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## The Structure of the ISAC Model

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**THE STRUCTURE OF THE ISAC MODEL**

by

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Preliminary version

May, 1981

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## Introduction -

A compact presentation of the formal structure of the ISAC model is given in this paper<sup>1</sup>. The model is presented block by block with special emphasis laid on those features which distinguishes ISAC from other growth models hitherto used in Sweden viz. the vintage approach to the formation of industrial capital, the "Phillips curve" - like way of dealing with wage determination, the explicit representation of energy substitution possibilities and the endogenous treatment of local government.

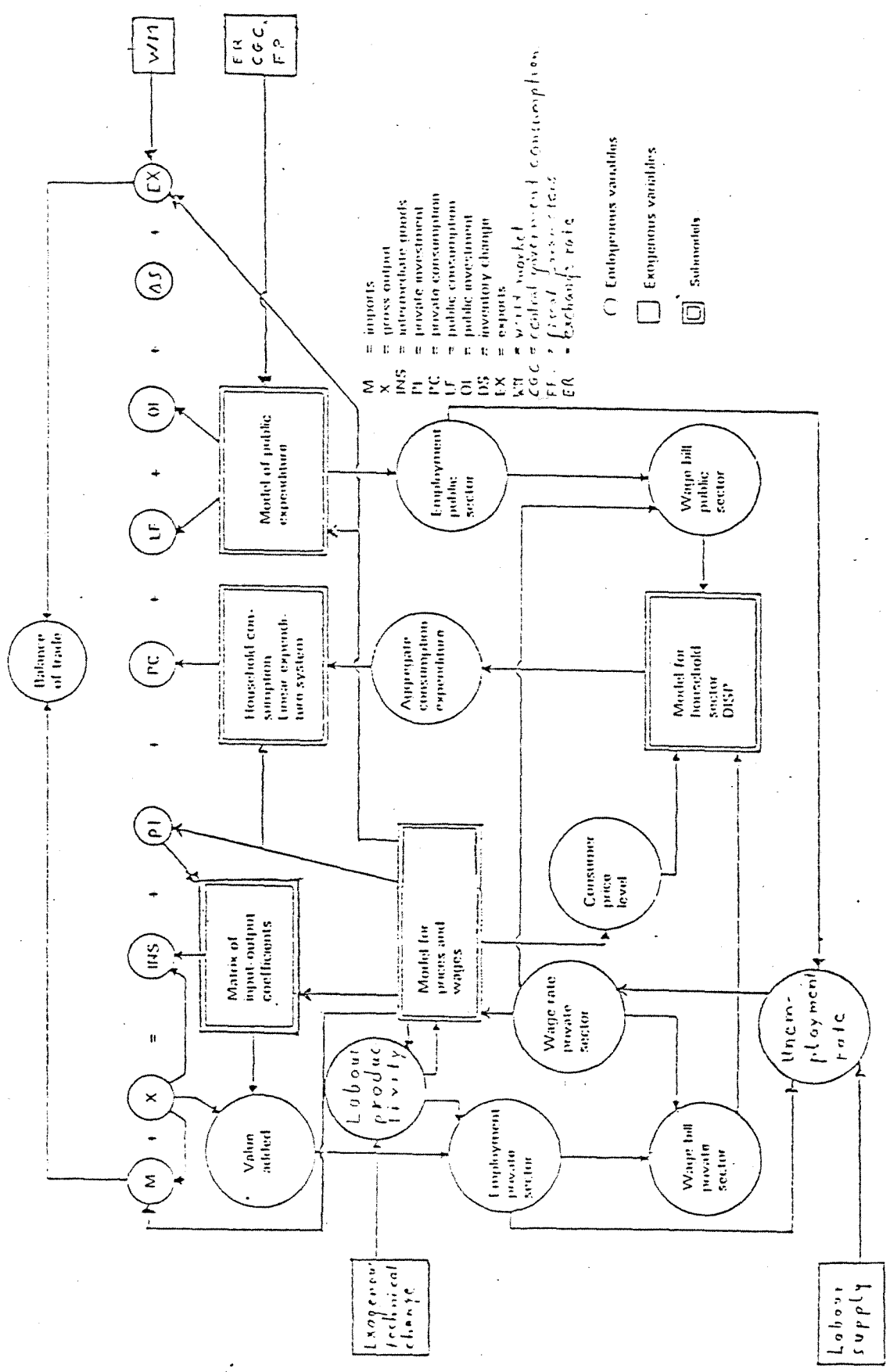
Figure 1 gives a synoptical view of the whole model. The balance equation at the top of the figure represents a 23 sector model where X (gross output), PI (private investment) etc. are column

<sup>1</sup> The growth model ISAC - Industrial Structure And Capital Growth - was developed on the basis of earlier macro-models used within IUI. The first model of this kind developed at the institute was designed for medium-term forecasting purposes and did indeed resemble the model used by the Swedish Government for its medium-term planning surveys, although including in addition a rather extensive specification of household income and payment. (For a detailed account of this model see IUI:s långtidsbedömning 1976. Bilagor, IUI 1977, in particular Chapt. 1-3 by Ulf Jacobsson, Göran Normann and Lars Dahlberg respectively.

