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Unemployment Benefits and Optimal Non-Linear Income Taxation^{*}

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Abstract

This paper explores the rationale for unemployment benefits as a complement to optimal non-linear income taxation. High-skilled workers and low-skilled workers face different exogenous risks of being unemployed. As long as the low-skilled workers face a higher unemployment risk, we find that there is a case for over-insuring the lowskilled, hence the unemployment benefits of the low-skilled should be higher than the pure insurance purpose would prescribe. This effect is likely to prevail in a model with a more realistic treatment of the labor market.

JEL-classification: H21, J22, J41, J64

Keywords: Optimal non-linear income taxation, unemployment benefits.

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1 Introduction

The primary purpose of unemployment benefits is the insurance aspect; risk averse individuals need to be insured against the wage loss during unemployment. There is however a cost of this insurance since it may result in adverse effects on the unemployed workers' search behavior; see Mortensen (1977). The seminal paper by Shavell and Weiss (1979) argues that this adverse behavioral effect may be reduced if the unemployment benefit is a decreasing function of the elapsed unemployment duration. This result is confirmed by a more recent study by Fredriksson and Holmlund (2001), who address the issue in a search unemployment framework (see e.g. Pissarides, 2000).

In the present study we will approach the unemployment insurance issue from a different angle. The model will be based on the Stiglitz (1982) model of optimal non-linear income taxation. We will introduce unemployment in the most simple fashion; high-skilled workers and low-skilled workers will face different exogenous risks of becoming unemployed. This means that we abstract from the adverse behavioral effects that the insurance system may have. Full insurance will thus be possible without any efficiency losses stemming from e.g. reduced job search.

The Stiglitz (1982) model of optimal income taxation has been applied to a large number of issues. Extensions have been made in many directions, ranging from heterogeneous preferences for leisure (Boadway et al, 2001) to the inclusion of commodity taxation (Edwards et al, 1994). However, papers that introduce unemployment into the model are very hard to come by. Two exceptions are Engström (2002) and Aronsson and Sjögren (2001). But Engström (2002) ignores unemployment benefits and Aronsson and Sjögren (2001) view the unemployed workers as a third type of workers, hence the unemployment benefits do not serve as an insurance tool.

We will highlight a new reason for having unemployment benefits for lowskilled workers. We find that in an optimal taxation setting, benefits may be an attractive way to transfer resources from the high-skilled to the lowskilled provided that the low-skill unemployment rate is at least as high as the high-skill rate.

A case for over-insurance of low-skilled workers arises when the so called self selection constraint (SSC) binds. The SSC ensures that the high-skilled workers prefer the consumption bundle that the government intends for them rather than the consumption bundle intended for the low-skilled workers. This constraint is due to the government's lack of information about each worker's skill type. If the high-skilled workers earn the same income as the low-skilled workers, the government cannot levy different tax rates, nor different unemployment benefits since the benefit level is based on the income. In the optimal taxation literature this is called "mimicking"; the high-skilled workers may mimic the low-skilled workers in order to avoid high taxes. The attractiveness of mimicking generally increases with the level of resources redistributed from high-skilled workers to low-skilled workers. But if unemployment is mostly a low-skilled phenomenon, increased low-skilled benefits will not increase the attractiveness of mimicking to a large extent. The reason for this is that the mimickers do not benefit from the low-skill unemployment benefits to the same extent as the low-skilled workers do.

The rationale for over-insuring the low-skilled workers can also be illustrated through the "economics of tagging" (see the seminal paper by Akerlof, 1978). If it is mostly the unskilled that are unemployed, the government can use the unemployment as a tagging device; being observed as an unemployed indicates that you probably belong to the "needy group".

The paper is outlined as follows. In section 2 the model is presented. We present the workers' optimization problems and the government's objectives and information set. In section 3 we derive analytical results for the optimal marginal tax rates as well as the optimal benefit levels. We proceed by discussing the intuition behind the results. Section 4 concludes.

