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ON THE ECONOMICS OF EFFLUENT CHARGES

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and  
Alfred Endres

Serie B — Nr. 17

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Serie A: Volkswirtschaftliche Beiträge

Serie B: Finanzwissenschaftliche Arbeitspapiere

Serie C: Betriebswirtschaftliche Beiträge (in Vorbereitung)

# ON THE ECONOMICS OF EFFLUENT CHARGES

Michael Braulke and Alfred Endres

## I. INTRODUCTION

In this paper the effects of effluent charges on the equilibrium effluent levels of a perfectly competitive industry as well as of individual, perfectly competitive polluting firms will be analyzed.

It will be shown that at the industry level, raising an effluent charge is both in the very short run and in the long run an effective means of controlling effluents. This is certainly not a surprising result. Its proof, however, will be shown to be a little more complicated than usually suggested in the literature if we deviate from the standard 'textbook assumption' of effluents being strictly proportional to output.

Moreover, the result is far from obvious for it turns out to be quite compatible with the paradoxical situation that the industry will initially react with an increase in emission. It will also be shown that, other than by brute force of assumption, it cannot be ruled out that individual firms which survive the adjustment process to the new industry equilibrium, may eventually emit a higher amount of effluents than they did originally.

## II. THE BASIC ASSUMPTIONS

Consider a perfectly competitive industry consisting of numerous firms which produce an output  $x$  jointly with an effluent  $e$  according to a simple technology that can be described by

$$(1) \quad x = x(L_x) \quad \text{with } \partial x / \partial L_x > 0$$

and

$$(2) \quad e = x \cdot y(L_y, x) \quad \text{with } \partial y / \partial L_y < 0 \quad ,$$

where  $L$  is the only input which can be devoted to the production of output ( $L_x$ ) or to activities which abate the effluent ( $L_y$ ). The two natural specifications listed in (1) and (2) aside, we will, in order to stay as general as possible, impose no further restrictions other than

- A1 the technology (1), (2) is compatible with profit maximization.

We will thus in particular allow the partial derivatives  $\partial y / \partial x$  and  $\partial^2 y / \partial L_y \partial x$  of the effluent per unit of output production function  $y(L_y, x)$  to have any sign they wish. Most readers may be tempted to assume that the amount of effluent per unit of output should not fall with the level of output implying the signature  $\partial y / \partial x \geq 0$ . We would like to point out, however, that even though  $\partial y / \partial x < 0$  cannot hold for all levels of output it may well do so over a relevant range of  $x$  and should consequently not be ruled out a priori. As to the other partial derivative mentioned,  $\partial^2 y / \partial L_y \partial x$ , which describes the effect of a rise in output on the marginal productivity of abatement activities, there is even less reason to insist beforehand on a particular sign. It is these two derivatives which will cause some problems in the comparative static analysis.

Since the remaining basic assumptions to be used below are obvious ones, we will simply state them without further comment:

- A2 Individual firms are pricetaking profit maximizers.  
 A3 The long run industry equilibrium is characterized by zero profits.  
 A4 The industry faces a downward sloping demand curve  $p = p(X)$  with  $\partial p / \partial X < 0$ .

A5 The industry is too small to affect the price of the input.

The comparative static framework we will have to work with is thus that of a firm which, given the product price  $p$ , the wage rate  $w$ , and the effluent charge  $t$ , solves the unconstrained problem

$$(3) \quad \max_{L_x, L_y} Q = px(L_x) - w(L_x + L_y) - tx(L_x)y(L_y, x(L_x)) \quad ,$$

which gives rise to the first order conditions

$$(4) \quad \frac{\partial Q}{\partial L_x} = \left[ p - t \left( y + x \frac{\partial y}{\partial x} \right) \right] \frac{\partial x}{\partial L_x} - w = 0$$

$$(5) \quad \frac{\partial Q}{\partial L_y} = -w - tx \frac{\partial y}{\partial L_y} = 0 \quad .$$