To the next IUI economic survey, in 1979, this model was further developed by including i.a. investment functions and price formation equations. (A description of this model and its solution algorithm is given in Jansson-Nordström-Ysander: "Utvecklingsvägar för svensk ekonomi 1978-1985 - en kalkylredovisning" i Kalkyler för 80-talet, IUI 1979.

Since then a major restructuring of the model has taken place. The model now incorporates adjustment mechanisms for wages prices and industrial capital and i.a. local government actions and part of the industrial productivity development are endogenously explained.

Figure 1. Main structure of the ISAC model 1981



vectors. The model can be characterized as a dynamic Keynes-Leontief model with an endogenously changing input-output matrix, with market rigidities and adjustments and with built-in multiplier mechanisms. The arrows emerging from the sector products indicate roughly the way in which an exogenously initiated change would work itself through the model. Specially marked (by single-line square frames) are the exogenous factors driving the model, i.e., the development of world markets, of labor supply, of the exogenous part of technical change and finally of the central government consumption and fiscal parameters. A number of time-lags in the model - i.a. in foreign trade, in private consumption and investment and related to wage and price formation and to tax yields - makes it dynamic and may produce oscillations of the kind associated with business cycles.

In the text the following definitions and conventions have been used.

Large romans = matrices

Small romans without subscripts = vectors

Greek or small romans with subscripts = scalars

Superscripts = indices of categories etc.

"Roofed" letters = vectors turned into diagonal matrices

(t-n) = n-year lag. When no time indication is given, time t is assumed.

Subscripts t-m = vintage t-m

$\bar{d}$  = gross change operator

$\Delta$  = net change operator.

Dotted variable = growth rate.

## THE EQUATIONS OF THE ISAC MODEL

### Commodity balances in fixed and current prices

The following two accounting identities merely state that total domestic demand (right side) is equal to total domestic supply (left side).

$$m + x = Ax + inv + pc + puc + e + \Delta s$$

$$\hat{p}^m m + \hat{p}^x x = \hat{p}^h (Ax + inv + pc + puc + \Delta s) + \hat{p}^e e$$

### The i/o matrix and employment

The i/o matrix for the industry sector is calculated from vintage model for the 14 branches. The i/o matrix thus becomes a slowly moving function of prices of intermediate goods, electricity, fuels, labor and capital (capital stocks together with investments and depreciations)<sup>1</sup>. The relationships are described in following formulas

$$A = DA[\widehat{dxcap/xcap}] + A(t-1)\{I - [\widehat{dxcap/xcap}]\}$$

where

$[\widehat{dxcap/xcap}]$  is a diagonal matrix with elements

$$dxcap_i / xcap_i$$

<sup>1</sup> A detailed account of the vintage approach as applied to the iron and steel industry, is given in L. Jansson: A Vintage Model of the Swedish Iron and Steel Industry, Working Paper No 41, IUI, 1981.

and

$$dxcap_i = inv_i / \varepsilon_{t,k,i}$$

$$xcap_i = dxcap_i - dv_i + xcap_i(t-1)$$

$$dv_i = \left( \sum_{\tau} \sum_{j \neq k} \varepsilon_{\tau,j,i} p_{j,i}(t-1) / p_i^x(t-1) \right) \delta_i \cdot xcap_i(t-1)$$

where

$j = el, fu, in, l, k$

$el = electricity$

$fu = fuel$

$in = intermediate goods$

$l = labor (hours)$

$k = capital$

$dxcap_i$  is total new installed capacity at time  $t$  in branch  $i$ ,  $inv$  is investments and  $\varepsilon_{\tau,k,i}$  is the capital output capacity ratio of vintage  $\tau$  installed in branch  $i$ .  $xcap_i$  stands for total capacity and  $dv_i$  denotes depreciation. The depreciation rate is assumed proportional to the operating cost ratio in each vintage (the term within large bracket) multiplied by a constant  $\delta_i$ .

The input shares for the new vintage installed at time  $t$  in branch  $i$  is calculated as

$$DA_i = \sum_j \varepsilon_{t,j,i} f_{j,i}$$

$DA_i$  thus denotes the  $i$ :th column of  $DA$ . The input shares  $\varepsilon_{t,j,i}$  are calculated from a constant elastic function as

$$\varepsilon_{t,j,i} = b_{j,i} \prod_m p_m(t-1)^{\alpha_{m,i}}$$

where  $m, j = el, fu, in, l, k$ .



The  $f_{j,i}$ 's are distribution vectors which convert the aggregate input shares to commodity input shares. Thus all elements in a distribution vector sum to unity. For instance in  $f_{el,i}$  all elements are zero except the element for the electricity generating branch which is equal to unity.

