitable and is not restricted by beliefs. Bargaining  $L$ -type applicants receive the actuarially fair price. Competition works here because information is revealed already in  $t = 1$ , which means that both firms are informed in  $t = 2$  and price offers are never restricted by beliefs on these types.

By contrast, consider  $\theta = H$  and an equilibrium in which the bargaining  $H$ -type applicant is offered the actuarially fair price by both firms. This can be an equilibrium only if a deviation on low risk consumers is not profitable in the first round, that is to say, if the firm will not earn higher profits on bargainers by rejecting all applicants in  $t = 1$  in order to make them believe that they are high risks. This in turn depends on the share of price takers in the market. A higher share implies that opportunity costs from rejecting low risk applicants are high. In particular, we say that the informed insurer is *credible* if an equilibrium in which both bargaining types are offered their fair prices exists. Credibility is given whenever  $\lambda \geq \overset{\circ}{\lambda}$ . The competitive outcome is unique whenever credibility is given. Consider a consumer who is rejected in the first stage but offered, by the same firm, a high price in the bargaining stage. If credibility is given, this person must believe that he is a high risk. This is inherent in the definition of credibility: since by employing this reject/accept strategy the informed party would earn less than equilibrium profits on the  $L$ -type, the consumer must not put positive probability on being a low risk.

On the other hand, if credibility is not given, all equilibria must be such that the informed firm earns nothing on high risk bargainers. This is because if a deviation is profitable if the

firm can hope to earn a price  $p_{i,2} = H$ , it is necessarily also profitable on higher prices. In fact, Proposition ?? states that if credibility is not given, in all equilibria, the uninformed firm earns strictly positive profits while her informed competitor earns nothing. All these equilibria are such that the consumer, in  $t = 2$ , upon observation of an offer that he would accept from the informed firm, puts strictly positive probability on  $\theta = L$ . These beliefs are always allowed by the definition of non-credibility: any deviation that would be profitable on the  $H$ -type, i.e.  $p_{j,2}^{I\Delta*} \geq p'_{i,2} \geq H$ , is also profitable on the  $L$ -type, albeit in the previous stage.

From Proposition ??, it follows that price taking consumers do not care whether credibility is given or not. Denote  $U_t$  ( $U_b$ ) a price taker's (a bargainer's) utility from buying insurance at equilibrium prices. A price taker has an expected utility of

$$\begin{aligned} E[U_t] &= q_L U(\tilde{p}_L) + q_H U(\tilde{p}_H) \\ &= q_L U^0(L) + q_H U^0(H), \end{aligned}$$

which is equal to no insurance and independent of  $\lambda$ .

However, price taking consumers impose a positive externality on bargainers. Denote  $p^H \in (H, \tilde{p}_H]$  the equilibrium price  $p_{j,2}^{I\Delta*}$  when credibility is not given. A bargainer's expected utility is given by

$$E[U_b] = \begin{cases} q_L U(L) + q_H U(H) & \text{if } \lambda \geq \hat{\lambda} \\ q_L U(L) + q_H U(p^H) & \text{otherwise.} \end{cases}$$

It remains to show that firm  $i$  indeed acquires information. This is given because firm profits are positive on the price taking  $L$ -type, while the firm would otherwise risk losses on the  $H$ -type.

For the sake of completeness we are also interested in the inverse case, in which the firstly approached firm does not acquire information while the second does. Since  $\tilde{p}_{L,H} < H$  if  $i$  is not informed,  $p_{i,1}^* = \tilde{p}_{L,H}$  and  $p_{i,1}^* \in \Delta$  are equilibrium candidates. Further, because of the firm's ignorance, any deviation does not shape consumer beliefs. Therefore, a deviation is always profitable. The following proposition summarizes the above.

**Proposition 3** *In any equilibrium, the firm to which the consumer has been initially assigned acquires information while the second firm does not. Such an equilibrium always exists. At the beginning of  $t = 1$  both firms expect to make strictly positive profits if and only if  $\lambda \leq \hat{\lambda}$ . Otherwise, the uninformed firm earns nothing.*

Clearly, the informed firm will always benefit from an increase in  $\lambda$ . On the other hand, for the given equilibria the uninformed firm will benefit from an increase in  $\lambda$  only up to a certain point,  $\hat{\lambda}$ .