Denoting the solution to (3) by  $L_x^*$  and  $L_y^*$  and differentiating (4) and (5) totally leads then directly to the basic comparative static equation

$$(6) \quad \begin{bmatrix} \frac{\partial^2 Q}{\partial L_x^2} & \frac{\partial^2 Q}{\partial L_x \partial L_y} \\ \frac{\partial^2 Q}{\partial L_y \partial L_x} & \frac{\partial^2 Q}{\partial L_y^2} \end{bmatrix} \begin{bmatrix} dL_x^* \\ dL_y^* \end{bmatrix} + \begin{bmatrix} \frac{\partial x}{\partial L_x} dp - \left( y + x \frac{\partial y}{\partial x} \right) \frac{\partial x}{\partial L_x} dt - dw \\ -x \frac{\partial y}{\partial L_y} dt - dw \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

which either has or doesn't have a unique solution  $dL_x^*$ ,  $dL_y^*$  depending on whether the Hessian matrix of  $Q$  on the LHS of (6) is regular or not. If it were not we could stop right here and consequently we are forced to assume ad hoc that this Hessian matrix is regular at  $L_x^*$ ,  $L_y^*$ . In view of A1 which requires that the second-order necessary conditions are met, this constitutes only a minor addition to the restrictiveness of our assumptions as it is tantamount to requiring<sup>1</sup>

A6 The maximization problem (3) meets the second-order sufficient conditions.

Writing

$$(7) \quad H = \begin{bmatrix} h_{11} & h_{12} \\ h_{12} & h_{22} \end{bmatrix} \quad \text{with} \quad h_{11} = \left[ p - t \left( y + x \frac{\partial y}{\partial x} \right) \right] \frac{\partial^2 x}{\partial L_x^2} - t \left( x \frac{\partial^2 y}{\partial x^2} + 2 \frac{\partial y}{\partial x} \right) \left( \frac{\partial x}{\partial L_x} \right)^2 \quad ,$$

$$h_{12} = -t \left( \frac{\partial y}{\partial L_y} + x \frac{\partial^2 y}{\partial x \partial L_y} \right) \frac{\partial x}{\partial L_x} \quad \text{and} \quad h_{22} = -tx \frac{\partial^2 y}{\partial L_y^2}$$

for the Hessian matrix of  $Q$  evaluated at  $L_x^*$ ,  $L_y^*$ , A6 thus requires the second differential of the profit function to be negative for any non-vanishing variation of inputs around  $L_x^*$ ,  $L_y^*$ , i.e.

$$(8) \quad d^2Q^* = (dL_x, dL_y)H(dL_x, dL_y)' < 0 \quad ,$$

whereas A1 would require merely  $d^2Q^* \leq 0$ . As is well known, (8) implies immediately

$$(9) \quad h_{11} < 0, \quad h_{22} < 0 \quad \text{and} \quad |H| > 0$$

which in turn allows us to conclude  $\partial^2 y / \partial L_y^2 > 0$  but says nothing about the signs of  $\partial y / \partial x$  and  $\partial^2 y / \partial x \partial L_y$  in particular. With the Hessian matrix now regular by A6, we can solve (6) for  $dL_x^*$  and  $dL_y^*$  to obtain explicitly

$$(10) \quad \begin{bmatrix} dL_x^* \\ dL_y^* \end{bmatrix} = \frac{1}{|H|} \begin{bmatrix} h_{22} & -h_{12} \\ -h_{12} & h_{11} \end{bmatrix} \begin{bmatrix} -\frac{\partial x}{\partial L_x} dp + \left( y + x \frac{\partial y}{\partial x} \right) \frac{\partial x}{\partial L_x} dt + dw \\ x \frac{\partial y}{\partial L_y} dt + dw \end{bmatrix}$$

as the basis for the comparative static analysis in the following section.

### III. THE VERY SHORT- AND THE LONG-RUN REACTIONS TO AN INCREASED EFFLUENT CHARGE

As the very short run we will denote the situation in which the firms already adjusted their input decisions to the rise in the charge but the market has not yet had the chance to respond to these reactions. The very short run is thus characterized by the constellation ( $dt > 0$ ,  $dp = 0$ ,  $dw = 0$ ). The long run, on the contrary, is meant to be the situation in which the industry has reached its zero profit equilibrium again. Now, starting from a long run industry equilibrium, the initial increase in the charge must have pushed all firms into a loss for  $dQ^*/dt = -xy$  by the envelope theorem. Since wages will not change by A5 it must be a price change which restores the zero profit equilibrium. More specifically, since  $dQ^*/dp = x$ , the price must rise by exactly  $dp = ydt$  if it is to neutralize the increase in the charge by



dt. Not interested at this point how the process may look that leads to the new industry equilibrium, we thus conclude simply that the long run is characterized by (dt>0, dp=ydt, dw=0).

