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# Endogenous Growth, Green Innovation and GDP Deceleration in a World with Polluting Production Inputs

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Work in progress

## Abstract

We study economic growth and pollution control in a model with endogenous rate and direction of technical change. Economic growth (growth of real GDP) results from growth in the quantity and productivity of polluting intermediates. Pollution can be controlled by reducing the pollution intensity of a given quantity through costly research (green innovation) and by reducing the share of polluting intermediate quantity in GDP. Without clean substitutes, saving on polluting inputs implies that the rate of GDP growth remains below productivity growth (deceleration). While neither green innovation nor deceleration is chosen under laissez-faire, both contribute to long-run optimal pollution control for reasonable parameter values. In our baseline model, there are no exhaustible resources. In an extension, we analyze the effects of resource scarcity on the environment, long-run growth and the direction of technical change.

**Keywords:** Endogenous Growth, Direction of Technical Change, Pollution, Green Innovation, Efficiency Improvements

**JEL Codes:** O31, O33, Q55

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# 1 Introduction

When it comes to the question of whether and how economic growth and environmental conservation should be reconciled, it is widely argued that technical change should be stimulated (see for instance the Stern Review, Stern (2007), IPCC AR4 Synthesis Report (2007)). By reducing the pollution intensity of production inputs and processes, by developing clean substitutes or by raising input productivity, technological development can help to decouple economic growth from pollution.

However, on the one hand, the ecological benefits of technical change are not undisputed. While in principle increased input productivity can reduce pollution as it allows to save on polluting inputs without giving up output, in practice it may mainly raise output or even stimulate the demand for polluting inputs. This so-called rebound effect<sup>1</sup> of technical progress is one reason for environmental activists like Greenpeace to believe that the world economy should give up economic growth and converge towards stationary levels of consumption and production.<sup>2</sup>

On the other hand, environment-friendly policy is often feared to imply large costs in terms of economic growth. First, controlling the rebound effect of productivity growth by saving on polluting inputs obviously comes at the cost of giving up potential consumption growth. Second, the larger the fraction of research directed towards reducing the polluting impact of inputs, the less the fraction that may be directed to raise input productivity.

To come to reliable conclusions concerning the prospects of reconciling economic growth with a clean environment, one has to analyze the interaction of productivity-enhancing and pollution-reducing technical change (green innovation) taking into account the possibility to control the rebound effect of productivity growth through input saving. To our best knowledge, this paper is the first to explicitly and analytically do so in a framework in which perfectly clean substitutes to the polluting input are not available and green innovation reduces the pollution intensity of the polluting input. In the few models which consider endogenous reductions in pollution intensity through technical change, the possibility to control the rebound effect of productivity growth is disregarded.

This omittance is not trivial: We show that for reasonable values of model parameters, the optimal growth rate of real GDP and consumption, driven by productivity growth, is strictly positive but must be persistently accompanied by green innovation and input saving. Neglecting the possibility to control the rebound effect of productivity growth weakens the prospects for persistent economic growth in the long-run optimal solution considerably.

We study the *laissez-faire* and the optimal relation between long-run economic growth and environmental quality in a model where research effort can be directed at increasing the productivity and/or decreasing the pollution intensity of polluting inputs. Reductions in pollution intensity are what we understand by **green**

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<sup>1</sup>A formal definition can be given along the lines of Berkhout et al. (2000): A rebound effect denotes the percentage of input-saving that is lost due to increased input use. A rebound effect of more than 100%, which implies a net increase in input-use, is sometimes referred to as 'backfire'.

<sup>2</sup>Convergence to a stationary economy as demanded by environmental activists usually goes beyond merely giving up long-run growth. Environmental activists believe the world economy to have surpassed sustainable levels of economic activity so that downsizing -'degrowth'- is unavoidable. This belief is shared by a political movement of the same name, which has its origin in France ('décroissance'), see for example Ariès (2005) and Latouche (2004).

**innovation** in this paper. As in standard endogenous growth models with vertical product differentiation, real GDP in our model can either be increased by raising the quantity of intermediate inputs or the productivity of a given amount of these inputs. Productivity growth has an indirect effect on pollution which is a priori ambiguous: Higher productivity allows to use polluting inputs more efficiently and reduce their ratio in production but it also increases their marginal product which stimulates intermediate demand. This encourages a rebound effect. We do therefore not classify productivity growth as ‘clean’ or ‘dirty’ a priori.

There are two ways then to decouple output- and pollution growth: The first is to partially direct R&D-effort towards green innovation to reduce the pollution intensity of intermediate quantity. The second is to restrict the rebound effect of productivity growth by keeping growth in polluting inputs below productivity growth. This implies a cost in terms of foregone potential growth in consumption and real GDP. Consumption and GDP per labor efficiency unit constantly decline. To emphasize its growth impact, we call this form of polluting input saving **deceleration**. If polluting quantity persistently falls in absolute terms and not only per labor efficiency unit we speak of **quantity degrowth**.<sup>3</sup>

Deceleration cannot occur on a balanced growth path where output, consumption and productivity grow at the same rate. The analysis of our model extends beyond balanced growth paths to solutions characterized by growth rates which converge towards constant values asymptotically. We call such solutions ‘asymptotically-balanced growth solutions’.<sup>4</sup>

While in the long-run laissez-faire equilibrium neither green innovation nor deceleration is chosen, we show that for the long-run social optimum, the following holds: (i) Long-run economic growth is optimal for a sufficiently patient household and it always goes along with persistent green innovation to reduce the pollution intensity of intermediate goods. This result is driven by the existence of fixed costs in each individual research unit. Once a research unit is opened up and the fixed costs are paid, making intermediates at least marginally cleaner while making them more productive generates almost no additional cost. (ii) If production is very elastic with respect to polluting inputs, the social planner relies exclusively on green innovation to control pollution. She keeps the share of polluting inputs in GDP constant (as under laissez-faire) to generate fast consumption growth. (iii) For reasonable parameter values, production is inelastic with respect to intermediate quantity compared to productivity. Deceleration then allows to gain from productivity growth in a relatively clean way without incurring a large loss in consumption growth. Further, the social return to green innovation rises in the marginal product of the polluting input which is comparatively small in this case. It is therefore optimal to choose deceleration along with reductions in the pollution intensity of intermediates.

Our results imply that policy may have to stimulate technical development and green innovation in particular while at the same time setting incentives to control the rebound effect which offsets efficiency gains in the laissez-faire equilibrium.

Quantity degrowth is a particularly strong form of deceleration. Even with quantity degrowth, productivity, consumption and GDP may still rise. GDP growth is driven by productivity improvements alone. In our model, persistent GDP degrowth towards stationary GDP and consumption levels in absolute terms is preferable to a

<sup>3</sup>If we interpret intermediate quantity as material used, quantity degrowth (and, in a weaker sense, deceleration) corresponds to what is sometimes called “**dematerialization**”, since it reduces the quantity in absolute terms (or its share in real GDP).

<sup>4</sup>Asymptotically-balanced growth paths in environmental economic models have been described in an Ak-model by Withaagen (1995) and in a general one-sector growth model with non-renewable resources (but without pollution) by Groth and Schou (2002).

path with unconstrained pollution growth as it is chosen in the entirely unregulated economy. However, giving up consumption growth altogether is never optimal (for a sufficiently patient household). Although our model remains too stylized to give conclusive answers, indicative parametrization of the model suggests that the option of quantity degrowth may very well be a sensitive long-run aim.

Quantity degrowth is unavoidable both in the market equilibrium and in the long-run social optimum if intermediate goods are produced from an exhaustible resource. Our baseline model does not consider resource scarcity. According to the IPCC, however, one of the major sources of pollution is energy<sup>5</sup>, the generation of which relies heavily on polluting non-renewable resources like fossil fuels. In an extension of our model, we therefore introduce a polluting non-renewable resource as input in intermediate production. We show that while resource scarcity slows growth in the *laissez-faire* equilibrium, the results of our baseline model still apply for the long-run social optimum if the optimal solution of the baseline model is characterized by quantity degrowth and the initial resource stock is large enough. The negative pollution externality of intermediate production then reduces optimal resource use in a way that a sufficiently large resource stock is never exhausted.

Not only does our paper support the view that pollution control is reconcilable with persistent economic growth, it allows for even more optimistic conclusions: In our model, environmental care need not be detrimental to long-run growth. Depending on model parameters, growth in consumption, production and productivity may be faster in the optimal solution of our model compared to the optimal growth rates in a model without negative environmental externality. The driving force behind this positive effect is that green innovation can boost the amount of labor used in R&D which fosters also productivity growth. Without the possibility of green innovation, consumption growth is unambiguously lower when household utility is negatively affected by pollution than in the model without environmental externality.

There are two different strands of literature on the direction of technological change, economic growth and pollution. Closest to our model are Hart (2004) and Ricci (2007) as they also consider the choice between a lower pollution intensity and greater productivity. However, they neglect deceleration and in general the possibility to save on polluting inputs. Ricci (2007) concentrates on the analysis of balanced growth paths along which, as indicated above, deceleration cannot occur. In Hart (2004), not only the quantity component of output but output itself has a negative effect on the environment. This assumption contradicts our intuition that higher productivity is not polluting in itself and implies that deceleration cannot reduce pollution.

While the understanding of green innovation in Hart (2004) and Ricci (2007) is similar to ours, a different definition of green innovation is given, e.g., in Grimaud and Rouge (2008) and Acemoglu, Aghion, Bursztyn and Hemous (2012). Building on Acemoglu (2002) both papers assume separate research sectors, one for a polluting and one for a non-polluting production input<sup>6</sup>. Green innovation increases the productivity of the clean good while leaving the pollution intensity of the dirty input unchanged. Given that the inputs are substitutable, pollution can be controlled by shifting the composition of GDP towards the clean sector. The share of the dirty input in production is not reduced through increases in input-efficiency, i.e., deceleration, as in our model, but

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<sup>5</sup>The energy share in total anthropogenic greenhouse gas emissions in 2004 in terms of CO<sub>2</sub>-equivalent was 25.9 % according to the IPCC Synthesis Report (IPCC (2007), p. 36).

<sup>6</sup>Several authors, among them Smulders and de Nooij (2003) and Hassler, Krusell and Olovsson (2012) use the framework by Acemoglu (2002) to analyze energy-saving technical change. As neither of the models takes pollution into account, we do not refer to these contributions in detail here.

through input substitution.<sup>7</sup>

In the models based on Acemoglu (2002), it depends crucially on the particular assumptions about the production function whether technical progress in the clean sector is indeed ‘green’ and does not increase the share of polluting inputs in GDP<sup>8</sup>. Further, while in the model there exists a clean substitute for the polluting input, no currently known production technology is completely clean in reality. Taking energy use as an example, even though there exist relatively clean alternatives to fossil fuels, transportation and storage of renewable energy and of course manufacturing of solar cells or wind turbines generate emissions. Hence, even within relatively clean sectors, technical progress can be directed towards reductions in pollution intensity. Also, more than 80% of today’s energy consumption is not produced from renewable energy but from oil, gas and coal and total energy consumption is growing for all forms of energy and in all regions.<sup>9</sup> We therefore think that both persistent within-sector green innovation and persistent within-sector efficiency improvements through deceleration should a priori be considered as potential instruments of world-wide pollution control.

The outline of our paper is as follows: Section 2 presents the model setup. We then determine the laissez-faire equilibrium in section 3. In section 4, we characterize the unique long-run optimum for different sets of parameters and illustrate in a numerical example that the optimal solution is likely to include both green innovation and deceleration to decouple output- and pollution growth. Further, we consider two different constrained optimal solutions where either green innovation or deceleration is not available to the social planner. Section 5 extends the baseline model to include a polluting non-renewable resource. Section 6 concludes.

## 2 The model

### 2.1 Setup

In each period, a representative household receives utility  $v(c_t) = \frac{\sigma_c}{\sigma_c - 1} c_t^{\frac{\sigma_c - 1}{\sigma_c}}$  from per-capita-consumption  $c_t = \frac{C_t}{L}$  and utility  $\phi^E(E_t) = \psi \frac{\sigma_E}{\sigma_E - 1} E_t^{\frac{\sigma_E - 1}{\sigma_E}}$  from environmental quality  $E_t$ . We assume, along the lines of, e.g., Stokey (1998) as well as Aghion and Howitt (1998, chapter 5), that utility is additively separable. Discounted intertemporal utility is given by

$$U = \int_0^\infty e^{-\rho t} \left( \frac{\sigma_c}{\sigma_c - 1} c_t^{\frac{\sigma_c - 1}{\sigma_c}} + \psi \frac{\sigma_E}{\sigma_E - 1} E_t^{\frac{\sigma_E - 1}{\sigma_E}} \right) L dt \quad (1)$$

where  $\rho$  is the rate of time preference,  $L$  total household labor-supply and  $\sigma_c, \sigma_E > 0$ ,  $\sigma_c, \sigma_E \neq 1$  are the intertemporal substitution elasticities of consumption and environmental quality respectively.  $\psi$  measures the weight of environmental quality in instantaneous utility. Utility is increasing and strictly concave in both arguments.

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<sup>7</sup>Input-saving through efficiency improvements in the dirty sector remains relevant in Grimaud and Rouge (2008) where pollution is generated by a non-renewable resource.

<sup>8</sup>This is also pointed out by Di Maria and Smulders (2004, 2012).

<sup>9</sup>See for instance the BP Statistical Review of World Energy June 2011 ([www.bp.com/statisticalreview](http://www.bp.com/statisticalreview)). Unexploited supply of fossil fuels is enough to sustain further growth for a long time and China and India alone plan to substantially raise fossil fuel production and consumption.

Environmental quality is inversely related to the stock of pollution originating from the intermediate sector:

$$E_t = \frac{1}{S_t} \quad (2)$$

While utility is concave in  $E_t$ , the relation between environmental quality and pollution is convex. Depending on  $\sigma_E$ , the disutility  $\phi^S(S_t) = -\phi^E(E_t) = \frac{\sigma_E}{1-\sigma_E} S_t^{\frac{1-\sigma_E}{\sigma_E}}$  of pollution can be concave or convex in  $S_t$ . We assume that it is convex, by restricting  $\sigma_E$  to the interval  $(0, 1/2)$ . The marginal impact of pollution on household wellbeing is often hardly felt as long as the environment is still comparatively clean, while the effects of pollution, e.g., on human health, become severe in a heavily polluted environment. It is therefore reasonable to assume that the marginal disutility of pollution increases in the pollution stock. The assumption of convex disutility also rules out parameter constellations for which the utility impact of pollution asymptotically becomes negligible relative to that of consumption in a growing economy. This is not an interesting case for the long-run analysis of questions arising out of the trade-off between economic growth and a clean environment. Not only the long-run laissez-faire equilibrium but also the long-run optimal solution would be similar to those in non-environmental models of growth through creative destruction.

The representative household allocates an amount  $L_{Yt}$  of its labor supply  $L$  to final-good production,  $L_{Xt}$  to intermediate production and an amount  $L_{Dt}$  to research:

$$L = L_{Yt} + L_{Xt} + L_{Dt} \quad (3)$$

Final output  $Y_t$  is produced from labor  $L_{Yt}$  and intermediate goods  $X_{it}$  of various productivity levels  $Q_{it}$ ,  $i \in [0, 1]$  with the production function

$$Y_t = L_{Yt}^{1-\alpha} \int_0^1 X_{it}^\alpha Q_{it}^{1-\alpha} di \quad (4)$$

where  $0 < \alpha < 1$ .  $Y_t$  is used for consumption only.

$$Y_t = c_t L. \quad (5)$$

Intermediate goods are produced with the production function

$$X_{it} = \varphi L_{Xit} Q_t \quad (6)$$

where  $\varphi > 0$  is a parameter and  $Q_t = \int_0^1 Q_{it} di$  measures aggregate productivity.<sup>10</sup>  $X_{it}$  can be interpreted as any kind of non-durable production input including energy. Polluting energy inputs are, however, usually associated with polluting non-renewable resources which we do not consider in the baseline specification of our model. We show in section 5 that for parameter constellations well in line with empirical evidence, the introduction of a non-renewable resource for intermediate production does not affect the long-run social optimum so that our main results still hold.

Pollution evolves according to equation of motion

$$\dot{S}_t = \frac{X_t}{B_t} - \delta S_t. \quad (7)$$

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<sup>10</sup>The dependence of  $X_{it}$  on aggregate productivity  $Q_t$  is needed to ensure that the allocation of labor supply and thus growth rates of the aggregate variables in our model are constant in the long run. Our results would not change if we assumed that instead of labor, a fraction of final output had to be spent on the production of intermediates.

In general, we use a dot above a variable to indicate its derivative with respect to time, while we mark growth rates with a circumflex.  $X_t/B_t$  is the pollution flow generated by the quantity of intermediates. The pollution intensity of  $X_t$  decreases in the aggregate cleanliness  $B_t = \int_0^1 B_{it} di$  of the inputs used. In every period, a fraction  $\delta$  of the pollution stock is cleaned up by natural regeneration processes. Pollution growth therefore eventually ceases if there is no growth in intermediate production. However, if  $X_t$  grows,  $S_t$  asymptotically grows at the same rate unless the pollution intensity of intermediates is reduced over time by green innovation. Still, even without green innovation pollution growth remains below its potential if there is deceleration so that the rebound effect of productivity growth is restricted.

We speak of deceleration whenever growth in intermediate quantity remains below productivity growth. This is expressed by the following, more formal definition:

**Definition 1** *There is **deceleration** whenever  $\hat{X}_t < \hat{Q}_t$  so that  $X_t/Q_t$  declines.*

The two sources of slow pollution accumulation (besides natural regeneration) become apparent when rewriting (7) as  $\dot{S}_t = \frac{X_t}{Q_t} \frac{Q_t}{B_t} - \delta S_t$ : First,  $\dot{S}_t$  is small whenever  $Q_t/B_t$  is small, which means a sufficiently large share of research must have been oriented towards green innovation in the past. Second, pollution accumulates more slowly with a smaller  $X_t/Q_t$ , brought about by deceleration.

The considerations in the previous paragraph suggest the definition of a natural benchmark for the direction of technical change:

**Definition 2** *The **direction of technical change** is ecologically **neutral** if and only if  $\hat{B}_t = \hat{Q}_t$ , **productivity-oriented** if and only if  $\hat{B}_t < \hat{Q}_t$ , and **green** if and only if  $\hat{B}_t > \hat{Q}_t$ .*

We say that the direction of technical change is ecologically neutral if pollution is perfectly controlled ( $\hat{S}_t = \hat{E}_t = 0$ ) without deceleration ( $\hat{X}_t = \hat{Q}_t$ ). Since  $\hat{S}_t = 0$  if  $X_t = \delta S_t B_t$  and  $\hat{X}_t = \hat{B}_t$ , the direction of technical change is ecologically neutral if and only if  $\hat{B}_t = \hat{Q}_t$ . Analogously, we say that the direction of technical change is productivity-oriented if in the absence of deceleration the stock of pollution would grow ( $\hat{S}_t > 0$ ) and environmental quality deteriorate ( $\hat{E}_t < 0$ ), which is the case for  $\hat{B}_t < \hat{Q}_t$ . And we say that the direction of technical change is green if the stock of pollution would fall ( $\hat{S}_t < 0$ ) and environmental quality rise ( $\hat{E}_t > 0$ ) in the absence of deceleration, which is the case for  $\hat{B}_t > \hat{Q}_t$ .

Both productivity  $Q$  and cleanliness  $B$  change over time due to innovations from a continuum of R&D-sectors. Entry to the research sector for any intermediate  $X_{it}$  is not restricted. For research unit  $j \in [0, \infty]$ , improving  $Q_{it}$  by a rate  $q_{ijt}$  and  $B_{it}$  by a rate  $b_{ijt}$  requires

$$l_{Dijt}(q_{ijt}, b_{ijt}, Q_{it}, B_{it}, Q_t, B_t) = q_{ijt}^2 \frac{Q_{it}}{Q_t} + b_{ijt}^2 \frac{B_{it}}{B_t} + d \frac{Q_{it}}{Q_t} \quad (8)$$

units of labor. We call  $q_{ijt}$  and  $b_{ijt}$  the step-size of an innovation with respect to productivity and cleanliness respectively. The wage rate is denoted by  $w_{Dt}$ . Then  $w_{Dt} d \frac{Q_{it}}{Q_t} > 0$  are fixed entry costs for unit  $j$  in sector  $i$ . Variable costs for each dimension of technology improvement are quadratic in the step-size. Total costs  $w_{Dt} l_{Dijt}$  rise with the level of sectoral relative to aggregate productivity  $Q_{it}/Q_t$  and cleanliness  $B_{it}/B_t$  respectively. The underlying assumption is that technology improvements in a given sector are increasingly difficult the more



advanced the technology in that sector is already while there are positive spillovers from the other sectors.<sup>11</sup> Given  $l_{Dijt}$ , a trade-off exists between making an intermediate more productive and making it cleaner. On the other hand, there is also an indirect positive relation between research orientations: Once fixed costs have been paid to innovate in one direction, a comparatively small additional labor-investment is needed to increase the other technology stock as well.

If a researcher  $j$  enters into the research sector for intermediate  $X_i$  at time  $t$ , he hires labor  $l_{Dijt}$  and chooses a step-size  $q_{ijt}$  and  $b_{ijt}$  for the improvement in productivity and cleanliness respectively. The wage rate  $w_{Dt}$  is taken as given. Innovations occur at the exogenous, constant Poisson arrival-rate  $\mu$  per unit of time for the individual researcher  $j$ . An innovation changes the sectoral productivity level by  $q_{ijt}Q_{it}$  and the cleanliness of production by  $b_{ijt}B_{it}$ . The innovator obtains a patent for the production of the improved intermediate good. He then receives a profit flow from selling the intermediate which eventually ceases when a new innovation arrives and the incumbent is replaced by another firm. If  $n_{it}$  units decide to enter research sector  $i$  in  $t$ , innovations arrive at rate  $\mu n_{it}$  in this sector. The expected development of  $Q_{it}$  and  $B_{it}$  is given by

$$E[\Delta Q_{it}] = \int_0^{n_{it}} \mu q_{ijt} Q_{it} dj \quad (9)$$

$$E[\Delta B_{it}] = \int_0^{n_{it}} \mu b_{ijt} B_{it} dj. \quad (10)$$

While the sectorial technology level faces discontinuous jumps, aggregate technology evolves continuously, because there is a continuum of sectors carrying out research. Arguing along the lines of the law of large numbers, the average rates of change  $\dot{Q}_t$  and  $\dot{B}_t$  of  $Q$  and  $B$  approximately equal the respective expected rates of change, which are derived by aggregating over sectors in (9) and (10). Accordingly, the aggregate equations of motion are

$$\dot{Q}_t = \int_0^1 \int_0^{n_{it}} \mu q_{ijt} Q_{it} dj di \quad (11)$$

$$\dot{B}_t = \int_0^1 \int_0^{n_{it}} \mu b_{ijt} B_{it} dj di. \quad (12)$$

## 2.2 Balanced and asymptotically-balanced growth

The subsequent analysis of our model in this and the following chapters extends beyond balanced growth paths to ‘asymptotically-balanced growth paths’. The following definition serves to clarify the terminology, where here and in the following,  $z_\infty$  refers to the limit  $\lim_{t \rightarrow \infty} z_t$  of a variable  $z$ :

**Definition 3** Assume that for some initial state  $(Q_0, B_0, S_0)$ , there exists a solution such that the sequence  $(\hat{Q}_t, \hat{B}_t, \hat{S}_t)_{t=0}^\infty$  converges towards the vector  $(\hat{Q}_\infty, \hat{B}_\infty, \hat{S}_\infty)$  for  $t \rightarrow \infty$ . We call such a solution an asymptotically-

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<sup>11</sup>Like the intermediate production function, labor required in R&D (equation (8)) must depend on the aggregate and additionally on the sectoral levels of technology to ensure asymptotically constant growth of the aggregate variables.

balanced growth (ABG) solution. We say that the model has an asymptotically unique ABG-solution if all ABG-solutions have the same limit vector  $(\widehat{Q}_\infty, \widehat{B}_\infty, \widehat{S}_\infty)$ .

