

Ibarra-Ramírez, Raúl

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**Raúl Ibarra-Ramírez**  
Banco de México

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# Stocks, Bonds and the Investment Horizon: A Spatial Dominance Approach\*

Raúl Ibarra-Ramírez<sup>†</sup>  
Banco de México

**Abstract:** Financial advisors typically recommend that a long-term investor should hold a higher percentage of his wealth in stocks than a short-term investor. However, part of the academic literature disagrees with this advice. We use a spatial dominance test which is suited for comparing alternative investments when their distributions are time-varying. Using daily data for the US from 1965 to 2008, we test for dominance of cumulative returns series for stocks versus bonds at different investment horizons from one to ten years. We find that bonds second order spatially dominate stocks for one and two year horizons. For horizons of nine years or longer, we find evidence that stocks dominate bonds. When different portfolios of stocks and bonds are compared, we find that for long investment horizons, only those portfolios with a sufficiently high proportion of stocks are efficient in the sense of spatial dominance.

**Keywords:** Investment decisions; Investment horizon; Stochastic dominance.

**JEL Classification:** C12, C14, G11.

**Resumen:** Los asesores financieros típicamente recomiendan que un inversionista de largo plazo debería mantener un mayor porcentaje de su riqueza en acciones que un inversionista de corto plazo. Sin embargo, parte de la literatura académica está en desacuerdo con esta recomendación. En este trabajo se utiliza una prueba de dominancia espacial que es apropiada para comparar inversiones alternativas cuando sus distribuciones varían a través del tiempo. Utilizando datos diarios para los Estados Unidos de 1965 a 2008, se realiza una prueba de dominancia para las series de retornos acumulados de acciones contra bonos para diferentes horizontes de uno a diez años. Se encuentra que los bonos dominan a las acciones en segundo orden para horizontes de inversión de uno y dos años. Para horizontes de nueve años o más, se encuentra evidencia que las acciones dominan a los bonos. Al comparar distintos portafolios de acciones y bonos, se encuentra que para horizontes largos solo aquellos portafolios con una proporción suficientemente alta de acciones son eficientes en el sentido de dominancia espacial.

**Palabras Clave:** Decisiones de inversión; Horizonte de inversión; Dominancia estocástica.

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<sup>†</sup> Dirección General de Investigación Económica. Email: ribarra@banxico.org.mx.

# 1 Introduction

Financial advisers typically recommend to allocate a greater proportion of stocks for long-term investors than for short-term investors.<sup>1</sup> The advice given by practitioners suggests that optimal investment strategies are horizon dependent and it is motivated by the idea that the risk of stocks decreases in the long run, which is called time diversification.<sup>2</sup> However, this conclusion is not supported in general by the academic literature. Merton and Samuelson (1974) conclude that lengthening the investment horizon should not reduce risk, which implies that the optimal portfolio of an investor should be independent of the planning holding period. According to Samuelson (1989, 1994), if equity prices follow a random walk, although the probability of the return falling below some minimal level falls with the investment horizon, the extent to which the actual outcome can fall short of this minimum level increases. Therefore, equity will never dominate bonds in the long run. These studies are based on a myopic utility function, for which the optimal asset allocation is independent of the investment horizon. On the other hand, Barberis (2000) finds that for a buy and hold investor, stocks dominate bonds for long investment horizons in the presence of mean reverting returns.

There is a large literature about the effects of optimal portfolio choice as a function of the investment horizon, including Jagannathan and Kocherlakota (1996), Viceira (2001), Wachter (2002), among others. Typically, these studies focus on an individual investor concerned about final wealth or who solves a life cycle consumption problem. In contrast, in this paper we will focus on evaluating the performance of stocks and bond returns, based on empirical data for the US. There are several approaches to

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<sup>1</sup>For example, the popular book on investment advice by Siegel (1994) recommends buying and holding stocks for long periods, given that the risk of stocks decreases with the investment horizon. In addition, Malkiel (2000) states that “The longer an individual’s investment horizon, the more likely is that stocks will outperform bonds”.

<sup>2</sup>Chung et al. (2009) make a distinction between time series diversification and cross sectional diversification. The former kind of diversification means that investors should reduce the holding of risky assets as they become older. Cross sectional diversification means that an older person should hold a smaller percentage of his wealth in risky assets than a younger person. Our paper is related with cross sectional diversification.

examine empirically the question of whether stocks should be preferred over bonds in the long run. One approach consists of directly calculate the terminal wealth distributions for various portfolios with different asset allocations, and to evaluate the expected utility for each portfolio. The drawback of this approach is that it requires one to assume a specific utility function, hence no general conclusions can be reached. Another possible approach is to employ the Markowitz (1952) mean variance analysis.<sup>3</sup> For example, Levy and Spector (1996) and Hansson and Persson (2000) use this method to find that the optimal allocation for stocks is significantly larger for long investment horizons than for a one-year horizon. The problem of using a mean variance approach is that it assumes that the investor preferences depend only on the mean and variance of portfolio returns over a single period. A more general approach is to employ a test for stochastic dominance. Stochastic dominance tests have been proposed by Mc Fadden (1989) and extended by Linton et. al (2005). This approach has the advantage of imposing less restrictive assumptions about the form of the investor utility function and hence it provides criteria for entire preference classes. Furthermore, this approach can be applied whether the returns distributions are normal or not.

One conclusion from previous research that employs dominance criteria is that stochastic dominance does not provide evidence that stocks dominate bonds as the investment horizon lengthens (Hodges and Yoder, 1996; Strong and Taylor, 2001). The standard stochastic dominance test is based on the assumption that stock and bond returns are independent and identically distributed. However, empirical evidence suggests that the assumption of iid stocks returns is not supported by the data. In particular, Campbell (1987) and Fama and French (1988b) show that there is strong evidence on the predictability of stock returns, which in turn implies that the optimal investment strategies are horizon dependent. Therefore, the time varying nature of stock returns creates a challenge in ranking alternative investments.