Consider now an individual firm in the very short run. With dp=dw=0 and dt>0, we have immediately from (10)

$$(11) \quad dL_x^* = \left[ h_{22} \left( y + x \frac{\partial y}{\partial x} \right) \frac{\partial x}{\partial L_x} - h_{12} x \frac{\partial y}{\partial L_y} \right] \frac{dt}{|H|}$$

$$(12) \quad dL_y^* = \left[ -h_{12} \left( y + x \frac{\partial y}{\partial x} \right) \frac{\partial x}{\partial L_x} + h_{11} x \frac{\partial y}{\partial L_y} \right] \frac{dt}{|H|}$$

and differentiating (2) totally and substituting these results we find

$$(13) \quad de^* = \left( y + x \frac{\partial y}{\partial x} \right) \frac{\partial x}{\partial L_x} dL_x^* + x \frac{\partial y}{\partial L_y} dL_y^*$$

$$= \left[ h_{11} \left( x \frac{\partial y}{\partial L_y} \right)^2 - 2h_{12} x \frac{\partial y}{\partial L_y} \left( y + x \frac{\partial y}{\partial x} \right) \frac{\partial x}{\partial L_x} + h_{22} \left( \left( y + x \frac{\partial y}{\partial x} \right) \frac{\partial x}{\partial L_x} \right)^2 \right] \frac{dt}{|H|}$$

$$= \left( -x \frac{\partial y}{\partial L_y}, \left( y + x \frac{\partial y}{\partial x} \right) \frac{\partial x}{\partial L_x} \right) H \left( -x \frac{\partial y}{\partial L_y}, \left( y + x \frac{\partial y}{\partial x} \right) \frac{\partial x}{\partial L_x} \right)' \frac{dt}{|H|} < 0$$

where the sign follows directly from (8) in conjunction with (9) since the quadratic form is nothing but a second differential of the profit function. The interesting point to note then is that even though there is in view of (11) and (12) no way to make definite statements as to where the output will move or whether the abatement activities will be stepped up or not we can be sure that the individual firm will, in the very short run, reduce its effluent output. With the number of firms in the industry not yet changed this means that in the very short run also the entire industry will reduce its emission.

Consider then an individual firm that has survived a perhaps painful adjustment process, in the new long run equilibrium. With the long run characterized by dw=0 and dt>0, dp=ydt, the total adjustments in inputs amount to

$$(14) \quad dL_x^* = \left( h_{22} x \frac{\partial y}{\partial x} \frac{\partial x}{\partial L_x} - h_{12} x \frac{\partial y}{\partial L_y} \right) \frac{dt}{|H|}$$

$$(15) \quad dL_y^* = \left( -h_{12}x \frac{\partial y}{\partial x} \frac{\partial x}{\partial L_x} + h_{11}x \frac{\partial y}{\partial L_y} \right) \frac{dt}{|H|}$$

and for the corresponding total adjustment in emission we find after some straightforward rearrangement

$$(16) \quad de^* = \left( -x \frac{\partial y}{\partial L_y}, x \frac{\partial y}{\partial x} \frac{\partial x}{\partial L_x} \right) H \left( -x \frac{\partial y}{\partial L_y}, x \frac{\partial y}{\partial x} \frac{\partial x}{\partial L_x} \right)' \frac{dt}{|H|} + y \frac{\partial x}{\partial L_x} dL_x^*$$

which in contrast to the short run reaction (13) cannot be signed since only the quadratic form is again clearly negative whereas the other term (compare (14)) may have any sign. Likewise, the input reactions (14) and (15) cannot be signed either.