If there exist initial states  $(Q_0, B_0, S_0)$  such that the corresponding solution paths are characterized by  $(\widehat{Q}_t, \widehat{B}_t, \widehat{S}_t) = (\widehat{Q}_\infty, \widehat{B}_\infty, \widehat{S}_\infty)$  for every  $t$ , we call the path defined by  $(\widehat{Q}_t, \widehat{B}_t, \widehat{S}_t)_{t=0}^\infty = (\widehat{Q}_\infty, \widehat{B}_\infty, \widehat{S}_\infty)$  the unique balanced growth (BG)-path.

In abuse of terminology, we sometimes refer to the unique limit of all ABG-solutions for  $t \rightarrow \infty$ , characterized by the unique vector  $(\widehat{Q}_\infty, \widehat{B}_\infty, \widehat{S}_\infty)$ , as *the* ABG-solution.

Note that a BG-solution, defined by constant growth rates of  $Q$ ,  $B$  and  $S$  for all  $t$ , is also an ABG-solution. The reverse is not true because there may not exist an initial state  $(Q_0, B_0, S_0)$  such that  $\widehat{Q}_t$ ,  $\widehat{B}_t$  and  $\widehat{S}_t$  are constant for all  $t$ . We will see that, while for any set of parameters the economy has a unique BG-equilibrium and a unique ABG-optimum, it need not have a BG-optimum. In particular, a BG-optimum does not exist in proposition 4 if the conditions for deceleration are satisfied or in proposition 5.

The distinction between ABG-paths and BG-paths is not necessarily important for interpreting the results of the social planner's solutions, as we concentrate on the long-run for both balanced and asymptotically-balanced growth solutions. From a more technical point of view an ABG-path which is not a BG-path is interesting for its own sake.

### 3 The laissez-faire equilibrium

The laissez-faire equilibrium is given by sequences of plans for per-capita consumption  $\{c_t\}_0^\infty$ , assets  $\{A_t\}_0^\infty$ , labor supply in production  $\{L_{Xit}, L_{Yt}\}_0^\infty$  and research  $\{L_{Dit}\}_0^\infty$ , demand for intermediates  $\{X_{it}^d\}_0^\infty$ , demand for labor in production  $\{L_{Xit}^d, L_{Yt}^d\}_0^\infty$  and research labor demand  $\{l_{Dijt}\}_0^\infty$ , plans for the step-size  $\{q_{ijt}\}_0^\infty$  in productivity and  $\{b_{ijt}\}_0^\infty$  in cleanliness, as well as sequences of intermediate prices  $\{p_{it}\}_0^\infty$  and wages  $\{w_{Xit}, w_{Yt}, w_{Dit}\}_0^\infty$  in intermediate production, final good production and research and a path  $\{r_t\}_0^\infty$  for the interest rate such that in every period  $t$ , (i) the representative household maximizes utility taking into account the budget constraint and the labor market constraint (3), (ii) profits from final and intermediate goods production as well as research profits are maximized, (iii) aggregate expected profits in each research sector  $i$  are zero (iv) the markets for intermediate goods, the three types of labor and assets clear (v) all variables with the possible exception of  $q_{ij}$  and  $b_{ij}$  are non-negative.

The solution of the model under laissez-faire follows closely that in standard endogenous growth models. We therefore refer to appendix A for the derivation of the laissez-faire solution and summarize our findings in the following proposition:

**Proposition 1 BG laissez-faire equilibrium**

Assume that  $\rho$  exceeds an upper bound  $\rho^{TVC, LF}$ , so that the transversality condition for assets is satisfied.

There exists a unique BG-laissez-faire equilibrium. Economic growth is strictly positive if and only if the rate of time preference is sufficiently small ( $\rho < \bar{\rho}^{LF}$ ). Productivity growth leads to equally fast expansion of polluting quantity ( $\widehat{X}_\infty^{LF} = \widehat{Q}_\infty^{LF}$ ). There is neither deceleration nor green innovation. Pollution grows at the same rate as

consumption, production and productivity. If the disutility of pollution is convex ( $\sigma_E < 1/2$ ), a solution without long-run growth is socially preferable.

**Proof.** See appendix A.5. ■

From the previous section it is obvious that in our model, there are no incentives for producers to self-restrict in polluting intermediate production or invest in cleaner intermediates. The rebound effect of productivity growth is particularly strong under laissez-faire as quantity grows at the same rate as productivity. In a growing economy, there is unconstrained pollution growth. This is clearly suboptimal if the disutility of pollution is convex but utility is concave in consumption: The marginal utility gain from an additional unit of consumption becomes negligible relative to the marginal utility loss generated by a unit increase in the pollution stock as consumption and pollution rise. Utility declines persistently without lower bound. If long-run consumption growth is given up, the pollution stock and utility converge to constant values. Stationary long-run levels of consumption and production as called for environmental activists are therefore welfare-improving over the laissez-faire equilibrium.

## 4 The Social Planner's solution

The social planner chooses the time paths of  $Q$ ,  $B$ ,  $Q_i$ ,  $B_i$  and  $S$  as well as consumption  $c$ , production  $Y$ ,  $X$ ,  $X_i$ ,  $x_{ij}$ , the allocation of labor  $L_{Yt}$ ,  $L_{Xt}$ ,  $L_{Xit}$ ,  $L_{Dt}$ ,  $l_{Dijt}$ , the number of research units<sup>12</sup>  $n_{it} = n_t$  and a step-size  $q_{ijt}$  and  $b_{ijt}$  for technology improvements in every period  $t$  so as to maximize utility (equation (1)). She takes into account the labor market constraint (3), the aggregate resource constraint (5), the effect of pollution on environmental quality (2), the equation of motion for pollution (7), the sectoral equations of motion for  $Q_i$  (9) and  $B_i$  (10) and the aggregate equations of motion for  $Q$  (11) and  $B$  (12).

Because all research units  $j$  are ex ante symmetric and research costs are convex in  $q_{ij}$  and  $b_{ij}$ , the social planner chooses the same  $q_{ijt}$ ,  $b_{ijt}$  and therefore  $l_{Dijt}$  for every  $j$  in sector  $i$ . Further, the planner allocates intermediate production in every sector  $i$  to the latest innovator because he is the most productive and cleanest and marginal costs are the same for all  $j$ . We therefore omit the index  $j$  from now on. In fact, it is optimal to choose the same  $q_{it} = q_t$  and  $b_{it} = b_t$  in every sector:

**Lemma 1** *The optimal  $q_{it}$  and  $b_{it}$  are the same for all sectors  $i$ :  $q_{it} = q_t$  and  $b_{it} = b_t$ .*

**Proof.** The social planner chooses the step-size in every sector  $i$  so as to reach a given rate of change  $\dot{Q}_t$  and  $\dot{B}_t$  in the respective aggregate technology level with a minimum labor investment. From the equations of motion (11) and (12) for  $Q$  and  $B$  together with the R&D-cost function (8) we can conclude that the marginal gain of an increase in  $b_i$  and  $q_i$ , in terms of faster technological progress, and the additional amount of labor required increase in the sectorial technology levels  $Q_{it}$  and  $B_{it}$  in the same way. Therefore sectorial differences are irrelevant for the optimal choice of  $q_i$  and  $b_i$ . ■

<sup>12</sup>To allow for an analytical solution to the planner problem we consider the constrained maximization problem with  $n_{it} = n_t$  for all  $i$ .

We show in appendix B.1, that the social planner's problem can then be solved by finding the utility-maximizing paths for the sector-independent variables  $Q, B, S, c, X, L_Y, n, q$  and  $b$  and we derive the first-order conditions.

Before analyzing how green innovation and deceleration are optimally used to control pollution growth in a growing economy (see section 4.3), we shortly describe the conditions for positive growth and the development of the pollution stock along the optimal path.

#### 4.1 Optimality of persistent economic growth

In standard models of endogenous growth, long-run growth is optimal for sufficiently patient households. This result carries over to our model with negative environmental externalities.

##### **Proposition 2** *Positive long-run consumption growth*

*Given a sufficiently small rate of time preference ( $\rho < \bar{\rho}$ ), long-run optimal consumption growth is positive.*

**Proof.** See appendix B.5. The upper bound  $\bar{\rho}$  differs depending on the parameter constellation considered. It is defined in appendix B.4. ■

The result is not surprising as pollution accumulation can be restricted without giving up consumption growth altogether. Persistent economic growth must however be accompanied by continuous pollution control.

#### 4.2 The optimal relation between economic growth and pollution accumulation

We show in this subsection, that optimal growth does not automatically require constant or decreasing pollution levels. More precisely, we find that for our assumption of convex disutility of pollution ( $\sigma_E < 1/2$ ) whether the pollution stock de- or increases in the long-run optimum depends on the intertemporal elasticity of substitution in consumption:

##### **Proposition 3** *Development of the pollution stock*

*Suppose instantaneous disutility of pollution is convex ( $\sigma_E < 1/2$ ).*

*Long-run growth must be accompanied by a persistent restriction of pollution growth. In a growing economy, the pollution stock  $S_t$  increases (decreases) for  $\sigma_c > 1$  ( $\sigma_c < 1$ ).*

**Proof.** The first statement follows as a corollary of proposition 1. As to the second, we show in appendix B.2 that in a solution with asymptotically-balanced growth, the following condition must hold:

$$\frac{\sigma_c - 1}{\sigma_c} \hat{c}_\infty = \frac{1 - \sigma_E}{\sigma_E} \hat{S}_\infty + \left( \hat{X}_\infty - \hat{B}_\infty - \hat{S}_\infty \right) \quad (13)$$

Given  $\hat{c}_\infty > 0$ , the left-hand side of (13) is positive whenever  $\sigma_c > 1$  while it is negative for  $\sigma_c < 1$ . Under the assumption of convex disutility of pollution,  $\frac{1 - \sigma_E}{\sigma_E}$  on the right-hand side is positive. The difference  $\hat{X}_\infty - \hat{B}_\infty - \hat{S}_\infty$  in parentheses is smaller than zero if and only if  $\hat{S}_\infty > \hat{X}_\infty - \hat{B}_\infty$  which implies that  $X/BS$  converges to zero so that  $\hat{S}_\infty = (-\delta)$  according to (7). Otherwise  $\hat{S}_\infty = \hat{X}_\infty - \hat{B}_\infty > (-\delta)$  and the difference is zero.

Therefore the right-hand side of equation (13) is negative if and only if  $\hat{S}_\infty < 0$  and positive if and only if  $\hat{S}_\infty > 0$ . It follows that the pollution stock must increase whenever  $\sigma_c > 1$  and decrease whenever  $\sigma_c < 1$ .<sup>13</sup> ■

The difference in parentheses in (13) is zero along a balanced growth path. The equation then is the balanced-growth condition described in Bovenberg and Smulders (1996) which has become standard in environmental endogenous growth models: It requires that the ratio of instantaneous marginal utility from consumption to instantaneous marginal disutility from pollution must develop proportionally to  $S/c$  so that the elasticity of substitution between  $c$  and  $S$  is unity.

To gain a better intuition, note that if instantaneous disutility  $\psi\phi^S(S_t)$  from pollution is convex, marginal instantaneous disutility converges to zero if and only if  $S_t$  decreases to zero and diverges to infinity if and only if  $S_t$  grows persistently.

For  $\sigma_c > 1$ , instantaneous marginal utility  $v'(c_t)$  of consumption falls in response to an increase in consumption, but underproportionally. If the pollution stock remained constant, the ratio of marginal (dis)utilities  $v'(c_t)/(\psi\phi'(S))$  would rise relative to  $S_t/c_t$  and it would be beneficial to invest less in pollution control. If, on the other hand, the pollution stock rose at the same rate as consumption,  $v'(c_t)/(\psi\phi'(S))$  would fall relative to  $S_t/c_t$  and it would be beneficial to restrict pollution growth. It follows that the pollution stock must rise in the long run but at a rate sufficiently below the consumption growth rate for the limit of the ratio  $\frac{v'(c_t)}{\psi\phi'(S)}/(\frac{S_t}{c_t})$  to be constant.

In the opposite case with  $\sigma_c < 1$ , instantaneous marginal utility of consumption falls overproportionally in response to an increase in the consumption level. In this case, the pollution stock must fall in the long run.

The pollution stock can at most decrease at the rate of natural regeneration ( $\hat{S}_\infty = -\delta$ ). To actually reach this rate of decrease, flow pollution would have to become zero and all economic activity would have to be given up. This path for pollution and environmental quality is clearly never optimal as a positive consumption level has to be maintained. Still, it can be optimal to approach it *asymptotically* by decreasing the pollution flow particularly fast<sup>14</sup>:

**Corollary 1** *Assume convex disutility of pollution ( $\sigma_E < 1/2$ ).*

*If and only if  $\sigma_c < 1$  and the representative household is sufficiently patient and the rate of natural regeneration is small ( $\rho \leq \rho^{\text{delta}} < \bar{\rho}$ ), so that the optimal consumption growth rate is equal to or larger than  $\frac{(1-\sigma_E)/\sigma_E}{(1-\sigma_c)/\sigma_c} \delta$ , the pollution stock decreases with the rate  $\hat{S}_\infty = -\delta$  asymptotically.*

**Proof.** Whenever  $\sigma_c < 1$ , the pollution stock decreases in the long-run optimal solution according to proposition 3. The pollution stock can at most decline at rate  $\hat{S}_\infty = -\delta$ . It then follows that for  $\sigma_c < 1$ , condition (13) can only hold with  $\hat{S}_\infty = \hat{X}_\infty - \hat{B}_\infty > (-\delta)$  for consumption growth rates  $\hat{c} < \frac{(1-\sigma_E)/\sigma_E}{(1-\sigma_c)/\sigma_c} \delta$ . This constraint defines

<sup>13</sup>Note that (13) also suggests that under more general assumptions concerning the utility function, whether the pollution stock de- or increases depends on  $\sigma_E$  being smaller or larger than one as well. For  $\sigma_E > 1$ , pollution is allowed to rise only if  $\sigma_C < 1$  while a falling pollution stock is required for  $\sigma_C > 1$ .

<sup>14</sup>As pollution falls, the amount  $(-\delta S)$  of the pollution stock which is cleaned up by natural processes also declines. As long as the pollution flow does not persistently decrease faster than  $(-\delta S)$ , the growth rate of the pollution stock cannot continuously fall and therefore not converge to its lower bound  $(-\delta)$ . The long-run growth rate of the pollution stock equals the growth rate of the flow in this case. For  $\hat{S}_\infty$  to converge towards  $(-\delta)$ , the constant growth rate of the pollution flow  $X/B$  must be below the asymptotic growth rate of  $-\delta S$ , which is  $(-\delta)$ .

the boundary value  $\rho^{\text{delta}}$  for the rate of time preference<sup>15</sup>. If  $\rho \leq \rho^{\text{delta}}$ , so that the optimal consumption growth rate is  $\hat{c}_\infty \geq \frac{(1-\sigma_E)/\sigma_E}{(1-\sigma_c)/\sigma_c} \delta$ , pollution growth must converge towards  $\hat{S}_\infty = (-\delta)$ . ■

### 4.3 Green innovation, deceleration and the direction of technical change

As shown in proposition 2, long-run growth in the optimal solution requires to persistently restrict pollution growth. It is intuitive that green innovation is always part of optimal pollution control: Once research units are opened up and the fixed costs (e.g., for equipment and fixed labor costs) have been paid, it is almost costless to make intermediates a little cleaner while making them more productive.

Unlike green innovation, deceleration is not always optimal in a growing economy as the costs in terms of foregone consumption growth may be substantial.

In the following, we characterize the long-run optimal solution for different sets of parameters. In proposition 4, we analyze in detail the long-run solution of the social planner's maximization problem for parameter constellations for which the conditions from corollary 1 are not satisfied so that pollution growth is given by  $\hat{S}_\infty = \hat{X}_\infty - \hat{B}_\infty$ . Given convex disutility of pollution ( $\sigma_E < 1/2$ ), this is always the case for  $\sigma_c > 1$  but requires the household to be sufficiently impatient and natural regeneration to be fast enough if  $\sigma_c < 1$ , as explained in corollary 1. The case with  $\hat{S}_\infty = (-\delta)$  yields similar results and is treated in proposition 5. Figure 1 gives an overview over the different cases in dependence on the rate of time preference,  $\rho$ .

For the following proposition, we define  $\underline{\rho} := \begin{cases} \rho^{\text{TVC}} & \sigma_c > 1 \\ \rho^{\text{delta}} & \sigma_c < 1 \end{cases}$  so that if  $\rho > \underline{\rho}$ , the transversality conditions are satisfied<sup>16</sup> and corollary 1 does not apply ( $\hat{S}_\infty > (-\delta)$ ).

#### Proposition 4 (A)BG optimum for $\hat{S}_\infty > (-\delta)$

Assume  $\sigma_E < 1/2$  so that the disutility of pollution is convex and  $\underline{\rho} < \rho < \bar{\rho}$ .

**(a) Green innovation without deceleration:** If  $\alpha/(1-\alpha) > 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}$ , there exists a unique optimal BG-solution.  $X_t, c_t$  and  $Q_t$  grow at the same constant rate. Thus there is no deceleration ( $(X/Q)_\infty$  constant). Growth in the pollution stock  $\hat{S}_\infty$  equals the growth rate of flow pollution,  $\hat{X}_\infty - \hat{B}_\infty = \hat{Q}_\infty - \hat{B}_\infty$ .  $\hat{S}_\infty$  is reduced below its potential rate  $\hat{Q}_\infty$  by green innovation ( $\hat{B}_\infty > 0$ ) only. The ratio of green relative to productivity-improving innovation is  $\hat{B}_\infty/\hat{Q}_\infty = 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}$ .

**(b) Green innovation with deceleration:** For  $\alpha/(1-\alpha) < 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}$ , a BG-solution does not exist. There exists an asymptotically unique ABG-optimum. Pollution growth  $\hat{S}_\infty$  equals the growth rate of flow pollution,  $\hat{X}_\infty - \hat{B}_\infty$ .  $\hat{S}_\infty$  is reduced below the potential rate  $\hat{Q}_\infty$  both by green innovation ( $\hat{B}_\infty > 0$ ) and through deceleration ( $\hat{X}_\infty < \hat{Q}_\infty$ ,  $\hat{c}_\infty < \hat{Q}_\infty$ ). The ratio of green relative to productivity-improving innovation is  $\hat{B}_\infty/\hat{Q}_\infty = \alpha/(1-\alpha)$ .

**Proof.** See appendix B.6. ■

We show in the appendix that  $\alpha/(1-\alpha)$  is an upper bound for the orientation of research towards green innovation as given by the ratio  $\hat{B}_\infty/\hat{Q}_\infty$ .

<sup>15</sup>  $\rho^{\text{delta}}$  again differs for the balanced-growth and the ABG-case. It is defined for each case in appendix B.4.

<sup>16</sup> The transversality conditions are given in equation (B.43) in the appendix. The critical value  $\rho^{\text{TVC}}$  is defined in appendix B.4.

The expression  $1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E}$ , on the other hand, is the ratio of green relative to productivity-improving innovation which yields the pollution growth rate reconcilable with asymptotically-balanced growth (according to equation (13)) when there is no deceleration. To gain a better understanding of the expression, assume - besides convex disutility of pollution - that the IES in consumption,  $\sigma_c$ , is smaller than one. The optimal pollution path in a growing economy must then be negatively sloped by proposition 3. The ratio  $\frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E}$  is strictly negative, so that  $1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E}$  is larger than one: Without deceleration, one percent growth in productivity leads to one percent growth in polluting quantity. For the pollution stock to fall, a more than one percent reduction in pollution intensity is needed. As  $\sigma_E$  increases from close to zero to  $1/2$ , the intertemporal elasticity of substitution in pollution ( $\sigma_E/(1 - 2\sigma_E)$ ) increases and the optimal pollution path becomes steeper.  $1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E}$  increases as well because relatively more green research is needed to achieve a faster pollution reduction.

If  $\sigma_c > 1$ , the pollution stock increases in the long run and research and technical change are oriented towards improving productivity ( $\hat{B}_\infty/\hat{Q}_\infty < 1$ ). As a response to an increase in  $\sigma_E$ , research is now shifted more strongly towards productivity improvements because there is less aversion towards pollution growth.

Reductions in pollution intensity are optimally combined with a restriction of polluting quantity growth, that is, deceleration, if the elasticity  $\alpha$  of final good production  $Y_t = X_t^\alpha (Q_t L_{Yt})^{1-\alpha}$  with respect to intermediate quantity is sufficiently small.

In this case, the cost of deceleration is rather low: A small elasticity implies that polluting quantity growth has only a minor effect on output growth compared to productivity growth. Quantity growth can be restricted without giving up too much consumption growth.

Further, with the relative unattractiveness of growth in polluting quantity for small  $\alpha$ , it becomes less important to reduce the pollution intensity of intermediate goods. The smaller  $\alpha$ , the lower therefore the social return to green as opposed to productivity-improving research. If  $\alpha/1 - \alpha < 1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E}$ , the relative return to green research is too small, so that it is not optimal to bring about the asymptotically-balanced pollution growth rate by green innovation alone. Research then remains rather productivity-oriented but deceleration lowers the rebound effect of productivity growth and thereby helps to restrict pollution growth.

A solution without deceleration becomes less likely as  $\sigma_E$  increases whenever the pollution stock falls ( $\sigma_c < 1$ ) and more likely if it rises ( $\sigma_c > 1$ ) because the steeper pollution path implies a stronger orientation of R&D towards green innovation in the former and a stronger orientation towards productivity-oriented innovation in the latter case.

We have characterized the social optimum in the long run only. The set of necessary conditions generates a complex dynamic system which does not allow to determine the transition path analytically. Numerical analysis suggests, however, that for any initial state of the economy, there exists a path leading towards the long-run optimal solution. This justifies the adoption of a long-run perspective in our analysis.

We now assume that natural regeneration is slow and the representative household is patient and desires a sufficiently high consumption growth rate so that corollary 1 holds and the pollution stock must decrease with the maximum rate  $\delta$  asymptotically. As before, depending on the specification of model parameters, we find two types of optima in appendix B.7 - one with and one without deceleration.