In this paper, we use a test for spatial dominance introduced by Park (2008) which is

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<sup>3</sup>For an empirical application of the expected utility and the mean variance approaches, see Thorley (1995).

suitable for comparing alternative investments when their distributions are time-varying. In particular, we test for dominance between the cumulative returns series of stocks and bonds at different investment horizons from one to ten years. Spatial dominance is a generalization of the concept of stochastic dominance to compare the performance of two assets over a given time interval. In other words, while the concept of stochastic dominance is static and it is only useful to compare two distributions at a fixed time, spatial dominance is useful to compare two distributions over a period of time. Roughly speaking, we say that one distribution spatially dominates another distribution when it gives a higher level of utility over a given period of time. Our analysis assumes pairwise comparisons between stock and bond portfolios in order to focus on the effect that the holding period has on the investor's preferences for stocks versus bonds.<sup>4</sup>

Our approach has several advantages over existing approaches to evaluate the performance between alternative investments. First, our methodology allows us to compare the entire distributions of two investments instead of just the mean or median returns used in most conventional studies. Second, the approach followed in this paper relaxes the parametric assumptions about preferences that are considered in other papers. Only a few restrictions on the form of utility function (i.e., nonsatiation, risk aversion and time separable preferences) are imposed. This is particularly important for financial institutions that represent the interests of numerous individuals with presumably different preferences. Third, the approach is valid for the nonstationary diffusion processes commonly used in finance. This is an important advantage of our approach, since the literature finds that asset prices tend to be nonstationary. Finally, the test employs information from the entire path of the asset price instead of using only the asset values at two fixed points in time.

The data for this study are daily U.S. stock and bond returns obtained from Datastream. The study period is from 1965 to 2008. The variable stock price refers to the

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<sup>4</sup>Recently, Post (2003) and Linton, Post and Whang (2005) have extended the standard pairwise stochastic dominance to compare a given portfolio with all possible portfolios constructed from a set of financial assets. This concept might be useful in our analysis, but we do not pursue that direction in this paper.

S&P 500 including dividends. Bond returns are based on the 10 year treasury bond, which we take as representative of the US bond market.<sup>5</sup>

The empirical results suggest that for investment horizons of two years or less, bonds second order spatially dominate stocks, which means that risk averse investors obtain higher levels of utility by investing in bonds. For horizons of nine years or more, stocks first order spatially dominate bonds. We also compare diversified portfolios of stocks and bonds. Overall, the results are consistent with the common advice that stocks should be preferred for long term investors.

This paper is organized as follows. The next section presents the econometric methodology. Section III discusses the test for spatial dominance. Section IV analyzes the empirical results. Concluding remarks are presented in Section V.

## 2 Econometric Methodology

The spatial dominance test used in this paper to compare the distributions of stocks and bond returns is based on spatial analysis (Park, 2008). Spatial analysis is based on the study of the distribution function of nonstationary time series. This methodology is designed for nonstationary time series, but the theory is also valid for stationary time series.

The spatial analysis consists of the study of a time series along the spatial axis rather than the time axis. Figure 1 is useful to explain the intuition behind spatial analysis. Usually we plot the data on the  $xy$  plane where  $x$  represents the time axis and  $y$  represents the space. For example, the left panel of Figure 1 shows the total return index for the S&P 500. However, this representation is meaningful only under the assumption of stationarity, as we can interpret these readings as repeated realizations from a common distribution. In contrast, for nonstationary data this representation is not appropriate since the distribution changes over time. Clearly, the data for stock

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<sup>5</sup>Another popular bond for long term investors is the 30-year Treasury bond. However, this bond was suspended by the U.S. Federal government for a four year period starting from February 18, 2002 to February 9, 2006.

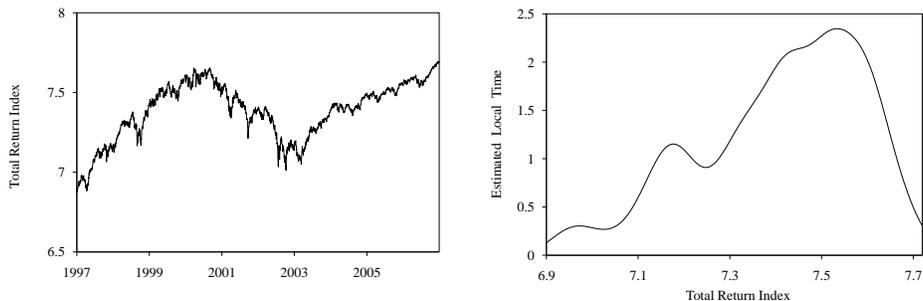


Figure 1: Spatial Analysis

prices are nonstationary. For this case, it is useful to read the data along the spatial axis. This is in particular useful for series that take repeated values over a certain range. The idea of spatial analysis is to calculate the frequency for each point on the spatial axis (right panel of Figure 1), that is, the local time of the process, which will be defined later and can be interpreted as a distribution function. The statistical properties of this distribution function are the main object of study in spatial analysis.

## 2.1 Preliminaries on Spatial Analysis

In order to explain the test for spatial dominance, it is necessary to introduce some important definitions. Let

$$X = (X)_t, t \in [0, T]. \quad (1)$$

be a stochastic process. The local time, represented as  $\ell(T, x)$ , is defined as the frequency at which the process visits the spatial point  $x$  up to time  $T$ . Notice that the local time itself is a stochastic process. It has two parameters, the time parameter  $T$  and the spatial parameter  $x$ . If the local time of a process is continuous, then we may deduce that,

$$\ell(T, x) = \lim_{\varepsilon \rightarrow 0} \frac{1}{2\varepsilon} \int_0^T 1\{|X_t - x| < \varepsilon\} dt. \quad (2)$$

Therefore, we may interpret the local time of a process as a density function over a given time interval.<sup>6</sup> The corresponding distribution function called integrated local time is defined as:

$$L(T, x) = \int_{-\infty}^x \ell(T, y) dy = \int_0^T 1\{X_t \leq x\} dt. \quad (3)$$

The local time is known to be well defined for a broad class of stochastic processes. Notice that the local time itself is a stochastic process and random. Taking the expectation of this random variable, we can define the spatial density function as:

$$\lambda(T, x) = E\ell(T, x) = \lim_{\varepsilon \rightarrow 0} \frac{1}{2\varepsilon} \int_0^T P\{|X_t - x| < \varepsilon\} dt. \quad (4)$$

The corresponding spatial distribution function is defined as:

$$\Lambda(T, x) = EL(T, x) = \int_0^T P\{X_t \leq x\} dt. \quad (5)$$

Thus, the spatial distribution function  $\Lambda(T, x)$  can be regarded as the distribution function of the values of  $X$ , which is nonstationary and time-varying, aggregated over time  $[0, T]$ .<sup>7</sup>

The spatial distribution is useful to analyze dynamic decision problems that involve utility maximization. Consider a continuous utility function  $u$  that depends on the value of the stochastic process  $X$ . By occupation times formula (see lemma 2.1 in Park

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<sup>6</sup>To understand this definition, recall that, for a density function  $f(x)$ ,

$$f(x) = \frac{dF(x)}{dx} = \frac{dP(X \leq x)}{dx} = \lim_{\varepsilon \rightarrow 0} \frac{1}{2\varepsilon} P\{|X_t - x| < \varepsilon\}.$$

<sup>7</sup>If the underlying process  $X$  is stationary, for each  $x$ ,  $P\{X_t \leq x\} = \Pi(x)$  is time invariant and identical for all  $t \in [0, T]$ . Therefore,  $X$  will have a time invariant continuous density function  $\Pi(x) = \frac{\Lambda(T, x)}{T}$ . In the spatial analysis used here,  $X$  is allowed to be a nonstationary stochastic process with time varying distribution. Park (2008) derives the asymptotics for processes with nonstationary increments and Markov processes, which include most models used in financial empirical applications.