All vintages are assumed to be used at the same intensity level. Since input-output ratios are supposed to be independent of the degree of utilization, this implies that the A matrix, defined above, describes the input-output relations not only at full capacity production but also at any other actual production level.

Employment  $l$  in branch  $i$  is determined as

$$l = [\varepsilon_{t,l,i} \cdot dxcap_i / xcap_i + \varepsilon_{l,i}(t-1) \cdot (1 - dxcap_i / xcap_i)] \cdot x_i$$

Investments, input shares and depreciations are all functions of only predetermined variables which means that they are not dependent on the model solution for year  $t$  and accordingly neither is A.

Fuel in the industry is mostly used to produce heat. Heat can be produced by different types of fuel. Ex post substitution between major types of fuels are allowed. Three fuels used for heating are distinguished: oil, coal and domestically produced fuel(dfu). The aggregate use of heat (fuel) in industry is simply the sum of heat produced by oil, coal and domestic fuel. Thus

$$x_{fu,i} = x_{oil,i} + x_{coal,i} + x_{dfu,i}$$

or

$$l = q_{oil,i} + q_{coal,i} + q_{dfu,i}$$

The amount of oil etc required to produce one unit of heat is assumed constant. The shares for coal and domestic fuel are dependent on prices and time according to following formula

$$q_j = a_j \cdot (b_j - p_j^{c_j})$$

where  $j$  = coal, domestic fuel.

The  $p$ 's are up to five year lagged energy prices, including capital cost, relative to oil produced energy. The share of oil is determined as a residual.

For heat to be separable in the production function of the various industrial branches choice of fuel must have negligible effect on the rest of the installed technique, i.e., the other input shares should be independent of the choice of fuel.

The switching between fuels can be dampened by an adjustment mechanism.

$$q_j = (1-c) q_j(t-1) + c q_j \quad 0 < c < 1$$

### Private investments

The investments in branch  $i$  of the industry depends on past profits.

$$inv_i = k_i(t-1) \left( \sum_{j=1}^4 \gamma_{j,i} \cdot ep(t-j)_i + d_i \right)$$

where the parameters  $\gamma_{j,i}$  are all  $> 0$  and

$$ep_i = vafa_i / (p_i^k k_i + w_i l_i)$$

$ep_i$  is an "excess profit" ratio fluctuating around unity.

### Disposable income in the household sector

The model of household income distinguishes between two kinds of individuals. "Pensioners" are people with most of their income from the social security system. The remainder is simply called "wage-earners" although it includes entrepreneurs as well as unemployed persons.

Individuals receive factor-income, capital-income and transfers from other sectors. After deduction of income- and payroll-taxes, and transfers to other sectors they are left with disposable income.

### Factor income

The main part of factor income is gross wages and salaries, including payroll taxes and other collective fees, but factor income also includes part of the net surplus in producing sectors.

Gross wages and salaries, bill, is the product of wage/hour and the number of hours worked in different sectors plus the public sector:

$$bill = w' \cdot l + obill$$

$$enet = he' \cdot \hat{x} \cdot vafa$$

where  $he$  = entrepreneurs' share of total hours worked in each branch and  $vafa$  = value added (i.e. their productivity is assumed equal to the employers).

According to accounting conventions  $enet$  also includes imputed income from owner-occupied houses. Thus:

$$fink = bill + enet.$$

### Capital income

Capital income consist of interest-payments calculated as a constant fraction of entrepreneurs' income:

$$ip = \alpha_{id} enet.$$

Other net capital income is calculated from financial assets which in turn are accumulated from the total financial surpluses and deficits of the household sector.

$$ii = \gamma_{if} fa$$

Thus:

$$cinc = ip + ii.$$

### Transfer income

This part of the model is fairly disaggregated and to a large extent exogenous except for inflation. There are six different types of transfer incomes:

- etra      1: National general pension (old age plus others)  
           2: Dito local  
           3: National supplementary pensions (old age plus others)  
           4: Private (collective) pensions (old age)  
           5: Other transfer income (non-taxable)  
           6: Other transfer income (taxable)

The first four items are calculated as number of persons times real income per capita with figures taken from various official sources

$$\text{etra}_i = n p_i \cdot r p_i \cdot \text{cpi}; \quad i=1, \dots, 4.$$

Other transfer income, which mainly goes to wage-earners, is divided according to whether it is taxable or not.

The non-taxable is set by an exogenous trend plus inflation:

$$\text{etra}_5 = \text{ec}_{51} \cdot \exp(\text{ec}_{52} \cdot t) \cdot \text{cpi}.$$

Taxable transfers are divided into sickness benefits, unemployment benefits and others:

$$\begin{aligned} \text{etra}_6 = & \text{ec}_{61} \cdot \text{bill} + \text{ec}_{62} \cdot \bar{w} \cdot \text{ue} + \text{ec}_{63} \\ & \cdot \exp(\text{ec}_{64} \cdot t) \cdot \text{cpi} \end{aligned}$$

Thus total transfer income is

$$\text{etra} = \sum_{i=1}^6 \text{etra}_i$$

Transfer payments

Five types of transfer payments are distinguished:

- btr 1: National income tax  
 2: Local income tax  
 3: Social security contributions  
 4: Other payroll fees  
 5: Other transfer payments.