## 4 List price competition

Up to now, we have assumed that the firm that makes the offer in  $t = 1$  incurs a monopoly position on price taking consumers. However, insurers sometimes compete on list prices, which in turn determine customer behavior. We are interested in how the credibility effect analyzed in the previous section affects competition on the price taking stage. For this purpose, we now adjust the game as follows: in  $t = 0$ , each firm sets a list price, denoted by  $p_A$  and  $p_B$ . Consumers then apply to the firm that offers the best list price conditions and randomize if offers are equal.<sup>8</sup> In the price taking stage, if approached by a consumer, a firm observes  $\theta^9$  and accepts or rejects the application. If accepted, the consumer is offered the list price. By modeling list price competition accordingly, we follow ? and ?.

Whether or not credibility arises in equilibrium now depends on the lowest list price which is offered in equilibrium. In the following, we show that credibility is never given in equilibrium.

Credibility can only be given if the informed firm makes strictly positive profits on price taking  $L$ -type consumers. Otherwise, rejecting low risks in the price taking stage and thereby hoping to convince consumers and the other firm that the type is  $H$  would always be profitable. The following lemma specifies this critical list price.

**Lemma 2** *There exists some  $\hat{p}(\lambda) > 0$  such that credibility is given if and only if  $p_i^* \geq \hat{p}(\lambda)$ . It is given by*

$$\hat{p}(\lambda) = \frac{1 - \lambda}{\lambda}(H - L) + L$$

As before, if credibility is not given the competitor makes strictly positive profits  $\pi_j = (p^H - H)$  on  $H$  bargaining type consumers.

Now consider some equilibrium price  $p_i^* \geq \hat{p}(\lambda)$ , such that credibility would be given. Because the uninformed firm earns nothing on bargainiers, both firms strive to be approached in the first place. Hence, competition works down to  $\hat{p}_L(\lambda)$ . It actually even works a little bit further: one firm will find it profitable to undercut this price, helping thereby her competitor to yield positive profits. As a result, credible equilibria cease to exist under list price

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<sup>8</sup>We abstract from the possibility that bargaining consumers may wish to employ a search strategy that differs from that of price taking consumers.

<sup>9</sup>This assumption makes the analysis tractable. For certain subgames following the price setting stage, subgame equilibria may not exist, or may be particularly difficult to track.

competition, independently of the share of price taking consumers in the market.

Therefore, all equilibria must exhibit non-credibility and  $p_i^* < \hat{p}(\lambda)$ . A 'natural' symmetric equilibrium price would be one where both firms earn the same. Define the according list price as

$$\mathring{p}(\lambda) \equiv \frac{1 - \lambda}{\lambda} \frac{q_H}{q_L} (p^H - H) + L.$$

It turns out that an equilibrium price  $p_i^*$  is in fact bound from above by  $\mathring{p}(\lambda)$ . The reason is simply that a deviation is always profitable for the uninformed firm if her competitor's profits exceeded her own. It should also not be lower, since otherwise, both firms will find it preferable to embrace the position of the uninformed firm. However, two restrictions apply: first,  $L$ -type consumers may not be willing to pay the corresponding price; second, as demonstrated above, the equilibrium is necessarily lower than the credibility threshold  $\hat{p}(\lambda)$ . The following proposition specifies equilibria.

**Proposition 4** *An equilibrium always exists. It has the following properties:  $p_{i,2}^{L*} = p_{j,2}^{IA*} = L$ ,  $p_{i,2}^{H*} > H$  and  $p_{j,2}^{I\Delta*} = \min\{p_{i,2}^{H*} - \varepsilon, \tilde{p}_H\}$  with  $\varepsilon$  being arbitrarily small.  $p_i^* = \min\{\tilde{p}_L; \hat{p}(\lambda) - \varepsilon; \mathring{p}(\lambda)\}$ ,  $p_j^* = \mathring{p}(\lambda)$  if  $p_i^* = \mathring{p}(\lambda)$ ,  $p_j^* = \hat{p}(\lambda)$  if  $p_i^* = \hat{p}(\lambda) - \varepsilon$ , and  $p_j^* \geq \hat{p}(\lambda)$  otherwise.*