2 The Model

2.1 The Workers

There are two types of workers in the economy: high-skilled (H) and lowskilled (L). We consider a very simple "semi-dynamic"¹ model with homogenous workers within each skill group. The workers are infinitely lived and forward looking. For simplicity we let the number of high-skilled workers equal the number of low-skilled workers, and we normalize this number to unity. The workers are either employed or unemployed. The rate at which unemployed workers of type j become employed is denoted α_j and the rate at which employed workers of type j lose their job is denoted ϕ_j , j = H, L. Both α_j and ϕ_j are exogenous. The workers derive utility from consumption (C) and disutility from work (L). The instantaneous utility of a worker of ability j is given by:

$$v_j = v(C_j, L_j)$$
 if employed, and (1)

$$v_{uj} = v(B_j, 0)$$
 if unemployed. (2)

where B_j is the unemployment benefit intended for the unemployed workers of ability j. The instantaneous utility function v(.) is assumed to be equal across different types of workers. We impose the standard assumptions that $v_{CC} < 0, v_{LL} < 0$. Furthermore we assume that consumption and leisure are weak complements, hence

$$v_{CL} \le 0. \tag{3}$$

Let E_j represent the total state value of being employed and U_j the state value of being unemployed. E_j and U_j are then determined by the following value functions:

$$rE_j = v_j + \phi_j (U_j - E_j), \tag{4}$$

$$rU_j = v_{uj} + \alpha_j \left(E_j - U_j \right), \tag{5}$$

¹The exact meaning of the term "semi-dynamic" will be revealed below.

where r is the time preference shared by all workers in the economy. We make the simplifying assumption that r approaches zero. This gives the following relationship:

$$\lim_{r \to 0} rE_j = \lim_{r \to 0} rU_j = \frac{\alpha_j}{\alpha_j + \phi_j} \upsilon_j + \frac{\phi_j}{\alpha_j + \phi_j} \upsilon_{uj}.$$
 (6)

In steady state the *j*-type unemployment rate is given by $u_j = \frac{\phi_j}{\alpha_j + \phi_j}$ and we can rewrite (6) as follows:

$$v_j^e \equiv \lim_{r \to 0} rE_j = \lim_{r \to 0} rU_j = (1 - u_j)v_j + u_j v_{uj},$$
(7)

where we have defined v_j^e as today's fraction of the total state value.² When there is no discounting (*r* approaches to zero), today's fraction of the total state value will simply be the state independent expected instantaneous utility for a representative worker.³ v_j^e is thus independent of the current state (employed or unemployed). The government is only concerned with v_H^e and v_L^e , which simplifies the optimal taxation analysis substantially since the government does not need to consider all 4 types/states of workers in the economy; all that matter to the government are the workers' types, not each worker's current state.⁴ These simplifying assumptions – steady state analysis and ignoring discounting – give very convenient static flavors to the model; hence the term "semi-dynamic".

Market wages (w) are exogenous with $w_H > w_L$. In the employed state the workers maximize $v(C_j, L_j)$ w.r.t. C_j and L_j , recognizing their budget restrictions and the tax function $T(w_jL_j)$. This maximization problem can thus be expressed as:

$$\max_{L_{j}} v(w_{j}L_{j} - T(w_{j}L_{j}), L_{j}).$$
(8)

²Since the unemployment rate is fully determined by the exogenous flow rates, we will treat the unemployment rates as exogenous in the subsequent analysis. α_j and ϕ_j will not matter apart from the equilibrium unemployment rate they imply.

³Note that a static one shot model, with u_j being the risk of unemployment, would generate the same expression (7) for the expected utility.

⁴In the subsequent analysis we will use the term "expected utility" instead of the longer term "today's fraction of the total state value".