National income tax is calculated from an aggregated progressive tax-function:

$$btr_1 = bc_1 \cdot skind \cdot n(besk/skind/n)^{bc_2}$$

Local tax is:

$$btr_2^1 = utd \cdot besk$$

Payroll taxes and fees are shares of total gross wage:

$$btr_3 = bc_3 \cdot bill$$

$$btr_4 = bc_4 \cdot bill$$

Other transfer payments are calculated as:

$$btr_5 = bc_5 \cdot \exp(bc_6 \cdot t) \cdot cpi.$$

<sup>1</sup> btr is the local tax payed in year t. As spelled out in the local government model below it, however, only reaches the local authorities in the year (t-2).

Disposable income

Summing up incomes and payments we get disposable income for the household sector

$$\text{disp} = \text{fink} + \text{cink} + \text{etra} - \text{btr}$$

**Private consumption**

Private consumption expenditure per capita equals disposable income less savings. Savings are set as a constant fraction of disposable income.

$$\text{pcons} = (1-s) \cdot \text{disp}/\eta$$

The total consumption per capita is distributed on 14 consumer goods by the following linear expenditure system

$$p_i^c \cdot c_i = \gamma_i p_i^c \cdot c_i(t-1) + \beta_i \left( \text{pcons} - \sum_{k=1}^{14} \gamma_k p_k^c c_i(t-1) \right)$$

$$\text{and } \sum_{j=1}^{14} \beta_j = 1 \quad i=1, \dots, 14$$

The price vector  $p^c$  represents the prices of domestic absorption converted to the consumer goods level

$$p^c = PKONV' \cdot p^h$$

Private consumption per capita is then transformed to total private consumption of commodities

$$pc = PKONC \cdot c \cdot \eta.$$

### Export

The general form of the export function for all branches except textile and paper and pulp industries is:

$$e_i = \beta_{i1} \cdot \exp(\beta_{i2} \cdot t) \cdot (p_i^{ew}/p_i^w)^{\beta_{i3}} \cdot (p_i^{ew}(t-1)/p_i^w(t-1))^{\beta_{i4}} w_{mi}^{\beta_{i5}}$$

for  $i = 1, \dots, 6, 9, \dots, 23$ .

where  $w_{mi}$  stands for the volume of the world market and  $p_i^{ew} = p_i^e/\theta$

For the textile and paper and pulp industries export is given by

$$e_i = \beta_{i1} \cdot (p_i^{ew}/p_i^w)^{\beta_{i2}} (p_i^{ew}(t-1)/p_i^w(t-1))^{\beta_{i3}} \cdot w_{mi}^1 + \\ + \beta_{i4} \cdot (p_i^{ew}/p_i^w)^{\beta_{i5}} (p_i^{ew}(t-1)/p_i^w(t-1))^{\beta_{i6}} w_{mi}^2$$

The export functions for textile and paper and pulp industries have been achieved from econometric studies, where the world market has been divided into two regions,  $w_{mi}^1$  and  $w_{mi}^2$ .

The rest of the export functions are based on econometric studies for only chemicals and engineering. The four branches thus covered in the studies were, however, responsible for more than 70 % of the total export of traded goods in 1977.



### Import

The import functions have the following general form

$$m_i = \gamma_{i1} \cdot h_i^{\gamma_{i2}} \cdot \prod_{j=0}^2 [p_i^w(t-j) \cdot \theta / p_i^{xh}(t-j)]^{\gamma_{i,3+j}}$$

The import function for the mining industry merely restates of the fact that all the coal used and all other mining products used outside the iron and steel industry are imported.

$$m_3 = \sum_j a_{3j} x_j + \sum_j \varepsilon_{coal,j} x_j$$

where the first term excludes deliveries of iron ore to the iron and steel industry and where  $\varepsilon_{coal,j}$  is the average input share of coal in sector  $j$ . Thus all imports in this sector is treated as complementary imports.

The same of course holds for oil. It is also assumed that a constant fraction of total supply of oil products is refined domestically. It thus becomes:

$$m_{12} = \gamma_{12,1} \cdot x_{12}$$

### Stock building

For goods producing branches the total change of inventories is assumed constant. The addition to inventories in each branch is proportional to its share of total production.

$$\Delta s_i = \Delta s \cdot Y_i / \sum_{j=1}^{18} Y_j \quad i=1, \dots, 18$$

where  $y_i$  is gross production in branch  $i$  and index  $i$  covers the goods producing part of the business sector.