Proposition ?? states that the lowest list price is the minimum of (1) the maximum willingness to pay for insurance for a low risk,  $\tilde{p}_L$ , (2) the credibility threshold  $\hat{p}(\lambda)$  and (3) the price at which both firms' expected profits are equal,  $\mathring{p}(\lambda)$ . It is notable that (infinitely many) truly asymmetric equilibria arise if the lowest list price is  $\tilde{p}_L$ . In that case, one firm - the future ignorant firm - will offer some list price that equals or exceeds the credibility threshold  $\hat{p}(\lambda)$ , ensuring thereby that the other firm - the future informed firm - has no incentives to seek to switch roles by offering a higher price and the ignorant firm earns strictly more than her competitor. Asymmetry is also given in (2), although list prices are arbitrarily close in equilibrium.

Like before, high risk consumers are rejected in  $t = 1$  and low risk bargainers receive the actuarially fair price. As opposed to the result of Proposition ??,  $H$  type bargainers now pay the higher price and price takers cease to impose a positive externality on them. A bargainer's expected utility is thereby independent of  $\lambda$ :

$$E[U_b^c] = q_L U(L) + q_H U(p^H).$$

By contrast, low risk price taking consumers are now offered (weakly) lower prices, albeit not the competitive contracts. A price taker's expected utility depends on the share of

bargaining consumers in the market. It is now given by

$$E[U_t^c] = q_L U(\min\{\tilde{p}_L; \hat{p}(\lambda) - \varepsilon; \mathring{p}(\lambda)\}) + q_H U^0(H),$$

a function that is weakly increasing in  $\lambda$ . In particular,  $\hat{p}(\lambda)$  and  $\mathring{p}(\lambda)$  are decreasing in  $\lambda$ , while  $\tilde{p}_L$  is independent of the parameter. Also note that the relationship of  $\hat{p}(\lambda)$  and  $\mathring{p}(\lambda)$  is independent of  $\lambda$ . To conclude, while price takers may benefit bargainers if prices are not listed, bargainers make price takers worse off if prices are listed.

*List price competition and industry profits.* From an industry point of view, setting list prices ex-ante may increase or decrease joint profits. Obviously, price competition is of no use or detrimental if  $\lambda \leq \mathring{\lambda}$ , i.e. if credibility is not given anyway. However, for larger values of  $\lambda$  there is a trade-off between the loss on price takers versus higher profits on bargaining consumers. To see that ex-ante price competition can sometimes be profitable, note that  $\tilde{p}_L = \hat{p}(\mathring{\lambda})$ . If  $\hat{p} < \mathring{p}$  there is an interval of  $\lambda$  with a lower bound at  $\mathring{\lambda}$  for which list price competition is profitable. The reason is that profits on a bargaining  $H$ -type exhibit a jump at  $\mathring{\lambda}$  while  $p_i^*$  is a continuous function of  $\lambda$ .

On the other hand, consider a value  $\lambda$  sufficiently close to 1. Both  $\hat{p}(1)$  and  $\mathring{p}(1)$  approach  $L$  which is clearly smaller than  $\tilde{p}_L$ , the profit on price taking  $L$ -type consumers when prices are not listed ex-ante. In that case, price competition is detrimental to the industry.

## 5 Discussion

Rejections are widely observed in insurance markets. We have studied a model which rationalizes rejections and we have investigated how consumer bargaining behavior affects the market. A rejection is due to the lack of instruments to signal high risk. Due to the monopoly position that a firm incurs on price takers, credible signaling is never possible in the first stage. However, if firms do not compete in list prices but, instead, make an individual offer upon application, an informed firm can credibly signal high risk in the bargaining stage if the share of bargaining consumers does not exceed some critical threshold. Firms can and might want to mitigate this effect - credibility reduces joint industry profits - by competing in list prices. This allows them to coordinate on non-credible equilibria, but decreases profits on price taking consumers.