The first order condition for (8) gives:

$$T'_{j} = \frac{1}{w_{j}} \frac{\frac{\partial v_{j}}{\partial L_{j}}}{\frac{\partial v_{j}}{\partial C_{i}}} + 1.$$
(9)

It turns out that it is more convenient for the subsequent analysis to rewrite (9) in terms of C_j and the gross income $Y_j \equiv L_j w_j$. We then get:

$$T'_{j} = \frac{\frac{\partial v_{j}}{\partial Y_{j}}}{\frac{\partial v_{j}}{\partial C_{i}}} + 1.$$
(10)

2.2 The Government

The government's only objective is redistribution; all the tax that is collected is redistributed back as transfers. The set of policy instruments at the government's disposal is $\{T(Y), B_H, B_L\}$, where B_j is the unemployment benefit intended for workers with ability j. The government seeks to find the set of Pareto optimal solutions to the tax problem. Technically this is done by maximizing the expected utility of a representative individual of one ability, under the restriction of a minimum utility constraint $(v_j^e = \overline{v}_j^e)$ on the representative individual of the other ability.

As in the basic Stiglitz (1982) model we assume that the government has imperfect information concerning each worker's ability. This means that the government can only tax income; each worker's ability is not revealed to the government. If the government would try to tax the high-skilled at a very high rate and transfer a lot to the low-skilled, the high-skilled workers may mimic the low-skilled workers. This imperfect information also holds in the unemployment state; the government can only base the level of the benefits to an unemployed worker of type j on the income he had when he was employed. The decision to mimic therefore also affects the future benefit level when being unemployed. In this model as in the basic Stiglitz (1982) model, the imperfect information leads to a *self selection constraint* (SSC) facing the government when finding the Pareto optimal path. The SSC ensures that the high-skilled workers do not have incentives to pretend that they are low-skilled workers.⁵ Formally this means that,

$$v^{e}(C_{H}, \frac{Y_{H}}{w_{H}}, B_{H}; u_{H}) \geq v^{e}(C_{L}, \frac{Y_{L}}{w_{H}}, B_{L}; u_{H}) \rightarrow$$
(11)

$$(1 - u_H) v(C_H, \frac{Y_H}{w_H}) + u_H v(B_H, 0) \geq (1 - u_H) v(C_L, \frac{Y_L}{w_H}) + u_H v(B_L, 0),$$

must hold.

Apart from the minimum utility constraint and the SSC, the government also needs to balance the budget. The budget constraint is:

$$(1 - u_H) Y_H + (1 - u_L) Y_L = (1 - u_H) C_H + (1 - u_L) C_L + u_H B_H + u_L B_L.$$
(12)

The optimal taxation problem can now be stated formally. We follow the usual practice (see e.g. Stiglitz, 1987) and substitute out the unobservable $L_j = \frac{Y_j}{w_j}$ when formulating the problem. This results in the following:

$$\max_{C_H, C_L, Y_H, Y_L, B_H, B_L} v^e(C_H, \frac{Y_H}{w_H}, B_H; u_H)$$
(13)

s.t.

$$v^{e}(C_{L}, \frac{Y_{L}}{w_{L}}, B_{L}; u_{L}) = \overline{v}_{j}^{e}$$

$$(1 - u_{H}) v(C_{H}, \frac{Y_{H}}{w_{H}}) + u_{H} v(B_{H}, 0) \geq (1 - u_{H}) v(C_{L}, \frac{Y_{L}}{w_{H}}) + u_{H} v(B_{L}, 0)$$

$$(1 - u_{H}) Y_{H} + (1 - u_{L}) Y_{L} = (1 - u_{H}) C_{H} + (1 - u_{L}) C_{L} + u_{H} B_{H} + u_{L} B_{L}$$

The Lagrange function for this problem can be written:

$$\Psi = v_{H}^{e} + \mu \left(v_{L}^{e} - \overline{v}_{j}^{e} \right) + \lambda_{H} \left(v_{H}^{e} - v_{m}^{e} \right) +$$

$$\gamma \begin{bmatrix} (1 - u_{H}) Y_{H} + (1 - u_{L}) Y_{L} \\ - (1 - u_{H}) C_{H} - (1 - u_{L}) C_{L} - u_{H} B_{H} - u_{L} B_{L} \end{bmatrix},$$
(14)

⁵In principle the low-skilled workers could have incentives to mimic the high-skilled workers, but we assume that the government wants to distribute from high-skilled to low-skilled and then this possibility never arises.