Thus, we cannot rule out that a firm which survived the adjustment process to the new long run equilibrium emits a higher amount of effluents than it did before the charge was raised. All we can say is that the individual firm's effluent production per unit of output declines:

We find by total differentiation of  $y(L_y, x(L_x))$  and substitution of the long run input adjustments (14) and (15)

$$(17) \quad dy^* = \frac{\partial y}{\partial x} \frac{\partial x}{\partial L_x} dL_x^* + \frac{\partial y}{\partial L_y} dL_y^*$$

$$= \left( -\sqrt{x} \frac{\partial y}{\partial L_y}, \sqrt{x} \frac{\partial y}{\partial x} \frac{\partial x}{\partial L_x} \right) H \left( -\sqrt{x} \frac{\partial y}{\partial L_y}, \sqrt{x} \frac{\partial y}{\partial x} \frac{\partial x}{\partial L_x} \right)' \frac{dt}{|H|} < 0$$

which says that a firm that survives produces less effluents per unit of output in the new equilibrium than it did in the old one. This finding, however, does not allow for the conclusion that the level of total emissions generated by the individual firm also falls. It is still possible that the firm increases output so as to overcompensate the effect of falling  $y$  such that  $e$  increases with the charge rate.

When looking at the entire industry, however, this ambiguity with respect to an individual firm is of no consequence. Since we know by A4 that total industry output must have fallen in view of the increase in the price by  $dp=ydt$ , and we know by (17) that emissions per unit of

output are reduced, we can easily conclude that the industry's long run emission will decrease with increasing charge rate. The effluent charge is consequently in the long run a quite effective means of fighting emissions.

#### IV. THE INTERMEDIATE RUN

There is little in our assumptions which allows us to make definite statements about how the industry's adjustment process towards the new zero profit equilibrium will look. Since the only thing we know about this new equilibrium is that total output will be smaller but don't know whether the industry will end up eventually with a larger or smaller number of firms sharing in this reduced output two basically different types of adjustment process are conceivable: first, one in which the new equilibrium price level is approached from above, which implies that during the adjustment process positive profits prevail signalling entry of additional firms. Intuitively, this case should arise only if the firms answer the increase in the effluent charge initially with such a drastic reduction in output that the ensuing rise in the market price, which is followed in turn by mitigating upward revisions in the firms' output, does not come to a halt before the long run equilibrium price is exceeded. Conversely, if the initial reduction in the firms' output is less marked or if the firms react to the increase in the effluent charge even with an increased output so that the market price remains initially below its long run equilibrium level we would have the second type of adjustment process in which all firms incur losses and the resulting exit of firms drives up the market price until it reaches its long run equilibrium level.

In either of these two cases<sup>2</sup> we cannot be sure what the industry's emission will do during the early phases of the adjustment process. By a continuity argument we know merely that during the later stages of the adjustment process industry-wide emission must be smaller than it was originally for the industry settles eventually with a strictly smaller emission level. But in earlier phases industry-wide emission

may well rise over and above what it was before the effluent charge was raised. Thus, for the intermediate run we cannot exclude the possibility that an increased effluent charge will bring about an increased rate of emission<sup>3</sup>.

## V. CONCLUSION

We have shown on the basis of rather commonplace and hardly restrictive assumptions that raising an effluent charge in a competitive industry may have unwarranted effects. Whereas there can be no question that the effluent charge will both in the very short and in the long run effectively reduce industry-wide emission, it cannot be ruled out that the industry will temporarily step up its emission after the charge has been raised nor that the firms which remain in the industry will raise their emission as compared to their original emission levels. In the latter case, the neighbours of the surviving firms would then be forced to pay a perhaps unreasonably high price for the reduction in industry-wide emission.

NOTES:

<sup>1</sup> See e.g. A. Takayama, Mathematical Economics, Hinsdale: The Dryden Press, 1974, p. 155.

<sup>2</sup> We disregard here the third but theoretically less interesting case where the industry moves to the new long run equilibrium price level in a single jump without new firms entering or old firms leaving the market.

<sup>3</sup> We should like to mention in passing that neither this ambiguity with regard to industry-wide emission during the early stages of the adjustment process nor the earlier mentioned ambiguity with regard to the long run emission rate at the firm's level can be resolved by assuming that the effluent per unit of output production function,  $y$ , is independent of the output level,  $x$ . The only ambiguity to disappear with this rather strong assumption is that the RHS of (14) and (15) become both positive, meaning that all firms that survived will eventually step up their abatement activities but at the same time also produce more than originally. It should also be noted that the main results of this paper are not changed by allowing for a more general production function in which  $x$  does not only depend upon  $L_x$  but also on  $L_y$ .