In short, the presence of (other) bargainers hurts some consumers: either the bargainers themselves, as in the case without list price competition, or the price takers, as is the case with list price competition. This result is in line with the literature on bargaining behavior, although the effect at work is different.

We have mentioned in the introduction that insurers usually exchange information about customer data through the means of an information sharing system. Alternatively, or additionally, they require consumers to reveal their application history. Our model helps to understand the use of such procedures: they help firms to gather information on the bargaining behavior of applicants. In particular, a firm that is not aware of the actual stage of the game may find it useful to always acquire information that leaves firms symmetrically informed in equilibrium. ? has demonstrated that two firms may make positive profits on bargaining consumers if they are symmetrically informed, these equilibria however appear unintuitive. This is owed to the fact that most equilibrium selection concepts, including the Intuitive Criterion by ?, were initially developed for a single signaling party. A more selective concept for this purpose has been proposed by ? which, in our setting, uniquely selects the fully competitive equilibrium. Thus, symmetric information, i.e. the non-existence of an information sharing system, weakly lowers the second moving firm's profits while it does not affect her competitor's well-being.

In the model studied here, the value of non-credibility is owed to the presence of only two firms in the market. If more firms compete on the bargaining stage, the effect vanishes due to the Bertrand structure. It can be restored, however, if competition is imperfect. In particular, it is visible if the informed firm can find no price at which her expected equilibrium profit on a low risk type is higher than the deviation profit on the same type. Otherwise, rejections are not expressed twice, although the credibility issue may still affect price setting behavior. We leave this issue for further research.

In order to rationalize rejections in the way we have done, it is necessary to consider a discrete risk type distribution. Alternatively, our model can be interpreted as a continuous type space with imperfect screening technology: consumer willingness to pay will change, but the logic remains the same.

Finally, firms may also offer partial insurance contracts while we have only taken into account full insurance. We believe that this is not so much of a concern here, as applicability seems limited in life or private occupational disability insurances. Furthermore, it is of no relevance to our analysis, as long as risk types are sufficiently distinct.

# A Appendix

## Proof of Lemma ??

See ?, Proposition 6.

## Proof of Proposition ??

First, there is no equilibrium in which both firms are informed. From Lemma ??,  $p_{i,2}^{L*} = p_{j,2}^{L*} = L$  and profits are zero on a bargaining  $L$  type. A firm that is not informed can always secure at least zero profits by deviating to  $p_{j,2}^{H*}$ . This will be profitable since information acquisition is costly.

Second, there is no equilibrium in which no firm is informed. Consider such an equilibrium. Then, by the standard Bertrand logic,  $p_{i,2}^* = p_{j,2}^* = E[\theta]$  in equilibrium. In that case information acquisition by firm  $j$  and  $p_{j,2}^{L*} < E[\theta]$  is a profitable deviation.

## Proof of Proposition ??

We first show that  $p_{i,1}^{L*} \neq p_{i,1}^{H*}$ .

By contrast, consider  $p_{i,1}^{L*} = p_{i,1}^{H*}$ .  $p_{i,1}^{L*} = p_{i,1}^{H*} \geq H$  can only be an equilibrium if  $p_{i,1}^{L*}, p_{i,1}^{H*} \in \Delta$  since  $\tilde{p}_{L,H} < H$ . In that case, consider some equilibrium price  $p_{i,2}^{L*}$  in the bargaining game. If  $p_{i,2}^{L*} > L + \gamma$ ,  $j$  will find it profitable to acquire information. Consider instead  $p_{i,2}^{L*} \leq L + \gamma$ . Then, upon observation of an  $L$  type, in  $t = 1$ , a deviation is always profitable if  $\lambda(\tilde{p}_L - L) > (1 - \lambda)\gamma$ . This is given since  $\gamma$  is arbitrarily small. On the other hand, consider  $p_{i,1}^{L*} = p_{i,1}^{H*} < H$ . But firm  $i$  never strictly gains on bargaining  $H$ -type consumers:  $j$  is not informed. Therefore,  $j$ 's offer is not restricted by beliefs and  $i$  cannot make positive profits on the bargaining  $H$ -type. A deviation to  $p_{i,1} \geq H$  is then always profitable.