where v_m^e is the mimicker's utility; hence $v_m^e = (1 - u_H) v(C_L, \frac{Y_L}{w_H}) + u_H v(B_L, 0)$. μ, λ_H and γ are Lagrange multipliers for which hold:

$$\mu > 0,$$

 $\lambda_H \ge 0 \text{ and}$
 $\gamma > 0.$

The first order conditions are:

$$\frac{\partial \Psi}{\partial C_H} = (1 + \lambda_H) \left(1 - u_H\right) \frac{\partial v_H}{\partial C_H} - \gamma \left(1 - u_H\right) = 0 \tag{15}$$

$$\frac{\partial \Psi}{\partial Y_H} = (1 + \lambda_H) \left(1 - u_H\right) \frac{\partial v_H}{\partial Y_H} + \gamma \left(1 - u_H\right) = 0 \tag{16}$$

$$\frac{\partial \Psi}{\partial B_H} = (1 + \lambda_H) \, u_H \frac{\partial v_{uH}}{\partial B_H} - \gamma u_H = 0 \tag{17}$$

$$\frac{\partial \Psi}{\partial C_L} = \mu (1 - u_L) \frac{\partial v_L}{\partial C_L} - \lambda_H (1 - u_H) \frac{\partial v_m}{\partial C_L} - \gamma (1 - u_L) = 0 \qquad (18)$$

$$\frac{\partial \Psi}{\partial Y_L} = \mu (1 - u_L) \frac{\partial v_L}{\partial Y_L} - \lambda_H (1 - u_H) \frac{\partial v_m}{\partial Y_L} + \gamma (1 - u_L) = 0$$
(19)

$$\frac{\partial \Psi}{\partial B_L} = \mu u_L \frac{\partial v_{uL}}{\partial B_L} - \lambda_H u_H \frac{\partial v_{uL}}{\partial B_L} - \gamma u_L = 0, \qquad (20)$$

where $v_m = v(C_L, \frac{Y_L}{w_H})$.

3 The Pareto Optimal Regimes

3.1 Optimal Marginal Tax Rates

We start by deriving the expressions for the optimal marginal tax rates. For this we need lemma 1 which gives the usual property that the indifference curves are convex in C and Y space.

Lemma 1. $\frac{d\frac{dC_j}{dY_j}}{dY_j}|_{v_j=\overline{v}_j} > 0$, hence the indifference curves are convex in C and Y space.

Proof See Appendix. \blacksquare

We can now show (proposition 1 below) that the results for the optimal marginal tax rates from Stiglitz (1982) are not changed when introducing exogenous unemployment. When the SSC does not bind we have a first best solution with zero marginal tax on both high and low-skilled workers. When the SSC binds we need a positive marginal tax on the low-skilled workers in order to reduce the attractiveness of mimicking.

Proposition 1 The optimal marginal tax rates consist of: $T'_H = 0$ and $T'_L \ge 0$.

Proof $T'_H = 0$: Combining (15), (16) and (10) gives:

$$\frac{\frac{\partial v_H}{\partial Y_H}}{\frac{\partial v_H}{\partial C_H}} + 1 = 0 \to T'_H = 0.$$
(21)

 $T'_H \ge 0$: Combining (18), (19) and (10) gives:

$$\frac{\frac{\partial v_L^e}{\partial Y_L}}{\frac{\partial v_L^e}{\partial C_L}} + 1 = \frac{1 - \left\lfloor -\frac{\frac{\partial v_m}{\partial Y_L}}{\frac{\partial v_m}{\partial C_L}} \right\rfloor}{1 + \frac{\gamma(1 - u_L)}{\lambda_H (1 - u_H)\frac{\partial v_m}{\partial C_L}}}.$$
(22)

Now note that lemma 1 gives:

$$\left[-\frac{\frac{\partial v_m}{\partial Y_L}}{\frac{\partial v_m}{\partial C_L}}\right] < \left[-\frac{\frac{\partial v_H}{\partial Y_H}}{\frac{\partial v_H}{\partial C_H}}\right] = 1,$$
(23)

since v_m and v_H are on the same indifference curve, and $Y_L < Y_H$. (23) in (22) finally gives:

$$T'_{L} = 0 \text{ for } \lambda_{H} = 0 \text{ and}$$

$$T'_{L} > 0 \text{ for } \lambda_{H} > 0.$$
(24)

3.2 Optimal Benefits

We now turn to the main objective of the paper: the optimal unemployment benefits. Proposition 2 shows that the high-skilled workers should be fully insured against unemployment and that there may be reasons to over-insure the low-skilled workers.