**Proposition 5 (A)BG optimum for  $\hat{S}_\infty = (-\delta)$**

Assume convex disutility of pollution ( $\sigma_E < 1/2$ ) and  $\sigma_c < 1$ . Assume further that the condition of corollary 1 holds ( $\rho \leq \rho^{\text{delta}}$ ):

**(a) Green innovation without deceleration:** If  $1/\sigma_c < \frac{\alpha}{1-\alpha}$  or if  $1/\sigma_c > \frac{\alpha}{1-\alpha}$  and  $\hat{c}_\infty < \frac{(1-2\sigma_E)/\sigma_E}{(1/\sigma_c) - \frac{\alpha}{1-\alpha}} \delta$ , any ABG-path without deceleration which solves the necessary first-order conditions for  $t \rightarrow \infty$  is characterized by a stronger orientation of research towards green innovation (larger  $\hat{B}_\infty/\hat{Q}_\infty$ ), compared to the corresponding case in proposition 4.

**(b) Green innovation with deceleration:** If  $1/\sigma_c > \frac{\alpha}{1-\alpha}$  and  $\hat{c}_\infty > \frac{(1-2\sigma_E)/\sigma_E}{(1/\sigma_c) - \frac{\alpha}{1-\alpha}} \delta$ , there exists an asymptotically unique ABG-optimum characterized by deceleration. The orientation of research is the same as in proposition 4(b).

**Proof.** See appendix B.8. ■

As in the preceding subsection, whether there is deceleration in the long-run optimal solution depends on how elastic the production function is with respect to intermediate quantity and productivity. For  $\frac{\alpha}{1-\alpha} < 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}$ , the conditions for deceleration are satisfied.<sup>17</sup> Additionally, there is deceleration in the long-run optimal solution also for  $\frac{\alpha}{1-\alpha} > 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}$  if  $1/\sigma_c > \frac{\alpha}{1-\alpha}$  and the desired consumption growth rate is sufficiently large ( $\hat{c}_\infty > \frac{(1-2\sigma_E)/\sigma_E}{(1/\sigma_c) - \frac{\alpha}{1-\alpha}} \delta$ ).

Because research orientation is green for  $\sigma_c < 1$ , the ratio  $\hat{B}_\infty/\hat{Q}_\infty$  must exceed one. At the same time, it must remain below the upper bound  $\frac{\alpha}{1-\alpha}$ . It follows that the case in proposition 5(a), where the long-run optimal solution is not characterized by deceleration, can only occur if  $\frac{\alpha}{1-\alpha} > 1$ , i.e., if the production elasticity of intermediate quantity exceeds 0.5. From empirical estimates, this is not a realistic range, as we argue in section 4.4.

An overview over the different cases for the long-run optimal solution outlined in propositions 4 and 5 is given in figure 1 below.

From the ratio  $\hat{B}_\infty/\hat{Q}_\infty$  in propositions 4 and 5, we can directly determine the direction of technical change:

**Corollary 2 Direction of technical change**

The direction of technical change is green (productivity-oriented), i.e.,  $\hat{B}_\infty > \hat{Q}_\infty$  ( $\hat{B}_\infty < \hat{Q}_\infty$ ), if and only if  $\sigma_c < 1$  ( $\sigma_c > 1$ ) if the conditions of propositions 4(a) and 5(a) hold and if and only if  $\alpha > 1/2$  ( $\alpha < 1/2$ ) given parameter constellations which satisfy propositions 4(b) and 5(b).

**Proof.** The ratio  $\frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}$  is negative (positive) so that  $\hat{B}_\infty/\hat{Q}_\infty > 1$  ( $\hat{B}_\infty/\hat{Q}_\infty < 1$ ) in proposition 4(a) whenever  $\sigma_c < 1$  ( $\sigma_c > 1$ ). Proposition 5 is only relevant for  $\sigma_c < 1$  and the ratio  $\hat{B}_\infty/\hat{Q}_\infty$  exceeds the value in proposition 4(a) if there is no deceleration. It follows that without deceleration, technological change is green ( $\hat{B}_\infty/\hat{Q}_\infty > 1$ ) if and only if  $\sigma_c < 1$  and productivity-oriented ( $\hat{B}_\infty/\hat{Q}_\infty < 1$ ) if and only if  $\sigma_c > 1$ . With deceleration,  $\hat{B}_\infty/\hat{Q}_\infty = \alpha/(1-\alpha)$  in propositions 4(b) and 5(b) so that technical change is green whenever  $\alpha > 1/2$  and productivity-oriented whenever  $\alpha < 1/2$ . ■

<sup>17</sup>First,  $1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E} < 1/\sigma_c$  if  $\sigma_c < 1$  and the disutility of pollution is convex ( $\sigma_E < 1/2$ ).  $\frac{\alpha}{1-\alpha} < 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}$  and  $1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E} < 1/\sigma_c$  implies that  $\frac{\alpha}{1-\alpha} < 1/\sigma_c$ . Given  $\frac{\alpha}{1-\alpha} < 1/\sigma_c$  and  $\frac{\alpha}{1-\alpha} < 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}$ , the condition for  $\hat{S}_\infty$  to converge to  $(-\delta)$ , which is  $\hat{c}_\infty > \frac{(1-\sigma_E)/\sigma_E}{(1/\sigma_c) - \frac{\alpha}{1-\alpha}} \delta$ , implies  $\hat{c}_\infty > \frac{(1-2\sigma_E)/\sigma_E}{(1/\sigma_c) - \frac{\alpha}{1-\alpha}} \delta$ .



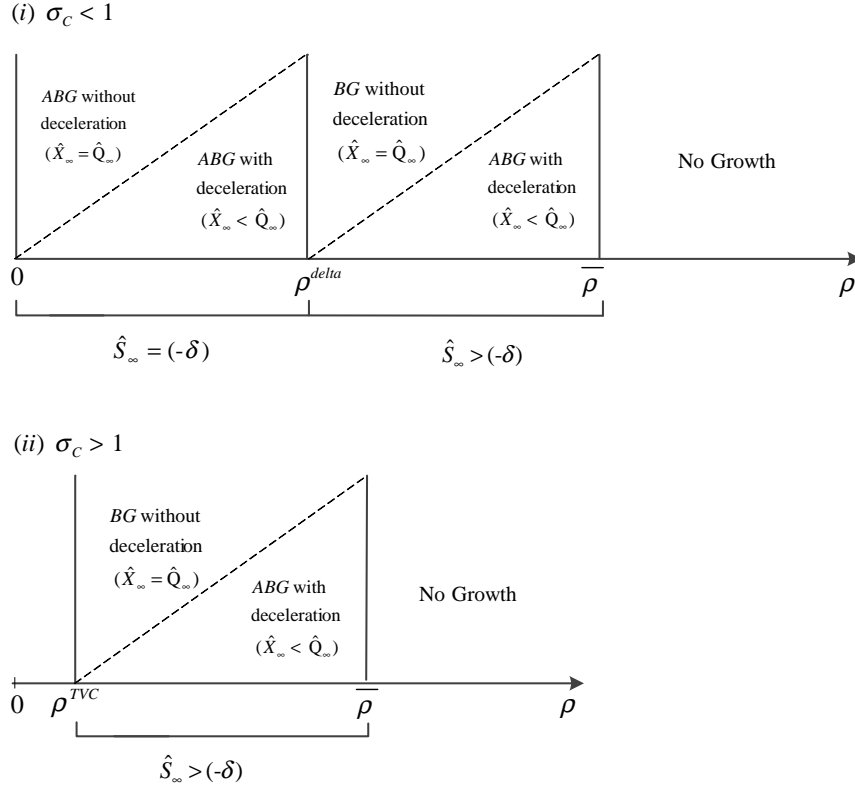


Figure 1: Case differentiation for the long-run optimal solution in dependence of the rate of time preference,  $\rho$ . Broken lines indicate a further partition of the parameter space in the relevant range of  $\rho$ , given by proposition 4 for  $\hat{S}_\infty > (-\delta)$  and by proposition 5 for  $\hat{S}_\infty = (-\delta)$ .

A very strong form of deceleration occurs if intermediate quantity falls in absolute terms, not only per labor efficiency unit. There is then degrowth in intermediate quantity (but not in GDP). Because quantity degrowth is deceleration in its extreme, it is optimal only if the ratio  $\alpha/(1 - \alpha)$  of production elasticities is particularly small. This result follows as a corollary from propositions 4 and 5:

### Corollary 3 *Quantity degrowth*

Assume that the disutility of pollution is convex ( $\sigma_E < 1/2$ ).

Given the conditions of proposition 4 ( $\hat{S}_\infty > (-\delta)$ ),  $X_t$  converges to zero as  $Q_t$  grows in an ABG-solution of the social planner's problem, i.e. there is quantity degrowth ( $\hat{X}_\infty < 0$ ), if and only if  $\frac{\alpha}{1-\alpha} < (1 - \alpha) \frac{(1-\sigma_c)/\sigma_c}{(1-\sigma_E)/\sigma_E}$ . Given the conditions of proposition 5 ( $\hat{S}_\infty = (-\delta)$ ), there is quantity degrowth if and only if  $\frac{\alpha}{1-\alpha} < (1 - \alpha) \frac{1-\sigma_c}{\sigma_c}$  and  $\hat{Q}_\infty$  is sufficiently large.

**Proof.** Proof follows directly from setting  $\hat{X}_\infty < 0$  in equation (B.55) and equation (B.73) in the appendix. ■

Note that quantity degrowth can only be optimal if the pollution stock is required to decline in the long-run optimum (for  $\sigma_c < 1$ ).

#### 4.4 A numerical example

Which of the cases in subsection 4.3 is more relevant empirically? Should green innovation be complemented by deceleration or even quantity degrowth in the long-run? Although our model is too stylized to give unambiguous quantitative answers, we think it allows to give a plausible indicative answer at least to these general qualitative questions. The relevant parameters are  $\alpha$ ,  $\sigma_c$  and  $\sigma_E$ . While there are little reliable empirical estimates of  $\sigma_E$ , we believe that disutility is convex in the pollution stock ( $\sigma_E < 1/2$ ) so that the marginal disutility of pollution is the larger, the more polluted the environment is. As for the IES of consumption  $\sigma_c$ , we restrict  $\sigma_c$  to the range  $\sigma_c \in (0, 1)$  as suggested by a large body of empirical literature (e.g. Hall (1988), Ogaki and Reinhart (1998)). Choosing  $\alpha$  is less straightforward. Setting  $\alpha$  to the capital share implies  $\alpha \approx 1/3$ . Interpreting  $X_t$  as energy,  $\alpha$  would be substantially smaller than the capital share: Energy expenditures as a share of GDP amounted to 8.9% in the U.S. in 2012 (EIA (2013)). On the other hand,  $\alpha$  is also the inverse of the mark-up in the intermediate sector. As Smulders and van der Meijden (2012) point out, estimates for the manufacturing sector in the U.S. (Roeger (1995)) suggest values of  $\alpha$  of at least 0.3. We consider values which do not exceed 0.5 as plausible.

**Corollary 4** *Assume convex disutility of pollution ( $\sigma_E < 1/2$ ), an intertemporal elasticity of substitution in consumption  $\sigma_c \in (0, 1)$  and  $0 < \alpha \leq 1/2$ .*

*If the representative household is patient enough to favor persistent growth, then productivity growth always has to be accompanied by both green innovation and deceleration. Research is productivity-oriented ( $\hat{Q}_\infty > \hat{B}_\infty > 0$ ), but the rebound effect of productivity growth is reduced by deceleration so that the share of polluting quantity in GDP decreases.*

**Proof.** We have shown in section 4.3, that for convex disutility of pollution, the condition  $\frac{\alpha}{1-\alpha} < 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}$  is necessary and sufficient for the long-run optimum to be characterized by deceleration in the case where  $\hat{S}_\infty > -\delta$  and a sufficient condition if  $\hat{S}_\infty$  converges towards  $(-\delta)$ . For  $0 < \alpha \leq 1/2$ , this condition holds for all  $\sigma_c \in (0, 1)$ . ■

If we choose a smaller range for  $\alpha$ , so that  $\alpha$  does not exceed the capital share of  $1/3$ , there will be deceleration in the optimal solution for  $\sigma_c < 2$  which covers most empirical estimates. Setting  $\alpha$  to the energy share in real GDP, even extremely high values of  $\sigma_c$  up to 4.4 as found by Fuse (2004) for Japan do not violate the condition.

For intermediate quantity to fall in absolute terms, so that there is quantity degrowth,  $\sigma_c$  must be smaller than one and  $\alpha$  should be substantially below the capital share. Quantity degrowth is likely to be optimal if  $\alpha$  is interpreted as the energy share in GDP: Setting  $\alpha \approx 0.09$ , the optimal solution is characterized by quantity degrowth for values of  $\sigma_c$  from almost the entire interval  $(0, 1)$  if  $1/3 \leq \sigma_E < 1/2$ .

We conclude that for reasonable assumptions about model parameters, both green innovation and deceleration (possibly in its extreme form, i.e., quantity degrowth) contribute to optimal pollution control. In the next subsection, we illustrate the adverse effects on long-run consumption growth if only one channel of pollution control is available to the social planner.

## 4.5 Constraining pollution control

We have shown in the previous subsection that for reasonable parameter values, the long-run optimal solution is characterized by green innovation and deceleration. We demonstrate in this subsection the importance of using both channels to control pollution growth simultaneously. For this purpose, we consider two constrained optimal solutions, where either green innovation or deceleration cannot be chosen by the social planner.

First, we rule out deceleration. More precisely, we assume that the social planner must keep intermediate quantity per efficiency unit at a constant, positive level in the long run which is equivalent to imposing the constraint  $\hat{X}_\infty = \hat{Q}_\infty$ . We are interested in the case where this constraint is binding, that is in the case where the social planner chooses  $\hat{X}_\infty < \hat{Q}_\infty$  in the unconstrained optimal solution.

Next, we allow for  $\hat{X}_\infty < \hat{Q}_\infty$  but set  $b_t$  exogenously to zero for all  $t$  so that green innovation is no longer available to the social planner. For ease of exposition, we focus on the parameter region where  $\rho > \rho^{\text{delta}}$ , which guarantees that  $\hat{S}_\infty > (-\delta)$  in the unconstrained and the constrained solution. The following proposition summarizes the results:

### Proposition 6 *Constrained (A)BG-optimum*

**(a) No deceleration:** Assume that the social planner is forced to choose balanced growth in the long run, i.e.,  $\hat{X}_\infty = \hat{Q}_\infty$ , so that pollution growth can only be restricted by green innovation.

Whenever the constraint is binding, the long-run growth rates of all variables in the constrained optimum are zero.

**(b) No green innovation:** Assume instead that the social planner is free to choose  $\hat{X}_\infty$  and  $\hat{Q}_\infty$  but  $b_t = 0$ ,  $\forall t$  is exogenously fixed so that there is no green innovation. Further, assume  $\rho > \rho^{\text{delta}}$ .

Then given positive long-run growth, there is always deceleration in the constrained optimal solution, i.e.,  $\hat{X}_\infty < \hat{Q}_\infty$ . Compared to the unconstrained solution in proposition 4, (i) the upper bound on the rate of time preference for positive long-run growth to be optimal is stricter, (ii) deceleration is faster ( $\hat{X}_\infty/\hat{Q}_\infty$  is smaller) and (iii) the growth rate of consumption per capita is slower.

**Proof.** See appendix B.9. ■

When the social planner is constrained to  $\hat{X}_\infty = \hat{Q}_\infty$ , bringing about the optimal relation between growth in consumption and pollution requires research to be rather strongly oriented towards green innovation. But the relative social return to green research is too small for such a strong orientation towards green research to be optimal. With less environmentally-oriented research, on the other hand, pollution growth is suboptimally fast. The only solution is to give up long-run growth.

The implications of this result are twofold: First, neglecting the possibility to control the rebound effect of productivity growth leads to the conclusion that long-run economic growth is not optimal for a larger parameter region than is actually the case. Second, the result reinforces the statement of proposition 1: We have shown in this proposition, that stationary consumption levels as demanded by certain green movements are socially preferable to the unregulated market solution. Proposition 6 indicates that stationary consumption levels may dominate long-run growth in terms of welfare even if there is government intervention so that the externalities

in green innovation in particular are internalized, if adequate policy-measures to control the rebound effect of productivity growth are not in place.

If green innovation is not available, deceleration is the only way to control pollution growth. Long-run growth in the constrained optimum is still positive for a sufficiently patient household, but stronger deceleration decreases the long-run consumption growth rate compared to the unconstrained solution.

## 4.6 Environmental care and the pace of economic growth

In our model, a stronger research orientation towards green innovation implies slower productivity growth for given total research effort. Further, deceleration requires to give up potential consumption growth. Intuitively, one might therefore expect environmental care to slow down economic growth relative to the case where the negative environmental externality of intermediate goods is not taken into account.

In appendix B.10, we derive the long-run optimal solution of our model assuming the weight of pollution in utility is zero ( $\psi = 0$ ), so that the representative household is not affected by pollution. Comparing the optimal solution of our baseline model to the optimum in the modified setting, we find that the above intuition is not necessarily correct. Economic growth is positive for larger rates of time preference in our framework and, depending on parameters, growth rates of consumption, production and productivity may in fact be higher than in the model without a negative external effect from pollution.<sup>18</sup>

Moreover, the degree of the household's preference for a clean environment and therefore the strength of the negative pollution externality, as reflected in the size of  $\psi$ , does not influence long-run growth rates at all (given  $\psi > 0$ ). The reason is that stronger environmental preference does not alter the social return to productivity-oriented research, which is the driver of economic growth. The long-run relation between productivity growth and growth in intermediate quantity, consumption and output is fixed independently of the environmental preference on an ABG-path.<sup>19</sup>

We prove our claim formally for parameter constellations for which the long-run optimal solution is characterized by deceleration:

### **Proposition 7** *Environmental care and the pace of economic growth*

*Assume that the disutility of pollution is convex ( $\sigma_E < 1/2$ ) and that the conditions for deceleration from propositions 4 (b) and 5 (b) are satisfied.*

*In the baseline setup with  $\psi > 0$ , compared to the optimal solution in a modified setting without negative external effect from pollution on utility ( $\psi = 0$ ), (i) the condition for growth in per capita consumption to be positive is less strict ( $\bar{\rho}^{\psi=0} < \bar{\rho}$ ) and (ii) optimal growth in per capita consumption is faster if and only if the rate of time preference is sufficiently large.*

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<sup>18</sup>A similar result can be obtained if the optimal solution for  $\psi > 0$  is compared not to the optimum for  $\psi = 0$  but to the laissez-faire equilibrium. It is however not possible to attribute faster growth to the environmental externality in particular in this case because equilibrium growth may be suboptimally slow as a result of several other externalities ('standing-on-shoulders' of previous innovators, firms cannot appropriate the whole consumer surplus).

<sup>19</sup>A similar result was found by Gradus and Smulders (1993) in a Lucas-Uzawa-model. While stronger environmental preference has no influence on long-run growth rates, it can be expected to affect the levels of the model variables along the long-run path. These effects can however not be analyzed without studying transitional dynamics.

*Given  $\psi > 0$ , the strength of the representative household's preference for a clean environment, as reflected in the size of  $\psi$ , has no influence on long-run optimal growth rates.*

**Proof.** See appendix B.11. ■

The driving force behind the result is a positive link between green and productivity-oriented research. Green innovation can lead to an increase in the optimal amount of labor devoted to research which fosters also productivity growth and therefore consumption growth. In fact, it can be shown that in the constrained optimum without green innovation ( $b_t = 0, \forall t$ ) from subsection 4.5, consumption growth is unambiguously slower than in the unconstrained optimal solution with  $\psi = 0$ . Not only is there no longer a positive effect of green innovation on total research labor, but pollution growth has to be controlled by deceleration only, which requires a rather strong restriction of GDP and consumption growth relative to productivity growth.

## 5 The model with a non-renewable resource

According to the IPCC, one of the major sources of pollution is energy supply, and the generation of energy relies heavily on non-renewable resources like fossil fuels. To reasonably interpret polluting intermediate inputs in our model as energy, it is therefore important to account for the exhaustibility of energy-resources along with the pollution externality. This section examines the robustness of the main results from the baseline model with respect to the consideration of a polluting non-renewable resource. More precisely, intermediate production is assumed to use a resource drawn from an exhaustible stock as the only input. It is shown that while resource scarcity slows growth in the laissez-faire equilibrium, the results of our baseline model still apply for the long-run social optimum if the optimal solution of the baseline model is characterized by quantity degrowth and the initial resource stock is large enough. We have argued in section 4.4 that judging by empirical estimates for the model parameters, a solution with quantity degrowth is reasonable. The negative pollution externality of intermediate production then reduces optimal resource use in a way that a sufficiently large resource stock is never exhausted.

### 5.1 Setup

We denote the resource stock in period  $t$  by  $F_t$ . Starting from a finite positive initial level  $F_0$ , the resource stock is depleted proportionally to resource use:

$$\dot{F}_t = -R_t. \quad (14)$$

We assume that the resource is owned by the representative household and, for simplification, that it can be extracted at zero cost (see also Barbier (1999), Schou (2000) and Groth and Schou (2002)).

The resource stock  $F_t$  must be non-negative for any  $t$ . Therefore total extraction must not exceed the initial stock  $F_0$ , a requirement which is formally represented in the condition

$$\int_0^\infty R_t dt \leq F_0. \quad (15)$$

Suppose that one unit of the intermediate good is produced by one unit of the non-renewable resource so that

$$X_{it} = R_{it} \quad (16)$$

is resource input in sector  $i$  and  $X_t = \int_{i=0}^1 X_{it} di = R_t$  aggregate resource use in period  $t$ . With a finite resource stock, it is obvious that resource use and therefore intermediate production must ultimately decline to zero in the long run, both in the socially optimal solution and the laissez-faire equilibrium. There has to be quantity degrowth.

**Lemma 2** *If intermediate goods are produced with a non-renewable resource according to equation (16), the growth rate  $\hat{X}$  of intermediate quantity is negative in the long run. Any solution path is characterized by quantity degrowth for  $t \rightarrow \infty$ .*

**Proof.** It follows from (16), that aggregate resource use is  $R_t = X_t$ . Substitution into equation (15) yields  $\int_0^\infty X_t dt \leq F_0$ . To satisfy the condition, the integral must converge, which requires  $\lim_{t \rightarrow \infty} \hat{X}_t = \hat{X}_\infty < 0$  as a necessary condition. ■

We now consider the equilibrium and the optimal outcome in more detail.<sup>20</sup>

## 5.2 Resource scarcity in the laissez-faire equilibrium

Instead of labor costs, firms in the intermediate sector have to pay a price  $p_{Rt}$  for the use of the non-renewable resource. This price increases over time according to the Hotelling rule<sup>21</sup>

$$\hat{p}_{Rt} = r_t, \quad (17)$$

while in the baseline specification without resources, labor costs for the production of one unit of intermediates were constant at equilibrium. Through the increasing resource price, resource scarcity leads to a restriction in the growth rate of intermediate quantity relative to productivity growth<sup>22</sup>:

$$\hat{X}^{\text{LF, R}} = \hat{Q}^{\text{LF, R}} - \frac{1}{1 - \alpha} r^{\text{LF, R}} \quad (18)$$

We know from lemma 2 that intermediate quantity must in fact fall along an equilibrium path. Formally, this is shown using the transversality condition in appendix C.1.4.