(2008)), we may deduce that:

$$E \int_0^T u(X_t) dt = \int_{-\infty}^{\infty} u(x) \lambda(T, x) dx. \quad (6)$$

The equation above implies that, for any given utility function, the sum of expected future utilities generated by a stochastic process over a period of time is determined by its spatial distribution.

Since we are interested in the sum of expected future utilities, we might consider a discount rate  $r$  for the level of utility. In this case, the discounted local time would be defined as:

$$\ell^r(T, x) = \int_0^T e^{-rt} \ell(dt, x).$$

The corresponding discounted integrated local time can be defined as:

$$L^r(T, x) = \int_{-\infty}^x e^{-rt} \ell(T, x) = \int_0^T e^{-rt} \mathbf{1}\{X_t \leq x\} dt.$$

Similarly, the discounted spatial density can be defined as:

$$\lambda^r(T, x) = E \ell^r(T, x) = \int_0^T e^{-rt} \lambda(dt, x).$$

The discounted spatial distribution is given by:

$$\Lambda^r(T, x) = E L^r(T, x) = \int_0^T e^{-rt} P\{X_t \leq x\} dt.$$

As it will be discussed later, the the discounted spatial distribution will be used to test for spatial dominance in a similar way as the usual distribution for stationary series is used to test for stochastic dominance.

We can show that the sum of discounted expected utilities is determined by its

discounted spatial density:

$$E \int_0^T e^{-rt} u(X_t) dt = \int_{-\infty}^{\infty} e^{-rt} u(x) \lambda^r(T, x) dx. \quad (7)$$

The equation above will be used later when we present the definition of spatial dominance.

## 2.2 Spatial Dominance

The usual approach to compare two distribution functions is to employ the concept of stochastic dominance. More specifically, if we have two stationary stochastic processes,  $X$  and  $Y$  with cumulative distribution functions  $\Pi^X$  and  $\Pi^Y$ , then we say that  $X$  first stochastically dominates  $Y$  if,

$$\Pi^X(x) \leq \Pi^Y(x) \quad (8)$$

for all  $x \in \mathbb{R}$  with strict inequality for some  $x$ . This is equivalent to:

$$Eu(X_t) \geq Eu(Y_t) \quad (9)$$

for every utility function  $u$  such that  $u'(x) > 0$ .<sup>8</sup> In other words, the process  $X$  stochastically dominates the process  $Y$  if and only if it yields a higher level of utility for any non decreasing utility function. Therefore, the notion of stochastic dominance is static and it is restricted to the study of stationary time series.

In this paper, the concept of stochastic dominance is generalized for dynamic settings, by introducing the notion of spatial dominance. Spatial dominance can be applied to compare the distribution function of two stochastic processes over a period of time. Suppose we have two nonstationary stochastic processes,  $X$  and  $Y$  defined over the same time interval with corresponding spatial distributions  $\Lambda^{r,X}$  and  $\Lambda^{r,Y}$ . Then, we

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<sup>8</sup>In what follows,  $u \in U$  will denote a set of admissible utility functions, where  $U$  is the class of all non decreasing utility functions which are assumed to have finite values for any finite value of  $x$ .

say that the stochastic process  $X$  first order spatially dominates the stochastic process  $Y$  if and only if,

$$\Lambda^{r,X}(T, x) \leq \Lambda^{r,Y}(T, x). \quad (10)$$

for all  $x \in \mathbb{R}$  with strict inequality for some  $x$ . This definition holds if and only if,

$$E \int_0^T e^{-rt} u(X_t) dt \geq E \int_0^T e^{-rt} u(Y_t) dt. \quad (11)$$

for any non decreasing utility function  $u$ , Equivalently,

$$\int_{-\infty}^{\infty} u(x)^X \lambda^r(T, x) dx \geq \int_{-\infty}^{\infty} u(x)^Y \lambda^r(T, x) dx. \quad (12)$$

for every utility function  $u(x)$  such that  $u'(x) > 0$ . This means that the stochastic process  $X$  provides at least the same level of expected utility than the stochastic process  $Y$  over a given period of time. This result is showed in Park (2008).

Several orders of spatial dominance can be defined, according to certain restrictions on the shape of the utility function. For the first four orders of spatial dominance, these restrictions consist of non satiation, risk aversion, preference for positive skewness and aversion to kurtosis, respectively (Levy, 2006). In our empirical application, we will focus on the first and second order spatial dominances.

The integrated local time of order  $s \geq 2$  can be defined as:

$$L^{r,X,s}(T, x) = \int_{-\infty}^x L^{r,X,s-1}(T, x) dz. \quad (13)$$

A stochastic process  $X$  spatially dominates  $Y$  at order  $s \geq 2$  if,

$$\Lambda^{r,X,s}(T, x) \leq \Lambda^{r,Y,s}(T, x). \quad (14)$$

where,

$$\Lambda^{r,X,s}(T, x) = \int_{-\infty}^x \Lambda^{r,X,s-1}(T, x) dz. \quad (15)$$

It can be shown that the definition of spatial dominance occurs if and only if the stochastic process  $X$  provides a higher level of expected utility than the stochastic process  $Y$  for every utility function  $u(x)$  such that  $u'(x) > 0$  and  $u''(x) < 0$ .<sup>9</sup>

## 2.3 Motivation for Spatial Dominance

The concept of spatial dominance consists of comparing the sum of expected utilities  $E \int_0^T e^{-rt} u(X_t) dt$  and  $E \int_0^T e^{-rt} u(Y_t) dt$  over a given period of time, where  $X_t$  and  $Y_t$  are the cumulative returns at time  $t$ .<sup>10</sup> We assume that the investor's wealth depends only on financial income. In reality, households derive income in the form of wages. For example, Jagannathan and Kocherlakota (1996) show that uncertainty over wage income can affect the investment proportions in stocks as people age. Viceira (2001), shows that the optimal allocation of stocks is larger for employed investors than for retired investors when labor income risk is uncorrelated with stock return risk. Only if labor income and stock return are sufficiently highly correlated, an employed investor will hold a lower allocation to stocks than a retired investor. We do not dispute the theoretical validity of the models that include labor income. However, it is also instructive to examine the case where the utility depends only on financial income.