Proposition 2 *i)* The optimal unemployment benefits for the high-skilled are given by:

$$\frac{\partial \upsilon_{uH}}{\partial B_H} = \frac{\partial \upsilon_H}{\partial C_H}.$$

ii) When $\lambda_H > 0$ the following hold for the low-skill unemployment benefits:

$$\begin{aligned} u_L &> u_H \to \frac{\partial \upsilon_{uL}}{\partial B_L} < \frac{\partial \upsilon_L}{\partial C_L}, \\ u_L &= u_H \to \frac{\partial \upsilon_{uL}}{\partial B_L} \le \frac{\partial \upsilon_L}{\partial C_L}. \end{aligned}$$

Proof i) The proof follows immediately from (15) and (17). ii) (18) and (20) gives:

$$\frac{\partial v_{uL}}{\partial B_L} = \frac{\mu}{\left[\mu - \lambda_H \frac{u_H}{u_L}\right]} \frac{\partial v_L}{\partial C_L} - \frac{\lambda_H}{\left[\mu - \lambda_H \frac{u_H}{u_L}\right]} \frac{(1 - u_H)}{(1 - u_L)} \frac{\partial v_m}{\partial C_L}$$
(25)

From (3) we have that,

$$\frac{\partial v_m}{\partial C_L} \ge \frac{\partial v_L}{\partial C_L},\tag{26}$$

since $L_m = \frac{Y_L}{w_H} < L_L$.

Combining (25) and (26) gives:

$$\frac{\partial \upsilon_{uL}}{\partial B_L} \leq \underbrace{\left[\frac{\mu - \lambda_H \frac{(1-u_H)}{(1-u_L)}}{\mu - \lambda_H \frac{u_H}{u_L}}\right]}_{<1 \text{ if } u_L > u_H \text{ and } = 1 \text{ if } u_L = u_H} \times \frac{\partial \upsilon_L}{\partial C_L},$$
(27)

which completes the proof. \blacksquare

The first part of the result says that the high-skilled workers should be fully insured against unemployment; the marginal utility of consumption is equalized between the two states. With the assumptions we made about the utility function this means that $B_H \ge C_H$, hence workers consume at least as much when unemployed as they do when employed. It is obvious that this result would not hold in a model featuring a more realistic treatment of the origins of unemployment. However, the focus of this paper is not to give a rigorous treatment of the unemployment benefit issue but rather to emphasize one important mechanism concerning the unemployment benefits of the low-skilled workers in an optimal taxation model.

The second result means that when the unemployment rate for the lowskilled is at least as high as the unemployment rate for the high-skilled and the SSC binds ($\lambda_H > 0$), there is a case for over-insurance against unemployment; the low-skilled workers' marginal utility of consumption should be lower in the unemployment state than in the employment state. The consumption in the unemployment state for the low-skilled is then higher than it would be if the government only used B_H as an insurance tool. This is the main result of this paper and we will spend some effort trying to pin down what drives it.