There also exists a simple stock building model which can be included when necessary. Only one aggregate inventory good is distinguished but demand for inventories is generated by four stock-holding branches which are simple subsums of the 18 goods producing branches. The four branches are foresting, agriculture and fishing, producers of intermediate goods, producers of finished goods and the merchandise trade. The stocks are proportional to branch production. The proportion might change over time.

$$s_i = sr_i y s_i \quad i=1, \dots, 4.$$

$$\Delta s = \sum_{i=1}^4 [s_i - s_i(t-1)]$$

The additions of inventory are distributed according to the formula above.

### Prices

Swedish producers' prices on the export and home markets are determined by:

$$p_i^e = b_{i1} (p_i^w)^{b_{i2}} \text{ucost}_i^{b_{i3}} \text{ur}_i(t-1)^{b_{i4}}$$

$$p_i^{xh} = c_{i1} (p_i^w)^{c_{i2}} \text{ucost}_i^{c_{i3}} \text{ur}_i(t-1)^{c_{i4}}$$

where

$$\text{ucost} = A'p^h + \text{vac}$$

and where

$$\text{vac} = [\hat{l} \cdot w + (\hat{r} + \delta) \cdot \hat{k} \cdot p^{\text{inv}}] \cdot \hat{x}^{-1}$$

that is, value added at normal profits and

$$\text{ur}_i = (x_i / \text{xcap}_i) / (x_i / \text{xcap}_i)_{1980}$$

The value of domestic absorption is

$$\hat{h}p^h = (\hat{x} - \hat{e})p^{xh} + \hat{m} + p^m$$

with

$$\hat{n} = \hat{x} - \hat{e} + \hat{m}.$$

Since  $p^h$  is appear in  $\text{ucost}$  the two blocks of equations for export prices and domestic producer prices on the home market are not in reduced. But the model is solved in such a way that equality is established between the two sides.

The value of domestic production is

$$\hat{x}p^x = \hat{e} \cdot p^e + (\hat{x} - \hat{e}) \cdot p^{xh}$$

For all business sectors the gross profit  $q_i$  is residually determined since a pure mark up pricing is not used.

$$q_i = (p_i^x - p^h A_i) x_i - w_i l_i$$

where  $A_i$  denotes column  $i$  of the matrix  $A$ .

### Wages

The changes in the wage rate is the same for all sectors though the level differs.

$$w_i = w_{conv_i} \cdot w_0 \quad i=1, \dots, 37$$

where index  $i$  includes both the 23 branches of the business sector and the 14 subbranches of the public sector.  $w_0$  is either exogenous or the percentage change of wage rate  $\dot{w}_0$  is endogenously determined by the following function:

$$\dot{w}_0 = d_0 + d_2(u - u_0) + cpi(t-1) + d_3[\Pi(t-1) - \Pi_0] + d_4\dot{\lambda}(t-1)$$

Thus the change in the wage rate is a function of current deviation of the unemployment rate from a "normal" unemployment rate  $(u - u_0)$ , past inflation,  $cpi(t-1)$ , past deviation from a "normal" level of aggregate gross profit margin in industry,  $(\Pi(t-1) - \Pi_0)$  and finally past change of labor productivity  $\lambda(t-1)$ . When  $w_0$  is determined endogeneously it is assumed that wage changes in the public sector is lagging one year compared to the business sector.

### Central government

The development of central government consumption is exogenously determined. Seven different consumption purposes are distinguished.

1. National defense.
2. Public order and safety.
3. Education.
4. Health.
5. Social security and welfare services.
6. Roads.
7. Other services.

The rate of growth of production and consumption for the various purposes is a constant proportion of an exogenously given common growth factor  $g_0$ .

$$sp = g_0 \cdot gr \cdot sp(t-1)$$

$$sc = (1-sale) \cdot sp.$$

The need for intermediate goods is related to production in each central government sector and then converted to demand from the 23 business branches;

$$sf = asf \cdot sp$$

$$lf = SGA \cdot sf.$$

Employment is derived from productivity assumption for each sector. Investment is exogeneously determined.

## Local government

Real current expenditure of local governments (consumption + market sales according to national accounting conventions) are split into five categories.

1. Education.
2. Health.
3. Social welfare.
4. Roads (total expenditures).
5. Central administration, fire service etc.

These expenditures are explained by linear expressions of the following form:

$$\lambda p_i = a_{1i} z_{1i} + a_{2i} z_{2i} + a_{3i} z_{3i} + a_{4i} z_{4i} + a_{5i} z_{5i}$$

where  $z_{1i}$  are shift variables,  $z_{2i}$  and  $z_{3i}$  stand for investment consequences and capacity restrictions, while  $z_{4i}$  and  $z_{5i}$  reflect the impact of changes in real income, local tax rates and relative prices<sup>1</sup>.

$$z_{2i} = \frac{1}{k_p} \left( \frac{\Delta k_p}{\Delta \lambda p_i} \right)^*$$

$$z_{3i} = z_{2i} \frac{\Delta k_p(t-1) - \Delta k_p^*(t-1)}{k_p}$$

$$z_{4i} = \phi_i \frac{b_{esk}}{b_{esk}(t-2)}$$

$$z_{5i} = z_{4i} b_{esk}(1-utd-av_0)$$

with  $\phi_i$  defined as  $\phi_i = (1-sb_i - av_i) \frac{p_i}{cpi}$

---

<sup>1</sup> All expressions explaining local authority behavior are derived from maximizing a quadratic goal-function under a budget restriction. A detailed account of the model is given in Ysander, B-C, An Econometric Model of Local Government Budgeting, IUI Working Paper, No 43, 1981.