Spatial dominance is based on buy and hold strategies. That is, an investor with an investment horizon of  $T$  years chooses an allocation at the beginning of the first year and does not touch his portfolio again until the  $T$  years are over. The investor is not allowed to rebalance his portfolio.<sup>11</sup> One possible motivation for this assumption is the existence of transaction costs (Liu and Loewenstein, 2002). In that paper, the presence of transaction costs together with a finite horizon imply a largely buy and

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<sup>9</sup>One difficulty of ranking two alternative strategies using spatial dominance relations is that their distributions often cross, implying that they are not comparable. However, the inability to infer a spatial dominance relation is also informative. Moreover, when first order dominance does not exist, we can find dominance relations using higher dominance orders such as the second order dominance which imposes additional restrictions on the form of utility function.

<sup>10</sup>Cumulative returns are defined as  $X_t = \sum_{\tau=1}^t r_\tau$ , where  $r_\tau$  is the daily return obtained at time  $\tau$ .

<sup>11</sup>Other studies such as Brennan et al. (1997), Campbell and Viceira (1999) and Jagannathan and Kocherlakota (1996) examine optimal portfolio choice as a function of the investment horizon under different assumptions such as rebalancing.

hold and horizon dependent investment strategy.<sup>12</sup> However, since transaction costs have decreased over time and we have two assets that are relatively liquid, it is worth to mention an alternative motivation for the buy and hold strategy based on the behavioral economics literature. In particular, Samuelson and Zeckhouser (1998) use survey results on retirement plans to show that individuals display a bias towards sticking with the status quo when choosing among alternatives. Moreover, Choi et al. (2002) and Agnew et al. (2003), find that investors tend to choose the “path of least resistance” by doing nothing to their asset allocations.

The spatial dominance employs information from the entire path of the value of the asset  $X_t$ . This is an appealing feature compared to the standard stochastic dominance which only depends on the value of the asset at two points in time,  $X_0$  and  $X_T$ . The standard stochastic dominance test ignores the important dynamics in between the end points. Therefore, the concept of spatial dominance allows to analyze the economic decision of an investor over a given period of time.

In our setup, utility is a function of the cumulative return at each point in time. We can think of this function as an indirect utility function, where the investor consumes a constant fraction of the price of the asset at each point in time. Another way to motivate this setup is a model in which the investor maximizes the expected utility of terminal wealth when the investment horizon is uncertain and follows an independent Poisson process with constant intensity (Merton, 1971). Ibarra (2009) extends the stochastic dominance test for situations that involve an uncertain time horizon.

The method of spatial dominance is valid to compare the time varying processes commonly used to model asset prices. The nonstationarity of asset prices is a widely accepted finding in the literature. For instance, Nelson and Plosser (1982), show empirically that the S&P 500 is a nonstationary process with no tendency to return to a trend line. In addition, the concept of spatial dominance is applicable to a wider range of economic variables since most economic and financial series are believed to

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<sup>12</sup>For example, Liu and Lowenstein (2002) find that, for investment horizons of three years or less, the optimal expected time to sale after a purchase in the presence of transaction costs is roughly equal to the investment horizon.

have time-varying distributions.

Since the asset price  $X_t$  is nonstationary, the distribution function of  $X_t$  for  $t \in [0, T]$  does not converge to the distribution function of a stationary random variable. For that reason, we cannot employ the standard stochastic dominance concept designed for stationary variables. Instead, this distribution converges to the local time distribution function. As it will be explained later, the spatial distribution employed in our paper will be estimated as an average of  $N$  observations of the local time distribution function.

## 2.4 Estimation Method

The estimation methods and the asymptotic theory for the spatial distribution are derived in Park (2008). The theory presented before is built for continuous time processes. In practice, we need a estimation method for data in discrete time. Suppose that we have discrete observations  $(X_{i\Delta})$  from a continuous stochastic process  $X$  on a time interval  $[0, T]$  where  $i = 1, 2, \dots, n$  and  $\Delta$  denotes the observation interval. The number of observations is given by  $n = T/\Delta$ . All the asymptotic theory assumes that  $n \rightarrow \infty$  via  $\Delta \rightarrow 0$  for a fixed  $T$ . Notice that, in contrast with the conventional approach, the theory is based on the infill asymptotics instead of the long span asymptotics that relies on  $T \rightarrow \infty$ . The infill asymptotics is more appropriate for the analysis, since the main focus of spatial analysis is the spatial distribution of a time series over a fixed time interval.

Under certain assumptions of continuity for the stochastic process, the integrated local time can be estimated as the frequency estimator of the spatial distribution,

$$\hat{L}(T, x) = \Delta \sum_{i=1}^n e^{-ri\Delta} \mathbf{1}\{X_{i\Delta} \leq x\}. \quad (16)$$

Park (2008) shows that the estimator above is consistent. For orders  $s > 1$  we have that,

$$\hat{L}^{X,r,s}(T, x) = \frac{\Delta}{(s-1)!} \sum_{i=1}^n e^{-ri\Delta} (x - X_{i\Delta})^{s-1} \mathbf{1}\{X_{i\Delta} \leq x\}. \quad (17)$$

To estimate the spatial distribution, we need to introduce a new process based on the original stochastic process. More precisely, a process with stationary increments is defined as:

$$X_t^k = X_{T(k-1)+t} - X_{T(k-1)} \quad (18)$$

for  $k = 1, 2, \dots, N$ . Roughly speaking, this stochastic process is defined in terms of the increment with respect to the first observation for each interval. The estimators for the spatial density and spatial distribution can be computed by taking the average of each of the  $N$  intervals:

$$\hat{\Lambda}_N^{r,s}(T, x) = \frac{1}{N} \sum_{k=1}^N \hat{L}_k^{r,s}(T, x). \quad (19)$$

The asymptotics for the estimators of the spatial density and the spatial distribution are developed in Park (2008). The framework requires very weak assumptions about the stochastic process. More specifically, the asymptotics are developed for two classes of models: processes with stationary increments and general Markov processes. These classes include most diffusion models that are used for the empirical research in finance and economics.

### 3 Testing for Spatial Dominance

The test for the null hypothesis that  $X$  first order spatially dominates  $Y$ , as defined in equation 10, can be written as:

$$H_0 : \delta(T) = \sup_{x \in \mathbb{R}} (\Lambda^{r,X}(T, x) - \Lambda^{r,Y}(T, x)) \leq 0. \quad (20)$$

against the alternative:

$$H_1 : \delta(T) > 0. \quad (21)$$

Under the null hypothesis, the spatial distribution of  $X$  is located to the right of the spatial distribution of  $Y$ , except at the lowest and highest values of the support,

where both distributions take the same value.