(1) Bargaining stage.

Given that first round prices are separating, it must be that  $p_{i,2}^{L*} = p_{j,2}^{L*} = L$ . This follows immediately from the standard Bertrand logic.

The informed party makes no positive profits on a bargaining  $H$ -type either. Consider by contrast the case in which a price  $\tilde{p}_H \geq p_{i,2}^{H*} > H$  is accepted by the bargainer with strictly positive probability. Then, it must hold that  $p_{j,2}^{I\Delta*} \geq p_{i,2}^{H*}$ . In any case, a deviation for  $j$  to a lower price is always profitable since  $j$ 's offer does not influence consumer beliefs.

In particular, it must be that either  $p_{i,2}^{H*} = H$  and  $p_{j,2}^{I\Delta*} = H$  or  $p_{i,2}^{H*} > H$  and  $p_{j,2}^{I\Delta*} = \min\{p_{i,2}^{H*} - \epsilon; \tilde{p}_H\}$  with  $\epsilon$  being arbitrarily small.

Consider the Bertrand outcome  $p_{i,2}^{H*} = p_{j,2}^{I\Delta*} = H$ . This can be an equilibrium only if firm  $i$  does not want to deviate in the first round and instead reject an  $L$ -type in the first place. Let  $\lambda$  be such that this deviation is profitable. Obviously, firm  $i$  will never offer a lower price. Therefore, an equilibrium must be such that an offer  $p_{i,2}^{H*}$  is never accepted by bargaining



applicants. This is possible for any price which is strictly greater than  $H$  because it can always be undercut by  $j$ .  $i$  has no incentive to deviate to a lower price if consumers then believe that  $\theta = L$ . This is always allowed since the deviation was profitable on the  $L$ -type. By contrast, let  $\lambda$  be such that the deviation on the  $L$ -type is not profitable and consider an equilibrium in which  $j$ 's offer is accepted with probability 1. In order for firm  $i$  not to deviate, we require that the belief  $L$  is allowed on a deviation by  $i$ . It is allowed if this deviation is admissible on the  $L$ -type. But we know that a deviation to a price  $H$  is not admissible on an  $L$ -type. Therefore,  $i$  will always find a profitable deviation from  $p_{i,2}^{H*}$  that is sufficiently close to  $H$  and that is not admissible on  $L$ .

$\overset{\circ}{\lambda}$  is uniquely determined by

$$\overset{\circ}{\lambda}(\tilde{p}_L - L) = (1 - \overset{\circ}{\lambda})(H - L)$$

which gives  $\overset{\circ}{\lambda} = \frac{H-L}{H-L+\tilde{p}_L-L}$  with  $\overset{\circ}{\lambda} \in [1/2, 1]$ .

(2) Price taking stage.

We now show that  $p_{i,1}^{H*} \in \Delta$ . By contrast, consider  $H \leq p_{i,1}^{H*} \leq \tilde{p}_H$ . This can be an equilibrium only if the offer is considered acceptable in equilibrium (i.e. the consumer believes  $H$ ) and if firm  $i$  does not want to deviate to this offer upon observation of an  $L$ -type. But in any equilibrium with  $p_{i,1}^{L*} \neq p_{i,1}^{H*}$ , the informed firm cannot make positive profits on the bargaining  $L$ -type. A deviation to a higher price is therefore always profitable.

Also,  $p_{i,1}^{H*} < H$  is not an equilibrium since firm  $i$  makes no positive profits on bargaining  $H$ -type consumers. A deviation to a higher price is therefore profitable whatever the beliefs of the other players.

Then, we show that  $p_{i,1}^{L*} = \tilde{p}_L$ . Assume not and consider  $p_{i,1}^{L*} > \tilde{p}_L$ . A deviation to  $p_{i,1} = \tilde{p}_L$  is clearly always profitable no matter how this deviation affects beliefs. Similarly, if  $p_{i,1}^{L*} < \tilde{p}_L$ , a deviation to  $p_{i,1} = \tilde{p}_L$  is profitable on a  $L$ -type, no matter how beliefs are affected.