While it is obvious that resource scarcity, through the increasing resource price, leads to persistent quantity degrowth, it is interesting to study the impact of resource scarcity on the laissez-faire equilibrium more extensively.

For any given initial resource stock  $F_0$ , the fact that the resource becomes scarcer over time is reflected in the positive growth rate of the resource price. Further, resource scarcity affects economic variables through the level of the resource price, which depends negatively on the size of the initial stock  $F_0$ .

While the size of  $F_0$  is exogenous, the growth rate of the resource price is endogenous. Still, it is possible to single out the effects not only of  $F_0$  but also the impact of the growing resource price on equilibrium growth and the levels of technology, production, consumption and pollution.

<sup>20</sup> As before, we focus on balanced and asymptotically-balanced growth solutions.

<sup>21</sup> See appendix C.1.1 for the derivation.

<sup>22</sup> For a derivation, see appendix C.1.2.

The following proposition summarizes the results:

**Proposition 8** *Effects of resource scarcity on the laissez-faire equilibrium*

**(a) Growth effects:** For any given initial resource stock  $F_0$ , the exhaustibility of the resource is reflected in a positive growth rate of the resource price. Resource scarcity thereby (i) leads to persistent quantity degrowth (ii) unambiguously decreases the equilibrium growth rates of  $Y$  and  $c$  as well as the long-run growth rate of  $S$ , (iii) lowers (increases) the productivity growth rate whenever  $\sigma_c > 1$  ( $\sigma_c < 1$ ) (given it is positive), and (iv) restricts the parameter range for which there is positive growth in per capita consumption.

**(b) Level effects:** The size of the initial resource stock  $F_0$  (i) does not affect growth rates of the economic variables and the long-run pollution growth rate, nor the paths of the technology stocks  $Q$  and  $B$ . However, the smaller  $F_0$ , (ii) the larger is the level of the resource price along the entire equilibrium path and (iii) the smaller are, accordingly, the levels of intermediate production, output, per capita consumption and pollution in every period  $t$ .

**Proof.** See appendix C.1.4. ■

The growth rate of the resource price has two opposing effects on the size of the productivity growth rate,  $\hat{Q}^{\text{LF,R}}$ : On the one hand, it depresses monopoly profits from intermediate production and thereby entry for a given interest rate. This tends to slow growth. On the other hand, the decrease in entry causes a countervailing general-equilibrium effect: The equilibrium interest rate is smaller, which slows the price increase, as can be seen from (17), and stimulates entry and productivity growth.

If  $\sigma_c < 1$ , the representative household desires to smooth consumption over time and reacts inelastically to changes in the interest rate. The decline in the interest rate is therefore more pronounced than for  $\sigma_c > 1$ , so that the positive effect on productivity growth predominates in the former, and the negative effect predominates in the latter case.

The growing resource price depresses consumption growth along the equilibrium path because it induces quantity degrowth. The increase in productivity growth for  $\sigma_c < 1$  dampens the decline in intermediate quantity and stimulates output and consumption growth also directly. Nevertheless, the overall effect of resource scarcity on consumption and output growth is unambiguously negative.<sup>23</sup>

On the other hand, resource scarcity has a beneficial effect on household utility through the pollution externality: The growing resource price ensures that the total amount of emissions at equilibrium is bounded and the pollution stock declines along the (asymptotically) balanced growth path.

The initial resource stock affects the laissez-faire equilibrium only through the level of the resource price (see equation (C.91) in the appendix). The price level leaves growth rates unaffected. The reason is that the level of the resource price does not influence research profits because profits from intermediate production and research costs decline in the price level in the same way. The implication of growth rates being unaffected by the size of the initial resource stock is that two economies with different initial resource endowments share the same long-run growth rates and the same technology paths.

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<sup>23</sup>Contrary to the equilibrium in the baseline model, it is even possible that the growth rates of consumption and output are negative (see also Schou (2002) for a similar result). Degrowth does then not only occur in polluting quantity but in consumption and output.

On the other hand, the price level is relevant for the determination of intermediate production levels in each period (see appendix C.1.2). The higher the resource price, the higher is the price firms in the consumption goods sector pay for intermediate goods and the lower are intermediate demand and the equilibrium quantity of intermediates. Because productivity and labor in the consumption goods sector are independent of the initial resource stock, it follows that the paths for output and consumption in an economy with small initial resource stock are below those of a more resource-abundant economy. At the same time, there is less pollution in every period as less of the polluting input is produced.

### 5.3 Resource scarcity in the long-run social optimum

We first characterize the long-run social optimum in case of a binding natural resource constraint, which is the case most commonly studied in the literature (Schou (2000, 2002), Grimaud and Rouge (2008)). The Lagrange-multiplier  $\lambda_{Rt}$  for the natural resource constraint reflects the social costs of producing one unit of intermediates, i.e., the social price of the non-renewable resource.  $\lambda_{Rt}$  increases over time according to the modified Hotelling rule<sup>24</sup>

$$\hat{\lambda}_{Rt} = \rho. \quad (19)$$

While the social price  $\lambda_{Rt}$  of the non-renewable resource increases with progressing resource scarcity, the shadow price  $v_{St}$  of pollution moves along with the marginal disutility of pollution on an asymptotically balanced growth path<sup>25</sup>. The shadow price therefore falls towards  $v_{S\infty}^R = 0$  as the stock of the polluting resource gets exhausted and the pollution stock declines. It is shown in the appendix that in this case, green innovation is no longer optimal in the long run, i.e.,

$$b_{\infty}^R = \hat{B}_{\infty}^R = 0.$$

However, we have suggested earlier that contrary to what is commonly assumed in the literature, the natural resource constraint need not be binding in the social planner's solution. We know from corollary 3 in subsection 4.3, that the social planner may choose to let the quantity of intermediates decrease in the long run even if there is no constraint imposed on intermediate production by resource scarcity. More precisely, this is the case if preferences are such that a declining pollution stock is desired and the factor elasticity of intermediates is particularly small so that quantity degrowth is not too costly in terms of foregone potential consumption growth. Whenever there is quantity degrowth in the long run, the integral  $\int_0^{\infty} X_t dt$  converges to a finite value.<sup>26</sup> In the modified setting where intermediates are produced from a non-renewable resource, the resource constraint is then not binding given that the initial resource stock is not too small. We prove in the appendix that the long-run optimal solution of the resource model is the same as in our baseline model without resources.

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<sup>24</sup>The first-order conditions are described in appendix C.2.1.

<sup>25</sup>See equation (B.45) in the appendix.

<sup>26</sup>The formal proof requires continuity of the path  $\{X_t\}_0^{\infty}$ , see appendix C.2.2(b).



We summarize our results in the following proposition:

**Proposition 9** *ABG-optimum with an exhaustible resource*

Besides  $\sigma_E < 1/2$ , assume that intermediates are produced with a non-renewable resource according to equation (16). Assume further that the path  $\{X_t\}_0^\infty$  for intermediate quantity is continuous.

There is always quantity degrowth in the long-run optimal solution. Further, the following holds:

**(a) Binding resource constraint** *If the resource constraint is binding, all labor in research and development is shifted to productivity improvements asymptotically and green innovation comes to a halt ( $\widehat{B}_\infty^R = 0$ ).*

**(b) Non-binding resource constraint** *Given that the conditions for quantity degrowth in the baseline model (see corollary 3) are satisfied and given a sufficiently large (but finite) initial resource stock  $F_0$ , the natural resource constraint is not binding for the social planner's problem. There exists an asymptotically unique ABG-solution which is identical to the ABG-solution with quantity degrowth described in section 4.3. More precisely, growth in output and consumption is positive, given a sufficiently small rate of time preference  $\rho$ , and entirely driven by productivity growth. The pollution stock  $S$  declines both due to quantity degrowth and because the pollution intensity of intermediate goods is reduced by green innovation. The orientation of research and technical change is given by  $\widehat{B}_\infty^R / \widehat{Q}_\infty^R = \alpha / (1 - \alpha)$ .*

**Proof.** See appendix C.2.2. ■

In case of a binding resource constraint, resource scarcity forces the social planner to save on polluting inputs to such an extent that investing in green innovation to bring about an even faster decline in pollution is not optimal in the long run. The depletion of the polluting resource automatically solves the pollution problem, while it poses an increasing threat to economic growth over time. Therefore, asymptotically, green innovation comes to a halt and all labor in the research sector is shifted towards productivity improvements. Productivity growth raises the productivity of intermediate goods and thereby dampens the adverse effects from resource scarcity on output and consumption growth.

Proposition 9 suggests, however, that the causality may work in the opposite direction as well: The preference for a clean environment may make it optimal to restrict resource use in a way that the resource stock is never exhausted and the resource constraint is not binding. In our numerical example in section 4.4, we have suggested that the parameter constellations for which there is quantity degrowth in the long-run optimal solution are well in line with empirical evidence. In particular, quantity degrowth has been shown to be a likely outcome of the social planner's optimization problem if the intermediate good is interpreted as energy input and its production elasticity  $\alpha$  as the energy share in GDP. We conclude that without too strong restrictions on the parameter range, the long-run results from the socially optimal solution of the baseline model extend to a model with a non-renewable resource.

## 6 Conclusion

We considered an endogenous growth model with polluting intermediate production where technical progress can increase the productivity and lower the pollution intensity of intermediate goods. Reductions in pollution intensity are called green innovation. Pollution accumulation can be controlled by green innovation and by

decreasing the ratio of polluting intermediate quantity in GDP. The latter requires to use intermediates more efficiently, keeping quantity growth below productivity growth, and goes along with a cost in terms of foregone potential growth in consumption and GDP. Not only intermediate quantity but also consumption and GDP fall persistently per labor efficiency unit. We therefore speak of ‘deceleration’. If intermediate quantity falls in absolute terms, we say that there is ‘quantity degrowth’ which is but a strong form of deceleration.

We have shown that under *laissez-faire*, neither green innovation nor deceleration is chosen at equilibrium. If utility is concave in consumption but the marginal disutility of pollution increases in the pollution stock, an equilibrium without long-run growth would be socially preferable. The social optimum allows for a constant positive long-run consumption growth rate whenever the representative household is sufficiently patient. However long-run growth has to be accompanied by pollution control. It is always optimal to allocate part of the labor supply to green innovation. For reasonable assumptions about model parameters, green innovation is optimally complemented by deceleration to reduce the share of polluting inputs in GDP. If either green innovation or deceleration is not available to the social planner, the prospects for long-run consumption growth are considerably more pessimistic in the long-run optimal solution.

Our baseline model does not include exhaustible resources although their use in production generates a substantial share of worldwide emissions. We have therefore extended the baseline model by assuming that a polluting non-renewable resource is the only input in intermediate production in order to analyze the robustness of the main results with respect to the consideration of resource scarcity. As the resource stock is depleted, resource use and therefore intermediate production must ultimately fall so that even in the *laissez-faire* equilibrium, the pollution stock declines persistently. The dependency of production on a scarce resource clearly alters the *laissez-faire* equilibrium. The literature on polluting non-renewable resources usually assumes that the resource constraint is binding in the socially optimal solution as well. Contrary to the literature, we have shown that the need to restrict pollution growth may reduce optimal resource use in a way that the resource is never exhausted if the initial stock is large enough. More precisely, if the socially optimal solution in the baseline model is characterized by quantity degrowth in the long run, the social planner’s solution is not affected by the resource constraint given a sufficiently large initial resource stock. The results from the socially optimal solution in the baseline model extend to the long-run social optimum with an exhaustible resource.

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## A Appendix to section 3 (Laissez-faire)

### A.1 The representative household

The representative household earns income from labor and asset holding which he spends on consumption and the acquisition of new assets. The budget constraint is

$$c_t L + \dot{A}_t = r_t A_t + \int_0^1 w_{Xit} L_{Xit} di + w_{Yt} L_{Yt} + w_{Dt} L_{Dt}, \quad (\text{A.1})$$

where  $A_t$  denotes asset holdings and  $w_{Xit}$ ,  $w_{Yt}$  and  $w_{Dt}$  the wage rates for labor in intermediate production, production of the consumption good and research. The interest rate is denoted by  $r_t$ .

For notational convenience, we rewrite the utility function (1) as function of the pollution stock  $S$  rather than environmental quality  $E$ , using (2):

$$U = \int_0^\infty e^{-\rho t} \left( \frac{\sigma_c}{\sigma_c - 1} c_t^{\frac{\sigma_c - 1}{\sigma_c}} - \psi \frac{\sigma_E}{1 - \sigma_E} S_t^{\frac{1 - \sigma_E}{\sigma_E}} \right) L dt \quad (\text{A.2})$$

The household maximizes (A.2) by choosing the paths for consumption, labor and asset holding while taking pollution accumulation as given. He takes into account the budget-constraint (A.1) and must satisfy the no-Ponzi condition

$$\lim_{t \rightarrow \infty} \left( e^{-\int_0^t r_v dv} A_t \right) \geq 0,$$

which rules out chain-letter finance, that is, schemes where the household borrows continuously without ever repaying his debt or interest. The current-value Hamiltonian function is:

$$\begin{aligned} H = & \left( \frac{\sigma_c}{\sigma_c - 1} c_t^{\frac{\sigma_c - 1}{\sigma_c}} - \psi \frac{\sigma_E}{1 - \sigma_E} S_t^{\frac{1 - \sigma_E}{\sigma_E}} \right) L \\ & + v_{At} \left( r_t A_t + \int_0^1 w_{Xit} L_{Xit} di + w_{Yt} L_{Yt} + w_{Dt} L_{Dt} - c_t L \right) \\ & + \lambda_{Lt} \left( L - \left( \int_0^1 L_{Xit} di + L_{Yt} + L_{Dt} \right) \right) \end{aligned}$$

$v_{At}$  is the current-value costate variable of assets  $A$  in  $t$  and  $\lambda_{Lt}$  the Lagrange-multiplier of the constraint on labor. The first-order conditions according to Pontryagin's maximum principle are:

$$\frac{\partial H}{\partial c_t} = 0 \Leftrightarrow v_{At} = c_t^{\frac{-1}{\sigma_c}} \quad (\text{A.3})$$

$$\frac{\partial H}{\partial L_{Xit}} = 0 \Leftrightarrow v_{At} w_{Xit} = \lambda_{Lt}$$

$$\frac{\partial H}{\partial L_{Yt}} = 0 \Leftrightarrow v_{At} w_{Yt} = \lambda_{Lt}$$

$$\frac{\partial H}{\partial L_{Dt}} = 0 \Leftrightarrow v_{At} w_{Dt} = \lambda_{Lt}$$

$$\frac{\partial H}{\partial A_t} = \rho v_{At} - \dot{v}_{At} \Leftrightarrow v_{At} r_t = \rho v_{At} - \dot{v}_{At} \quad (\text{A.4})$$

$$\frac{\partial H}{\partial v_{At}} = \dot{A}_t \Leftrightarrow \dot{A}_t = r_t A_t + \int_0^1 w_{Xit} L_{Xit} di + w_{Yt} L_{Yt} + w_{Dt} L_{Dt} - c_t L \quad (\text{A.5})$$

$$\frac{\partial H}{\partial \lambda_{Lt}} = 0 \Leftrightarrow L = \int_0^1 L_{Xit} di + L_{Yt} + L_{Dt}$$

The first-order conditions for the different types of labor can only be satisfied simultaneously, if firms in the different sectors of intermediate production as well as firms in final good production and research all offer the same wage. The household is then indifferent about the allocation of his labor supply.

The first-order condition for assets,  $A_t$ , can be restated as  $\hat{v}_{At} = \rho - r_t$ . Log-differentiating both sides of the first-order condition for consumption yields

$$\frac{-1}{\sigma_c} \hat{c}_t = \hat{v}_{At}.$$

By substituting the expression for  $\hat{v}_{At}$ , we obtain the standard Euler-equation for per capita consumption:

$$\hat{c}_t = \sigma_c \cdot (r_t - \rho) \quad (\text{A.6})$$

## A.2 Production

The production function for the consumption good is given by (4). Firms maximize profits over  $L_Y$  and  $X_i$ , taking the wage rate  $w_{Yt}$  and the prices  $p_{it}$  of the intermediates in sectors  $i \in [0, 1]$  as given. We normalize the price of the consumption good to one. The first order condition for  $L_Y$  yields the implicit labor demand function

$$w_{Yt} = (1 - \alpha) L_{Yt}^{-\alpha} \int_0^1 X_{it}^\alpha Q_{it}^{1-\alpha} di. \quad (\text{A.7})$$

From the first-order condition for  $X_i$ , the following demand function for intermediate  $i$  is derived:

$$X_{it}^d(p_{it}) = \left( \frac{\alpha}{p_{it}} \right)^{\frac{1}{1-\alpha}} Q_{it} L_{Yt} \quad (\text{A.8})$$

Each unit of the intermediate is produced with the production function (6):

$$X_{it} = \varphi L_{Xit} Q_t$$

At equilibrium, the wage in intermediate production must be the same in every sector  $i$ , so that marginal costs  $MC_t = (1/\varphi) \cdot (w_{Xt}/Q_t)$  are the same for goods with different productivity levels. On the other hand, final good producers' demand is larger for more productive intermediates. It follows that only the owner of the patent for the intermediate design with the highest productivity will be producing in sector  $i$ , as he can always choose a price so that the firm with the next highest productivity level cannot break even. For the rest of this subsection, the firm index  $j$  is therefore omitted.

The intermediate good in sector  $i$  is sold at a price  $p_{it}$  to firms in the final good sector. The monopoly producer chooses  $p_{it}$  to maximize profits

$$\pi_{it}^X(p_{it}) = (p_{it} - MC_t) X_{it},$$

taking into account  $MC_t = (1/\varphi) \cdot (w_{Xt}/Q_t)$  and the demand function (A.8). The profit maximizing monopoly price is given by a constant mark-up over marginal costs for all  $i$ <sup>27</sup>:

$$p_t = \frac{1}{\alpha\varphi} \cdot (w_{Xt}/Q_t)$$

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<sup>27</sup> Monopoly pricing prevails under certain restrictions on model parameters which we derive in section A.3.

The wage rate at equilibrium is obtained by substituting (A.8) in (A.7), using the fact that wages from intermediate and final good production as well as research must be equal in equilibrium if all three types of labor are to be provided by the household. The equilibrium wage is:

$$w_{Xt} = w_{Yt} = w_{Dt} = (1 - \alpha)^{1-\alpha} \alpha^{2\alpha} (\varphi)^\alpha Q_t \quad (\text{A.9})$$

We then derive the quantity of intermediates produced in sector  $i$  as function of the amount of labor employed in final good production, for any given sectoral level of productivity, from (A.8):

$$X_{it} = \frac{\alpha^2}{1 - \alpha} \varphi L_{Yt} Q_{it} \quad (\text{A.10})$$

Monopoly profits in sector  $i$  in period  $t$  are:

$$\pi_{it}^X = \frac{(1 - \alpha)^{1-\alpha}}{\alpha} \alpha^{2(1+\alpha)} \varphi^\alpha L_{Yt} \cdot Q_{it} \quad (\text{A.11})$$

The aggregate quantity  $X_t$  of intermediates is

$$X_t = \int_0^1 X_{it} di = \frac{\alpha^2}{(1 - \alpha)} \varphi L_{Yt} Q_t, \quad (\text{A.12})$$

where we used the definition  $\int_0^1 Q_{it} di := Q_t$  of aggregate productivity.

### A.3 Research

At time  $t$ , researcher  $j$  in sector  $i$  chooses  $l_{Dijt}$ ,  $q_{ijt}$  and  $b_{ijt}$  to maximize expected profits from R&D. These consist of the profit flow he expects to receive as a monopolist in intermediate production less of research labor costs.

In every period and every sector, the exogenous arrival rate of innovations for the individual researcher is  $\mu$ . If researcher  $j$  succeeds in innovating, he changes the productivity level in sector  $i$  from  $Q_{it}$  to  $(q_{ijt} + 1) \cdot Q_{it}$ . After the innovation, the productivity level remains constant until the next innovation occurs and the monopoly producer is replaced by the new innovator.

The probability per unit of time of being replaced as the monopolist in sector  $i$  is exogenously given from the perspective of researcher  $j$  in every period  $v > t$  and increases in the mass  $n_{iv}$  of research units active in sector  $i$  at time  $v$ . More precisely, innovations in every sector  $i$  follow a Poisson-process with arrival-rate  $\mu_{iv} = \mu n_{iv}$ . The probability that the incumbent monopolist is still producing in period  $s > t$  is then given by  $P(s) = e^{-\int_t^s \mu_{iv} dv}$ . His profits in period  $s$  can be deduced from (A.11), substituting the after-innovation productivity level  $(q_{ijt} + 1) \cdot Q_{it}$  for  $Q_{it}$ .

Expected discounted lifetime-profits are:

$$\begin{aligned} E[V_{ijt}(q_{ijt})] &= \int_t^\infty \pi_{ijs}^X(q_{ijt}) \cdot P(s) e^{-\int_t^s r_v dv} ds \\ &= \frac{(1 - \alpha)^{1-\alpha}}{\alpha} \alpha^{2(1+\alpha)} \varphi^\alpha (q_{ijt} + 1) \cdot Q_{it} \int_t^\infty L_{Ys} e^{-\int_t^s (r_v + \mu_{iv}) dv} ds \end{aligned} \quad (\text{A.13})$$

Expected research profits are obtained by subtracting research costs  $w_{Dt} l_{Dijt}$ :

$$E[\pi_{ijt}^D(q_{ijt}, b_{ijt})] = \mu E[V_{ijt}(q_{ijt})] - w_{Dt} l_{Dijt}(q_{ijt}, b_{ijt}), \quad (\text{A.14})$$

Labor  $l_{Dijt}$  is given by (8) and the wage  $w_{Dt}$  by (A.9).

Researcher  $j$  maximizes (A.14) by choosing  $q_{ijt}$  and  $b_{ijt}$ . Reducing the pollution intensity of intermediates by increasing  $B_i$  is costly but does not increase profits  $E[V_{ijt}]$ . Therefore  $b_{ijt} = 0$  for all  $i, j, t$  so that the pollution intensity of intermediates is constant under laissez-faire. The first-order condition for  $q_{ij}$  can, after simplification, be written as:

$$\alpha\mu\int_t^\infty L_{Ys}e^{-\int_t^s(r_v+\mu_{iv})dv}ds - 2q_{ijt} = 0 \quad (\text{A.15})$$

The equation still depends on  $n_i$  through the sectoral arrival rate  $\mu_i$ . To determine  $q_{it}$  and  $n_{it}$ , it must be taken into account that expected research profits in every sector  $i$  have to be zero at equilibrium. Otherwise, further research units would enter into sector  $i$  so that  $n_i$  would rise as long as the expectation value of profits was positive, while  $n_i$  would decrease if expected discounted profits were negative. The zero profit condition is:

$$\int_t^\infty L_{Ys}e^{-\int_t^s(r_v+\mu_{iv})dv}ds = \frac{q_{ijt}^2 + d}{(1 + q_{ijt})\alpha\mu} \quad (\text{A.16})$$

From (A.15) with (A.16), we determine the equilibrium value

$$q_{ijt}^{\text{LF}} = q^{\text{LF}} = \sqrt{1+d} - 1 \quad (\text{A.17})$$

of  $q_{ijt}$ .  $q^{\text{LF}}$  is constant over time and across sectors. It increases in the entry cost parameter  $d$  because less entry lowers the probability of being replaced by the next innovator and therefore increases marginal profits from productivity improvements<sup>28</sup>.