As proposed in the stochastic dominance literature (Mc Fadden, 1989), the Kolmogorov-Smirnov statistics are used to test for spatial dominance. The Kolmogorov-Smirnov statistic can be written as:

$$D_N(T) = \sqrt{N} \sup_{x \in R} (\hat{\Lambda}_N^{r,X}(T, x) - \hat{\Lambda}_N^{r,Y}(T, x)). \quad (22)$$

Park (2008) shows that assuming continuity and controlling for dependencies, then, under the null hypothesis,

$$D_N(T) \rightarrow_d \sup_{x \in R} (U^X(T, x) - U^Y(T, x)). \quad (23)$$

where  $(U^X(T, x), U^Y(T, x))'$  is a mean zero vector Gaussian process.<sup>13</sup>

If we are interested in testing for spatial dominance of order  $s > 1$ , then we need to replace  $\hat{\Lambda}_N^{r,X}(T, x)$  in equation 22 by  $\hat{\Lambda}_N^{r,X,s}(T, x)$  from equation 19.

Notice that the distribution of  $D_N$  depends upon the unknown probability law of the unknown stochastic processes  $X, Y$ . Thus, the asymptotic critical values cannot be tabulated. There are three alternatives to obtain the critical values: simulation, bootstrapping and subsampling. The results presented here are based on subsampling methods to obtain the critical values. Subsampling methods are well suited for financial data that typically exhibit dependencies such as conditional heteroskedasticity or stochastic volatility and serial correlations. The general theory for subsampling methods is explained in Politis, Romano and Wolf (1999). In the stochastic dominance literature, subsampling methods have been proposed by Linton, Massoumi and Whang (2005), who prove that subsampling provides consistent critical values under very weak conditions allowing for cross sectional dependency among the series and weak temporal dependency. They also provide simulation evidence on the sample performance of their statistics in a variety of sampling schemes.

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<sup>13</sup>Discussions about the statistical power of this test can be found in Park and Shintani (2008).

Let  $N_s$  denote the subsample size. Then, we will have  $N - N_s + 1$  overlapping subsamples. For each of these subsamples  $i$ , we calculate the test statistic for the spatial dominance test,  $D_{N_s,i}$ , where  $i = 1, \dots, N - N_s + 1$ . Then, we approximate the sampling distribution of  $D_N$  using the distribution of the values of  $D_{N_s,i}$ . Therefore, the critical value can be approximated as

$$g_{N_s,\alpha} = \inf_w \left( \frac{1}{N - N_s - 1} \sum_{i=1}^{N-N_s-1} 1 \{D_{N_s,i} \leq w\} \geq 1 - \alpha \right) \quad (24)$$

Thus, we reject the null hypothesis at the significance level  $\alpha$  if  $D_N > g_{N_s,\alpha}$ .

## 4 Empirical Results

This section applies the test of spatial dominance to a data set of daily returns for the S&P 500 index and the 10 year government bond from 1965 to 2008.<sup>14</sup> The descriptive statistics for horizons from 1 to 10 years are reported in Table 1. The cumulative returns are calculated using overlapping with a moving step of 1 month (i.e., 21 days). For all investment horizons, the mean and the standard deviation of stock returns are greater than those of bond returns. Stock returns are found to be negatively skewed, while bond returns are found to be positively skewed. Return distributions are leptokurtic only for short investment horizons. As documented in the literature, these distributions are found to be non-normal (the critical value for the Jarque Bera test at the 5% significance level is 5.99). Notice that the mean returns increase proportionally with the investment horizons, but the standard deviation increases less than proportionally with the investment horizon. When the horizon increases from one to ten years, the standard deviation for stock returns increases about three times. However, the standard deviation for bond returns increases about 7.5 times.

The result above seems to suggest that stocks become relatively more attractive as the investment horizon lengthens. As Barberis (2000) and Campbell and Viceira

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<sup>14</sup>Returns are expressed in nominal terms, given that the consumer price index is not available on a daily basis.

Table 1: Descriptive Statistics

Horizon	Mean	Std. dev.	Skewness	Kurtosis	Jarque-Bera
Stocks					
1	0.091	0.157	-0.690	3.809	57.021
2	0.192	0.219	-0.737	3.810	61.614
3	0.294	0.263	-0.483	3.336	22.260
4	0.393	0.303	-0.121	2.558	5.266
5	0.497	0.350	-0.042	2.188	13.461
6	0.603	0.377	-0.119	1.854	27.004
7	0.709	0.408	-0.149	1.729	32.740
8	0.823	0.435	-0.245	1.855	28.951
9	0.948	0.455	-0.362	1.982	28.345
10	1.082	0.459	-0.423	2.036	29.090
Bonds					
1	0.073	0.025	0.967	3.472	88.282
2	0.148	0.048	0.892	3.174	69.994
3	0.224	0.070	0.809	2.949	55.725
4	0.301	0.091	0.732	2.788	45.371
5	0.379	0.111	0.643	2.601	36.639
6	0.459	0.129	0.537	2.374	30.474
7	0.539	0.147	0.435	2.157	28.212
8	0.622	0.162	0.340	1.979	28.088
9	0.705	0.177	0.246	1.849	28.455
10	0.789	0.190	0.148	1.767	28.387

Note: The sample period is from 1/7/1965 to 1/6/2009.

(1999) show, this result can be explained from the mean reverting property of stock returns. There is related evidence that stock returns exhibit mean reversion. Fama and French (1988a) and Poterba and Summers (1988) demonstrate that the variance of stocks is reduced at longer horizons. If the investment opportunity set remains constant over time, the investor decision will not depend on the investment horizon (Samuelson, 1989, 1994). If, on the other hand, stock returns are mean reverting, the variance of cumulative returns decreases over long investment horizons.

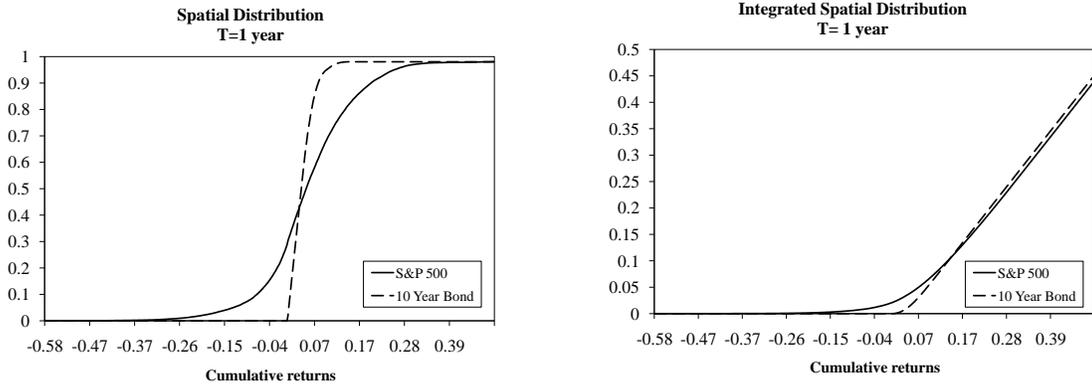


Figure 2: Estimated Spatial Distribution and Integrated Spatial Distribution for a 1 year Investment Horizon