### Proof of Proposition ??

First, we show that  $i$  acquires information. The ex-ante profit from acquiring information is  $\pi_i^{*,e} = \lambda(\tilde{p}_L - L) > 0$  whereas the deviation profit is  $\pi_i^{d,e} = 0$  if the firm rejects the consumer in the first place, and  $\pi_i^{d,e} = \lambda(q_L(\tilde{p}_L - L) - q_H(H - \tilde{p}_L)) < \pi_i^{*,e}$  if he offers  $\tilde{p}_L$ . Any other offer is either not considered acceptable and yields no profits, or yields even lower profits than the offer  $\tilde{p}_L$ .

Second, there is no equilibrium in which  $i$  is not informed while  $j$  is. In such an equilibrium,  $i$  offers, in the price taking stage some price  $p_{i,1}^*$  to all types. The price is acceptable if and only if  $p_{i,1}^* \leq \tilde{p}_{L,H}$ . Consider  $p_{i,1}^* > \tilde{p}_{L,H}$ . In that case, no consumer accepts. To acquire

information is always profitable because  $i$  can make an offer to a  $L$ -type that is considered acceptable, yields strictly positive profits, and does not influence consumer beliefs. By contrast, consider  $p_{i,1}^* \leq \tilde{p}_{L,H}$ . All types accept this offer, but profits are strictly negative on  $H$ -type consumers. Therefore, acquiring information and rejecting the  $H$ -type is a profitable deviation.

### Proof of Lemma ??

From Proposition ??, Part (1), if actions are separating on the price taking stage,  $p_{i,2}^{L*} = p_{j,2}^{IA*} = L$ .

In analogy to the proof of Proposition ??, it then follows that the  $H$ -type is rejected in the price taking stage. The credibility logic does not change, therefore, credibility is given if and only if

$$\lambda(p_i^* - L) \geq (1 - \lambda)(H - L)$$

$\hat{p}(\lambda)$  is the price at which the equation is strict, i.e.  $\hat{p}(\lambda) = \frac{1-\lambda}{\lambda}(H - L) + L$ .

### Proof of Proposition ??

$p_{i,2}^{L*}, p_{j,2}^{IA*}$  have been shown. Now consider  $p_i^*, p_j^*$  and consider the following first stage beliefs for a given  $p_i^*$ :  $\hat{\theta}^1(\text{accepted}) = L, \hat{\theta}^1(\text{rejected}) = H$  if  $p_i^* < H$  and  $\hat{\theta}^1(\text{accepted}) = E[\theta], \hat{\theta}^1(\text{rejected}) = H$  otherwise. We first show that there is no equilibrium in which  $p_i^* > \min\{\tilde{p}_L, \hat{p}(\lambda) - \varepsilon, \hat{p}(\lambda)\}$ . Let  $i$  be a firm which is approached in the first place with positive probability, i.e.  $p_i^* \leq p_j^*$ .

- (a)  $p_j^* \geq p_i^* > \tilde{p}_L$ . Then,  $\pi_i^e = 0$ . For given beliefs about investments, a deviation to  $p_i = \tilde{p}_L$  is profitable.
- (b)  $p_j^* \geq p_i^* \geq \hat{p}(\cdot)$ . Credibility is not given and  $\pi_j^{*,e} = 0$  if  $p_j^* > p_i^*$  and  $\pi_j^{*,e} = \frac{1}{2}0 + \frac{1}{2}q_L(p_j^* - L)$  if  $p_j^* = p_i^*$ . A deviation to an arbitrarily smaller price  $p_j$  yields  $\pi_j^e = q_L(p_j - L)$ .
- (c)  $p_j^* \geq p_i^* > \hat{p}(\cdot)$ . In that case,  $\pi_i^{*,e} \geq \pi_j^{*,e}$ . Therefore, a deviation to a lower price  $p_j < p_i^*$  is strictly profitable.

Second, we show that there is no equilibrium in which  $p_i^* < \min\{\tilde{p}_L, \hat{p}(\lambda), \hat{p}(\lambda)\}$ .