Because  $q^{\text{LF}}$  is constant, the integral  $\int_t^\infty L_{Ys}e^{-\int_t^s(r_v+\mu_{iv})dv}ds$  on the left-hand side of the free-entry condition (A.16) must be independent of  $t$ . Setting the time derivative of the integral to zero shows that the integral must be equal to  $\frac{L_{Yt}}{r_t+\mu_{it}}$ . This suggests that  $L_Y$ ,  $n_i$  and  $r$  must be constant at equilibrium even if there is no balanced growth, which we prove in the next section. After substituting  $\frac{L_{Yt}}{r_t+\mu_{it}}$  for the integral in equation (A.16) and the equilibrium value of  $q$ , (A.17), on the right-hand side, we can solve the free-entry condition for  $n_{it}$ :

$$n_{it} = n_t = \frac{1}{2} \frac{1}{\sqrt{1+d} - 1} \cdot \alpha L_{Yt} - \frac{r_t}{\mu}. \quad (\text{A.18})$$

## A.4 General equilibrium

### A.4.1 The market value of firms

Every unit of assets  $A$  in our model corresponds to a share of the market value of firms in the intermediate sector. The total stock of the representative household's assets at the beginning of period  $t$  must therefore equal the aggregate market value of firms before innovation. In each sector  $i$ , only the firm with the highest productivity level  $Q_{it}$  is active in production. The before-innovation market value of this firm can be derived from (A.13), substituting  $Q_{it}$  for the after-innovation productivity level  $(q_{ijt} + 1) \cdot Q_{it}$ . To obtain the aggregate market value  $V_t$  of firms, we take the integral over all sectors and use (A.16) with (A.17) to replace  $\int_t^\infty L_{Ys}e^{-\int_t^s(r_v+\mu_{iv})dv}ds$ :

$$\begin{aligned} V_t &= \int_0^1 E[V_{ijt}] di \\ &= 2 \frac{(1-\alpha)^{1-\alpha} \alpha^{2\alpha} \varphi^\alpha}{\mu} (\sqrt{1+d} - 1) Q_t \end{aligned} \quad (\text{A.19})$$

<sup>28</sup>In the analysis, it has been assumed that the monopoly price is smaller than the limit price. This will be the case whenever  $p^{\text{mon}} < (q^{\text{LF}} + 1) \cdot (1/\varphi) (w_{Xt}/Q_t)$  which is equivalent to choosing fixed costs  $d > \frac{1}{\alpha^2} - 1$ .



The market value is proportional to the economy-wide productivity level  $Q_t$ .

#### A.4.2 Labor market clearing

We use (A.18) along with the labor market constraint (3) and equation (A.12) to find the allocation of labor between final good production, intermediate production and research ( $L_{Yt}$ ,  $L_{Xt}$ ,  $L_{Dt}$ ) and determine the mass  $n_t$  of research units in sector  $i$  for any given interest rate  $r_t$ . The equilibrium  $n_t$  is:

$$n_t = \frac{\frac{1}{2}L - \left(\frac{1}{\alpha} + \frac{\alpha}{1-\alpha}\right) (\sqrt{1+d} - 1) \frac{r_t}{\mu}}{\left(\frac{1-\alpha}{\alpha} + \frac{\alpha}{1-\alpha}\right) (\sqrt{1+d} - 1) + d} \quad (\text{A.20})$$

The mass of research units is the same in every sector. It increases in the arrival rate  $\mu$  for innovations and decreases in the interest rate  $r_t$  and the fixed labor requirement  $d$ .

#### A.4.3 Equilibrium growth

Taking into account that  $n_t$  and  $q^{\text{LF}}$  are the same for all research sectors and using the definition of the aggregate productivity index  $Q$ , the equation of motion (11) for  $Q$  simplifies to

$$\dot{Q}_t = \mu n_t q^{\text{LF}} Q_t.$$

Substituting (A.17) for  $q^{\text{LF}}$  and (A.20) for  $n_t$ , we obtain the productivity growth rate in period  $t$  as a function of the interest rate  $r_t$ :

$$\hat{Q}_t = \mu \frac{\frac{1}{2}L - \left(\frac{1}{\alpha} + \frac{\alpha}{1-\alpha}\right) (\sqrt{1+d} - 1) \frac{r_t}{\mu}}{\left(\frac{1-\alpha}{\alpha} + \frac{\alpha}{1-\alpha}\right) (\sqrt{1+d} - 1) + d} (\sqrt{1+d} - 1) \quad (\text{A.21})$$

It follows from (A.12) that  $X_t$  and  $Q_t$  grow at the same rate at equilibrium because labor must be constant. From the resource constraint, it is obvious that  $c_t$  then also grows at the rate  $\hat{Q}_t$ . We set (A.21) equal to (A.6) and solve for the equilibrium interest rate.

$$r^{\text{LF}} = \frac{\frac{1}{2} \frac{1}{\sigma_c} \mu L (\sqrt{1+d} - 1) + \left(\left(\frac{1-\alpha}{\alpha} + \frac{\alpha}{1-\alpha}\right) (\sqrt{1+d} - 1) + d\right) \rho}{\left(\frac{1-\alpha}{\alpha} + \frac{\alpha}{1-\alpha}\right) (\sqrt{1+d} - 1) + d + \frac{1}{\sigma_c} \left(\frac{1}{\alpha} + \frac{\alpha}{1-\alpha}\right) (\sqrt{1+d} - 1)^2} \quad (\text{A.22})$$

With  $r^{\text{LF}}$ , equation (A.21) yields the equilibrium productivity growth rate

$$\hat{Q}^{\text{LF}} = \frac{\frac{1}{2} \mu L - \left(\frac{1}{\alpha} + \frac{\alpha}{1-\alpha}\right) (\sqrt{1+d} - 1) \rho}{\left(\frac{1-\alpha}{\alpha} + \frac{\alpha}{1-\alpha}\right) (\sqrt{1+d} - 1) + d + \frac{1}{\sigma_c} \left(\frac{1}{\alpha} + \frac{\alpha}{1-\alpha}\right) (\sqrt{1+d} - 1)^2} (\sqrt{1+d} - 1). \quad (\text{A.23})$$

$\hat{Q}^{\text{LF}}$  is positive if and only if the representative household is sufficiently patient, i.e., if and only if  $\rho < \bar{\rho}^{\text{LF}} := \frac{1}{2} \frac{\mu L}{\left(\frac{1}{\alpha} + \frac{\alpha}{1-\alpha}\right) (\sqrt{1+d} - 1)}$ .

The growth rates of  $c$ ,  $X$ ,  $Q$  and  $B$  are constant for any set of initial values for the state variables. Therefore growth in  $c$ ,  $X$ ,  $Q$  and  $B$  is balanced without transitional dynamics.

It follows from (7) that the pollution stock must increase at the same rate  $\hat{Q}^{\text{LF}}$  as intermediate quantity, productivity and consumption in the long run:

$$\hat{S}_{\infty}^{\text{LF}} = \hat{Q}^{\text{LF}}$$

However, contrary to the growth rates of the other variables, the growth rate of the pollution stock does not adjust to its balanced-growth level instantly if the relation between the state variables is not reconcilable with constant growth of the pollution stock initially.

## A.5 Proof of proposition 1

1. **Existence and Uniqueness:** The path defined by initial states  $Q_0$ ,  $B_0$  and  $S_0$  so that the pollution accumulation function (7) yields the balanced growth rate  $\widehat{S}_\infty^{\text{LF}}$ , the initial values  $X_0$  from (6) and  $c_0 = Y_0$  from (5) as well as the growth rates  $\widehat{c}^{\text{LF}} = \widehat{Y}^{\text{LF}} = \widehat{X}^{\text{LF}} = \widehat{Q}^{\text{LF}}$ ,  $\widehat{B}^{\text{LF}} = 0$  in every period  $t$  satisfies all the necessary conditions for an equilibrium as defined in section 3. The Hamiltonian function for the intertemporal maximization problem of the representative household is strictly concave in consumption and linear in all other variables. It follows that the household's maximization problem has a unique solution. The same is true for the static maximization problems in the R&D-sector as well as the production sectors for the consumption good and intermediates, which are concave as well. The path described in the text is therefore the unique laissez-faire equilibrium for  $\rho^{\text{TVC,LF}} < \rho < \bar{\rho}^{\text{LF}}$ . For  $\rho^{\text{TVC,LF}} < \bar{\rho}^{\text{LF}} \leq \rho$ , it still needs to be shown that  $n_t = n_{it} = 0$  for all  $t$  is an equilibrium.  $n = 0$  implies  $\widehat{Q}^{\text{LF}} = \widehat{c}^{\text{LF}} = 0$ . Setting  $\widehat{c}^{\text{LF}}$  in the Euler-equation and solving for  $r$  yields  $r = \rho$ . For  $r = \rho$ , equation (A.7) and the maximization problem in the R&D-sector yield  $w_{Yt} > w_{Dt}$ , which proves that  $n = 0$  is an equilibrium for this value of  $r$ .
2. **Derivation of  $\rho^{\text{TVC,LF}}$ :** The critical value  $\rho^{\text{TVC,LF}} := \frac{1}{2}\alpha(1-\alpha)\left(1 - \frac{1}{\sigma_c}\right)\mu L(1+d)^{-1/2}$  is derived from the transversality condition  $\lim_{t \rightarrow \infty} (e^{-\rho t} v_{At} A_t) = 0$ . Using (A.4), substituting  $A_t = A_0 e^{\widehat{Q}^{\text{LF}} \cdot t}$  with  $A_0 = 2 \frac{(1-\alpha)^{1-\alpha} \alpha^{2\alpha} \varphi^\alpha}{\mu} (\sqrt{1+d} - 1) Q_0$  from (A.19) and taking into account that  $r_t = r^{\text{LF}}$  for all  $t$  shows that the condition can be simplified to  $v_{A0} A_0 \lim_{t \rightarrow \infty} e^{-(r^{\text{LF}} - \widehat{Q}^{\text{LF}})t} = 0$ . The transversality condition is satisfied if and only if  $r^{\text{LF}} - \widehat{Q}^{\text{LF}} > 0$ . With (A.22) and (A.23), the critical value  $\rho^{\text{TVC,LF}}$  follows.
3. **Welfare comparison:** To prove that for convex disutility of pollution, a path without long-run growth would be welfare-improving, consider the utility function (A.2). For convex disutility of pollution ( $\sigma_E < 1/2$ ),  $\frac{1-\sigma_E}{\sigma_E}$  is at least one while  $\frac{\sigma_c-1}{\sigma_c}$  is smaller than one. In the long run,  $\widehat{S}_\infty^{\text{LF}} = \widehat{c}^{\text{LF}}$ . Instantaneous utility  $u_t = \frac{\sigma_c}{\sigma_c-1} c_t^{\frac{\sigma_c-1}{\sigma_c}} - \psi \frac{\sigma_E}{1-\sigma_E} S_t^{\frac{1-\sigma_E}{\sigma_E}}$  converges to  $-\phi^S(S_t) = -\psi \frac{\sigma_E}{1-\sigma_E} S_t^{\frac{1-\sigma_E}{\sigma_E}}$  and declines persistently towards  $(-\infty)$  at the rate  $\frac{1-\sigma_E}{\sigma_E} \widehat{S}_\infty$ . Now assume instead that economic growth is given up in a period  $s$ : Consumption growth drops to zero instantly while pollution growth converges to zero over time. Initially, there is a loss in per-period-utility compared to the laissez-faire equilibrium. This loss is only transitory: In the long-run, the pollution stock is constant and so is utility, while utility decreases in the laissez-faire equilibrium. Therefore, from a certain time onwards, not growing yields a utility-gain in each period which increases as  $t \rightarrow \infty$ . Because of the concavity of the utility from consumption and convexity of the disutility from pollution, the transitional welfare-loss is smaller, the later in time the regime-switch occurs and converges to zero as  $s \rightarrow \infty$ . Giving up economic growth in the long-run therefore yields an increase in intertemporal welfare.

## B Appendix to section 4 (Social Planner)

### B.1 Maximization problem and first-order-conditions

We can use lemma 1 to express the dynamic optimization problem in aggregate variables only: From (8), with  $\int_0^1 Q_{it} di = Q_t$ ,  $\int_0^1 B_{it} di = B_t$  and  $n_{it} = n_t$ , the amount of labor allocated to research in period  $t$  is  $L_{Dt} = n_t(q_t^2 + b_t^2 + d)$ . To produce  $X_t$  units of intermediates requires  $L_{Xt} = \frac{1}{\varphi} \frac{X_t}{Q_t}$  units of labor. The labor market constraint can be written as

$$L = \frac{1}{\varphi} \frac{X_t}{Q_t} + L_{Yt} + n_t(q_t^2 + b_t^2 + d). \quad (\text{B.24})$$

The equations of motion (11) for  $Q$  and (12) for  $B$  are:

$$\dot{Q}_t = \mu n q_t Q_t \quad (\text{B.25})$$

$$\dot{B}_t = \mu n b_t B_t \quad (\text{B.26})$$

Given aggregate intermediate production  $X_t$  the decision over  $X_{it}$  is static. The planner optimally allocates a higher share of aggregate intermediate production to the sectors with higher productivity level so as to maximize  $Y_t$ . The optimal  $X_{it}$  is:

$$X_{it} = X_t \frac{Q_{it}}{Q_t} \quad (\text{B.27})$$

With (B.27), the aggregate resource constraint can be rewritten as:

$$L_{Yt}^{1-\alpha} X_t^\alpha Q_t^{1-\alpha} = c_t L \quad (\text{B.28})$$

The maximization problem can then be solved by finding the optimal paths for  $Q$ ,  $B$ ,  $S$ ,  $c$ ,  $X$ ,  $L_Y$ ,  $n$ ,  $q$  and  $b$  subject to (7), (B.24), (B.25), (B.26) and the resource constraint (B.28). The social planner maximizes the current-value Hamiltonian

$$\begin{aligned} H = & \left( \frac{\sigma_c}{\sigma_c - 1} c_t^{\frac{\sigma_c - 1}{\sigma_c}} - \psi \frac{\sigma_E}{1 - \sigma_E} S_t^{\frac{1 - \sigma_E}{\sigma_E}} \right) L \\ & + v_{St} \left( \frac{X_t}{B_t} - \delta S_t \right) \\ & + v_{Qt} \mu n_t q_t Q_t \\ & + v_{Bt} \mu n_t b_t B_t \\ & + \lambda_{Yt} (X_t^\alpha Q_t^{1-\alpha} L_{Yt}^{1-\alpha} - c_t L) \\ & + \lambda_{Lt} (L - \frac{1}{\varphi} \frac{X_t}{Q_t} - L_{Yt} - n_t(q_t^2 + b_t^2 + d)) \end{aligned}$$

where  $v_{St}$ ,  $v_{Qt}$  and  $v_{Bt}$  are the shadow-prices of  $S_t$ ,  $Q_t$  and  $B_t$  respectively and  $\lambda_{Yt}$  and  $\lambda_{Lt}$  are Lagrange-multipliers.

The first-order conditions are:

$$\frac{\partial H}{\partial c_t} = 0 \Leftrightarrow \lambda_{Yt} = c_t^{-1/\sigma_c} \quad (\text{B.29})$$

$$\frac{\partial H}{\partial X_t} = 0 \Leftrightarrow \frac{v_{St}}{B_t} + \lambda_{Yt} \alpha X_t^{\alpha-1} L_{Yt}^{1-\alpha} Q_t^{1-\alpha} - \lambda_{Lt} \frac{1}{\varphi Q_t} = 0 \quad (\text{B.30})$$

$$\frac{\partial H}{\partial q_t} = 0 \Leftrightarrow v_{Qt} \mu n_t Q_t = 2 \lambda_{Lt} n_t q_t \quad (\text{B.31})$$

$$\frac{\partial H}{\partial b_t} = 0 \Leftrightarrow v_{Bt} \mu n_t B_t = 2 \lambda_{Lt} n_t b_t \quad (\text{B.32})$$

$$\frac{\partial H}{\partial n_t} = 0 \Leftrightarrow v_{Qt} \mu q_t Q_t + v_{Bt} \mu b_t B_t = \lambda_{Lt} (q_t^2 + b_t^2 + d) \quad (\text{B.33})$$

$$\frac{\partial H}{\partial L_{Yt}} = 0 \Leftrightarrow \lambda_{Yt} (1 - \alpha) X_t^\alpha Q_t^{1-\alpha} L_{Yt}^{-\alpha} = \lambda_{Lt} \quad (\text{B.34})$$

$$\frac{\partial H}{\partial S_t} = \rho v_{St} - \dot{v}_{St} \Leftrightarrow -\psi S_t^{(1-2\sigma_E)/\sigma_E} L - \delta v_{St} = \rho v_{St} - \dot{v}_{St} \quad (\text{B.35})$$

$$\frac{\partial H}{\partial Q_t} = \rho v_{Qt} - \dot{v}_{Qt} \quad (\text{B.36})$$

$$\Leftrightarrow v_{Qt} \mu n_t q_t + \lambda_{Yt} (1 - \alpha) X_t^\alpha Q_t^{-\alpha} L_{Yt}^{1-\alpha} + \lambda_{Lt} \frac{X_t}{\varphi} \frac{1}{Q_t^2} = \rho v_{Qt} - \dot{v}_{Qt}$$

$$\frac{\partial H}{\partial B_t} = \rho v_{Bt} - \dot{v}_{Bt} \Leftrightarrow -v_{St} \frac{X_t}{B_t^2} + v_{Bt} \mu n_t b_t = \rho v_{Bt} - \dot{v}_{Bt} \quad (\text{B.37})$$

$$\frac{\partial H}{\partial v_{St}} = \dot{S}_t \Leftrightarrow \frac{X_t}{B_t} - \delta S_t = \dot{S}_t \quad (\text{B.38})$$

$$\frac{\partial H}{\partial v_{Qt}} = \dot{Q}_t \Leftrightarrow \mu n_t q_t Q_t = \dot{Q}_t \quad (\text{B.39})$$

$$\frac{\partial H}{\partial v_{Bt}} = \dot{B}_t \Leftrightarrow \mu n_t b_t B_t = \dot{B}_t \quad (\text{B.40})$$

$$\frac{\partial H}{\partial \lambda_{Yt}} = 0 \Leftrightarrow X_t^\alpha Q_t^{1-\alpha} L_{Yt}^{1-\alpha} = c_t L \quad (\text{B.41})$$

$$\frac{\partial H}{\partial \lambda_{Lt}} = 0 \Leftrightarrow L = \frac{1}{\varphi} \frac{X_t}{Q_t} + L_{Yt} + n_t (q_t^2 + b_t^2 + d) \quad (\text{B.42})$$

Further, the transversality conditions

$$\begin{aligned} \lim_{t \rightarrow \infty} (e^{-\rho t} v_{Qt} Q_t) &= 0 \\ \lim_{t \rightarrow \infty} (e^{-\rho t} v_{Bt} B_t) &= 0 \\ \lim_{t \rightarrow \infty} (e^{-\rho t} v_{St} S_t) &= 0 \end{aligned} \quad (\text{B.43})$$

and the non-negativity constraints

$$Q_t, B_t, S_t, c_t, X_t, L_{Yt}, n_t \geq 0, \forall t$$

must be satisfied.

## B.2 Derivation of equation (13)

The first-order condition (B.30) for  $X$  yields a relation between the growth rates of the marginal utility  $c_t^{-1/\sigma_c}$  of consumption and the shadow price  $v_S$  of pollution. For  $t \rightarrow \infty$ , it is given by:

$$\hat{v}_{S\infty} = -(1/\sigma_c) \hat{c}_\infty + \hat{B}_\infty - (1 - \alpha) (\hat{X}_\infty - \hat{Q}_\infty)$$

Taking into account that  $\hat{c}_\infty = \alpha \hat{X}_\infty + (1 - \alpha) \hat{Q}_\infty$  is required for the resource constraint to be satisfied in the long run, an equivalent equation is

$$\hat{v}_{S\infty} = (1 - 1/\sigma_c) \hat{c}_\infty + \hat{B}_\infty - \hat{X}_\infty. \quad (\text{B.44})$$

Along an ABG-path,  $v_S$  must grow at a constant rate. From the first-order condition (B.35) for the pollution stock, it follows that the ratio  $S_t^{(1-2\sigma_E)/\sigma_E}/v_{St}$  must be asymptotically constant, so that in the long run,  $v_S$  must grow at the same rate as the (instantaneous) marginal disutility  $\psi S^{(1-2\sigma_E)/\sigma_E}$  of pollution, i.e.:

$$\hat{v}_{S\infty} = ((1 - 2\sigma_E)/\sigma_E) \hat{S}_\infty \quad (\text{B.45})$$

Substituting (B.44) for  $\hat{v}_{S\infty}$  and rearranging yields (13) in the proof of proposition 3.

### B.3 Long-run solution to the necessary conditions for parameter constellations in proposition 4 ( $\hat{S}_\infty > (-\delta)$ )

We assume  $n_\infty > 0$ . Solving (B.31) and (B.32) for  $v_Q$  and  $v_B$  respectively, substituting in the first-order condition (B.33) for  $n$  and taking the limit for  $t \rightarrow \infty$  yields

$$q_\infty^2 + b_\infty^2 = d \quad (\text{B.46})$$

Condition (B.46) is an *indifference condition*. It guarantees that the social planner is indifferent between all possible values for  $n$ .

In the optimal solution, the social net return to a higher level of  $Q$  must compensate the household for postponing consumption into the future. Dividing by  $v_{Qt}$ , setting  $t = \infty$  and rearranging, (B.36) can be written as:

$$(1/\sigma_c) \hat{c}_\infty + \rho = \frac{1}{2} \mu q_\infty^{-1} \left( L_{Y\infty} + \frac{1}{\varphi} \left( \frac{X}{Q} \right)_\infty \right) + \alpha \hat{X}_\infty + (1 - \alpha) \mu n_\infty q_\infty \quad (\text{B.47})$$

Equation (B.47) is a version of the *consumption Euler-equation*, where we replaced the shadow-prices and Lagrange-multipliers as well as their growth rates using (B.31), (B.34) and (B.29).