Aggregate investments by local authorities are explained by a gradual adjustment to desired capital stock levels, with the rate of adjustment depending on capital good prices, interest rates and real income development.

$$\Delta kp = a_{16}z_{16} + a_{26}z_{26} + a_{36}z_{36} + a_{46}z_{46} + a_{56}z_{56}$$

$$z_{16} = kp; \quad z_{26} = \Delta kp^*; \quad z_{36} = \Delta liq(t-1) \cdot z_{16};$$

$$z_{46} = \phi_6 \cdot kp^2 \cdot r \cdot \frac{besk}{besk(t-2)}$$

with  $\phi_6$ , net real price of capital goods, defined in the same way as  $\phi_i$ .

$$z_{56} = (1-utd-av_0)besk \cdot z_{46} \cdot z_{16}$$

The depreciation is assumed to be a constant fraction of existing capital stock. Gross investments then becomes

$$lp_6 = \Delta kp + a_{66}z_{66}$$

where  $a_{66}$  is the depreciation rate and  $z_{66} = kp(t-1)$ .

Investments may also be computed in a simplified manner as equal to desired capital stock changes plus reinvestments.

Sales made by local authorities of goods and services at market prices are assumed to be a constant fraction of local production. The local governments' final demand of commodities from the business sector can be summarized:

$$\lambda p_i = AL'_i \cdot Z_i$$

$$\lambda c = LGA \cdot \lambda p;$$

where  $AL'_i$  and  $Z_i$  are the  $i$ :th column of  $AL$  and  $Z$  respectively. The matrix  $LGA$  converts local governments' final demand to commodities.

Employment is derived from productivity assumption for each sector.

Transfer payments are split into two categories, - subsidies to public utilities and direct transfers to the household sector. In the model the explanation of these payments is derived from the idea that the provision of housing space and of public utilities are arguments in the local governments' goal function, pursued indirectly by way of "price subsidies".

$$t_i = a_{2i}\zeta_i + a_{3i}\mu_i + a_{4i}\gamma_i$$

$i=1,2$     1 = public utilities subsidies  
               2 = direct household (housing) subsidies,

where  $\zeta_i$  represents the cost for the households relative to disposable income and  $\mu_i$  and  $\gamma_i$  express the impact of developments in real income, local tax rate and relative prices.

$$\bar{w}_1 = 1; \quad \bar{w}_2 = z_{13}; \quad \varepsilon_i = \frac{\tau_i/cpi}{(1-utd-av_0)(t-1)besk} \quad \zeta_i = \bar{w}_i \varepsilon_i$$

$$\mu_i = \varepsilon_i \frac{\tau_i}{cpi} \frac{besk}{besk(t-2)}; \quad \gamma = \mu_i (1-utd-av_0)besk$$

with  $\tau_i$  defined as  $\tau_i = p_i^c(1-sb_i)$ .



The transfer payments can alternatively be treated in a simplified manner. The net amount of subsidies to public utilities are then approximated as a constant fraction of production in sector 18 (electricity, heat etc.). Direct household transfers are set as a linear function of households' housing expenditures ( $z_{13}$ ).

Local government expenditures are financed by taxes and state grants with liquidity changes acting as a buffer against planning failures. Given state grants the local tax rate is determined residually by way of the budget restriction:

$$\text{utd} = \frac{\text{besk}}{\text{besk}(t-2)} [-\text{utd}(t-2)(\text{besk}(t-2) - \text{besk}(t-4)) - \text{sb}_0 - \Delta \text{dt} + e_0 + \text{rdt}]$$

To simulate possible restrictions or inertia in local government political behavior, the model can alternatively be supplemented with a floor restriction on the tax rate complemented with a rule, prescribing that planned surpluses that increase liquidity above a certain relative level are used to scale up current expenditure.

### The system of equations

The list of equations on the next page summarizes the formal structure of the ISAC model<sup>1</sup>. At the bottom the two main disequilibria in the model - in foreign trade and in the aggregate labor market - are listed. Clearance also of these markets requires a "steering" by way of exogenous policy parameters like the exchange rate or state consumption. (Financial entities like tax rates and transfers are excluded from this very compressed list.) To ensure also a "normal" rate of capacity utilization in all branches and periods would obviously require a large and special set of policy parameters. To the left of the equation list in the table the number of variables are given, while the corresponding number of equations are registered in the right-most column. Excluded from the count are exogenous variables not used as instruments, predetermined variables and identities.

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<sup>1</sup> The Gauss-Seidel algorithm is used to compute the solution. This algorithm, according to our experience, is easy to program and implement, its speed of convergence is mostly satisfactory and it has rarely failed to converge, though convergence is not theoretically assured.