#### 4.1 Spatial Dominance Results from undiversified stock/bond portfolios

Figure 2 plots the estimated discounted spatial distribution  $\hat{\Lambda}^r(T, x)$  and integrated discounted spatial distribution  $\hat{\Lambda}^{r,2}(T, x)$  of the two series for an investment horizon of one year, that is,  $T=1$ .<sup>15</sup> Following the standard macroeconomics literature (Kydland and Prescott, 1982), the annual discount rate  $r$  is set to 4%.<sup>16</sup> As can be seen, the distributions cross in both cases, suggesting no evidence of spatial dominance over this time period.

Figure 3 presents the case of a ten year horizon. The estimated spatial distribution and integrated spatial distribution for a ten year investment horizon suggest that the S&P 500 first and second order spatially dominate the 10-year government bond.<sup>17</sup>

The first order spatial dominance tests are reported in Table 2. For the FOSD test, the null hypothesis is that  $H_0 : \Lambda^{r,X}(T, x) \leq \Lambda^{r,Y}(T, x)$  for all  $x$ . The first column shows

<sup>15</sup>The support of the estimated distributions is based on the range of data of cumulative returns with 500 intermediate points. The results are not sensitive to the number of intermediate points for the estimation of the spatial distribution.

<sup>16</sup>For estimating the spatial distribution, we allow for overlapping with a moving step of one month. We have tried different moving steps and found similar results than those reported in the paper.

<sup>17</sup>The estimated spatial distribution appears to cross for very low values since returns on government bonds are non negative. However, as it will be discussed later, the test fails to reject the null hypothesis that stocks dominate bonds.

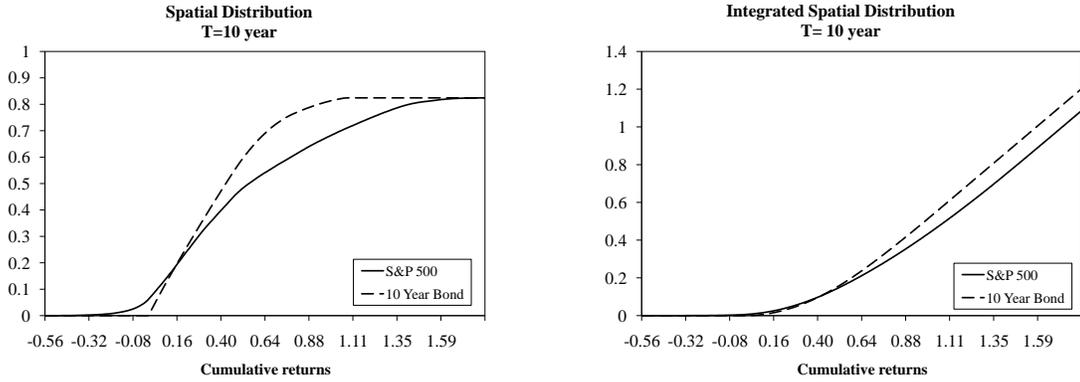


Figure 3: Estimated Spatial Distribution and Integrated Spatial Distribution for a 10 year Investment Horizon

the investment horizon (in years), while the test statistic is showed in the second column. The next column reports the subsample size which is based on the minimum volatility method. The last two columns report the critical value and the p-value respectively.

The sampling distribution of the test statistic is based on subsampling methods with overlapping subsamples. To obtain the critical values, we use the subsampling approach for sub-sample sizes ranging between  $N^4$  and  $N^7$ . For choosing the optimal subsample size, the minimum volatility method is employed, as suggested by Politis et al. (1999). This method consists of calculating the local standard deviation of the critical value and then selecting the subsample size that minimizes this volatility measure. The local standard deviation is based on the critical values in the range  $[N_s - b, N_s - b + 1, \dots, N_s + b]$ .<sup>18</sup> This method ensures that the critical values are relatively stable around the optimal subsample size.

For investment horizons of eight years or less, we reject the null hypothesis of first order spatial dominance at the 10% significance level. However, we cannot reject the null hypothesis of first order spatial dominance of stocks over bonds for horizons of 9 and 10 years. This result implies that a buy and hold investor with preferences characterized by nonsatiation will attain a higher expected utility by investing in S&P

<sup>18</sup>The results presented here are for  $b = 5$ . Sensitivity analysis for different values of  $b$  yield similar results.

Table 2: First Order Spatial Dominance Test

Horizon	KS	$N_s$	CV	PV
a) $H_0$ : S&P 500 FOSD Government Bond				
1	3.78	26.00	1.40	0.00
2	3.01	19.00	1.24	0.00
3	2.49	31.00	1.09	0.00
4	2.13	28.00	1.19	0.00
5	1.94	30.00	1.22	0.00
6	1.74	27.00	1.31	0.00
7	1.52	29.00	1.32	0.00
8	1.26	29.00	1.40	0.09
9	0.94	27.00	1.37	0.21
10	0.72	27.00	1.34	0.28
b) $H_0$ : Government Bond FOSD S&P 500				
1	3.79	19.00	1.65	0.00
2	3.35	33.00	1.31	0.00
3	2.99	18.00	1.56	0.00
4	2.68	31.00	1.48	0.00
5	2.41	23.00	1.63	0.00
6	2.18	23.00	1.62	0.00
7	1.94	24.00	1.64	0.00
8	1.92	29.00	1.59	0.00
9	1.90	26.00	1.62	0.00
10	1.88	27.00	1.60	0.00

<sup>b</sup> Note: The number of subsamples  $N_s$  is based on the minimum volatility method. The p values are based on critical values at the 5% level.

500 rather than Government Bond.<sup>19</sup>

The second order spatial dominance (SOSD) test is reported in Table 3. For the SOSD test, the null hypothesis is that  $H_0 : \Lambda^{r,X,2}(T, x) \leq \Lambda^{r,Y,2}(T, x)$  for all  $x$ . For investment horizons between two and eight years, we reject the null hypothesis of SOSD. This result implies that there are no spatial dominance relationships between the S&P 500 and the 10-year Treasury Bond at those investment horizons. However, for investment horizons of 1 and 2 years, the 10 year bond dominates the S&P 500 at the 10% significance level. This result implies that the buy and hold investor with monotonic preferences will obtain a higher level of expected utility by investing in bonds.<sup>20</sup>

These results are robust across different subsample sizes ( $N_s$ ). Figure 4 plots the p-value for the null hypothesis of spatial dominance, for investment horizons of one and ten years against subsample size ( $N_s$ ). The p-values support the results suggested by the estimated spatial distributions. For a one year investment horizon bonds second order spatially dominate stocks, while for a ten year investment horizon, the S&P 500 index FOSD the government bond.