Both research directions, that is, increasing  $Q$  and increasing  $B$ , must yield the same social net return. We manipulate the first-order condition (B.37) for  $B$  similarly to the one for  $Q$ , using (B.32) as well as the expression  $v_{St} = \left( \lambda_{Lt} \frac{1}{\varphi Q_t} - \lambda_{Yt} \alpha X_t^{\alpha-1} L_{Yt}^{1-\alpha} Q_t^{1-\alpha} \right) B_t$  from (B.30), and equations (B.34) and (B.29):

$$(1/\sigma_c) \hat{c}_\infty + \rho = \frac{1}{2} \mu b_\infty^{-1} \left( \frac{\alpha}{1-\alpha} L_{Y\infty} - \frac{1}{\varphi} \left( \frac{X}{Q} \right)_\infty \right) + \alpha \hat{X}_\infty + (1 - \alpha) \mu n_\infty q_\infty. \quad (\text{B.48})$$

Setting equal the right-hand sides of (B.47) and (B.48), we obtain the *research-arbitrage condition*

$$\frac{1}{2} \mu q_\infty^{-1} \left( L_{Y\infty} + \frac{1}{\varphi} \left( \frac{X}{Q} \right)_\infty \right) = \frac{1}{2} \mu b_\infty^{-1} \left( \frac{\alpha}{1-\alpha} L_{Y\infty} - \frac{1}{\varphi} \left( \frac{X}{Q} \right)_\infty \right). \quad (\text{B.49})$$

The left hand side of (B.49) shows the sum of the marginal return to  $Q$  (in utility units) in final good production and the marginal social benefit from lower costs in intermediate production. The term on the right-hand side denotes the social return to green innovation from slower pollution accumulation (in utility units). This return is larger if the production elasticity  $\alpha$  of  $X$  is large and the production costs,  $\frac{1}{\varphi} (X/Q)$ , in terms of labor are small because the incentive to produce intermediate quantity  $X$  is stronger.

The three conditions (B.46), (B.47) and (B.49) together with the ABG-condition (13) are key equations for the determination of the long-run optimum.

**(a) Balanced growth** ( $\hat{X}_\infty = \hat{Q}_\infty$ )

If growth rates are to be constant asymptotically, the allocation of labor and the step-size  $q$  and  $b$  must be constant in the long run. In this case, equation (B.49) requires intermediate quantity in efficiency units, more precisely the ratio  $(X/Q)_\infty$ , to be constant in the limit as well.

A balanced growth path, along which productivity and cleanliness grow at constant rates not only asymptotically, must be characterized by a strictly positive  $(X/Q)_\infty$ <sup>29</sup> and therefore equal growth in intermediate quantity and productivity. It follows from the resource constraint that consumption  $c$  will also grow at the same rate. Equation (13) then directly yields a ratio  $\hat{B}_\infty/\hat{Q}_\infty$  (equivalently  $b/q$ ): Substituting  $\hat{c}_\infty = \hat{Q}_\infty$  and  $\hat{S}_\infty = \hat{Q}_\infty - \hat{B}_\infty$ , we find that

$$\hat{B}_\infty/\hat{Q}_\infty = 1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E}. \quad (\text{B.50})$$

However, it follows from the research arbitrage condition (B.49) that this ratio is only reconcilable with a positive value of  $(X/Q)_\infty$  if it does not exceed  $\alpha/(1 - \alpha)$ .

For convex disutility of pollution ( $\sigma_E < 1/2$ ),  $\frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E}$  is smaller than one. Therefore  $1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E} > 0$  and the long-run optimal solution described in proposition 4(a) is characterized by green innovation ( $\hat{B}_\infty > 0$ ) whenever there is productivity growth ( $\hat{Q}_\infty > 0$ ).

To derive  $\hat{Q}_\infty$ , we need to find  $n_\infty$  and  $q_\infty$ . The solution for  $q_\infty$  (and also  $b_\infty$ ) follows directly from the indifference equation (B.46) together with (B.50):

$$\begin{aligned} q_\infty &= \left( 1 + \left( 1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E} \right)^2 \right)^{-1/2} \cdot d^{1/2} \\ b_\infty &= \left( 1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E} \right) \left( 1 + \left( 1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E} \right)^2 \right)^{-1/2} \cdot d^{1/2} \end{aligned}$$

We determine  $n_\infty$  from the consumption Euler-equation (B.47), where we replace  $L_{Y_\infty}$  from the labor market constraint (B.42) and make use of (B.46) and  $\hat{c}_\infty = \hat{X}_\infty = \hat{Q}_\infty = \mu n_\infty q_\infty$ .

$$n_\infty = \frac{\frac{1}{2} \mu q_\infty^{-1} L - \rho}{(d - (1 - 1/\sigma_c) q_\infty^2) \mu q_\infty^{-1}}.$$

With the solutions for  $n_\infty$  and  $q_\infty$  we find that the growth rate of productivity  $Q$  is

$$\begin{aligned} \hat{Q}_\infty &= \mu n_\infty q_\infty \\ &= \frac{1}{1/\sigma_c + \left( 1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E} \right)^2} \left( \frac{1}{2} \left( 1 + \left( 1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E} \right)^2 \right)^{1/2} d^{-1/2} \mu L - \rho \right). \end{aligned} \quad (\text{B.51})$$

Economic growth is positive if and only if:

$$\rho < \bar{\rho}_{\text{BG}} = \frac{1}{2} \left( 1 + \left( 1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E} \right)^2 \right)^{1/2} d^{-1/2} \mu L \quad (\text{B.52})$$

Note that the upper bound for  $\rho$  does not depend on the rate of natural regeneration,  $\delta$ , or the weight  $\psi$  of the pollution stock in utility.

<sup>29</sup> On a balanced growth path,  $(X/Q)_\infty = 0$  implies  $X_t/Q_t = 0$  for all  $t$ . This is only possible if  $X_t = 0$  for all  $t$  which, as explained before, cannot be an optimal path for  $X$  because of the Inada-conditions for consumption.

As there is no deceleration, the growth rates of intermediate quantity, final output and consumption equal the productivity growth rate,  $\hat{X}_\infty = \hat{Y}_\infty = \hat{c}_\infty = \hat{Q}_\infty$ . The growth rate of intermediate cleanliness is given by (B.50). We derive the pollution growth rate from (13):

$$\begin{aligned}\hat{S}_\infty &= \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E} \hat{c}_\infty \\ &= \frac{\frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E}}{1/\sigma_c + \left(1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E}\right)^2} \cdot \left(\frac{1}{2} \sqrt{1 + \left(1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E}\right)^2} d^{-1/2} \mu L - \rho\right)\end{aligned}$$

It remains to be proven that  $\hat{S}_\infty$  is indeed larger than  $(-\delta)$  for  $\sigma_c < 1$ . The condition on model parameters to ensure  $\hat{S}_\infty > (-\delta)$  for  $\sigma_c < 1$  is:

$$\begin{aligned}\rho &> \rho_{\text{BG}}^{\text{delta}} = \frac{1}{2} \sqrt{1 + \left(1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E}\right)^2} d^{-1/2} \mu L \\ &\quad - \left(\frac{1}{\sigma_c} + \left(1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E}\right)^2\right) \frac{(1 - \sigma_E)/\sigma_E}{(1 - \sigma_c)/\sigma_c} \delta\end{aligned}\tag{B.53}$$

$\hat{S}_\infty > (-\delta)$  is satisfied for any  $\rho < \bar{\rho}_{\text{BG}}$  whenever  $\sigma_c > 1$  so that  $\hat{S}_\infty > 0$ .

#### (b) ABG with deceleration ( $\hat{X}_\infty < \hat{Q}_\infty$ )

If  $\alpha/(1 - \alpha) > 1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E}$ , a balanced growth solution to the social planner's problem does not exist, because the ratio  $\hat{B}_\infty/\hat{Q}_\infty$  from (13) is not reconcilable with equation (B.49) for any nonnegative  $(X/Q)_\infty$ . As  $X/Q < 0$  has no sensible interpretation, the optimal solution is to let  $X/Q$  converge to zero asymptotically by choosing  $\hat{X}_\infty < \hat{Q}_\infty$ . According to (B.49), the optimal ratio  $\hat{B}_\infty/\hat{Q}_\infty$  corresponds to

$$\hat{B}_\infty/\hat{Q}_\infty = \frac{\alpha}{1 - \alpha}.\tag{B.54}$$

To compute the relation between the growth rates  $\hat{X}_\infty$  and  $\hat{Q}_\infty$ , we use (13), substituting  $\hat{X}_\infty - \hat{B}_\infty = \hat{X}_\infty - \frac{\alpha}{1 - \alpha} \hat{Q}_\infty$  for  $\hat{S}_\infty$  and  $\alpha \hat{X}_\infty + (1 - \alpha) \hat{Q}_\infty$  from the resource constraint for  $\hat{c}_\infty$ . After some manipulation, we obtain:

$$\hat{X}_\infty = \frac{1 + \left(\frac{\alpha}{1 - \alpha}\right)^2 - \left(1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E} - \frac{\alpha}{1 - \alpha}\right)}{1 + \frac{\alpha}{1 - \alpha} \left(1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E}\right)} \hat{Q}_\infty\tag{B.55}$$

For  $\frac{\alpha}{1 - \alpha} < 1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E}$ , it is obvious that  $\hat{X}_\infty \leq \hat{Q}_\infty$  where equality is given if and only if  $\hat{Q}_\infty = 0$ .

We substitute  $\hat{X}_\infty$  from (B.55) into  $\hat{c}_\infty = \alpha \hat{X}_\infty + (1 - \alpha) \hat{Q}_\infty$  to find  $\hat{c}_\infty$  as function of  $\hat{Q}_\infty$ :

$$\hat{c}_\infty = \frac{1 + \left(\frac{\alpha}{1 - \alpha}\right)^2}{1 + \frac{\alpha}{1 - \alpha} \left(1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E}\right)} \hat{Q}_\infty\tag{B.56}$$

For  $\frac{\alpha}{1 - \alpha} < 1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E}$ , it is obvious that  $\hat{c}_\infty \leq \hat{Q}_\infty$  where equality is given if and only if  $\hat{Q}_\infty = 0$ . Also note that for any  $\hat{Q}_\infty > 0$ ,  $\hat{X}_\infty < \hat{c}_\infty$ .

To derive the long-run optimal growth rate of  $Q$ , we proceed in the same way as in the previous paragraph. We first use the indifference condition (B.46) with (B.54) to determine  $q_\infty$  (and thereby also  $b_\infty$ ):

$$q_\infty = \left(1 + \left(\frac{\alpha}{1-\alpha}\right)^2\right)^{-1/2} d^{1/2} \quad (\text{B.57})$$

$$b_\infty = \frac{\alpha}{1-\alpha} \left(1 + \left(\frac{\alpha}{1-\alpha}\right)^2\right)^{-1/2} d^{1/2} \quad (\text{B.58})$$

We then solve the Euler-equation (B.47) for  $n_\infty$  once more, taking into account (B.55) and (B.56) as well as  $(X/Q)_\infty = 0$ . With the solution

$$n_\infty = \frac{\frac{1}{2}\mu q_\infty^{-1}L - \rho}{\left(d - \frac{(1-1/\sigma_c)\left(1 + \left(\frac{\alpha}{1-\alpha}\right)^2\right)}{1 + \frac{\alpha}{1-\alpha} \left(1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}\right)} q_\infty^2\right) \mu q_\infty^{-1}}$$

and  $q_\infty$  from (B.57) we find that

$$\begin{aligned} \hat{Q}_\infty &= \mu n_\infty q_\infty \\ &= \frac{1 + \frac{\alpha}{1-\alpha} \left(1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}\right)}{1/\sigma_c + \frac{\alpha}{1-\alpha} \left(1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}\right)} \frac{1}{1 + \left(\frac{\alpha}{1-\alpha}\right)^2} \left(\frac{1}{2} \left(1 + \left(\frac{\alpha}{1-\alpha}\right)^2\right)^{1/2} d^{-1/2} \mu L - \rho\right). \end{aligned} \quad (\text{B.59})$$

$\hat{Q}_\infty > 0$  which implies that  $\hat{c}_\infty$  is positive, if and only if:

$$\rho < \bar{\rho}_{\text{Dec}} = \frac{1}{2} \left(1 + \left(\frac{\alpha}{1-\alpha}\right)^2\right)^{1/2} d^{-1/2} \mu L \quad (\text{B.60})$$

Note that the upper bound for  $\rho$  does again not depend on the rate of natural regeneration,  $\delta$ , or the weight  $\psi$  of the pollution stock in utility.

With (B.59), the growth rates of intermediate cleanliness, intermediate quantity and consumption (and GDP) can be found from (B.54), (B.55) and (B.56) respectively.

$$\hat{B}_\infty = \frac{1 + \frac{\alpha}{1-\alpha} \left(1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}\right)}{1/\sigma_c + \frac{\alpha}{1-\alpha} \left(1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}\right)} \frac{\frac{\alpha}{1-\alpha}}{1 + \left(\frac{\alpha}{1-\alpha}\right)^2} \left(\frac{1}{2} \left(1 + \left(\frac{\alpha}{1-\alpha}\right)^2\right)^{1/2} d^{-1/2} \mu L - \rho\right) \quad (\text{B.61})$$

$$\hat{X}_\infty = \frac{1 + \frac{\alpha}{1-\alpha}^2 - \left(1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E} - \frac{\alpha}{1-\alpha}\right)}{1/\sigma_c + \frac{\alpha}{1-\alpha} \left(1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}\right)} \frac{1}{1 + \left(\frac{\alpha}{1-\alpha}\right)^2} \left(\frac{1}{2} \left(1 + \left(\frac{\alpha}{1-\alpha}\right)^2\right)^{1/2} d^{-1/2} \mu L - \rho\right) \quad (\text{B.62})$$

$$\hat{c}_\infty = \hat{Y}_\infty = \frac{1}{1/\sigma_c + \frac{\alpha}{1-\alpha} \left(1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}\right)} \left(\frac{1}{2} \left(1 + \left(\frac{\alpha}{1-\alpha}\right)^2\right)^{1/2} d^{-1/2} \mu L - \rho\right) \quad (\text{B.63})$$

From (13), the growth rate of the pollution stock is:

$$\hat{S}_\infty = \frac{\frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}}{1/\sigma_c + \frac{\alpha}{1-\alpha} \left(1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}\right)} \cdot \left(\frac{1}{2} \sqrt{1 + \left(\frac{\alpha}{1-\alpha}\right)^2} d^{-1/2} \mu L - \rho\right). \quad (\text{B.64})$$



The condition on the rate of time preference needed to ensure  $\hat{S}_\infty > (-\delta)$  for  $\sigma_c < 1$  is

$$\begin{aligned} \rho > \rho_{\text{Dec}}^{\text{delta}} = & \frac{1}{2} \sqrt{1 + \left( \frac{\alpha}{1-\alpha} \right)^2} d^{-1/2} \mu L \\ & - \left( 1/\sigma_c + \frac{\alpha}{1-\alpha} \left( 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E} \right) \right) \frac{(1-\sigma_E)/\sigma_E}{(1-\sigma_c)/\sigma_c} \delta. \end{aligned} \quad (\text{B.65})$$

Similar to the previous paragraph,  $\hat{S}_\infty > (-\delta)$  is satisfied for any  $\rho < \bar{\rho}_{\text{Dec}}$  if  $\sigma_c > 1$ .

#### B.4 Definition of boundary values for the rate of time preference

From (B.52), (B.53), (B.60) and (B.65), we define the following boundary values for the rate of time preference:

$$\bar{\rho} := \begin{cases} \frac{1}{2} \left( 1 + \left( 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E} \right)^2 \right)^{1/2} d^{-1/2} \mu L, & \frac{\alpha}{1-\alpha} > 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E} \\ \frac{1}{2} \left( 1 + \left( \frac{\alpha}{1-\alpha} \right)^2 \right)^{1/2} d^{-1/2} \mu L, & \frac{\alpha}{1-\alpha} < 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E} \end{cases} \quad (\text{B.66})$$

$$\rho^{\text{delta}} := \begin{cases} \frac{1}{2} \left( 1 + \left( 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E} \right)^2 \right)^{1/2} d^{-1/2} \mu L - \kappa_1 \frac{(1-\sigma_E)/\sigma_E}{(1-\sigma_c)/\sigma_c} \delta, & \frac{\alpha}{1-\alpha} > 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}, \sigma_c < 1 \\ \frac{1}{2} \left( 1 + \left( \frac{\alpha}{1-\alpha} \right)^2 \right)^{1/2} d^{-1/2} \mu L - \kappa_2 \frac{(1-\sigma_E)/\sigma_E}{(1-\sigma_c)/\sigma_c} \delta, & \frac{\alpha}{1-\alpha} < 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}, \sigma_c < 1 \end{cases} \quad (\text{B.67})$$

From the transversality conditions in (B.43), it follows that

$$\rho^{\text{TVC}} := \begin{cases} \frac{1}{2} \frac{1-1/\sigma_c}{1 + \left( 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E} \right)^2} \left( 1 + \left( 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E} \right)^2 \right)^{1/2} d^{-1/2} \mu L, & \frac{\alpha}{1-\alpha} > 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E} \\ \frac{1}{2} \frac{1-1/\sigma_c}{1 + \frac{\alpha}{1-\alpha} \left( 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E} \right)} \left( 1 + \left( \frac{\alpha}{1-\alpha} \right)^2 \right)^{1/2} d^{-1/2} \mu L, & \frac{\alpha}{1-\alpha} < 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E} \end{cases}$$

$\kappa_1 = \frac{1}{\sigma_c} + \left( 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E} \right)^2$  and  $\kappa_2 = \frac{1}{\sigma_c} + \frac{\alpha}{1-\alpha} \left( 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E} \right)$  are positive constants.

#### B.5 Proof of proposition 2

Given that  $q_\infty > 0$  and  $\hat{c}_\infty$  is proportional to  $\hat{Q}_\infty$  in both cases of appendix B.3,  $\hat{c}_\infty > 0 \Leftrightarrow n_\infty > 0$ . It follows that for parameter constellations as assumed in proposition 4, long-run consumption growth is positive if and only if  $\rho < \bar{\rho}$ .

The cases in proposition 5 only occur for sufficiently fast consumption growth so that whenever proposition 5 applies,  $\hat{c}_\infty > 0$  is necessarily satisfied and no additional restriction of the parameter range is needed.

#### B.6 Proof of proposition 4

The long-run growth path derived in appendix B.3 for  $\frac{\alpha}{1-\alpha} > 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}$  and  $\frac{\alpha}{1-\alpha} < 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}$  respectively satisfies all the necessary conditions, given the parameter restriction  $\underline{\rho} < \rho < \bar{\rho}$  in proposition 4. For  $\frac{\alpha}{1-\alpha} <$

$1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E}$ , the solution is characterized by green innovation and deceleration, while for  $\frac{\alpha}{1 - \alpha} > 1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E}$ , there is only green innovation.

It remains to be shown that the solution is unique. The only other solution candidate which has so far been excluded by the assumption of an interior solution is a solution with  $n_\infty = 0$ . To prove that  $n_\infty = 0$  cannot be an optimal choice for  $n$  under the parameter restriction  $\rho < \bar{\rho}$ , we show that, given  $n_\infty = 0$  and  $\rho < \bar{\rho}$ , the partial derivative of the Hamiltonian-function with respect to  $n$  is positive in the limit, i.e.  $\lim_{t \rightarrow \infty} \frac{\partial H}{\partial n} \big|_{n_\infty=0} > 0$ . This condition is satisfied, if and only if

$$v_{Q\infty} \mu q_\infty Q_\infty + v_{B\infty} \mu b_\infty B_\infty > \lambda_{L\infty} (q_\infty^2 + b_\infty^2 + d). \quad (\text{B.68})$$

Given  $n_\infty = 0$ , the first-order conditions (B.31) and (B.32) for  $q$  and  $b$  are always satisfied and the social planner is indifferent between any levels of  $q_\infty$  and  $b_\infty$ . Because every choice of  $q_\infty$  and  $b_\infty$  must yield the same level of intertemporal welfare, any particular pair can be selected as solution. We define the limits  $\lim_{n_\infty \rightarrow 0} q(n_\infty)$  and  $\lim_{n_\infty \rightarrow 0} b(n_\infty)$ , obtained from the first-order conditions given  $n_\infty > 0$  as the solutions in this case. The limit for  $q$  can be derived by solving the Euler-equation (B.47) for  $q$  instead of  $n$ . The limit for  $b$  follows from (B.50) or (B.54) respectively. It differs between the balanced-growth case and the case with deceleration.

#### (a) Balanced growth

Substituting the labor market constraint (B.24) into the Euler-equation (B.47) and taking the limit for  $n_\infty \rightarrow 0$  on both sides yields  $\lim_{n_\infty \rightarrow 0} q(n_\infty) = \frac{\mu}{2} L / \rho$ . Accordingly, the limit for  $b$  is  $\lim_{n_\infty \rightarrow 0} b(n_\infty) = \left(1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E}\right) \frac{\mu}{2} L / \rho$  from (B.50). The research-arbitrage condition (B.49) requires that in the limit,  $X/Q$  equals  $\lim_{n_\infty \rightarrow 0} \frac{X}{Q}(n_\infty) = (1 - \alpha) \varphi \left( \frac{\alpha}{1 - \alpha} - \frac{b_\infty}{q_\infty} \right) L$ .

We then determine the values of the shadow prices  $v_{Q\infty}$  and  $v_{B\infty}$  for  $n_\infty = 0$  from (B.36) and (B.37) with (B.30) and (B.42), taking into account that  $X$ ,  $c$ ,  $Q$ ,  $B$  and  $S$  are constant in the long run. We obtain the expressions  $v_{Q\infty} = \lambda_{L\infty} Q_\infty^{-1} \left( L_{Y\infty} + \frac{1}{\varphi} \left( \frac{X}{Q} \right)_\infty \right) \frac{1}{\rho}$  and  $v_{B\infty} = \lambda_{L\infty} B_\infty^{-1} \left( \frac{\alpha}{1 - \alpha} L_{Y\infty} - \frac{1}{\varphi} \left( \frac{X}{Q} \right)_\infty \right) \frac{1}{\rho}$ .

Substituting  $v_{Q\infty}$ ,  $v_{B\infty}$ ,  $\lim_{n_\infty \rightarrow 0} q(n_\infty)$ ,  $\lim_{n_\infty \rightarrow 0} b(n_\infty)$  and  $\lim_{n_\infty \rightarrow 0} \frac{X}{Q}(n_\infty)$  as well as  $L_{Y\infty} = L - \frac{1}{\varphi} \left( \frac{X}{Q} \right)_\infty$  in (B.68) and simplifying yields

$$\begin{aligned} \lim_{t \rightarrow \infty} \frac{\partial H}{\partial n} &> 0 \\ \Leftrightarrow \rho &< \frac{1}{2} \left( 1 + \left( 1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E} \right)^2 \right)^{1/2} d^{-1/2} \mu L. \end{aligned}$$

Because  $\frac{1}{2} \left( 1 + \left( 1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E} \right)^2 \right)^{1/2} d^{-1/2} \mu L = \bar{\rho}_{\text{BG}}$ , which is the upper limit for  $\frac{\alpha}{1 - \alpha} > 1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E}$ , we have shown that given  $\rho < \bar{\rho}$  and  $\frac{\alpha}{1 - \alpha} > 1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E}$ , no solution to the set of necessary first-order conditions with  $n_\infty = 0$  exists.

#### (b) ABG with deceleration

In the asymptotically-balanced growth case, it can readily be verified from (B.47) that  $\lim_{n_\infty \rightarrow 0} q(n_\infty) = \frac{\mu}{2} L / \rho$  as before. The limit for  $b$  changes to  $\lim_{n_\infty \rightarrow 0} b(n_\infty) = \frac{\alpha}{1 - \alpha} \frac{\mu}{2} L / \rho$  and  $\lim_{n_\infty \rightarrow 0} \frac{X}{Q}(n_\infty) = 0$ .