- (a)  $p_j^* \geq p_i^* < \tilde{p}_L$  and  $\tilde{p}_L \leq \min\{\hat{p}(\cdot), \hat{p}(\cdot)\}$ . Then, a deviation to a higher price is always profitable: if  $p_j^* > p_i^*$ , a deviation to  $p_i = \tilde{p}_L$  is profitable. If  $p_j^* = p_i^*$ , a deviation to some higher price is profitable because credibility is not given and the uninformed firm earns more than the informed party.
- (b)  $p_j^* \geq p_i^* < \hat{p}(\cdot) - \varepsilon$  and  $\hat{p}(\cdot) - \varepsilon \leq \min\{\tilde{p}_L, \hat{p}(\cdot)\}$ . Then, credibility is given and the uninformed firm earns more than the informed firm. Therefore, a deviation to a higher price  $p_i = \hat{p}(\cdot)$  is always profitable.

- (c)  $p_j^* \geq p_i^* < \hat{p}(\cdot)$  and  $\hat{p}(\cdot) \leq \min\{\tilde{p}_L, \hat{p}(\cdot)\}$ . Then, the informed firm earns less than the uninformed party and credibility is given. A deviation to  $p_i^* = \hat{p}(\cdot)$  is always profitable.

Next, we investigate deviation incentives for  $p_i^* = \min\{\tilde{p}_L, \hat{p}(\lambda) - \varepsilon, \hat{p}(\lambda)\}$ .

- (a)  $\min\{\tilde{p}_L, \hat{p}(\cdot) - \varepsilon, \hat{p}(\cdot)\} = \tilde{p}_L$ .

Consider some  $p_j^* \geq \hat{p}(\cdot)$ .  $j$  does not want to deviate since  $\pi_j^e > \pi_i^e$ .  $i$ 's best response is  $p_i(p_j^*) = \tilde{p}_L$ . Therefore,  $p_i^* = \tilde{p}_L$  and any price  $p_j^* \geq \hat{p}(\cdot)$  is an equilibrium.

By contrast, consider some equilibrium price  $p_j^* < \hat{p}(\cdot)$ . But  $i$ 's best response is  $p_i(p_j^*) > p_j^*$ .

- (b)  $\min\{\tilde{p}_L, \hat{p}(\cdot) - \varepsilon, \hat{p}(\cdot)\} = \hat{p}(\cdot) - \varepsilon$ .

Consider  $p_j^* = \hat{p}(\cdot)$ .  $j$  does not want to deviate since  $\pi_j^e > \pi_i^e$ .  $i$ 's best response is  $p_i(p_j^*) = \hat{p}(\cdot) - \varepsilon$ . Therefore,  $p_i^* = \hat{p}(\cdot) - \varepsilon$ ,  $p_j^* = \hat{p}(\cdot)$  is an equilibrium.

By contrast, consider  $p_j^* = \hat{p}(\cdot) - \varepsilon$ .  $i$ 's best response is  $p_i(p_j^*) > p_j^*$ .

Further, consider  $p_j^* > \hat{p}(\cdot)$ . This cannot be an equilibrium.  $i$ 's best response to  $p_j^*$  is  $p_i(p_j^*) = \min\{\tilde{p}_L; p_j^* - \varepsilon\}$ .

- (c)  $\min\{\tilde{p}_L, \hat{p}(\cdot) - \varepsilon, \hat{p}(\cdot)\} = \hat{p}(\cdot)$ .

Consider  $p_j^* = \hat{p}(\lambda)$ .  $j$  does not want to deviate because  $\pi_j^e = \pi_i^e$ .  $i$ 's best response is  $p_i(p_j^*) = p_j^*$ . Therefore,  $p_i^* = p_j^* = \hat{p}_L$  is an equilibrium.

By contrast, consider  $p_j^* > \hat{p}(\lambda)$ .  $i$ 's best response is  $p_i(p_j^*) = \min\{p_j^* - \varepsilon; \tilde{p}_L\}$ .

Last, we check that the belief structure for given list prices  $\hat{\theta}^1(\text{accepted}) = L, \hat{\theta}^1(\text{rejected}) = H$  forms an equilibrium. This holds true if  $L \leq p_i^* < H$  which is given since  $\tilde{p}_L < H$ .