Overall, our results suggest that equities dominate bonds for long investment horizons. Samuelson (1994) examines the risk of stocks at longer horizons, which might justify our empirical results. He finds that if returns are mean reverting, stocks will become less risky the longer the investment horizon is. Returns are negatively correlated so that volatility is reduced, because a positive or negative price movement tends to be followed by a price movement in the negative direction. Notice that Samuelson proves this result for an investor who optimally rebalances his portfolio at regular intervals, rather than the buy and hold investor that we consider here. Barberis (2000) finds that, assuming a buy and hold investment horizon with utility defined over terminal wealth, predictability in stock returns implies that long term investors allocate more to

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<sup>19</sup>Levy and Spector (1996) find results that are consistent with ours in a model where borrowing and lending are not allowed or when borrowing takes place at a higher rate than lending. Using data for annual returns from 1926 to 1990, the authors find that investors having a log utility function and facing a long term horizon should invest all wealth in stocks.

<sup>20</sup>Liu and Loewenstein (2002) find that in a model with transaction costs, a short term investor might optimally hold only bonds even when there is a positive risk premium.

Table 3: Second Order Spatial Dominance Test

Horizon	KS	$N_s$	CV	PV
a) $H_0$ : S&P 500 SOSD Government Bond				
1	0.33	29.00	0.15	0.00
2	0.37	30.00	0.32	0.02
3	0.38	30.00	0.45	0.13
4	0.37	30.00	0.57	0.24
5	0.36	31.00	0.69	0.24
6	0.34	29.00	0.78	0.26
7	0.32	29.00	0.87	0.26
8	0.27	29.00	0.99	0.28
9	0.18	27.00	1.10	0.31
10	0.11	27.00	1.21	0.41
b) $H_0$ : Government Bond SOSD S&P 500				
1	0.15	15.00	0.27	0.14
2	0.32	19.00	0.49	0.10
3	0.46	19.00	0.71	0.10
4	0.57	20.00	0.87	0.11
5	0.67	22.00	0.99	0.13
6	0.75	22.00	1.08	0.13
7	0.83	29.00	1.14	0.16
8	0.97	28.00	1.17	0.14
9	1.17	27.00	1.16	0.04
10	1.36	27.00	1.12	0.00

<sup>c</sup> Note: The number of subsamples  $N_s$  is based on the minimum volatility method. The p values are based on critical values at the 5% level.

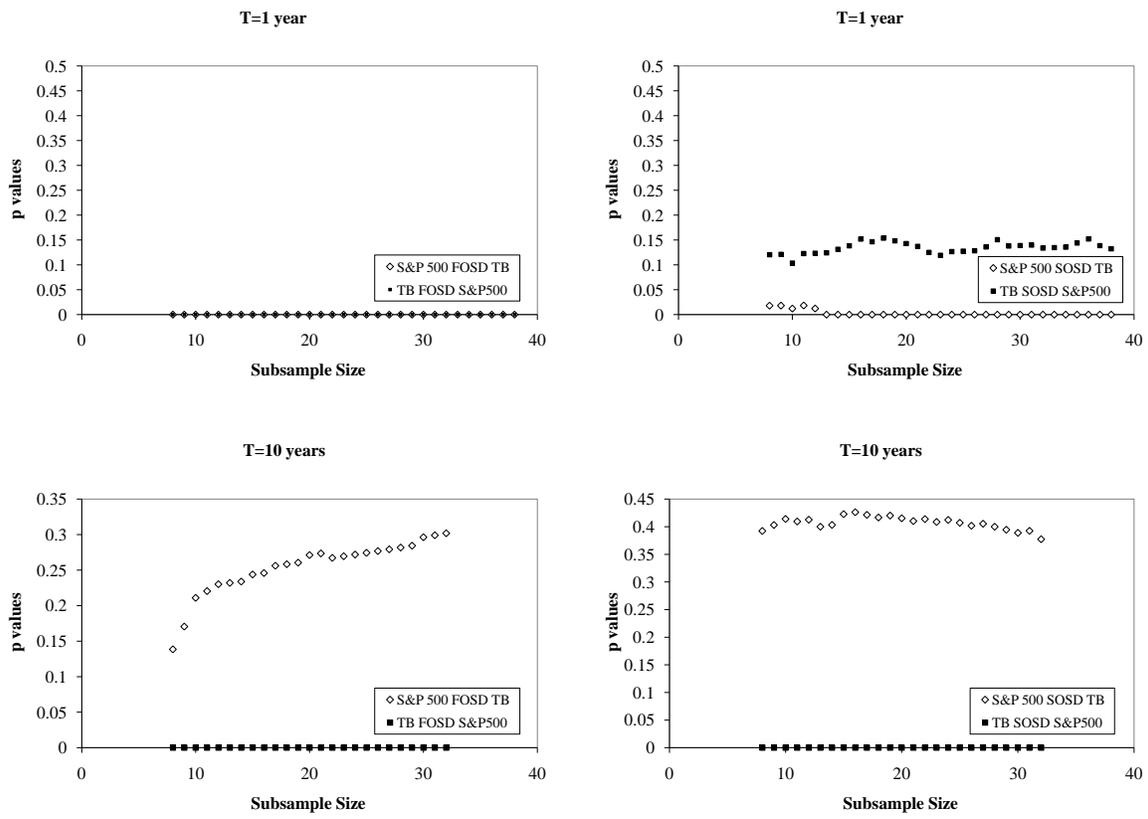


Figure 4: P values for Spatial Dominance Test

equities than short term investors.

## 4.2 Spatial Dominance Results for diversified stock/bond portfolios

We have presented results for spatial dominance between portfolios consisting of 100% stocks and 100% bonds and found that stocks dominate bonds in long horizons. However, the advice of practitioners is to allocate a higher proportion of stocks for longer investment horizon. In this subsection, we will test for spatial dominance between diversified stock and bond portfolios.