Proceeding as in the balanced-growth case, we find that  $\lim_{t \rightarrow \infty} \frac{\partial H}{\partial n} > 0 \Leftrightarrow \rho < \frac{1}{2} \left( 1 + \left( \frac{\alpha}{1-\alpha} \right)^2 \right)^{1/2} d^{-1/2} \mu L$ . The right-hand side corresponds to the upper bound  $\bar{\rho}$  for  $\frac{\alpha}{1-\alpha} < 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}$ . Again,  $\lim_{t \rightarrow \infty} \frac{\partial H}{\partial n} > 0$  proves that  $n_\infty = 0$  cannot be an optimal solution in the given parameter range.

## B.7 Long-run solution to the necessary conditions for parameter constellations in proposition 5 ( $\hat{S}_\infty = (-\delta)$ )

### (a) ABG without deceleration ( $\hat{X}_\infty = \hat{Q}_\infty$ )

It still follows from the resource constraint that

$$\hat{c}_\infty = \hat{X}_\infty = \hat{Q}_\infty.$$

Substituting into the ABG-condition (13) with  $\hat{S}_\infty = (-\delta)$  and solving for  $\hat{B}_\infty$ , we obtain

$$\hat{B}_\infty = (1/\sigma_c) \hat{Q}_\infty - ((1 - 2\sigma_E)/\sigma_E) \delta. \quad (\text{B.69})$$

As  $\sigma_c < 1$ , the ratio  $\hat{B}_\infty/\hat{Q}_\infty$  in (B.69) is larger than  $\hat{B}_\infty/\hat{Q}_\infty = 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}$  in (B.50) for every constellation of parameters that satisfies the condition  $\hat{Q}_\infty > \frac{(1-\sigma_E)/\sigma_E}{(1-\sigma_c)/\sigma_c} \delta$  for  $\hat{S}_\infty = (-\delta)$ .

Like in proposition 4, the ratio  $\hat{B}_\infty/\hat{Q}_\infty$  implied by (B.69) is reconcilable with the research arbitrage equation (B.49) if it is smaller than  $\alpha/(1-\alpha)$ . If  $(1/\sigma_c) < \frac{\alpha}{1-\alpha}$ , this is true for any  $\hat{c}_\infty > 0$ . If  $(1/\sigma_c) > \frac{\alpha}{1-\alpha}$ ,  $\hat{B}_\infty/\hat{Q}_\infty < \frac{\alpha}{1-\alpha}$  is guaranteed by the condition

$$\hat{Q}_\infty < \frac{(1 - 2\sigma_E)/\sigma_E}{(1/\sigma_c) - \frac{\alpha}{1-\alpha}} \delta, \quad (\text{B.70})$$

which implies a lower bound for the rate of time preference  $\rho$ .

Using the indifference condition (B.46),  $q_\infty^2 + b_\infty^2 = d$ , to express  $b_\infty$  as function of  $q_\infty$ , equation (B.69) can also be written as

$$\mu n_\infty \sqrt{d - q_\infty^2} = (1/\sigma_c) \mu n_\infty q_\infty - ((1 - 2\sigma_E)/\sigma_E) \delta \quad (\text{B.71})$$

The consumption Euler-equation (B.47) does not depend on  $\hat{S}_\infty$  or the ratio  $\hat{B}_\infty/\hat{Q}_\infty$ . It yields the same relation

$$n_\infty = \frac{\frac{1}{2} \mu q_\infty^{-1} L - \rho}{(d - (1 - 1/\sigma_c) q_\infty^2) \mu} q_\infty \quad (\text{B.72})$$

between  $n_\infty$  and  $q_\infty$  as in the balanced-growth case.

Equations (B.71) and (B.72) form a system of two equations in the two unknowns  $q_\infty$  and  $n_\infty$ . However, after substituting for  $n_\infty$  in (B.71), it is not possible to solve (B.71) for  $q_\infty$  analytically due to the mixture of exponents.

Depending on parameters, there may be a unique solution, two solutions or no solution. To prove this claim, consider equation (B.71), where  $n_\infty = n_\infty(q_\infty)$  is given by (B.72). We divide both sides of equation (B.71) by  $n_\infty(q_\infty)$ :

$$\mu \sqrt{d - q_\infty^2} = (1/\sigma_c) \mu q_\infty - n_\infty(q_\infty)^{-1} ((1 - 2\sigma_E)/\sigma_E) \delta$$

The left-hand side of the modified equation is non-negative as well as decreasing and concave in  $q_\infty$ . The right-hand side is positive whenever the condition for  $\hat{S}_\infty = (-\delta)$  is satisfied, because  $\mu n_\infty q_\infty = \hat{Q}_\infty > \frac{(1-2\sigma_E)/\sigma_E}{1/\sigma_c} \delta$  is a weaker condition than  $\hat{Q}_\infty > \frac{(1-\sigma_E)/\sigma_E}{(1-\sigma_c)/\sigma_c} \delta$ . For  $\sigma_c < 1$ , in the relevant range with  $\hat{Q}_\infty > \frac{(1-\sigma_E)/\sigma_E}{(1-\sigma_c)/\sigma_c} \delta$ , the right-hand side is concave and first increasing, then decreasing in  $q_\infty$  because the first term is linear and  $n_\infty(q_\infty)^{-1}$  is decreasing and convex in  $q_\infty$  whenever  $\sigma_c < 1$ .

A unique solution exists if and only if the value  $q_\infty = \sqrt{d}$ , which sets the left-hand side of the equation to zero, lies between the two zeros of the right-hand side. An equivalent condition is that at  $q_\infty = \sqrt{d}$ , the right-hand side is positive. This is true if and only if

$$\rho < \frac{1}{2} \mu d^{-1/2} L - ((1 - 2\sigma_E) / \sigma_E) \delta.$$

### (b) ABG with deceleration ( $\hat{X}_\infty < \hat{Q}_\infty$ )

If  $(X/Q)_\infty = 0$  asymptotically, equation (B.49) again yields the ratio

$$\hat{B}_\infty / \hat{Q}_\infty = \frac{\alpha}{1 - \alpha}$$

from proposition 4(b). From (B.46) and  $\hat{B}_\infty / \hat{Q}_\infty = \frac{\alpha}{1 - \alpha}$ , we obtain the same solutions for  $q$  and  $b$  as in (B.57) and (B.58).

With  $\hat{S}_\infty = (-\delta)$ , the relation between  $\hat{X}_\infty$  and  $\hat{Q}_\infty$  differs from the one in (B.55): Substituting  $\hat{c}_\infty = \alpha \hat{X}_\infty + (1 - \alpha) \hat{Q}_\infty$ ,  $\hat{B}_\infty = \frac{\alpha}{1 - \alpha} \hat{Q}_\infty$  as well as  $\hat{S}_\infty = (-\delta)$  in the ABG-condition (B.47), we obtain the following expression for  $\hat{X}_\infty$  as function of  $\hat{Q}_\infty$ :

$$\hat{X}_\infty = \frac{1}{1 - \alpha} \frac{(1 - 2\sigma_E) / \sigma_E}{\frac{\alpha}{1 - \alpha} (1/\sigma_c) + 1} \delta + \frac{1 + \left( \frac{\alpha}{1 - \alpha} \right)^2 - \left( 1/\sigma_c - \frac{\alpha}{1 - \alpha} \right)}{\frac{\alpha}{1 - \alpha} (1/\sigma_c) + 1} \hat{Q}_\infty. \quad (\text{B.73})$$

Whenever the two conditions  $1/\sigma_c > \frac{\alpha}{1 - \alpha}$  and  $\hat{Q}_\infty > \frac{(1-2\sigma_E)/\sigma_E}{(1/\sigma_c) - \frac{\alpha}{1 - \alpha}} \delta$ , which together are necessary and sufficient for deceleration, are satisfied, the growth rate  $\hat{X}_\infty$  is smaller than  $\hat{Q}_\infty$ .

With  $\hat{X}_\infty$  from (B.73),  $\hat{c}_\infty = \alpha \hat{X}_\infty + (1 - \alpha) \hat{Q}_\infty$  yields

$$\hat{c}_\infty = \frac{\alpha}{1 - \alpha} \frac{(1 - 2\sigma_E) / \sigma_E}{\frac{\alpha}{1 - \alpha} (1/\sigma_c) + 1} \delta + \frac{1 + \left( \frac{\alpha}{1 - \alpha} \right)^2}{\frac{\alpha}{1 - \alpha} (1/\sigma_c) + 1} \hat{Q}_\infty. \quad (\text{B.74})$$

In the same way as in the previous subsection, we derive  $n_\infty$  from the Euler-equation. The solution is:

$$n_\infty = \frac{\frac{1}{2} \mu q_\infty^{-1} L - \rho + (1 - 1/\sigma_c) \frac{\alpha}{1 - \alpha} \frac{(1-2\sigma_E)/\sigma_E}{\frac{\alpha}{1 - \alpha} (1/\sigma_c) + 1} \delta}{\left( d - \frac{(1-1/\sigma_c) \left( 1 + \left( \frac{\alpha}{1 - \alpha} \right)^2 \right)}{\frac{\alpha}{1 - \alpha} (1/\sigma_c) + 1} q_\infty^2 \right) \mu q_\infty^{-1}}$$

And the productivity growth rate is given by:

$$\begin{aligned} \hat{Q}_\infty &= \mu n_\infty q_\infty \\ &= (1 - \alpha) \left( \frac{\alpha}{1 - \alpha} + \sigma_c \right) \left( 1 + \left( \frac{\alpha}{1 - \alpha} \right)^2 \right)^{-1} \\ &\quad \cdot \left( \frac{1}{2} \left( 1 + \left( \frac{\alpha}{1 - \alpha} \right)^2 \right)^{1/2} d^{-1/2} \mu L - \rho + (1 - 1/\sigma_c) \frac{\alpha}{1 - \alpha} \frac{(1 - 2\sigma_E) / \sigma_E}{\frac{\alpha}{1 - \alpha} (1/\sigma_c) + 1} \delta \right). \end{aligned} \quad (\text{B.75})$$

The growth rate of intermediate cleanliness is the same as in (B.61). The growth rates of intermediate quantity and consumption (and GDP) follow from (B.73) and (B.74).

$$\begin{aligned}\hat{X}_\infty &= \frac{1}{1-\alpha} \frac{(1-2\sigma_E)/\sigma_E}{\frac{\alpha}{1-\alpha}(1/\sigma_c)+1} \delta + (1-\alpha) \sigma_c \frac{1 + \left(\frac{\alpha}{1-\alpha}\right)^2 - \left(1/\sigma_c - \frac{\alpha}{1-\alpha}\right)}{1 + \left(\frac{\alpha}{1-\alpha}\right)^2} \\ &\quad \cdot \left( \frac{1}{2} \left( 1 + \left( \frac{\alpha}{1-\alpha} \right)^2 \right)^{1/2} d^{-1/2} \mu L - \rho + (1-1/\sigma_c) \frac{\alpha}{1-\alpha} \frac{(1-2\sigma_E)/\sigma_E}{\frac{\alpha}{1-\alpha}(1/\sigma_c)+1} \delta \right)\end{aligned}\tag{B.76}$$

$$\begin{aligned}\hat{c}_\infty &= \hat{Y}_\infty = \frac{\alpha}{1-\alpha} \frac{(1-2\sigma_E)/\sigma_E}{\frac{\alpha}{1-\alpha}(1/\sigma_c)+1} \delta + (1-\alpha) \sigma_c \\ &\quad \cdot \left( \frac{1}{2} \left( 1 + \left( \frac{\alpha}{1-\alpha} \right)^2 \right)^{1/2} d^{-1/2} \mu L - \rho + (1-1/\sigma_c) \frac{\alpha}{1-\alpha} \frac{(1-2\sigma_E)/\sigma_E}{\frac{\alpha}{1-\alpha}(1/\sigma_c)+1} \delta \right)\end{aligned}\tag{B.77}$$

## B.8 Proof of Proposition 5

In the case without deceleration, depending on parameters, one, two or no interior solution to the set of first-order conditions with the properties described in proposition 5 may exist as shown in appendix B.7(a).

The solution derived in appendix B.7(b) for the case with deceleration satisfies all the necessary first-order conditions given  $\rho \leq \rho^{\text{delta}}$ . Uniqueness follows from appendix B.6. No further proof has to be provided for the case with deceleration.

## B.9 Proof of proposition 6

### (a) No deceleration

We consider a constrained long-run optimum in which the social planner is forced to choose  $\hat{X}_\infty = \hat{Q}_\infty$ . If aggregate variables are to grow at constant rates asymptotically, the ABG-condition (13), i.e.

$$\frac{\sigma_c - 1}{\sigma_c} \hat{c}_\infty = \frac{1 - \sigma_E}{\sigma_E} \hat{S}_\infty + \left( \hat{X}_\infty - \hat{B}_\infty - \hat{S}_\infty \right)$$

must still hold. Further, the no-arbitrage condition for research directions (B.49)

$$\frac{1}{2} \mu q_\infty^{-1} \left( L_{Y\infty} + \frac{1}{\varphi} \left( \frac{X}{Q} \right)_\infty \right) = \frac{1}{2} \mu b_\infty^{-1} \left( \frac{\alpha}{1-\alpha} L_{Y\infty} - \frac{1}{\varphi} \left( \frac{X}{Q} \right)_\infty \right)$$

is still relevant.

The constraint is binding whenever parameters are such that the unconstrained long-run optimum is characterized by deceleration, i.e.,  $\hat{X}_\infty < \hat{Q}_\infty$ . With  $\hat{X}_\infty = \hat{Q}_\infty$ , equation (13) yields the ratio  $\hat{B}_\infty/\hat{Q}_\infty$  from (B.50) which is however not compatible with  $\left( \frac{X}{Q} \right)_\infty > 0$  and (B.49) to be satisfied simultaneously. The only solution to (13) and (B.49) with  $\hat{X}_\infty = \hat{Q}_\infty$  is  $\hat{X}_\infty = \hat{Q}_\infty = \hat{B}_\infty = \hat{S}_\infty = 0$  and  $q_\infty = b_\infty = 0$ . Then the social return to  $n$  is smaller than the costs and  $\lim_{t \rightarrow \infty} \frac{\partial H}{\partial n_t} < 0$  so that  $n_\infty = 0$ . Conditions (B.31) and (B.32) are satisfied for any  $q_\infty$  and  $b_\infty$  whenever  $n_\infty = 0$ . The consumption Euler-equation (B.47) and the first-order conditions for  $B$  and  $S$  determine the long-run levels of the shadow prices.

As all variables and shadow prices are constant in the long run when  $n_\infty = q_\infty = b_\infty = 0$ , time discounting ( $\rho > 0$ ) guarantees that the transversality conditions for  $Q$ ,  $B$  and  $S$  are satisfied as well.

**(b) No green innovation**

**Solution of the first-order conditions** If  $b_t = 0$  for all  $t$ , the cleanliness  $B$  of intermediates stays constant. The Hamiltonian simplifies to:

$$\begin{aligned}
H = & \left( \frac{\sigma_c}{\sigma_c - 1} c_t^{\frac{\sigma_c - 1}{\sigma_c}} - \psi \frac{\sigma_E}{1 - \sigma_E} S_t^{\frac{1 - \sigma_E}{\sigma_E}} \right) L \\
& + v_{St} \left( \frac{X_t}{B} - \delta S_t \right) \\
& + v_{Qt} \mu n_t q_t Q_t \\
& + \lambda_t (X_t^\alpha Q_t^{1 - \alpha} L_Y^{1 - \alpha} - c_t L) \\
& + \gamma_t (L - L_Y - \frac{1}{\varphi} \frac{X_t}{Q_t} - n_t (q_t^2 + d))
\end{aligned}$$

With  $b_t = 0$  for all  $t$ , the analogue to equation (B.46) is

$$q_\infty^2 = d.$$

It follows that:

$$q_\infty^{b=0} = \sqrt{d}$$

As before, we use (13) to derive the relation between the growth rates  $\hat{X}_\infty$  and  $\hat{Q}_\infty$ . Without green innovation, the pollution stock grows at the same rate as intermediate quantity, i.e.,  $\hat{S}_\infty = \hat{X}_\infty$ . The relation  $\hat{c}_\infty = \alpha \hat{X}_\infty + (1 - \alpha) \hat{Q}_\infty$  from the resource constraint still holds. After some manipulation, we obtain:

$$\hat{X}_\infty^{b=0} = \frac{\frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E}}{1 + \frac{\alpha}{1 - \alpha} \left( 1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E} \right)} \hat{Q}_\infty^{b=0} \quad (\text{B.78})$$

With our assumption that  $\sigma_E < 1/2$ , the numerator is less than one while the denominator exceeds one so that  $\hat{X}_\infty < \hat{Q}_\infty$ . Because according to corollary 1 the pollution stock must decline for  $\sigma_c < 1$ , there is quantity degrowth ( $\hat{X}_\infty < 0$ ) in this case.

Consumption growth as a function of  $\hat{Q}_\infty$  is

$$\begin{aligned}
\hat{c}_\infty^{b=0} &= \alpha \hat{X}_\infty^{b=0} + (1 - \alpha) \hat{Q}_\infty^{b=0} \\
&= \frac{1}{1 + \frac{\alpha}{1 - \alpha} \left( 1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E} \right)} \hat{Q}_\infty^{b=0}.
\end{aligned} \quad (\text{B.79})$$

From (B.47) with  $(X/Q)_\infty = 0$  and  $q_\infty^{b=0} = \sqrt{d}$ , substituting  $\hat{X}_\infty^{b=0}$  and  $\hat{c}_\infty^{b=0}$  from (B.78) and (B.79) and  $L_Y$  from the labor market constraint (B.42) and using (B.46) we find that

$$n_\infty^{b=0} = \frac{1 + \frac{\alpha}{1 - \alpha} \left( 1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E} \right)}{(1/\sigma_c) + \frac{\alpha}{1 - \alpha} \left( 1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E} \right)} \frac{1}{\sqrt{d}\mu} \left( \frac{1}{2} \mu d^{-1/2} L - \rho \right).$$

Therefore

$$\begin{aligned}
\hat{Q}_\infty^{b=0} &= \mu n_\infty^{b=0} q_\infty^{b=0} \\
&= \frac{1 + \frac{\alpha}{1 - \alpha} \left( 1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E} \right)}{(1/\sigma_c) + \frac{\alpha}{1 - \alpha} \left( 1 - \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E} \right)} \left( \frac{1}{2} \mu d^{-1/2} L - \rho \right)
\end{aligned}$$

is the long-run growth rate of productivity and the consumption growth rate is

$$\hat{c}_\infty^{b=0} = \frac{1}{(1/\sigma_c) + \frac{\alpha}{1-\alpha} \left(1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}\right)} \left(\frac{1}{2}\mu d^{-1/2}L - \rho\right) \quad (\text{B.80})$$

Consumption growth is positive if and only if  $\rho < \bar{\rho}^{b=0} = \frac{1}{2}\mu d^{-1/2}L$ .

The pollution growth rate is given by

$$\begin{aligned} \hat{S}_\infty^{b=0} &= \frac{(\sigma_c - 1)/\sigma_c}{(1 - \sigma_E)/\sigma_E} \hat{c}_\infty^{b=0} \\ &= \frac{\frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}}{1/\sigma_c + \frac{\alpha}{1-\alpha} \left(1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}\right)} \left(\frac{1}{2}\mu d^{-1/2}L - \rho\right). \end{aligned}$$

The condition for the rate of time preference needed to ensure  $\hat{S}_\infty > (-\delta)$  for  $\sigma_c < 1$  is  $\rho > \bar{\rho}^{\text{delta}, b=0} = \frac{1}{2}\mu d^{-1/2}L + \left(1/\sigma_c + \frac{\alpha}{1-\alpha} \left(1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}\right)\right) \frac{(1-\sigma_E)/\sigma_E}{(\sigma_c-1)/\sigma_c} \delta$ . Comparison to  $\rho^{\text{delta}}$  from the unconstrained model shows that  $\rho^{\text{delta}, b=0} < \rho^{\text{delta}}$ , so that the parameter restriction  $\rho > \rho^{\text{delta}}$  guarantees that  $\hat{S}_\infty > (-\delta)$  in the unconstrained and the constrained solution.

#### Comparison to the unconstrained solution in proposition 4

1. **Parameter restriction for positive long-run growth:** Comparing  $\bar{\rho}^{b=0}$  with  $\bar{\rho}$  from the unconstrained solution shows that  $\bar{\rho}^{b=0} < \bar{\rho}$ , as  $\left(1 + \left(1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}\right)^2\right)^{1/2}$  and  $\left(1 + \left(\frac{\alpha}{1-\alpha}\right)^2\right)^{1/2}$  both exceed one.
2. **Comparison of  $\hat{X}_\infty/\hat{Q}_\infty$ :** If there is no deceleration in the unconstrained solution of the baseline model, it follows straightforwardly that  $\hat{X}_\infty^{b=0}/\hat{Q}_\infty^{b=0} < \hat{X}_\infty/\hat{Q}_\infty = 1$ .  
If there is deceleration and  $\hat{S}_\infty > (-\delta)$  in the baseline model, comparison of (B.78) and (B.55) shows that  $\hat{X}_\infty^{b=0}/\hat{Q}_\infty^{b=0} < \hat{X}_\infty/\hat{Q}_\infty$  if and only if  $\frac{\alpha}{(1-\alpha)^2} > 0$ , which is true.
3. **Comparison of  $\hat{c}_\infty$ :** Comparing (B.80) to (B.51) if  $\frac{\alpha}{1-\alpha} > \left(1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}\right)$  and (B.56) if  $\frac{\alpha}{1-\alpha} < \left(1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}\right)$ , it is obvious that consumption growth is slower when the option of green innovation is not available: In both cases, the first term of the difference in parentheses is smaller in (B.80). Moreover, for  $\frac{\alpha}{1-\alpha} > \left(1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}\right)$ , the denominator of the fraction which precedes the term in parentheses is larger in (B.80).

#### B.10 Long-run growth in the model without pollution externality

Without the pollution externality, utility depends on consumption only:

$$U = \int_{t=0}^{\infty} e^{-\rho t} \frac{\sigma_c}{\sigma_c - 1} c_t^{\frac{\sigma_c-1}{\sigma_c}} L dt$$

The first-order condition for  $S$  becomes

$$\frac{\partial H}{\partial S_t} = \rho v_{St} - \dot{v}_{St} \Leftrightarrow -\delta v_{St} = \rho v_{St} - \dot{v}_{St}. \quad (\text{B.81})$$

This condition can only be satisfied if  $v_{St} = 0$  and  $\dot{v}_{St} = 0$  for all  $t$ . The second solution  $\hat{v}_{1t} = \rho + \delta$  violates the transversality condition for  $S$  for all possible long-run growth rates  $\hat{S}_\infty \geq -\delta$ .

The first-order condition (B.30) for  $X_t$  then directly yields aggregate intermediate production

$$X_t = \frac{\alpha}{(1-\alpha)} \varphi Q_t L_{Yt} \quad (\text{B.82})$$

for any given labor supply  $L_{Yt}$  and productivity level  $Q_t$ .