LIST OF EQUATIONS<sup>1</sup>

Number of variables

Number of equations

n=23

n=23

$$4n \quad m + x = Ax + inv + pc + puc + \Delta s + e \quad n$$

**Prices**

$$2n+1 \quad p^e = p^e[\theta p^w, ucost, ur(t-1)] \quad n$$

$$n \quad p^{xh} = p^{xh}[\theta p^w, ucost, ur(t-1)] \quad n$$

$$p^h = (\hat{x} - \hat{e})p^{xh} + \theta \cdot \hat{m}p^w$$

$$n+1 \quad ucost = p^h A + w \hat{\epsilon}_\lambda + \quad n$$

$$+ p^K(p^h, r, \delta) \hat{\epsilon}_K \cdot \hat{u}r^{-1}$$

**Wages**

$$w = [1 + \dot{w}(cpi(t-1), u, \Pi(t-1), \epsilon_\lambda(t-1))] \cdot w(t-1) \quad 1$$

**Production capacity and technique**

$$ur_i = x_i / xcap_i \quad n$$

$$A(t) = DA(t) \cdot \widehat{dxcap} \cdot \widehat{xcap}^{-1} +$$

$$A(t-1) \cdot (I - \widehat{dxcap} \cdot \widehat{xcap}^{-1})$$

$$dxcap = inv \cdot \hat{\epsilon}_{t,k}^{-1}$$

$$inv = inv[ep \cdot \Lambda(1, 4), \delta, k(t-1)]$$

**Privat consumption and labour demand**

$$(n+14) \quad pc = pc(c_0 w' \lambda - c_1, p^h) \quad n$$

$$\lambda = (\epsilon_\lambda \hat{x}, \epsilon_{puc} \cdot puc)^2 \quad (n+14)$$

<sup>1</sup> Financial streams are excluded from the brief list of equations above.

<sup>2</sup>  $\lambda$  includes employment in both business and public sectors.

### Government production and investment

$$2n+1 \quad puc = sc + lc \quad n$$

$$lc = LGA \cdot lp \quad n$$

### Foreign trade

$$e = e[(p^e_\theta, p^w) \Lambda(0,1), wm] \quad n$$

$$m = m[(p^w_\theta, p^{xh}) \Lambda(0,2), h]; \quad h=x-e+m \quad n$$

### Unemployment

$$\sum \lambda_i - \lambda_s = \text{disequilibrium 1} \quad 1$$

### Balance of trade

$$\sum [p^e e - p^m m] = \text{disequilibrium 2} \quad 1$$

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10n+3

10n+3

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<sup>1</sup> In the equations above the lagoperator operates on the left hand variabel in the following way

,  $x \Lambda(i,j), \quad + \quad , x(t-i), \dots, x(t-j).$

## LIST OF VARIABLES

## Commodity balances

$m$	= imported goods and services
$x$	= domestically produced goods and services in the business sector
$inv$	= private investments
$puc$	= public use of intermediate goods and public investments
$e$	= exported goods and services
$\Delta s$	= changes in inventory stocks (exogenous)
$p^e$	= Swedish producers export prices
$p^m$	= import prices
$p^h$	= prices of domestic absorption

## The i/o matrix and employment

$dxcap$	= total new capacity in branch $i$
$inv_i$	= investments in branch $i$
$\epsilon_{\tau,k,i}$	= capital output ratio of vintage $\tau$ installed in branch $i$
$xcap_i$	= total capacity in branch $i$
$dv_i$	= depreciation in branch $i$
$A$	= matrix of i/o coefficients
$DA$	= i/o matrix for the new vintages
$\epsilon_{\tau,j,i}$	= the i/o ratio of vintage $\tau$ for input $j$ in branch $i$
$f_{j,i}$	= distribution vector which converts aggregate input shares to commodity input shares
$q_j$	= share of aggregate use of fuel
$l_i$	= employment in branch $i$

### Private investments

$k_i$	= capital stock
ep	= excess profits
$w_i h_i$	= gross wages and salaries in branch i
$p_i^k$	= $p^i(r+\delta_i)$ user cost of capital
r	= pretax discount rate
$\delta_i$	= depreciation rate (historical average)
$A_j$	= the j:th column of the i/o matrix A
$p_i^{xh}$	= domestic producers prices on the home market

### Disposable income in the household sector

bill	= gross wages and salaries
$w' \cdot l$	= gross wages and salaries in the household sector
obill	= gross wages and salaries in the public sector
enet	= entrepreneurs' income
vafa	= value added in current prices (factor values)
fink	= factor income

### Capital income

ip	= interest payments
$\alpha_{id}$	= interest payment as ratio of entrepreneurs' income
ii	= other net capital income
$\gamma_{it}$	= average yield on financial assets
fa	= net financial assets
cinc	= capital income