We consider 11 portfolios consisting of 100% bonds, 90% bonds and 10% stocks, ..., and 100% stocks. Table 4 shows the results for investment horizons of  $T = 1, 5,$  and 8 years. Percentage values at the left and top of the table indicate the proportion of stocks held in the portfolio. An entry in the table of 1 (2) indicates that the portfolio in the left dominates the portfolio across the top at the 5% significance level at the horizon indicated above in first (second) order sense. An entry of 0 indicates no dominance.<sup>21</sup>

The results of Table 4 are consistent with practitioners advice. In general, for a short investment horizon of 1 year, portfolios with higher proportion of bonds second order spatially dominate portfolios with lower proportion of bonds (the entries above the diagonals are 2 in almost all cases). The implication is that an investor near to retirement will obtain a higher level of expected utility by allocating a greater proportion of bonds in the portfolio. In contrast, for investment horizons of 5 and 10 years, the portfolios with higher proportion of bonds never dominate the portfolios with smaller proportion of bonds (the entries above the diagonal are zeros in all cases). For investment horizons of five years, the portfolios with 0% stocks is dominated by the 40% stocks, and the portfolio with 10% stocks is dominated by the portfolios with 20% and 30% stocks. Therefore, only the portfolios that consist of 20% or more stocks are efficient in the sense of first order spatial dominance. For investment horizons of 8

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<sup>21</sup>We report the results for a subsample size of  $N^8$ . The moving step for the overlapping periods used to estimate the spatial distribution is 3 months.

Table 4: Spatial Dominance of diversified stock/bond portfolios

	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
1 Year horizon											
0%	-	0	0	2	2	2	2	2	2	2	2
10%	1	-	2	2	2	2	2	2	2	2	2
20%	0	0	-	2	2	2	2	2	2	2	2
30%	0	0	0	-	2	2	2	2	2	2	2
40%	0	0	0	0	-	2	2	2	2	2	2
50%	0	0	0	0	0	-	2	2	2	2	2
60%	0	0	0	0	0	0	-	2	2	2	2
70%	0	0	0	0	0	0	0	-	2	2	2
80%	0	0	0	0	0	0	0	0	-	2	2
90%	0	0	0	0	0	0	0	0	0	-	2
100%	0	0	0	0	0	0	0	0	0	0	-
5 Year horizon											
0%	-	0	0	0	0	0	0	0	0	0	0
10%	0	-	0	0	0	0	0	0	0	0	0
20%	0	1	-	0	0	0	0	0	0	0	0
30%	0	1	0	-	0	0	0	0	0	0	0
40%	1	0	0	0	-	0	0	0	0	0	0
50%	0	0	0	0	0	-	0	0	0	0	0
60%	0	0	0	0	0	0	-	0	0	0	0
70%	0	0	0	0	0	0	0	-	0	0	0
80%	0	0	0	0	0	0	0	0	-	0	0
90%	0	0	0	0	0	0	0	0	0	-	0
100%	0	0	0	0	0	0	0	0	0	0	-
8 Year horizon											
0%	-	0	0	0	0	0	0	0	0	0	0
10%	0	-	0	0	0	0	0	0	0	0	0
20%	0	0	-	0	0	0	0	0	0	0	0
30%	0	0	1	-	0	0	0	0	0	0	0
40%	0	1	1	1	-	0	0	0	0	0	0
50%	0	1	1	1	0	-	0	0	0	0	0
60%	1	1	0	0	0	0	-	0	0	0	0
70%	1	1	0	0	0	0	0	-	0	0	0
80%	1	0	0	0	0	0	0	0	-	0	0
90%	1	0	0	0	0	0	0	0	0	-	0
100%	1	0	0	0	0	0	0	0	0	0	-

Note: Percentage values indicate proportion of stocks held in a portfolio. An entry of 1 (2) in the table means that the portfolio in the left spatially dominates the portfolio across the top in first (second) order sense. All the tests use a 5% significance level.

years, the portfolios with 0%, 10%, 20% and 30% stocks are dominated by portfolios with a higher proportions of stocks. Therefore, only the portfolios with more than 40% stocks are efficient in the sense of first order spatial dominance at the 8-year investment horizon. Thus, our results are consistent with the practices of life cycle fund managers of allocating a greater proportion of stocks in their portfolios.<sup>22</sup>

## 5 Concluding Remarks

This paper employs a spatial dominance test to compare the distributions of stocks and bonds for different investment horizons. There are several advantages of using the concept of spatial dominance. First, we are able to rank investments without imposing restrictive assumptions on the form of the utility function as most theoretical studies. Second, we compare the entire distribution of returns rather than only the mean or the median return as used in the traditional studies. Third, this methodology is valid for either stationary or nonstationary time series. This is an important advantage considering the time varying nature of the returns distributions found in the literature.

Using a daily data set from 1965-2008, it is found that the spatial dominance relations between S&P 500 and bonds depend on the investment horizon. First, for investment horizons of two year or less, bonds second order spatially dominate stocks, which means that risk averse investors obtain higher levels of utility by investing in bonds. In contrast, for investment horizons longer than eight years, stocks first order spatially dominate bonds. An explanation of this result is the empirical evidence on mean reversion of stock returns (Poterba and Summers, 1988). This makes stocks less risky to long horizon investors, thus making stocks dominate bonds at long investment horizons.

The spatial dominance results for comparing diversified portfolios show that for investment horizons of one year, a portfolio with a higher proportion of bonds domi-

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<sup>22</sup>It is important to note that the results are for a p-value of 5%. If higher p-values are used, some dominances might disappear, but the conclusions will still be consistent with practitioner's advice.

nates a portfolio with a lower proportion of bonds. On the other hand, for investment horizons of five years, portfolios with a higher proportion of stocks dominate portfolios with a lower proportion of stocks. For a five year investment horizon, only the portfolios consisting of 20% stocks or more are efficient, while for an eight year investment horizon only the portfolios with 40% stocks or more are efficient in the sense of spatial dominance.

To be conservative, our results are applicable to the sample period, the type of buy and hold investment strategy and the preferences considered in this paper. The spatial analysis is based on the framework of expected utility, and it assumes non-satiation, risk aversion and time-separable preferences. Other types of preferences that have appeared in the literature to explain important puzzles on finance, such as habit formation (Constantinides, 1990), relative consumption (Abel, 1990), recursive utility (Epstein and Zin, 1991), prospect theory (Kahneman and Tversky, 1979) and models with labor income (Viceira, 2001), are not included in our framework. Moreover, portfolio rebalancing strategies that could potentially introduce horizon effects (Merton, 1971) are not considered in this paper. In spite of this, the spatial dominance results provide a useful evaluation of the relative performance of stocks and bonds based on the empirical data, and it sheds light about the role of the investment horizon to study optimal allocations.

The empirical results can be extended in a number of ways. The assumption of expected utility theory can be relaxed by considering an extension of the prospect dominance test of Linton et al. (2005) for nonstationary series. It would also be interesting to extend the spatial dominance test to compare all possible portfolios available to investors. We leave those extensions for further research.

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