With  $v_{St} = 0$ , it follows from the first-order condition

$$v_{Bt} \mu n_t b_t = \rho v_{Bt} - \dot{v}_{Bt}$$

for  $B_t$  that  $v_{Bt} = 0$  and  $\dot{v}_{Bt} = 0$  for all  $t$ <sup>30</sup>.

If  $v_{Bt} = 0$ , it is optimal to set  $b_t = 0$  for all  $t$  as can be seen from (B.32). Then the optimal long-run level of  $q$  is

$$q_\infty^{\psi=0} = \sqrt{d} \quad (\text{B.83})$$

from (B.46).

As  $L_{Y\infty}$  is constant, we conclude from (B.82) and the resource constraint that  $\hat{X}_\infty = \hat{c}_\infty = \hat{Y}_\infty = \hat{Q}_\infty$ . We can still determine  $n_\infty$  from (B.47) using  $\hat{X}_\infty = \hat{c}_\infty = \hat{Q}_\infty$ ,  $q_\infty^{\psi=0} = \sqrt{d}$ , the labor market constraint (B.42) and (B.46):

$$n_\infty^{\psi=0} = \frac{\sigma_c}{\sqrt{d}\mu} \left( \frac{1}{2} \mu d^{-1/2} L - \rho \right)$$

The consumption growth rate of the economy is

$$\begin{aligned} \hat{c}_\infty^{\psi=0} &= \hat{Q}_\infty^{\psi=0} = \mu n_\infty q_\infty \\ &= \frac{1}{1/\sigma_c} \left( \frac{1}{2} \mu d^{-1/2} L - \rho \right) \end{aligned} \quad (\text{B.84})$$

which is positive whenever  $\rho < \bar{\rho}^{\psi=0} = \frac{1}{2} \mu d^{-1/2} L$ .

## B.11 Proof of proposition 7

### B.11.1 Comparison with consumption growth under propositions 4(b) and 5(b)

- (i) **Parameter restriction for positive long-run growth:** Comparison of  $\bar{\rho} = \frac{1}{2} \mu \left( 1 + \left( \frac{\alpha}{1-\alpha} \right)^2 \right)^{1/2} d^{-1/2} L$  and  $\bar{\rho}^{\psi=0}$  shows that positive consumption growth is optimal for larger values of the rate of time preference with  $\psi > 0$  than with  $\psi = 0$  because  $\left( 1 + \left( \frac{\alpha}{1-\alpha} \right)^2 \right)^{1/2} > 1$ .
- (ii) **Comparison of  $\hat{c}_\infty$ :** If proposition 4(b) applies ( $\hat{S}_\infty > (-\delta)$ ), the long-run consumption growth rate in the baseline model is given by (B.63). If proposition 5(b) is relevant ( $\hat{S}_\infty = (-\delta)$ ), equation (B.77) displays the consumption growth rate. Comparison of (B.63) and (B.77) respectively to the growth rate in (B.84) proves the claim in the proposition.

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<sup>30</sup> Again, there is a second solution,  $\hat{v}_{Bt} = \rho - \mu n_t b_t$ , but like the non-zero solution for  $v_{St}$ , it does not satisfy the transversality condition for the associated state-variable ( $B$ ).



### B.11.2 Influence of the size of $\psi$

Given  $\widehat{S}_\infty > (-\delta)$ , it follows from equations (B.59) and (B.61) to (B.64) that long-run growth rates are not affected by the parameter  $\psi$ . If  $\widehat{S}_\infty = (-\delta)$ , the relevant equations are (B.75) to (B.77) and  $\widehat{B}_\infty = \frac{\alpha}{1-\alpha}\widehat{Q}_\infty$ .

## C Appendix to section 5 (Non-renewable resource)

### C.1 Appendix to section 5.2 (Laissez-faire)

#### C.1.1 Derivation of the Hotelling rule (equation (17))

As we assume that the representative household owns the resource stock, the budget constraint in period  $t$  becomes

$$C_t + \dot{A}_t = r_t A_t + w_{Yt} L_{Yt} + w_{Dt} L_{Dt} + p_{Rt} R_t.$$

The household maximizes intertemporal utility subject to the budget constraint and the natural resource constraint (15). Denote the Lagrange-multiplier for the natural resource constraint by  $\lambda_{Rt}$ . Two new conditions are added to the set of necessary first-order conditions:

$$\frac{\partial H}{\partial R_t} = 0 \Leftrightarrow v_{At} p_{Rt} = \lambda_{Rt} \quad (\text{C.85})$$

$$\frac{\partial H}{\partial \lambda_{Rt}} \leq 0 \Leftrightarrow \int_0^\infty R_t dt \leq F_0 \quad \lambda_{Rt} \geq 0 \quad \lambda_{Rt} \left( F_0 - \int_0^\infty R_t dt \right) = 0 \quad (\text{C.86})$$

The new first-order condition (C.86) for  $\lambda_{Rt}$  together with the first-order condition (C.85) for  $R_t$  states that either the resource stock is fully exhausted asymptotically, or the price  $p_{Rt}$  of the resource must be zero for all  $t$ : If  $\int_0^\infty R_t dt < F_0$ ,  $\lambda_{Rt}$  must be zero by the complementary-slackness condition  $\lambda_{Rt} (F_0 - \int_0^\infty R_t dt) = 0$ . But if  $\lambda_{Rt} = 0$  for some  $t < \infty$ , then  $\lambda_{Rt} = 0$  for all  $t < \infty$  since  $\lambda_{Rt} = 0$  can only be satisfied if  $\tilde{\lambda}_R = 0$ . By (C.85), it follows that  $p_{Rt} = 0$  for all  $t$ . We conclude that in the laissez-faire equilibrium, the resource stock is always fully depleted asymptotically and  $\lambda_{Rt} > 0$ .

Using  $\widehat{\lambda}_R = \rho$ , the first-order condition for  $R_t$  together with the first-order-condition for  $A_t$  (equation (A.4)) yields the Hotelling rule:

$$\begin{aligned} \widehat{p}_{Rt} &= \widehat{\lambda}_R - \widehat{v}_A \\ &= \rho - (\rho - r_t) \\ &= r_t \end{aligned}$$

#### C.1.2 Derivation of equation (18)

The demand function (A.8) for intermediate goods remains unchanged vis-à-vis the baseline model:

$$X_{it}^d(p_{it}, L_{Yt}, Q_{it}) = \left( \frac{\alpha}{p_{it}} \right)^{\frac{1}{1-\alpha}} Q_{it} L_{Yt}$$

In the profit function

$$\pi_{it}^X = (p_{it} - MC_t) X_{it}$$

it has to be taken into account that marginal production costs correspond to the price  $p_{Rt}$  for the resource instead of marginal labor costs, as one unit of intermediate goods is now produced from one unit of the non-renewable resource. Marginal costs are still the same for every firm  $j$  so that again, only the firm with the latest patent will be active in production. The profit-maximizing monopoly price, given by the constant mark-up  $\frac{1}{\alpha}$  over marginal costs, is

$$p_{it} = p_t = \frac{1}{\alpha} \cdot p_{Rt} \quad (\text{C.87})$$

Along the lines of the baseline model, it can be shown that the economy adjusts to its balanced-growth path without transitional dynamics. The allocation of labor is constant for all  $t$ . Integrating (A.8) over  $i$ , computing the growth rate and using (17) then yields (18).

### C.1.3 Resource market clearing

Because the resource stock is fully exhausted, total resource demand  $\int_0^\infty R_t dt = \int_0^\infty X_t^d dt$  must equal total supply  $F_0$ . Integrating (A.8) over all sectors  $i$  and using the Hotelling-rule to describe the development of the resource price, the condition can be written as

$$\left( \frac{\alpha^2}{p_{R0}} \right)^{\frac{1}{1-\alpha}} \int_0^\infty e^{-\frac{1}{1-\alpha} \int_0^t r_v dv} L_{Yt} Q_t dt = F_0. \quad (\text{C.88})$$

Given the paths for productivity  $Q$ , labor  $L_Y$  and the interest rate, condition (C.88) fixes the resource price at  $t = 0$  for any given initial resource stock  $F_0$ . It thereby determines the level of the path  $\{p_{Rt}\}_0^\infty$ . The more resource-abundant the economy is, the smaller is the resource price in every period  $t$ .

### C.1.4 Proof of proposition 8

#### (a) Growth effects

- (i) **Quantity degrowth:** As  $\frac{1}{1-\alpha} > 1$ , the transversality condition is sufficient for  $\widehat{X}^{\text{LF,R}}$  in (18) to be negative.
- (ii) **Growth rates of  $c$ ,  $Y$ ,  $S$ :** As to the effect of the growth rate  $\widehat{p}_R$  of the resource price on the equilibrium growth rate of  $S$ , it is sufficient to note that the long-run pollution growth rate is negative only because the price grows over time.

From the consumption Euler-equation

$$\widehat{c}_t = \sigma_c \cdot (r_t - \rho),$$

taking into account  $\widehat{Y}_t = \widehat{c}_t$ , it follows that  $\widehat{p}_R$  lowers output and consumption growth if and only if it decreases the equilibrium interest rate. The equilibrium interest rate can be determined in the same way as in appendix A.4. The solution is

$$r^{\text{LF,R}} = \frac{\frac{1}{2} \frac{1}{\sigma_c} \mu L (\sqrt{1+d} - 1) + \left( \frac{1-\alpha}{\alpha} (\sqrt{1+d} - 1) + d \right) \rho}{\frac{1}{\alpha} \frac{1}{\sigma_c} (\sqrt{1+d} - 1)^2 + \frac{1-\alpha}{\alpha} (\sqrt{1+d} - 1) + d + \kappa_{R,1}} \quad (\text{C.89})$$

where  $\kappa_{R,1} := \frac{\alpha}{1-\alpha} \frac{1}{\sigma_c} \left( \frac{1}{\alpha} (\sqrt{1+d} - 1)^2 + \frac{1-\alpha}{\alpha} (\sqrt{1+d} - 1) + d \right)$ . The term  $\kappa_{R,1}$  in the denominator reflects the influence of  $\widehat{p}_R$ . As  $\kappa_{R,1} > 0$ , the fact that the price rises over time decreases the equilibrium interest rate and therefore growth in output and consumption.

- (iii) **Growth rate of  $Q$ :** Setting equal the consumption Euler-equation and the relation  $\hat{c} = \alpha \hat{X} + (1 - \alpha) \hat{Q}$  with  $\hat{X}$  given by (18),  $\hat{Q}^{\text{LF,R}}$  can be determined. After some manipulation, the growth rate is found to be

$$\hat{Q}^{\text{LF,R}} = \frac{\frac{1}{2}\mu L - \frac{1}{\alpha}(\sqrt{1+d}-1)\rho + k_{R,2}}{\frac{1}{\alpha}\frac{1}{\sigma_c}(\sqrt{1+d}-1)^2 + \frac{1-\alpha}{\alpha}(\sqrt{1+d}-1) + d + \kappa_{R,1}}(\sqrt{1+d}-1), \quad (\text{C.90})$$

with  $\kappa_{R,2} = \frac{\alpha}{1-\alpha} \left( \frac{1}{\sigma_c} \frac{1}{2} \mu L - \frac{1}{\alpha} (\sqrt{1+d}-1) \rho \right)$ . Both  $\kappa_{R,1}$  and  $\kappa_{R,2}$  are attributable to the growth rate of the resource price. Setting  $\kappa_{R,1}$  and  $\kappa_{R,2}$  to zero and comparing the resulting expression to  $\hat{Q}^{\text{LF,R}}$  proves that the growing resource price decreases the productivity growth rate if and only if

$$k_{R,2} \left( \frac{1}{\alpha} \frac{1}{\sigma_c} (\sqrt{1+d}-1)^2 + \frac{1-\alpha}{\alpha} (\sqrt{1+d}-1) + d \right) < \left( \frac{1}{2} \mu L - \frac{1}{\alpha} (\sqrt{1+d}-1) \rho \right) \kappa_{R,1}.$$

Substituting the expressions for  $\kappa_{R,1}$  and  $k_{R,2}$ , the condition becomes

$$\frac{1}{\alpha} \frac{1-\sigma_c}{\sigma_c} (\sqrt{1+d}-1) \left( \frac{1}{2} \frac{1}{\sigma_c} \mu L (\sqrt{1+d}-1) + \left( \frac{1-\alpha}{\alpha} (\sqrt{1+d}-1) + d \right) \rho \right) < 0$$

which is equivalent to  $\sigma_c > 1$ . For  $\sigma_c < 1$ , the growth rate of the resource price increases the productivity growth rate.

- (iv) **Parameter restriction for positive long-run growth:** Substituting the solution for  $r^{\text{LF,R}}$  in the Euler-equation, the upper bound on  $\rho$  which guarantees positive consumption growth can be shown to equal  $\bar{\rho}_c^{\text{LF,R}} = \frac{1}{2} \mu L \frac{1}{\frac{1}{\alpha}(\sqrt{1+d}-1) + \kappa_{R,3}}$ . The expression  $\kappa_{R,3}$ , defined as  $\kappa_{R,3} := \frac{1}{1-\alpha} (\sqrt{1+d}-1) + \frac{1}{\alpha^2} + d/(\sqrt{1+d}-1)$ , results from the growth rate of the resource price. As  $\kappa_{R,3} > 0$ , the increasing resource price lowers the upper bound on  $\rho$ .

## (b) Level effects

- (i) **Growth rates and technology path:** It is obvious from the previous paragraph along with  $\hat{B}^{\text{LF,R}} = 0$  and  $\hat{S}_\infty^{\text{LF,R}} = \max[\hat{X}^{\text{LF,R}}, -\delta]$  that the initial resource stock  $F_0$  does not influence the growth rates of  $c$ ,  $Y$ ,  $X$ ,  $Q$ ,  $B$  and  $S$  along the equilibrium path. Because the initial values for  $Q$  and  $B$  are given and the growth rates of  $Q$  and  $B$  jump to their respective ABG-levels directly, it follows that the entire paths of  $Q$  and  $B$  do not depend on  $F_0$ .
- (ii) **Level of  $p_R$ :** Taking into account that  $r = r^{\text{LF,R}}$  and  $L_Y$  are constant along the equilibrium path and  $Q$  grows at the constant rate  $\hat{Q}^{\text{LF,R}}$  for all  $t$ , solving equation (C.88) for the resource price in  $t = 0$  yields

$$p_{R0} = \alpha^2 \left( \frac{Q_0 L_Y}{F_0} \right)^{1-\alpha} \left( \frac{1}{1-\alpha} r^{\text{LF,R}} - \hat{Q}^{\text{LF,R}} \right)^{1-\alpha}, \quad (\text{C.91})$$

with  $r^{\text{LF,R}}$  and  $\hat{Q}^{\text{LF,R}}$  given by (C.89) and (C.90). Using the Hotelling-rule (17), the resource price can be determined at any point in time. A decline in  $F_0$  increases the price for all  $t$ .

- (iii) **Levels of  $X$ ,  $c$ ,  $Y$ ,  $S$ :** It has been shown in (i) that the path for  $Q$  is unaffected by a variation in  $F_0$ . It can be shown that the same is true for the constant  $L_Y$ . Intermediate demand in every period  $t$  decreases

in the resource price according to equation (A.8) and (C.87). It follows that by increasing the resource price for all  $t$ , a decline in  $F_0$  shifts the path for intermediate quantity downwards.

Because  $\{L_Y\}_0^\infty$  and  $\{Q_t\}_0^\infty$  are independent of  $F_0$ , the path for output and consumption shifts downwards with the path for  $X$ .

Further, because  $\{B_t\}_0^\infty$  is not affected by  $F_0$ , emissions  $X_t/B_t$  are lower for all  $t$ . The path for the pollution stock  $S_t$  is given by the solution to the differential equation (7),  $\dot{S}_t = X_t/B_t - \delta S_t$ . From the general solution, it can be concluded that due to the decline in emissions, the pollution stock  $S$  is lower in every period.

## C.2 Appendix to section 5.3 (Social Planner)

### C.2.1 First-order conditions

Four changes occur in the set of necessary first-order conditions compared to the baseline model: First, the shadow price  $\lambda_R$  of the non-renewable resource occurs in the first-order condition for intermediates,

$$\frac{\partial H}{\partial X_t} = 0 \Leftrightarrow \frac{v_{St}}{B_t} + \lambda_{Yt} \alpha X_t^{\alpha-1} L_{Yt}^{1-\alpha} Q_t^{1-\alpha} - \lambda_{Rt} = 0. \quad (\text{C.92})$$

Second, the first-order conditions are complemented by a complementary slackness condition:

$$\frac{\partial H}{\partial \lambda_{Rt}} \leq 0 \Leftrightarrow F_0 - \int_0^\infty X_t dt \geq 0 \quad \lambda_{Rt} \geq 0 \quad \lambda_{Rt} \left( F_0 - \int_0^\infty X_t dt \right) = 0 \quad (\text{C.93})$$

Third, labor is only allocated to research and output production which changes the first order condition for  $\lambda_{Lt}$  to:

$$\frac{\partial H}{\partial \lambda_{Lt}} = 0 \Leftrightarrow L = L_{Yt} + n_t(q_t^2 + b_t^2 + d) \quad (\text{C.94})$$

Finally, in the first-order condition (B.36) for  $Q$ , the last term on the left-hand side  $(\lambda_{Lt} (1/\varphi) (X_t/Q_t^2))$  drops out because  $Q$  no longer affects the production of intermediate goods.

As the set of first-order conditions is otherwise unaffected by the modifications in the model setup, the indifference equation

$$q_\infty^2 + b_\infty^2 = d$$

and equation (B.45), i.e.,

$$\hat{v}_{S\infty} = ((1 - 2\sigma_E)/\sigma_E) \hat{S}_\infty$$

must still hold in the long run.

The research-arbitrage equation becomes:

$$\frac{\mu}{2q_\infty} L_{Y\infty} = \frac{\mu}{2b_\infty} L_{Y\infty} \left( \frac{\alpha}{1-\alpha} - \frac{1}{1-\alpha} \left( \frac{\lambda_R}{\lambda_Y} \right)_\infty \left( \frac{X}{Q} \right)_\infty^{1-\alpha} L_{Y\infty}^{\alpha-1} \right) \quad (\text{C.95})$$

### C.2.2 Proof of proposition 9

#### (a) Binding constraint

- (i) **Quantity degrowth:** If there is quantity degrowth,  $\hat{S}_\infty = 0$  so that  $v_{S\infty} = 0$  according to (B.45), while  $\lambda_R$  grows also for  $t \rightarrow \infty$ . To satisfy the first-order condition (C.92) for  $X$ , the social marginal product of  $X$  in production must equal  $\lambda_R$  asymptotically:

$$c_\infty^{-1/\sigma_c} \alpha X_\infty^{\alpha-1} L_{Y\infty}^{1-\alpha} Q_\infty^{1-\alpha} = \lambda_{R\infty} \quad (\text{C.96})$$

Computing growth rates on both sides of (C.96) yields  $(-1/\sigma_c \cdot \hat{c}_\infty) - (1-\alpha)(\hat{X}_\infty - \hat{Q}_\infty) = \rho$ . From this equation, using  $\hat{c}_\infty = \alpha \hat{X}_\infty + (1-\alpha)\hat{Q}_\infty$ , we derive the growth rate  $\hat{X}_\infty^R$  for any given  $\hat{Q}_\infty^R$ :

$$\hat{X}_\infty^R = \frac{1}{\frac{\alpha}{1-\alpha} \frac{1}{\sigma_c} + 1} \left( \left(1 - \frac{1}{\sigma_c}\right) \hat{Q}_\infty^R - \frac{1}{1-\alpha} \rho \right) \quad (\text{C.97})$$

If  $\sigma_c < 1$ , it can be seen directly that  $\hat{X}_\infty^R < 0$ . For  $\sigma_c > 1$  the transversality conditions, which require  $\rho > \left(1 - \frac{1}{\sigma_c}\right) \hat{Q}_\infty^R$ , together with  $(1-\alpha) < 1$  guarantee that indeed  $\hat{X}_\infty^R < 0$ .

- (ii) **Green Innovation:** Substituting (C.96) in (C.95) shows that investing in the cleanliness of technology is not optimal in the long run:

$$\begin{aligned} \frac{\mu}{2b_\infty} L_{Y\infty} \left( -\frac{\alpha}{1-\alpha} + \frac{\alpha}{1-\alpha} \right) &= (\rho - (1 - 1/\sigma_c) \hat{c}_\infty) \\ \Leftrightarrow b_\infty^R &= 0 \end{aligned}$$

From  $q_\infty^2 + b_\infty^2 = d$  it follows that  $q_\infty^R = \sqrt{d}$  so that labor in the R&D-sector is entirely used for productivity-oriented innovation.

#### (b) Unbinding constraint

- (i) **Convergence of  $\int_0^\infty X_t dt$ :** The integral  $\int_0^\infty X_t dt$  can be written as the sum of the two integrals  $\int_0^T X_t dt$  and  $\int_T^\infty X_t dt$ . It converges if and only if both integrals in the sum converge.

Because  $X_t$  is finite for every  $t$ , the definite integral  $\int_0^T X_t dt$  assumes a finite value.

Consider the second integral: In any solution to the social planner's problem for which growth rates converge to the growth rates of the asymptotically-balanced growth solution with quantity degrowth from section 4.3, the sequence  $\{\hat{X}_t\}_0^\infty$  converges to the constant  $\hat{X}_\infty < 0$ . Assuming continuity, convergence implies that there exists a time  $T$  such that  $\hat{X}_t < \bar{\hat{X}} < 0$  for all  $t > T$ . Therefore if the integral  $\int_T^\infty X_T e^{\bar{\hat{X}} \cdot t} dt$  converges, then so does the integral  $\int_T^\infty X_t dt$ . The limit of the integral  $\int_T^\infty X_T e^{\bar{\hat{X}} \cdot t} dt$  is  $X_T [1/\bar{\hat{X}} \cdot e^{\bar{\hat{X}} \cdot t}]_T^\infty = -X_T/\bar{\hat{X}} \cdot e^{\bar{\hat{X}} \cdot T} > 0$  as  $\bar{\hat{X}} < 0$ . Because  $X_T < \infty$ , the limit is finite. It follows that the integral  $\int_T^\infty X_t dt$  converges.

We have thus proven that  $\int_0^\infty X_t dt = \int_0^T X_t dt + \int_T^\infty X_t dt$  converges.

- (ii) **Equality of solutions:** Because the integral  $\int_0^\infty X_t dt$  converges,  $\int_0^\infty X_t dt < F_0$  for a sufficiently large  $F_0$ . In this case, the natural resource constraint is not binding and it follows from (C.93) that  $\lambda_{Rt} = 0, \forall t$ . If

$\lambda_R = 0$ , differences in the first-order conditions compared to the baseline model only arise because labor is no longer used in intermediate production in the model of this section. But for parameter constellations such that there is quantity degrowth in the baseline model, labor use in intermediate production converges to zero in the baseline model as well, so that the first-order conditions and therefore the long-run solutions are identical for  $t \rightarrow \infty$ .