### Transfer income

np = number of pensioners  
rp = real pension income per capita  
cpi = consumer price index  
etra = transfer income  
ec<sub>51...</sub> = constants  
ec<sub>63</sub>  
ue = unemployment

### Transfer payments

btr<sub>i</sub> = transfer payments of type i  
bc<sub>1...</sub>  
bc<sub>6</sub> = constants  
skind = tax index (one year lagged cpi)  
besk = assessed income  
n = number of taxpayers  
utd = local tax rate  
disp = disposable income

### **Private consumption**

pcons = private consumption expenditures  
per capita  
s = net savings ratio  
η = total population  
c<sub>i</sub> = per capita demand for consumption of  
goods i  
PKONV = matrix of conversion  
pc<sub>i</sub> = consumption of commodity i  
p<sub>i</sub><sup>c</sup> = consumer price of commodity i

**Export**

- $e_i$  = export from branch  $i$   
 $\beta_{ij}$  = parameters  
 $p_i^{ew}$  = export prices of domestic producers in foreign currency  
 $wm_i$  = world market demand of commodity  $i$   
 $\theta$  = the exchange rate

**Import**

- $m_i$  = import of commodity  $i$   
 $\gamma_{ij}$  = parameter  
 $h_i$  = domestic absorption of commodity  $i$

**Stock building**

- $\Delta s_i$  = the change of stock in branch  $i$   
 $\Delta s$  = the aggregate change of stock  
 $Y_i$  = gross production in producers prices in branch  $i$   
 $s_i$  = stock in stock holding branch  $i$   
 $sr_i$  = stock output ratio  
 $ys_i$  = aggregated output in stock holding branch  $i$

**Prices**

- $p_i^{xh}$  = Swedish producers price of commodities  $i$  on the domestic market  
 $b_{ij}$  = parameters  
 $c_{ij}$   
 $p_i^w$  = average world market prices in foreign currency  
 $\theta$  = the exchange rate



$ucost$  = unit cost of production  
 $u$  = relative utilisation rate  
 (index=1 at normal capacity use)  
 $vac$  = value added computed with a normalized  
 pretax discount rate  
 $w$  = wage rate  
 $l$  = employment in the business sector  
 $p^{inv}$  = price index for investment goods  
 $r$  = pretax discount rate  
 $\delta$  = depreciation rate  
 $k$  = capital stock  
 $q_i$  = gross profit in branch

#### Wages

$w_i$  = wage rate in branch  $i$   
 $wconv_i$  = branch wage - average wage rate ratio  
 $w_0$  = the average wage rate  
 $d_j$  = constants  
 $u$  = unemployment rate  
 $u_0$  = "normal" unemployment rate  
 $\Pi$  = average gross profit margin in  
 the industrial sector  
 $\Pi_0$  = the "normal" gross profit margin in  
 the industrial sector  
 $\lambda$  = average labor productivity in  
 the industrial sector

**Central government consumption**

- gr = vector of relative growth numbers of state consumption purposes
- $g_0$  = the common growth factor of state consumption
- sp = central government production volume (market prices)
- sc = central government consumption volume
- sale = share of production sold
- sf = demand for intermediate goods for each state sector
- asf = share of intermediate goods
- gf = demand for intermediate goods from the business branch
- SGA = conversion matrix (23×7)

## Local government

- $lp_i$  = volume of local government production, category i  
 $z_{11}$  = index of population change, ages 7-19 years  
 $z_{12}$  = index of population change, weighted with average number of hospital days for the various age-brackets  
 $z_{13}$  = index of population change, ages over 71 years  
 $z_{14}$  = value added of business sector  
 $z_{15}$  = total population index

The rest of the z variables are explained in the text.

- $\Delta kp$  = aggregate net investments  
 $kp$  = aggregate capital stock in local government  
 $av_i$  = local government fees as share of total production costs for category i  
 $sb_i$  = categorical state grants as share of production costs for category i  
 $p_i$  = production cost index, category i  
 $\Delta kp^x$  = desired change of capital stock  
 $\Delta liq$  = net change of short-term assets  
 $r$  = rate of interest on local government bonds

$lp_6$  = gross investments  
 $AL$  = matrix of the parameters  $a_{ij}$   
 $Z$  = matrix of the variables  $z_{ij}$   
 $LGA$  = conversion matrix  
 $lp_c$  = diagonal matrix that transforms local government production to local government consumption

$\left(\frac{\Delta kp}{\Delta lp_i}\right)^x$  = marginal capital output ratio, category i

$av_0$  = local government fees as share of total production costs

$utd$  = local tax rate

$t_i$  = subsidies as share of net expenditures for category i by the households

$\phi_i, \phi_I, Y_i, \bar{w}_i, \varepsilon_i, \mu_i, \gamma_i$  and  $\tau_i$  = cf definitions in the text

$sb_0$  = general state grant

$dt$  = net financial assets

$\Delta dt$  = net change of financial assets

$e_0$  = total expenditure for production investment and transfer payments net of state categorical grants and fees

$lc$  = local government production and investment