The result arises when the SSC is binding; when λ_H is zero the benefits to the low-skilled are only used for insurance purposes. When trying to understand results intuitively it is often instructive to consider the extreme special cases. We therefore start by considering the case when unemployment is a pure low-skilled phenomenon. Think of the case when $u_H = 0$ and the SSC binds ($\lambda_H > 0$). Equation (27) then becomes:

$$\frac{\partial v_{uL}}{\partial B_L} \le \underbrace{\left[1 - \frac{\lambda_H}{\mu \left(1 - u_L\right)}\right]}_{<1} \frac{\partial v_L}{\partial C_L},\tag{28}$$

and B_L is obviously larger than the insurance purpose would prescribe. The government wants to distribute resources to the low-skilled, without inducing the high-skilled to mimic the low-skilled. When $u_H = 0$ this can easily be achieved by increasing B_L , since the high-skilled cannot take advantage of the increased benefits, due to their lack of unemployment spells. Increased lowskilled benefits therefore only increase the utility of the low-skilled. There is of course a cost associated with this redistribution, stemming from an efficiency loss caused by over-insurance of the low-skilled. But this example explains why the benefits to the low-skilled should be higher than the efficient insurance level when unemployment is mostly a low-skilled phenomenon. On the margin, the government needs to balance two different efficiency losses: the loss from over-insurance of the low-skilled and the loss from increasing the marginal tax on the low-skilled. In the model without unemployment (see e.g. Stiglitz, 1982), the only way the government could distribute more to the low-skilled was by increasing the marginal tax facing the low-skilled, and thereby making mimicking less attractive.⁶

The above example does not explain why there still may be over-insurance of the low-skilled when $u_H = u_L$ and $\lambda_H > 0$. In this case the mimicker will be unemployed to the same extent as a low skilled worker and thus benefit from the high level of unemployment insurance. However, we need to compare this to the alternative. The government has two choices – provided that it must increase the utility of the low-skilled workers – of how to transfer resources. The government could either increase the low-skilled workers' unemployment benefits, or it could increase the in-work transfers to the lowskilled. In the former case the mimickers and the low-skilled workers benefit to the exactly same extent, since the unemployment rates are the same and the instantaneous utilities in the unemployment state are the same. But in the latter case the mimicker will benefit more if the marginal utility of consumption is decreasing in labor input (hence in the case when (3) holds with strict inequality), and the mimicker – due to his high wage – puts in less labor than the low-skilled. The government will therefore choose the least bad thing of two bad things and increase the unemployment benefits for the low-skilled until the efficiency loss from over insurance gets too high.

4 Conclusions

The paper has exposed a new reason for having high unemployment benefits for low-skilled workers. We found that in an optimal taxation setting, benefits

⁶See Engström (2002) for an intuitive explanation of why increased distribution to the low-skilled needs to be accompanied by an increase in the low-skill marginal tax.

may be a preferable way to transfer resources from the high-skilled to the low-skilled provided that the low-skill unemployment rate is at least as high as the high-skill rate. The reason for this is that the mimickers do not benefit from the unemployment benefits intended for the low-skilled workers to the same extent as the low-skilled workers do. If the mimickers do not face unemployment spells, they will not benefit from this transfer at all, which makes the unemployment benefit an attractive tool for redistribution.

In future work it would be interesting to introduce a more realistic treatment of the labor market into this model. Since the unemployment benefits have adverse effects on job search, the optimal unemployment benefit levels would be lower with a more serious treatment of the labor market. But the main mechanism described in this paper would prevail as long as unemployment is mostly a low-skilled phenomenon.

Appendix

We want to prove

$$\frac{d\frac{dC_j}{dY_j}}{dY_j}\Big|_{v_j=\overline{v}_j} > 0.$$
(A1)

$$\overline{v}_j = v(C_j, \frac{Y_j}{w_j}) \tag{A2}$$

Differentiate (A2) to get:

$$\frac{dC_j}{dY_j} = -\frac{v_{jL}}{v_{jC}}\frac{1}{w_j} > 0.$$
(A3)

Taking the derivative of $\frac{dC_j}{dY_j}$ w.r.t. Y_j finally gives:

$$\frac{d\frac{dC_j}{dY_j}}{dY_j} = \left[\frac{v_{jLL}}{v_{jL}}\frac{1}{w_j} - \frac{v_{jCL}}{v_C}\left(1 + \frac{1}{w_j}\right) + \frac{v_{jL}v_{jCC}}{v_C^2}\frac{1}{w_j}\right]\frac{dC_j}{dY_j} > 0, \quad (A4)$$

which completes the proof.

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